The Microcosm Principle and Compositionality of GSOS-Based Component Calculi

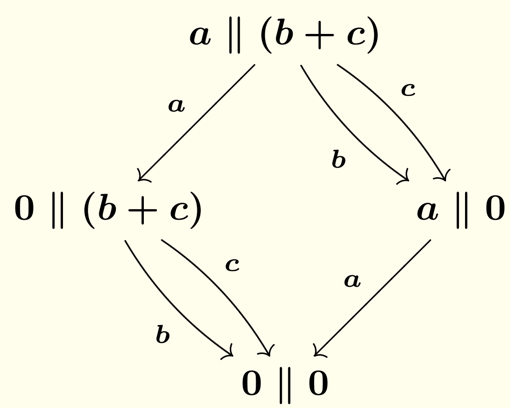
Ichiro Hasuo
University of Tokyo (JP)

東京大学
THE UNIVERSITY OF TOKYO

SOS: Variations

(Conventional) Process SOS

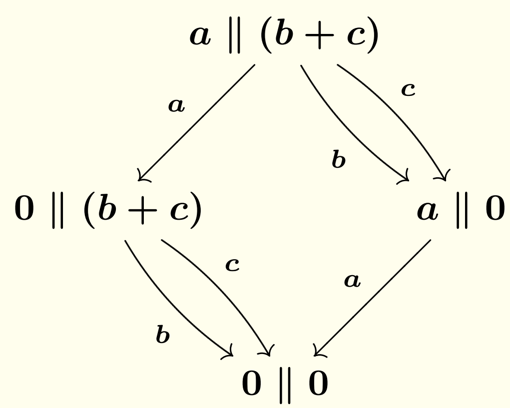
$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$



SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$



categorically

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

$$\begin{array}{c} \Sigma I \\ \big\downarrow \text{initial} \\ I \\ \big\downarrow ! \\ FI \end{array}$$

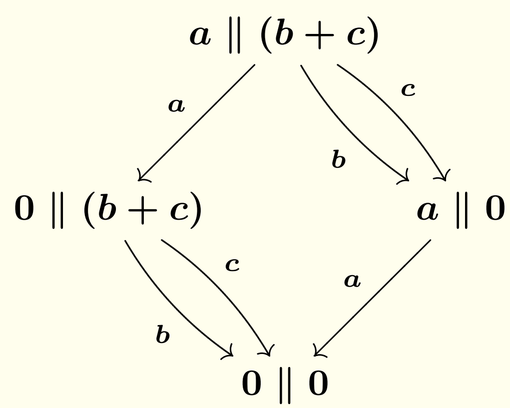
Hasuo (Tokyo)

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥



➡ categorically

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥

$$\begin{array}{c} \Sigma I \\ \big\downarrow \text{initial} \\ I \\ \big\downarrow ! \\ FI \end{array}$$

Component SOS

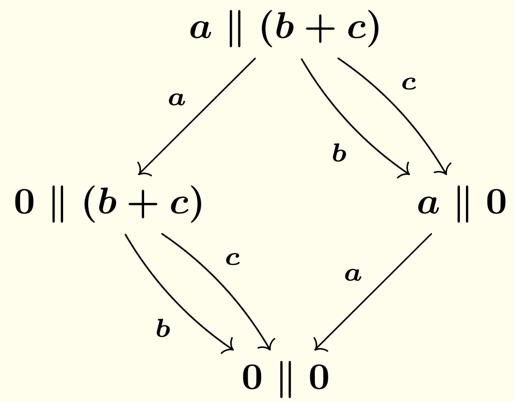
$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥ parallel composition of LTSs

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\|\textsc{Sync})$$



Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\|\textsc{Sync})$$

➡ parallel composition of LTSs

categorically

Bialgebraic SOS

[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➡ $$\begin{array}{c} \Sigma I \\ \big\downarrow \text{initial} \\ I \\ \big\downarrow ! \\ FI \end{array}$$

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\|} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

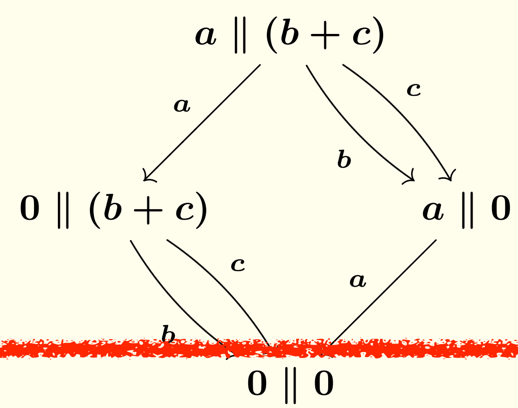
SOS: Variations

Part 1

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥



Bialgebraic SOS

[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥

$$\begin{array}{c} \Sigma I \\ \downarrow \text{initial} \\ I \\ \downarrow ! \\ FI \end{array}$$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥ parallel composition of LTSs

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

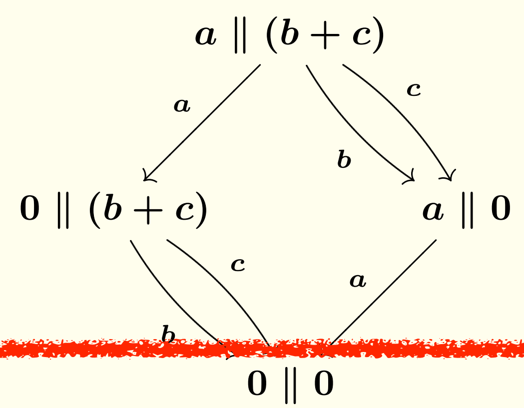
for any GSOS-specified σ

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\|\textsc{Sync})$$

➥



Bialgebraic SOS
[Turi&Plotkin, LICS'97]

Part 1

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥

$$\begin{array}{c} \Sigma I \\ \big\downarrow \text{initial} \\ I \\ \big\downarrow ! \\ FI \end{array}$$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\|\textsc{Sync})$$

➥ parallel composition of LTSs

Microcosm SOS

Part 2

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathrm{Coalg}_F)^2 \xrightarrow{\;\parallel\;} \mathrm{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathrm{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathrm{Coalg}_F$$

for any GSOS-specified σ

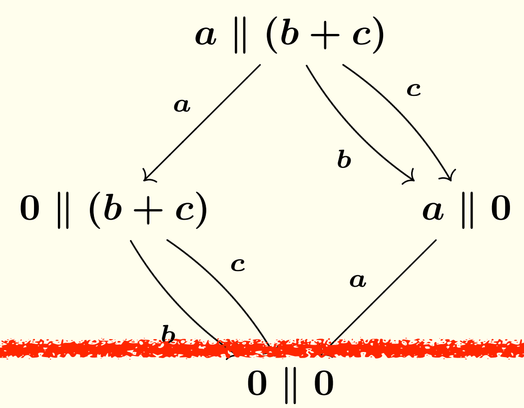
SOS: Variations

Part 1

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\|\textsc{Sync})$$

➥



Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥ $$\begin{array}{c} \Sigma I \\ \downarrow \text{initial} \\ I \\ \downarrow ! \\ FI \end{array}$$

categorically

Part 2

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\|\textsc{Sync})$$

➥ parallel composition of LTSs

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Part 3

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

Part 1

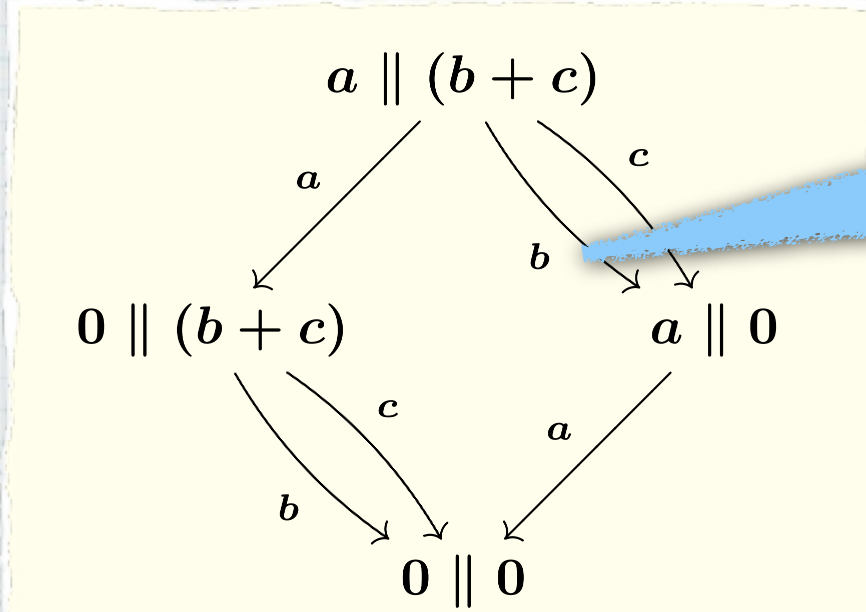
(Conventional) Process SOS & Bialgebraic SOS

Process SOS

* Given SOS rules

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\bar{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync}) \qquad \cdots$$

* Derive transitions between process terms

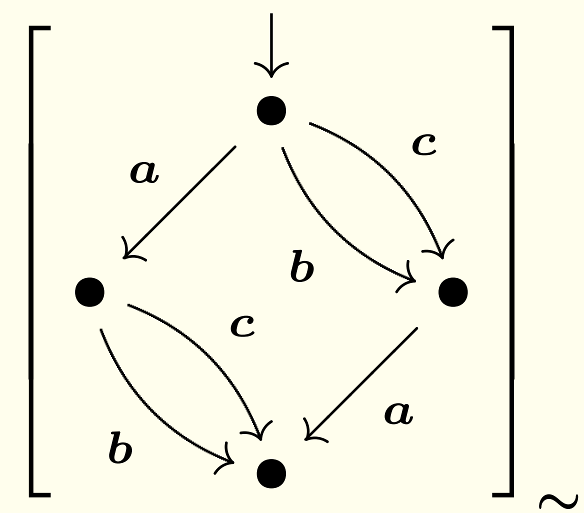


$$\cfrac{\cfrac{\cfrac{}{b \xrightarrow{b} 0}\ (\textsc{AtomAct})}{b + c \xrightarrow{b} 0}\ (+\mathrm{L})}{a \parallel (b+c) \xrightarrow{b} a \parallel 0}\ (\parallel \mathrm{R})$$

Process SOS

- Modulo bisimilarity ➜ "semantics"

$[\![a \parallel (b+c)]\!] =$

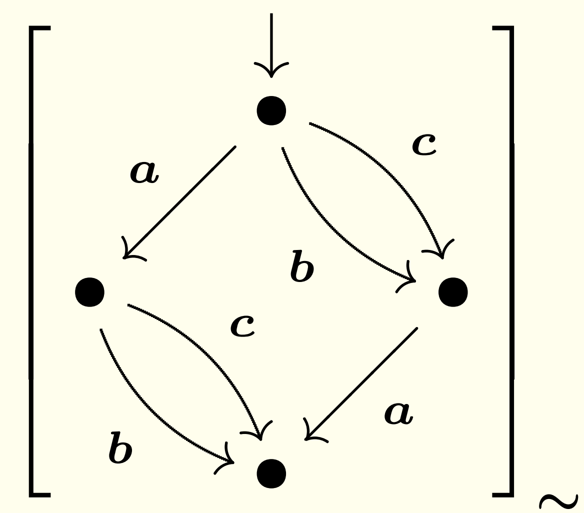


- Compositionality
  - $s \sim s',\ t \sim t' \quad \Longrightarrow \quad s \parallel t \sim s' \parallel t'$
  - that is, $[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$

Process SOS

* Modulo bisimilarity ➜ "semantics"

$$[\![a \parallel (b+c)]\!] = $$



* Compositionality
  * $s \sim s',\ t \sim t' \implies s \parallel t \sim s' \parallel t'$
  * that is, $[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$

well-dfd opr. on bisim classes!

Bialgebraic SOS

[Turi&Plotkin, LICS'97]

- Bialgebra

Σ-algebra. Typically:

$$\Sigma = \coprod_{\sigma \in \Sigma} (\_)^{\mathrm{arity}(\sigma)}$$

(algebraic signature)

$$\begin{array}{c} \Sigma X \\ \downarrow \\ X \\ \downarrow \\ FX \end{array}$$

F-coalgebra. E.g.

$$F = \mathcal{P}_{\mathrm{fin}}(A \times \_)$$

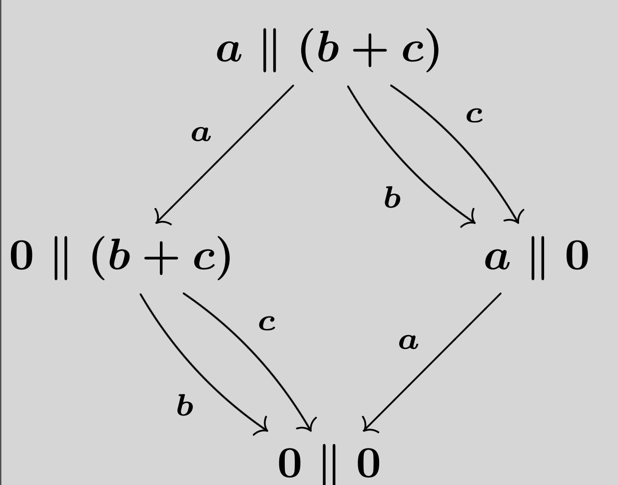
(functor for LTSs)

Process SOS

* From

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$

* Derive



Bialgebraic SOS

* Given SOS rules
  (Categorical SOS rule)

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

* Derive

$$\Sigma I$$
$$\downarrow \text{initial}$$
{process terms} = I
$$\downarrow$$
$$FI$$

!!

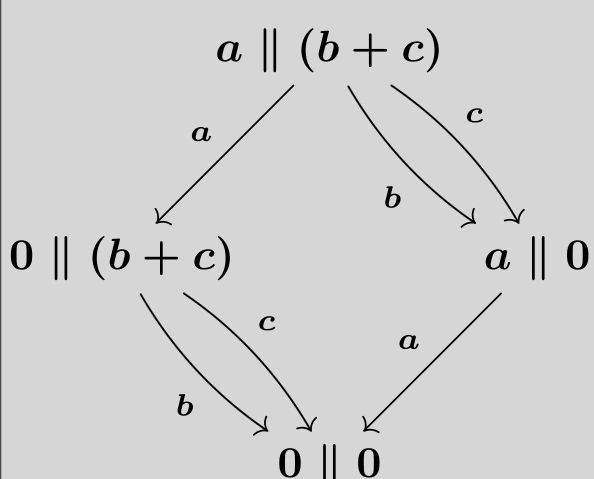
Hasuo (Tokyo)

Process SOS

* From

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\parallel\textsc{Sync})$$

* Derive



Bialgebraic SOS

* Given SOS rules (Categorical SOS rule)

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

* Derive

$$\Sigma I \xrightarrow{\text{initial}} I \longrightarrow FI$$

{process terms} = I

!!

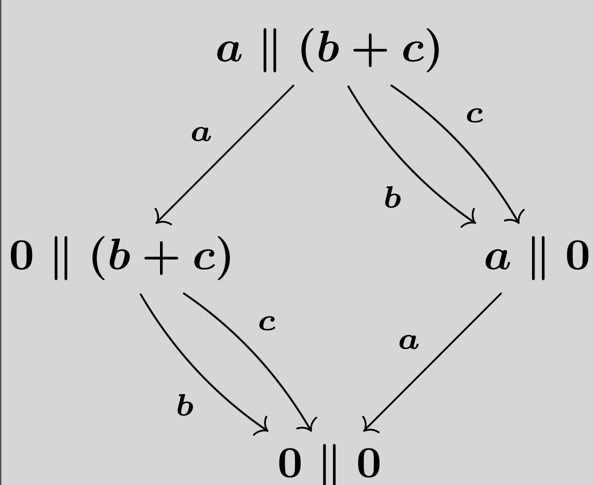
$$\begin{array}{c} \Sigma I \\ \downarrow \text{initial} \\ I \dashrightarrow FI \end{array}$$

Process SOS

* From

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\text{SYNC})$$

* Derive



Bialgebraic SOS

* Given SOS rules
(Categorical SOS rule)

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

* Derive

ΣI

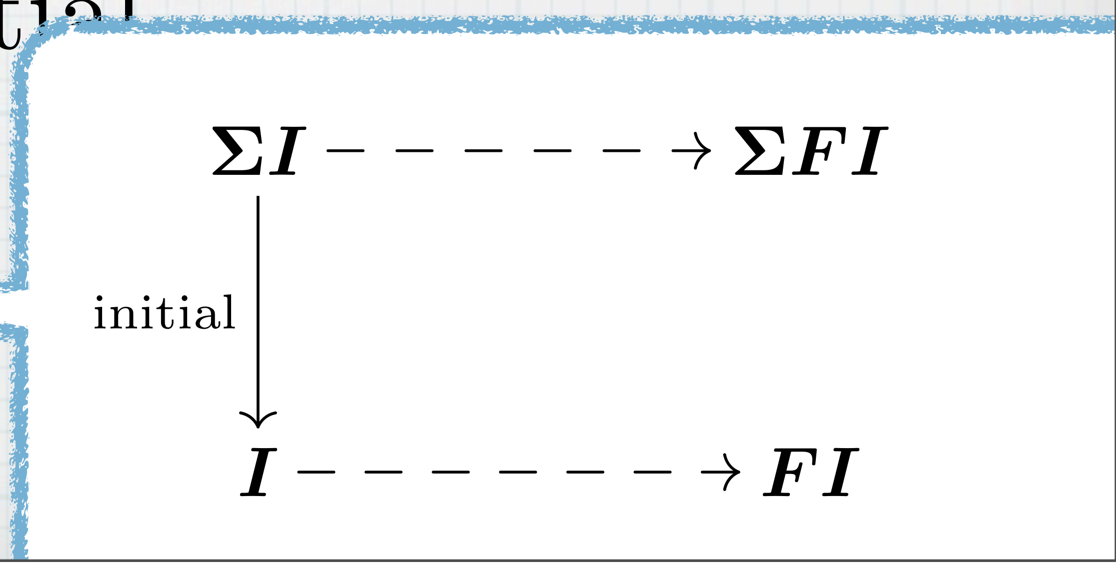
$\downarrow$ initial

{process terms} = I

!!

$\downarrow$

FI

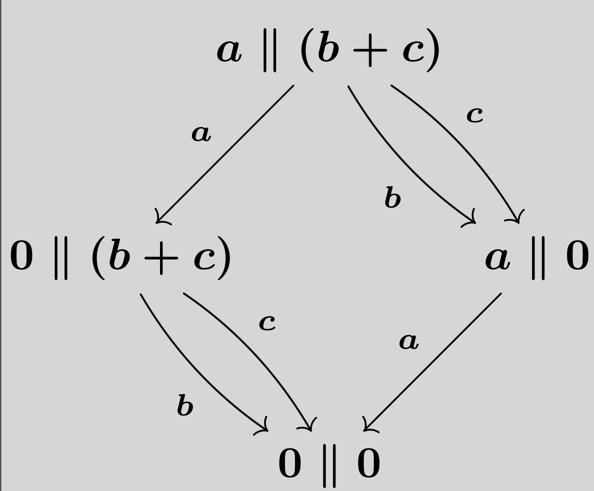


Process SOS

- From

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\parallel\textsc{Sync})$$

- Derive



Bialgebraic SOS

- Given SOS rules (Categorical SOS rule)

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

- Derive

$$\Sigma I \xrightarrow{\text{initi}} I \longrightarrow FI$$

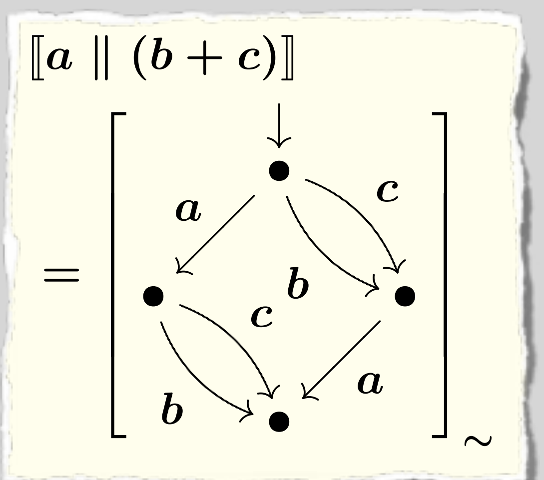
{process terms} = I

!!

$$\begin{array}{ccc} \Sigma I & \dashrightarrow & \Sigma F I \\ & & \downarrow \lambda_I \\ \text{initial} \downarrow & & F\Sigma I \\ & & \downarrow F(\text{initial}) \\ I & \dashrightarrow & FI \end{array}$$

Process SOS

* Modulo bisimilarity ➜ "semantics"



Bialgebraic SOS

* "Semantics" $[\![\_]\!]$ by coinduction

$$
\begin{array}{c}
\Sigma I \\
\big\downarrow \text{initial} \\
\{\text{process terms}\} = I \\
\big\downarrow \\
FI
\end{array}
$$

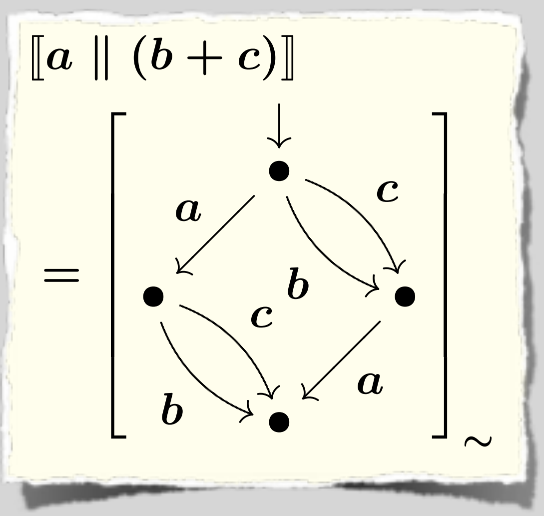
Bialgebraic SOS

* “Semantics” ⟦_⟧ by coinduction

$$
\begin{array}{ccc}
\Sigma I & & \\
\text{initial}\downarrow & & \\
I & \overset{[\![\_]\!]}{- - - - - \rightarrow} & Z \\
\downarrow & & \downarrow \text{final} \\
FI & \underset{F[\![\_]\!]}{- - - - - \rightarrow} & FZ
\end{array}
$$

Process SOS

* Modulo bisimilarity ➜ “semantics”



Process

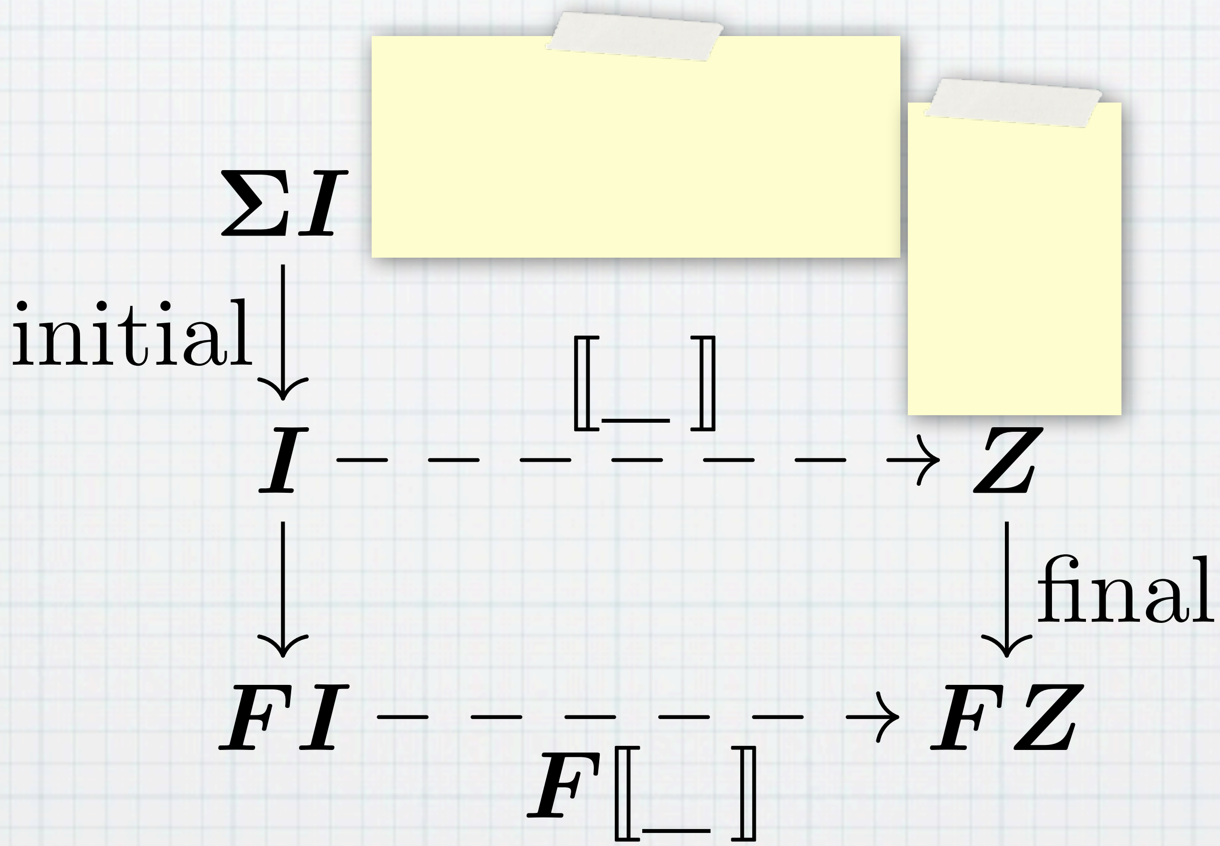
SOS

* Compositionality

$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$

Bialgebraic SOS

* Bialgebraic compositionality

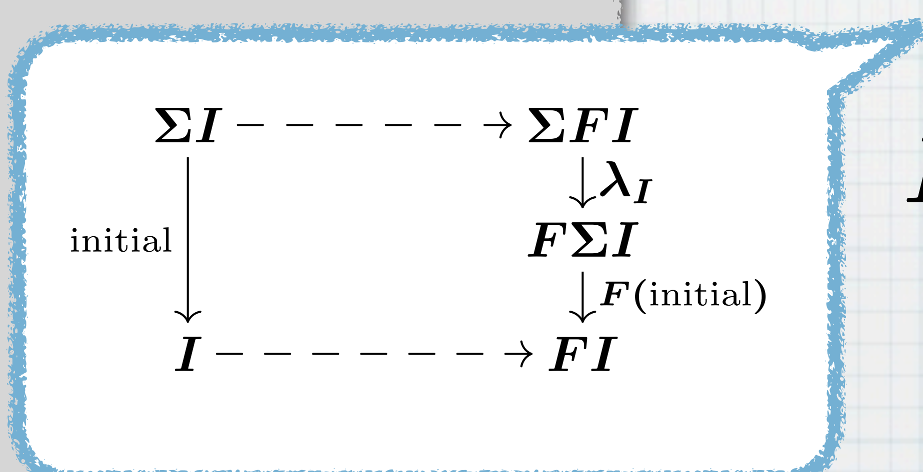


Hasuo (Tokyo)

Process SOS

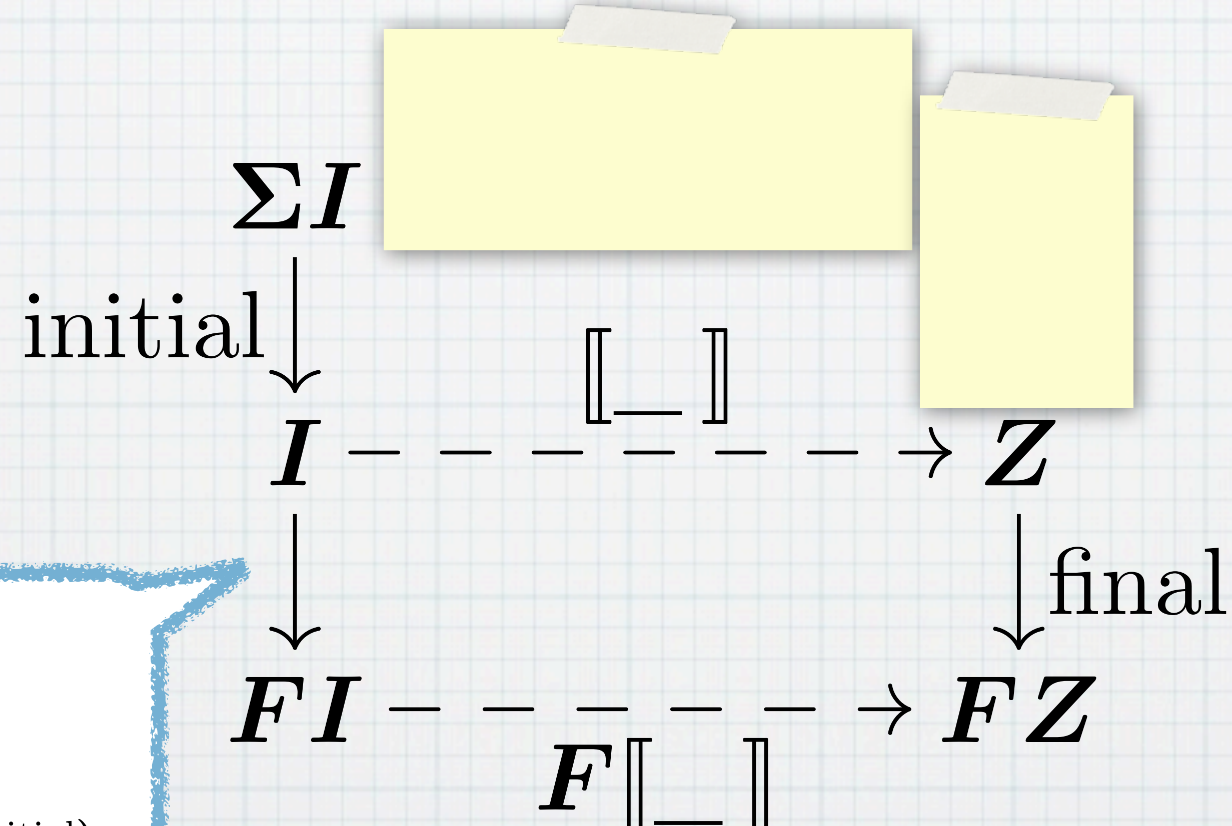
* Compositionality

$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$



Bialgebraic SOS

* Bialgebraic compositionality



Hasuo (Tokyo)

Process SOS

* Compositionality

$$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$$

Bialgebraic SOS

* Bialgebraic compositionality

$$\begin{array}{ccc}
\Sigma I & & \Sigma Z \\
{\scriptstyle \text{initial}}\downarrow & & \downarrow \\
I & \xrightarrow{[\![_]\!]} & Z \\
\downarrow & & \downarrow{\scriptstyle \text{final}} \\
FI & \xrightarrow[F[\![_]\!]]{} & FZ
\end{array}$$

$$\begin{array}{ccc}
\Sigma I & \dashrightarrow & \Sigma F I \\
 & & \downarrow{\scriptstyle \lambda_I} \\
{\scriptstyle \text{initial}}\downarrow & & F\Sigma I \\
 & & \downarrow{\scriptstyle F(\text{initial})} \\
I & \dashrightarrow & FI
\end{array}$$

$$\begin{array}{ccc}
\Sigma Z & \dashrightarrow & Z \\
{\scriptstyle \Sigma(\text{final})}\downarrow & & \\
\Sigma F Z & & \downarrow{\scriptstyle \text{final}} \\
{\scriptstyle \lambda_Z}\downarrow & & \\
F\Sigma Z & \dashrightarrow & FZ
\end{array}$$

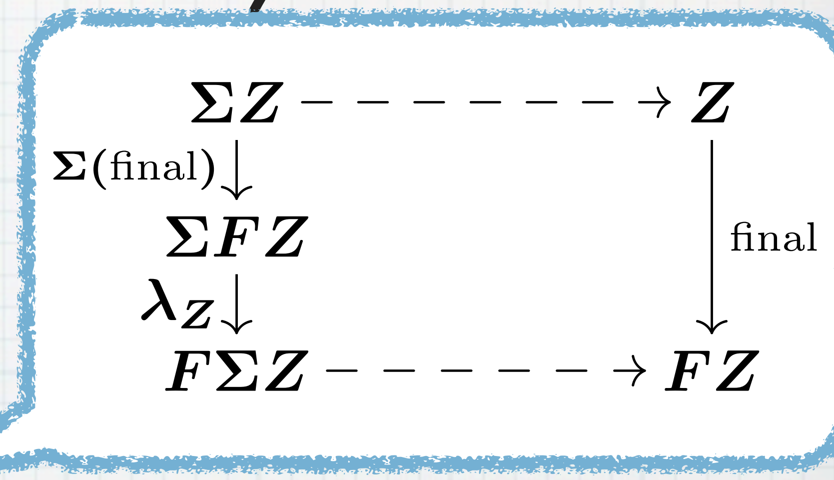
Process SOS

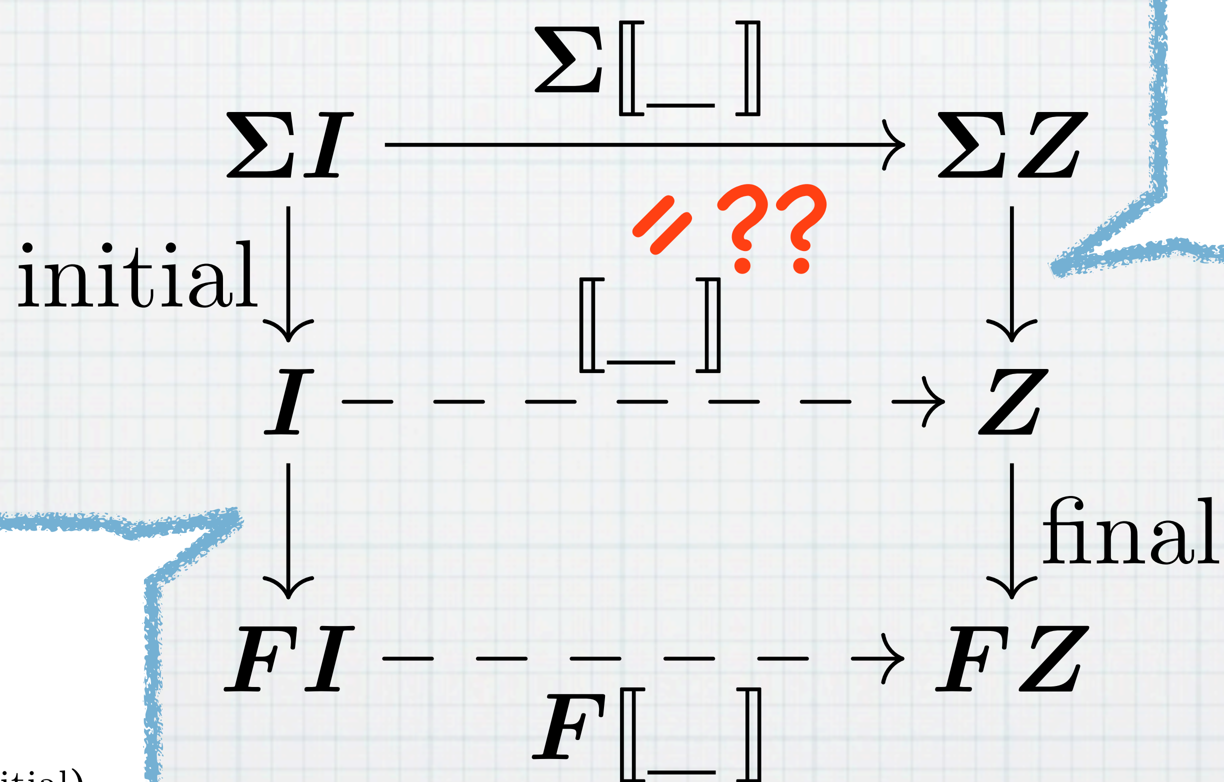
* Compositionality

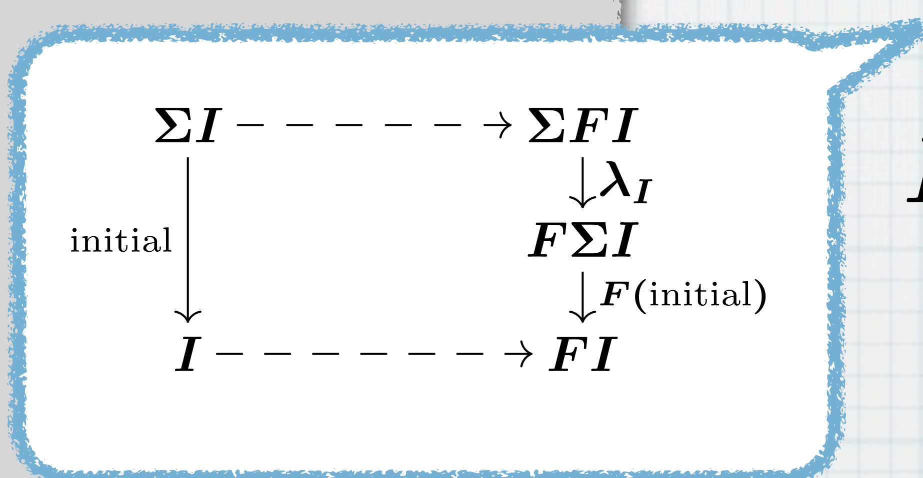
$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$

Bialgebraic SOS

* Bialgebraic compositionality







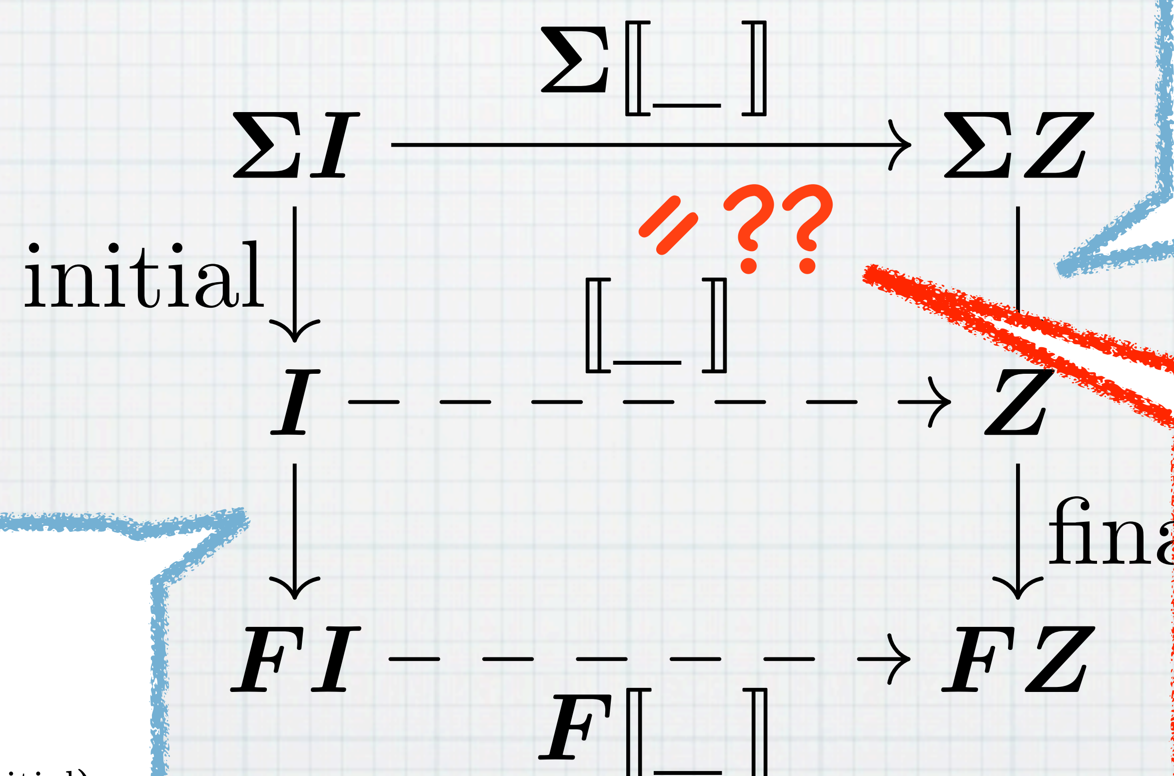
Process
SOS

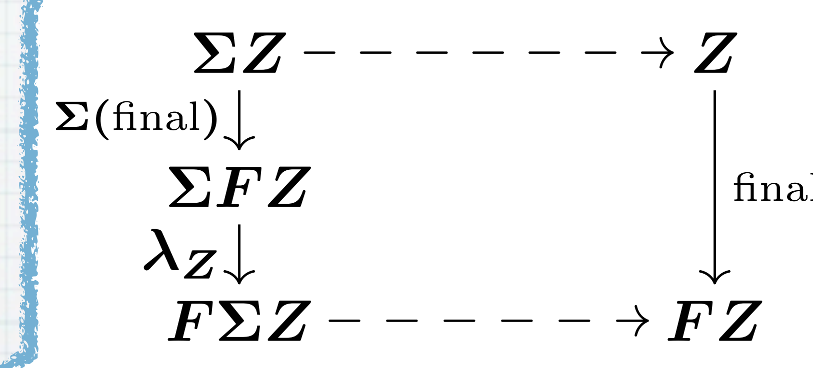
- Compositionality

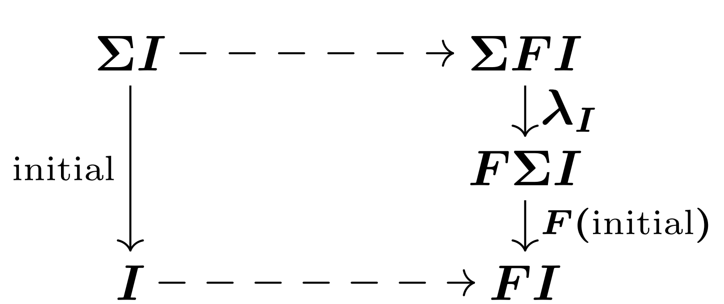
$$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$$

Bialgebraic SOS

- Bialgebraic compositionality







Yes! By routine diagram chase

Process SOS

* Compositionality

$$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$$

$$\begin{array}{ccc}
\Sigma I & \dashrightarrow & \Sigma F I \\
 & & \downarrow \lambda_I \\
\text{initial} \downarrow & & F \Sigma I \\
 & & \downarrow F(\text{initial}) \\
I & \dashrightarrow & F I
\end{array}$$

Bialgebraic SOS

$$\begin{array}{ccc}
\Sigma I & \xrightarrow{\Sigma[\![_]\!]} & \Sigma Z \\
\text{initial} \downarrow & & \downarrow \\
I & \overset{[\![_]\!]}{\dashrightarrow} & Z \\
\downarrow & & \downarrow \text{final} \\
F I & \underset{F[\![_]\!]}{\dashrightarrow} & F Z
\end{array}$$

$$\begin{array}{ccc}
\Sigma Z & \dashrightarrow & Z \\
\Sigma(\text{final}) \downarrow & & \\
\Sigma F Z & & \downarrow \text{final} \\
\lambda_Z \downarrow & & \\
F \Sigma Z & \dashrightarrow & F Z
\end{array}$$

Thm. (Compositionality)
The diagram commutes.

Process

SOS

* Compositionality

$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$

$$\begin{array}{ccc}
\Sigma I & \dashrightarrow & \Sigma F I \\
 & & \downarrow \lambda_I \\
\text{initial} \downarrow & & F\Sigma I \\
 & & \downarrow F(\text{initial}) \\
I & \dashrightarrow & F I
\end{array}$$

Bialgebraic SOS

$$\begin{array}{ccc}
\Sigma Z & \dashrightarrow & Z \\
\Sigma(\text{final}) \downarrow & & \\
\Sigma F Z & & \downarrow \text{final} \\
\lambda_Z \downarrow & & \\
F \Sigma Z & \dashrightarrow & F Z
\end{array}$$

$$\begin{array}{ccc}
\Sigma I & \xrightarrow{\Sigma[\![_]\!]} & \Sigma Z \\
\text{initial} \downarrow & & \downarrow \\
I & \overset{[\![_]\!]}{\dashrightarrow} & Z \\
\downarrow & & \downarrow \text{final} \\
F I & \underset{F[\![_]\!]}{\dashrightarrow} & F Z
\end{array}$$

$$\begin{array}{c}
\left(\coprod_{\sigma \in \Sigma} Z^{\text{arity}(\sigma)} \right) \\
\downarrow [\sigma]_{\sigma \in \Sigma} \\
Z
\end{array}$$

process opr. acting on **behaviors**

Thm. (Compositionality)
The diagram commutes.

Process SOS

* Compositionality

$$[\![s \parallel t]\!] = [\![s]\!] \parallel [\![t]\!]$$

$$\begin{array}{ccc} \Sigma I & \dashrightarrow & \Sigma F I \\ & & \downarrow \lambda_I \\ \text{initial} \downarrow & & F\Sigma I \\ & & \downarrow F(\text{initial}) \\ I & \dashrightarrow & FI \end{array}$$

Bialgebraic SOS

$$\begin{array}{ccc} \Sigma I & \xrightarrow{\Sigma[\![\_]\!]} & \Sigma Z \\ \text{initial} \downarrow & & \downarrow \\ I & \overset{[\![\_]\!]}{\dashrightarrow} & Z \\ \downarrow & & \downarrow \text{final} \\ FI & \underset{F[\![\_]\!]}{\dashrightarrow} & FZ \end{array}$$

$$\begin{array}{ccc} \Sigma Z & \dashrightarrow & Z \\ \Sigma(\text{final}) \downarrow & & \\ \Sigma F Z & & \downarrow \text{final} \\ \lambda_Z \downarrow & & \\ F\Sigma Z & \dashrightarrow & FZ \end{array}$$

$$\Big(\coprod_{\sigma \in \Sigma} Z^{\text{arity}(\sigma)}\Big) \xrightarrow{[\sigma]_{\sigma\in\Sigma}} Z$$

process opr. acting on **behaviors**

Thm. (Compositionality)
The diagram commutes.

Slogan Bialgebraic SOS is to derive

$$Z^{\text{arity}(\sigma)} \xrightarrow{[\sigma]} Z$$

Hasuo (Tokyo)

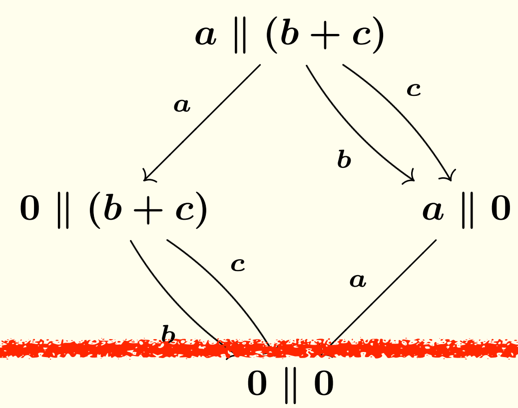
SOS: Variations

Part 1

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➡



Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➡

$$\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$$

categorically

Part 2

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➡ parallel composition of LTSs

Microcosm SOS
[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Part 3

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

Part 2

SOS in Component Calculi

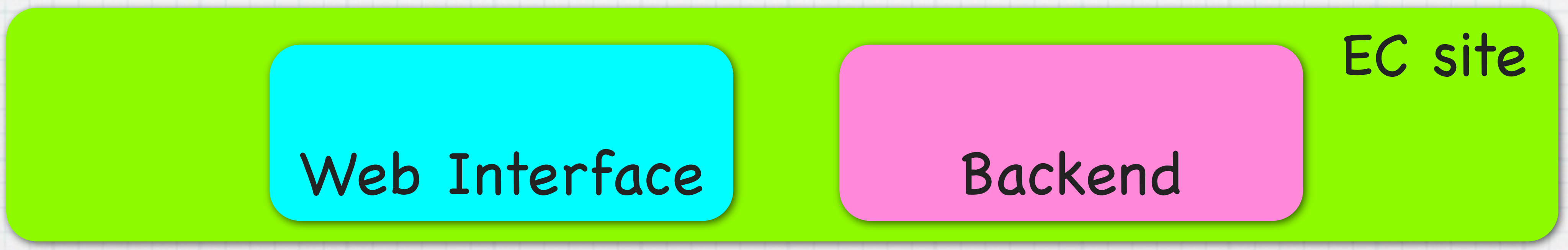
Component-Based Design

EC site

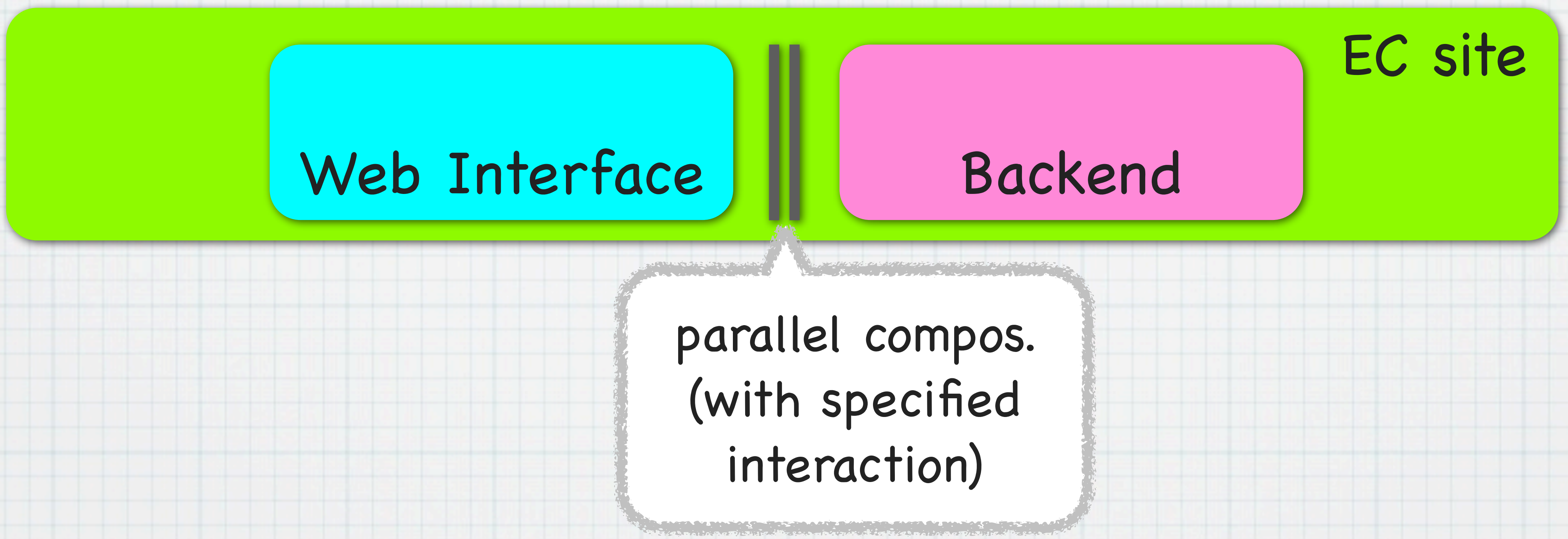
Hasuo (Tokyo)

Component-Based Design

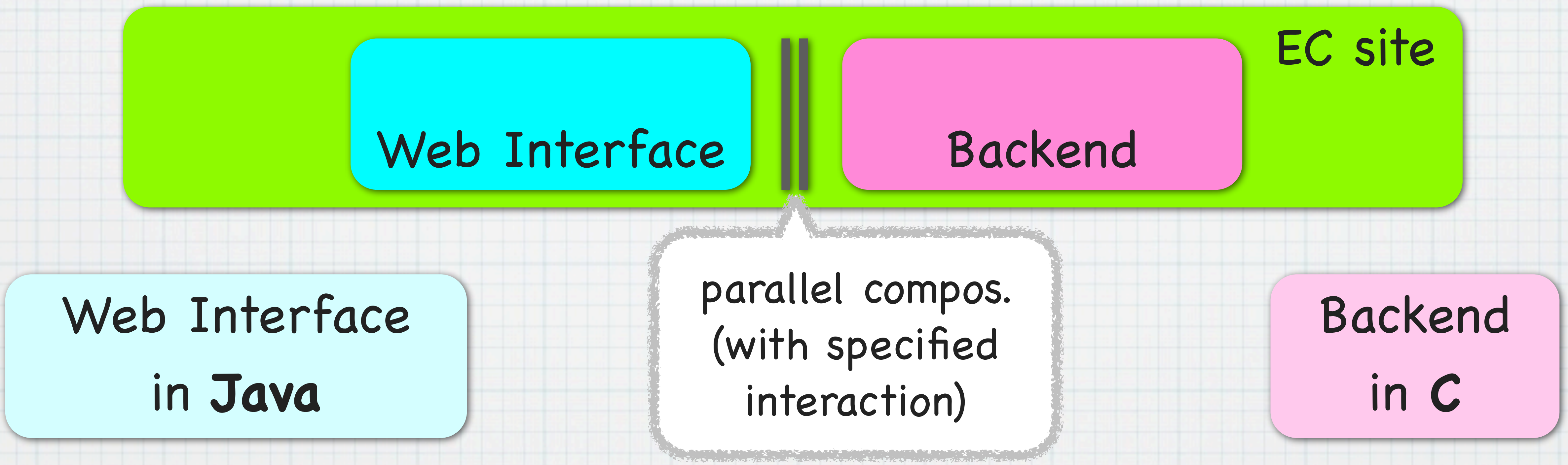
Component-Based Design



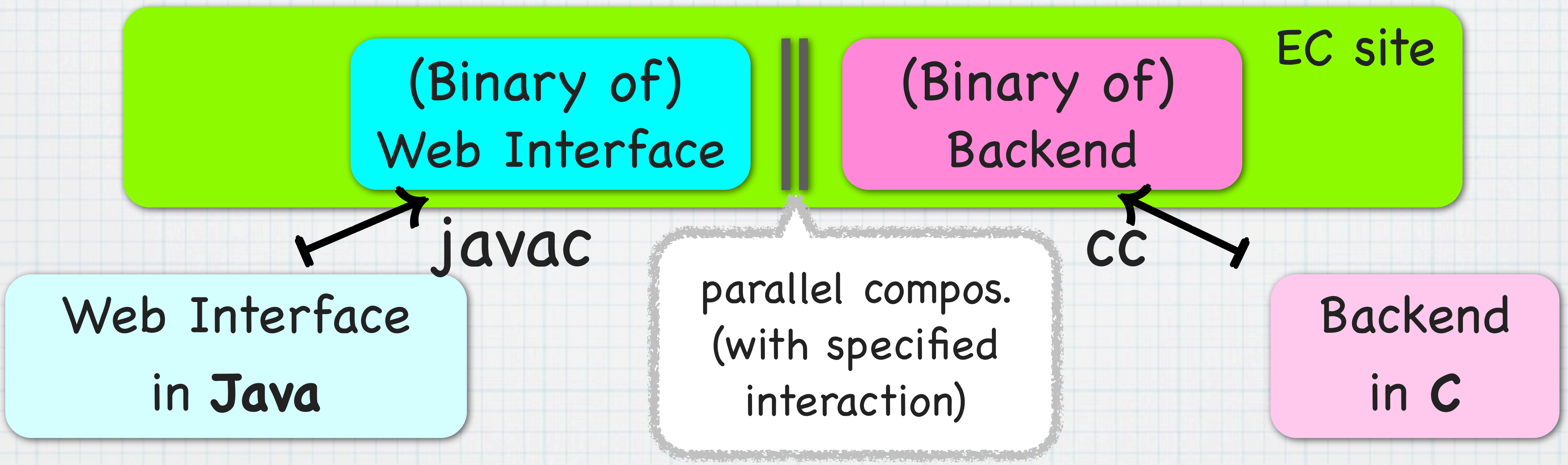
Component-Based Design



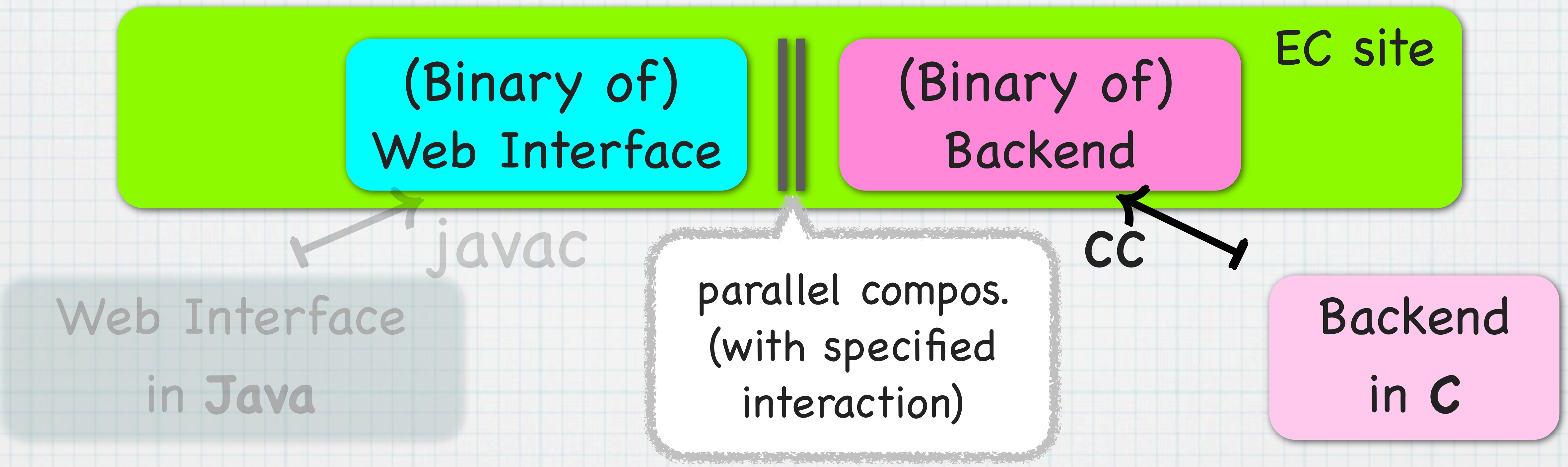
Component-Based Design



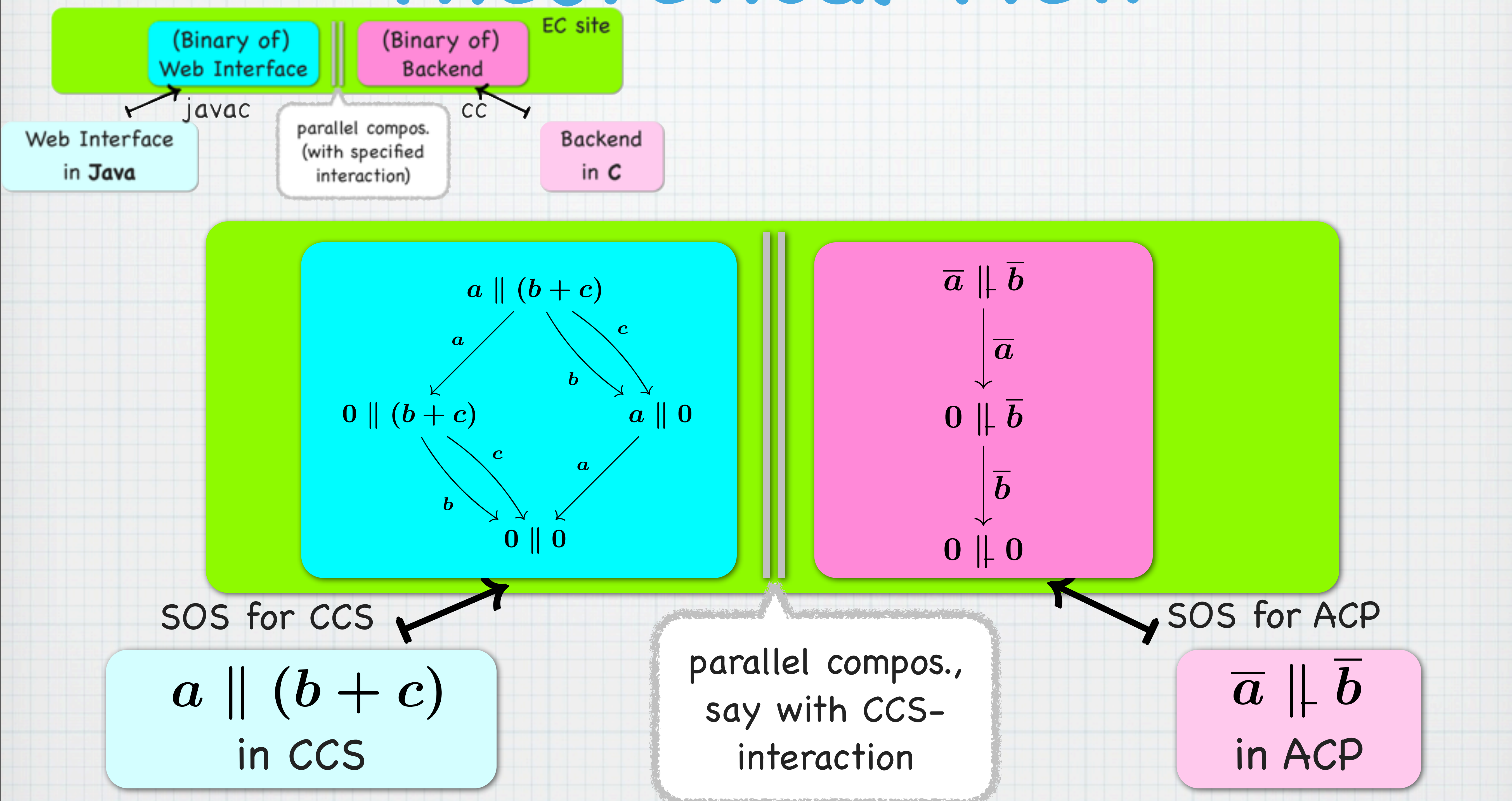
Component-Based Design



Component-Based Design



Component-Based Design: Theoretical View



Mathematical Theory of Components

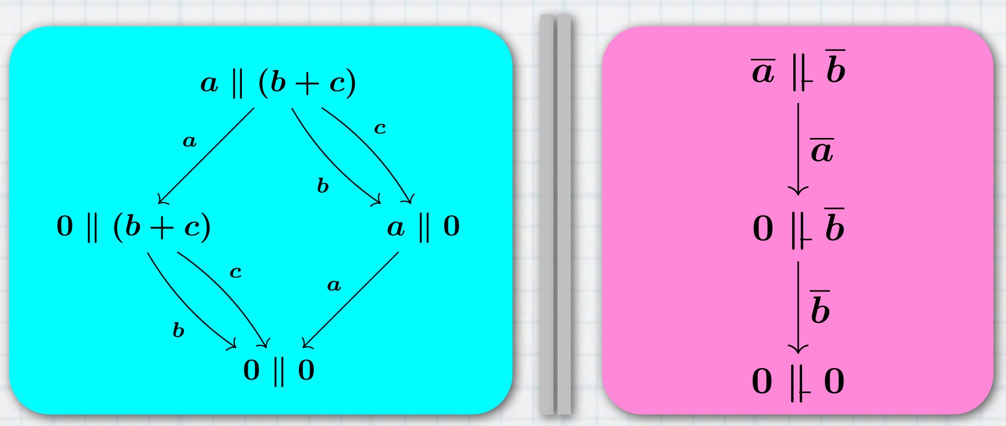
Hasuo (Tokyo)

Mathematical Theory of Components

* **Aim.**
  A model, for verification to be based on

Mathematical Theory of Components

- **Aim.**
  A model, for verification to be based on
- **That is**
  (Concrete/behavioral) description of



Component SOS

Idea also in:
[Bliudze&Krob,Fund.Inf.'09]
[Bruni,Lanese&Montanari,TCS'06]

$$\frac{x \xrightarrow{a} x' \qquad\qquad y \xrightarrow{\overline{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'} \;(\|\textsc{Sync})$$

Component SOS

Idea also in:
[Bliudze&Krob,Fund.Inf.'09]
[Bruni,Lanese&Montanari,TCS'06]

$$\frac{x \xrightarrow{a} x' \ \text{ in } \mathcal{S} \qquad\qquad y \xrightarrow{\overline{a}} y' \ \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \ \text{ in } \mathcal{S} \parallel \mathcal{T}} \ (\|\textsc{Sync})$$

Component SOS

Idea also in:
[Bliudze&Krob,Fund.Inf.'09]
[Bruni,Lanese&Montanari,TCS'06]

$$\frac{x \xrightarrow{a} x' \ \text{in } \mathcal{S} \qquad\qquad y \xrightarrow{\overline{a}} y' \ \text{in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \ (\|\textsc{Sync})$$

Variables x, y, ...

- in Process SOS: process terms
- in Component SOS: states of LTSs

Component SOS

Idea also in:
[Bliudze&Krob,Fund.Inf.'09]
[Bruni,Lanese&Montanari,TCS'06]

* More generally:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t}$$

Component SOS

Idea also in:
[Bliudze&Krob,Fund.Inf.'09]
[Bruni,Lanese&Montanari,TCS'06]

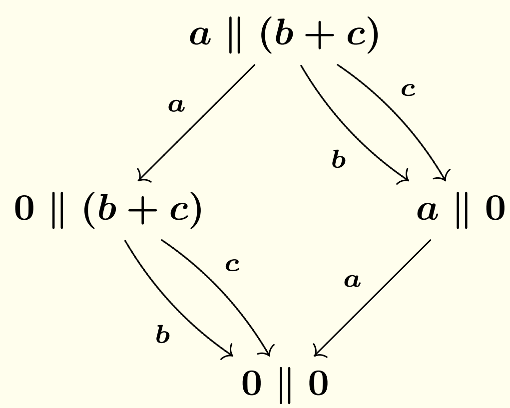
- More generally:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \stackrel{b}{\not\to} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t \quad \text{in } \sigma(\mathcal{S}_1,\dots,\mathcal{S}_m)}$$

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$



Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

$$\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

parallel composition of LTSs

Microcosm SOS
[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

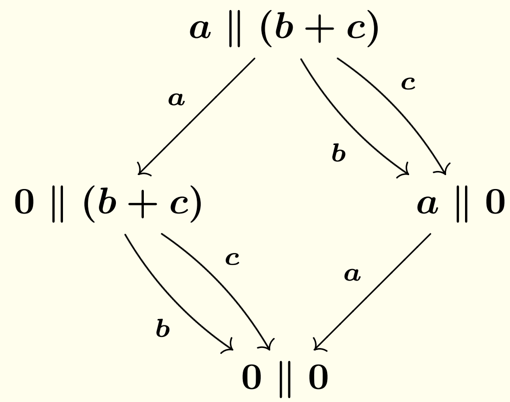
for any GSOS-specified σ

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\parallel\textsc{Sync})$$

➥



Bialgebraic SOS

[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥

$$\Sigma I$$
$$\downarrow \text{initial}$$
$$I$$
$$\downarrow !$$
$$FI$$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\parallel\textsc{Sync})$$

➥ parallel composition of LTSs

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\bar{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\parallel\textsc{Sync})$$

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* We focused on || (synchr. par. comp.)

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\bar{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \ (\|\text{SYNC})$$

Bialg.
SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\text{SYNC})$$

➥ $\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$

➥ $Z^{\text{arity}(\sigma)} \xrightarrow{[\sigma]} Z$

process opr.
on behaviors

Bialg. SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥ $\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$

➥ $Z^{\mathrm{arity}(\sigma)} \xrightarrow{[\sigma]} Z$

process opr. on behaviors

Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* We focused on || (synchr. par. comp.)

$$\frac{x \stackrel{a}{\to} x' \text{ in } \mathcal{S} \qquad y \stackrel{\bar{a}}{\to} y' \text{ in } \mathcal{T}}{x \parallel y \stackrel{\tau}{\to} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \ (\|\textsc{Sync})$$

➥ $$\Big(FX \times FY \xrightarrow{\mathsf{sync}} F(X \times Y) \Big)_{X,Y \in \mathbf{Sets}}$$

Bialg.
SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$

➥ $\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$

➥ $Z^{\mathrm{arity}(\sigma)} \xrightarrow{[\sigma]} Z$

process opr.
on behaviors

Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* We focused on || (synchr. par. comp.)

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \ (\parallel\textsc{Sync})$$

➥ $\Big(FX \times FY \xrightarrow{\mathsf{sync}} F(X \times Y) \Big)_{X,Y \in \mathbf{Sets}}$

➥ $\mathbf{Coalg}_F \times \mathbf{Coalg}_F \xrightarrow{\parallel} \mathbf{Coalg}_F$

Bialg. SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\|\textsc{Sync})$$

➥ $\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$

➥ $Z^{\mathrm{arity}(\sigma)} \xrightarrow{[\sigma]} Z$

process opr. on behaviors

Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

- We focused on || (synchr. par. comp.)

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\bar{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

➥ $$\Big(FX \times FY \xrightarrow{\mathsf{sync}} F(X \times Y) \Big)_{X,Y \in \mathbf{Sets}}$$

➥ $$\mathbf{Coalg}_F \times \mathbf{Coalg}_F \xrightarrow{\|} \mathbf{Coalg}_F$$

$$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \mathsf{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$

Bialg.
SOS

Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \; (\|\text{SYNC})$$

➥ $\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$

➥ $Z^{\text{arity}(\sigma)} \xrightarrow{[\sigma]} Z$

process opr. on behaviors

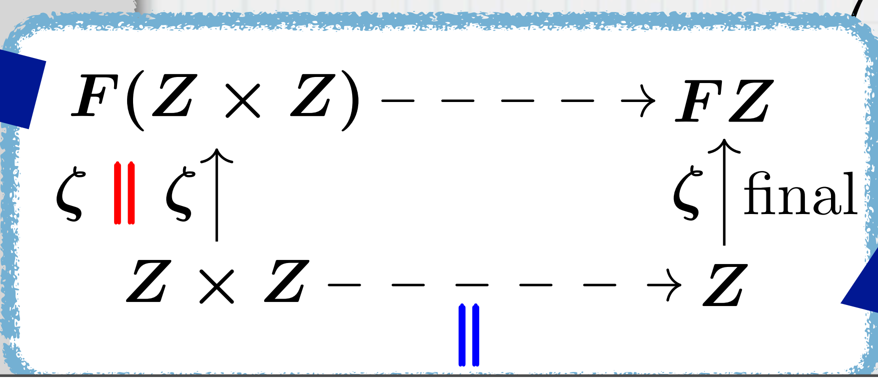
* We focused on || (synchr. par. comp.)

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \; (\|\text{SYNC})$$

➥ $$\Big(FX \times FY \xrightarrow{\text{sync}} F(X \times Y) \Big)_{X,Y \in \mathbf{Sets}}$$

➥ $$\mathbf{Coalg}_F \times \mathbf{Coalg}_F \xrightarrow{\|} \mathbf{Coalg}_F$$

$$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \text{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$



Hasuo (Tokyo)

Compositionality for Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* **Thm.** Given $\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y) \right)_{X,Y \in \mathbf{Sets}}$

1. Induces $\mathbf{Coalg}_F \times \mathbf{Coalg}_F \overset{\|}{\longrightarrow} \mathbf{Coalg}_F$

2. Induces $Z \times Z \overset{\|}{\longrightarrow} Z$ (coinduction)

3. The diagram commutes:

$$\begin{array}{ccc} \mathbf{Coalg}_F \times \mathbf{Coalg}_F & \overset{\|}{\longrightarrow} & \mathbf{Coalg}_F \\ {\scriptstyle \mathsf{beh} \times \mathsf{beh}} \downarrow & & \downarrow {\scriptstyle \mathsf{beh}} \\ \mathbf{Sets}/Z \times \mathbf{Sets}/Z & \underset{\|}{\longrightarrow} & \mathbf{Sets}/Z \end{array}$$

Compositionality for Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* **Thm.** Given $\left(FX \times FY \stackrel{\text{sync}}{\longrightarrow} F(X \times Y) \right)_{X,Y \in \mathbf{Sets}}$

1. Induces $\mathbf{Coalg}_F \times \mathbf{Coalg}_F \stackrel{\|}{\longrightarrow} \mathbf{Coalg}_F$

$$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \text{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$

2. Induces $Z \times Z \stackrel{\|}{\longrightarrow} Z$ (coinduction)

3. The diagram commutes:

$$\begin{array}{ccc} \mathbf{Coalg}_F \times \mathbf{Coalg}_F & \stackrel{\|}{\longrightarrow} & \mathbf{Coalg}_F \\ \mathbf{beh} \times \mathbf{beh} \downarrow & & \downarrow \mathbf{beh} \\ \mathbf{Sets}/Z \times \mathbf{Sets}/Z & \underset{\|}{\longrightarrow} & \mathbf{Sets}/Z \end{array}$$

Compositionality for Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* **Thm.** Given $\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y) \right)_{X,Y \in \mathbf{Sets}}$

  1. Induces $\mathrm{Coalg}_F \times \mathrm{Coalg}_F \xrightarrow{\parallel} \mathrm{Coalg}_F$

     $$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \text{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$

  2. Induces $Z \times Z \xrightarrow{\parallel} Z$ (coinduction)

  3. The diagram commutes:

$$\begin{array}{ccc} \mathrm{Coalg}_F \times \mathrm{Coalg}_F & \xrightarrow{\parallel} & \mathrm{Coalg}_F \\ \mathbf{beh} \times \mathbf{beh} \downarrow & & \downarrow \mathbf{beh} \\ \mathbf{Sets}/Z \times \mathbf{Sets}/Z & \xrightarrow{\parallel} & \mathbf{Sets}/Z \end{array}$$

by coinduction:

$$\begin{array}{ccc} FX & \dashrightarrow & FZ \\ c \uparrow & & \zeta \uparrow \text{final} \\ X & \underset{\mathbf{beh}(c)}{\dashrightarrow} & Z \end{array}$$

Hasuo (Tokyo)

Compositionality for Microcosm SOS

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* **Thm.** Given $\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y) \right)_{X,Y \in \mathbf{Sets}}$

 1. Induces $\mathrm{Coalg}_F \times \mathrm{Coalg}_F \xrightarrow{\;\|\;} \mathrm{Coalg}_F$

    $$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \text{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$

 2. Induces $Z \times Z \xrightarrow{\;\|\;} Z$ (coinduction)

 3. The diagram commutes:

$$\begin{array}{ccc} \mathrm{Coalg}_F \times \mathrm{Coalg}_F & \xrightarrow{\;\|\;} & \mathrm{Coalg}_F \\ \text{beh} \times \text{beh} \downarrow & & \downarrow \text{beh} \\ \mathbf{Sets}/Z \times \mathbf{Sets}/Z & \xrightarrow{\;\|\;} & \mathbf{Sets}/Z \end{array}$$

$$\left(\begin{array}{c} X \\ \downarrow f \\ Z \end{array} , \begin{array}{c} Y \\ \downarrow g \\ Z \end{array} \right) \longmapsto \begin{array}{c} X \times Y \\ \downarrow f \times g \\ Z \times Z \\ \downarrow \| \\ Z \end{array}$$

by coinduction:

$$\begin{array}{ccc} FX & \dashrightarrow & FZ \\ c \uparrow & & \zeta \uparrow \text{final} \\ X & \underset{\text{beh}(c)}{\dashrightarrow} & Z \end{array}$$

Hasuo (Tokyo)

Compositionality for Microcosm SOS

Categorically general, but limited expressivity

[Hasuo,Jacobs&Sokolova,FoSSaCS'08]

* **Thm.** Given $\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y) \right)_{X,Y \in \mathbf{Sets}}$

1. Induces $\mathbf{Coalg}_F \times \mathbf{Coalg}_F \xrightarrow{\parallel} \mathbf{Coalg}_F$

$$\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array} , \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \right) \longmapsto \begin{array}{c} F(X \times Y) \\ \uparrow \text{sync} \\ FX \times FY \\ \uparrow c \times d \\ X \times Y \end{array}$$

2. Induces $Z \times Z \xrightarrow{\parallel} Z$ (coinduction)

3. The diagram commutes:

$$\begin{array}{ccc} \mathbf{Coalg}_F \times \mathbf{Coalg}_F & \xrightarrow{\parallel} & \mathbf{Coalg}_F \\ \mathsf{beh} \times \mathsf{beh} \downarrow & & \downarrow \mathsf{beh} \\ \mathbf{Sets}/Z \times \mathbf{Sets}/Z & \xrightarrow{\parallel} & \mathbf{Sets}/Z \end{array}$$

$$\left(\begin{array}{c} X \\ \downarrow f \\ Z \end{array} , \begin{array}{c} Y \\ \downarrow g \\ Z \end{array} \right) \longmapsto \begin{array}{c} X \times Y \\ \downarrow f \times g \\ Z \times Z \\ \downarrow \parallel \\ Z \end{array}$$

by coinduction:

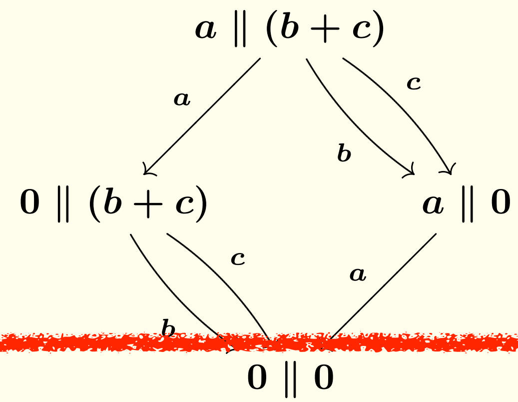
$$\begin{array}{ccc} FX & \dashrightarrow & FZ \\ c \uparrow & & \zeta \uparrow \text{final} \\ X & \underset{\mathsf{beh}(c)}{\dashrightarrow} & Z \end{array}$$

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\parallel\textsc{Sync})$$

➡



Bialgebraic SOS

[Turi&Plotkin, LICS'97]

Part 1

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➡ $\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\parallel\textsc{Sync})$$

➡ parallel composition of LTSs

Microcosm SOS

Part 2

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Part 3

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

Part 3

Microcosm SOS
for the full GSOS format

GSOS-Based Component Calculi

$F = \mathcal{P}_{\mathrm{fin}}(A \times \_\,)$

* **Aim:**

  $(\Sigma, \mathcal{R})$, GSOS-specification

  $\mapsto$

  For each σ∈Σ,

  $[\![\sigma]\!] \;:\; (\mathbf{Coalg}_F)^n \to \mathbf{Coalg}_F$

GSOS-Based Component Calculi

$F = \mathcal{P}_{\mathrm{fin}}(A \times \_\,)$

* **Aim:**

$(\Sigma, \mathcal{R})$, GSOS-specification

* Σ: algebraic signature
* R: set of GSOS rules

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t \quad \text{in } \sigma(\mathcal{S}_1,\dots,\mathcal{S}_m)}$$

$\longmapsto$

For each $\sigma \in \Sigma$,

$[\![\sigma]\!] \;:\; (\mathrm{Coalg}_F)^n \to \mathrm{Coalg}_F$

GSOS-Based Component Calculi

$F = \mathcal{P}_{\mathrm{fin}}(A \times \_\,)$

- **Aim:**

$(\Sigma, \mathcal{R})$, GSOS-specification

- Σ: algebraic signature
- R: set of GSOS rules

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t \quad \text{in } \sigma(\mathcal{S}_1,\dots,\mathcal{S}_m)}$$

$\longmapsto$

For each $\sigma \in \Sigma$,

$[\![\sigma]\!] \; : \; (\mathbf{Coalg}_F)^n \to \mathbf{Coalg}_F$

par. comp., seq. comp., replication, Kleene star, ...

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow c \parallel d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow \\ X \times Y \end{array}$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow \\ X \times Y \end{array}$$

$$\frac{x \overset{a}{\not\to} \;(\forall a \in A) \quad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'} \;(;\mathrm{R})$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\bar{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\text{SYNC})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \\ X \times Y \end{array}$$

$$\frac{x \overset{a}{\not\to} \;(\forall a \in A) \qquad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'} \;(;\text{R})$$

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow c \parallel d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \ (\|\textsc{Sync})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \\ X \times Y \end{array} \qquad \begin{array}{c} F(X \times Y + Y) \\ \uparrow \\ X \times Y + Y \end{array} \ ??$$

$$\frac{x \stackrel{a}{\not\to} \ (\forall a \in A) \qquad y \xrightarrow{b} y'}{x ; y \xrightarrow{b} y'} \ (;\mathrm{R})$$

Hasuo (Tokyo)

Sunday, September 11, 2011

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\textsc{Sync})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \quad = \quad \begin{array}{c} F(X \times Y) \\ \\ X \times Y \end{array} \qquad \begin{array}{c} F(X \times Y + Y) \\ \uparrow \\ X \times Y + Y \end{array} \;\; ??$$

$$\frac{x \overset{a}{\not\to} \;(\forall a \in A) \quad y \xrightarrow{b} y'}{x ; y \xrightarrow{b} y'} \;(;\mathrm{R})$$

$$!\left(\begin{array}{c} FX \\ \uparrow c \\ X \end{array}\right) \quad = \quad \begin{array}{c} F\,\boxed{??} \\ \uparrow \\ \boxed{??} \end{array}$$

Hasuo (Tokyo)

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow c \parallel d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\parallel\textsc{Sync})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \\ X \times Y \end{array} \qquad \begin{array}{c} F(X \times Y + Y) \\ \uparrow \\ X \times Y + Y \end{array} \;??$$

$$\frac{x \stackrel{a}{\not\to} \;(\forall a \in A) \qquad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'} \;(;\mathrm{R})$$

$$\frac{x \xrightarrow{a} x'}{!x \xrightarrow{a} x' \parallel !x} \;(!)$$

$$!\begin{pmatrix} FX \\ \uparrow c \\ X \end{pmatrix} \;=\; \begin{array}{c} F\,\boxed{??} \\ \uparrow \\ \boxed{??} \end{array}$$

Hasuo (Tokyo)

The State Space Problem

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;\Big\|\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \uparrow c \,\|\, d \\ X \times Y \end{array}$$

$$\frac{x \xrightarrow{a} x' \text{ in } \mathcal{S} \qquad y \xrightarrow{\overline{a}} y' \text{ in } \mathcal{T}}{x \parallel y \xrightarrow{\tau} x' \parallel y' \quad \text{in } \mathcal{S} \parallel \mathcal{T}} \;(\|\text{SYNC})$$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array} \;;\; \begin{array}{c} FY \\ \uparrow d \\ Y \end{array} \;=\; \begin{array}{c} F(X \times Y) \\ \\ X \times Y \end{array} \qquad \begin{array}{c} F(X \times Y + Y) \\ \uparrow \\ X \times Y + Y \end{array} \;??$$

$$\frac{x \overset{a}{\not\to} \;(\forall a \in A) \qquad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'} \;(;\text{R})$$

$$\frac{x \xrightarrow{a} x'}{!x \xrightarrow{a} x' \,\|\, !x} \;(!)$$

$$!\begin{pmatrix} FX \\ \uparrow c \\ X \end{pmatrix} = \begin{array}{c} F\,\boxed{??} \\ \uparrow \\ \boxed{??} \end{array}$$

$X \to X^2 \to X^3 \to \ldots$

X^ω ? X^* ? X^+ ?

In Other Words...

(Conventional) Process SOS

* **Syntactic format:**
  * One of the simplest:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} \sigma'(y_{i_1}^{a_1,j_1},\dots,y_{i_n}^{a_n,j_n})}$$

  * The GSOS format [Bloom,Istrail&Meyer,JACM'95]

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t}$$

Bialgebraic SOS [Turi&Plotkin, LICS'97]

* **Categorical format:**
  * Natural transformation

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

  * Categorical GSOS format [Turi&Plotkin, LICS'97]

$$\Sigma(F \times \mathrm{id}) \stackrel{\lambda}{\Rightarrow} F\Sigma^*$$

Microcosm SOS

* **Categorical format:**
  * Natural transformation

$$\Big(FX \times FY \xrightarrow{\mathsf{sync}} F(X \times Y)\Big)_{X,Y\in\mathbf{Sets}}$$

  * Categorical GSOS format

??

GSOS-Compatible State Space

* We want, for each process opr. σ∈Σ,

$$(\!|\sigma|\!) \;:\; \mathbf{Sets}^m \longrightarrow \mathbf{Sets}$$

* that generalizes "par. comp.":

$$(\!|\,\|\,|\!) \;:\; (X, Y) \longmapsto X \times Y$$

* that is functorial
* that supports

$$[\![\sigma]\!]\begin{pmatrix} FX_1 & & FX_m \\ \uparrow c_1 & ,\ldots, & \uparrow c_m \\ X_1 & & X_m \end{pmatrix} = \begin{array}{c} F\Big((\!|\sigma|\!)(X_1,\ldots,X_m)\Big) \\ \uparrow \\ (\!|\sigma|\!)(X_1,\ldots,X_m) \end{array}$$

Candidates

* $(\!|\sigma|\!)(X_1, \ldots, X_m)$
  $= \{\text{all } \Sigma\text{-terms with } x_i \in X_i \text{ as variables}\}$??

* → Too big for $(\!|\,\|\,|\!) \; : \; (X, Y) \longmapsto X \times Y$

*

Candidates

* $(\!|\sigma|\!)(X_1, \ldots, X_m)$
  $= \{\text{all } \Sigma\text{-terms with } x_i \in X_i \text{ as variables}\}$??
  * ➜ Too big for $(\!|\,\|\,|\!) \;:\; (X, Y) \longmapsto X \times Y$
* $(\!|\sigma|\!)(X_1, \ldots, X_m)$
  $= \left\{ \begin{array}{c} \text{all } \Sigma\text{-terms with } x_i \in X_i \text{ as variables} \\ \text{that are “reachable”} \end{array} \right\}$??
  * ➜ involves dynamics, not $(\!|\sigma|\!) \;:\; \mathbf{Sets}^m \longrightarrow \mathbf{Sets}$

Proposed Solution

- **Syntactic** approximation of actual reachable part:

$$\frac{\{x_i \stackrel{a}{\to} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \stackrel{e}{\to} t}$$

- ... formally via **term lineage graphs**

Proposed Solution

* **Syntactic** approximation of actual reachable part:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A,\, j\in[1,N_i^a]} \qquad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t}$$

* … formally via **term lineage graphs**

Proposed Solution

* **Syntactic** approximation of actual reachable part:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \overset{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t}$$

"σ evolves into the term t"

* … formally via **term lineage graphs**

Proposed Solution

- **Syntactic** approximation of actual reachable part:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t}$$

"σ evolves into the term t"

- ... formally via **term lineage graphs**

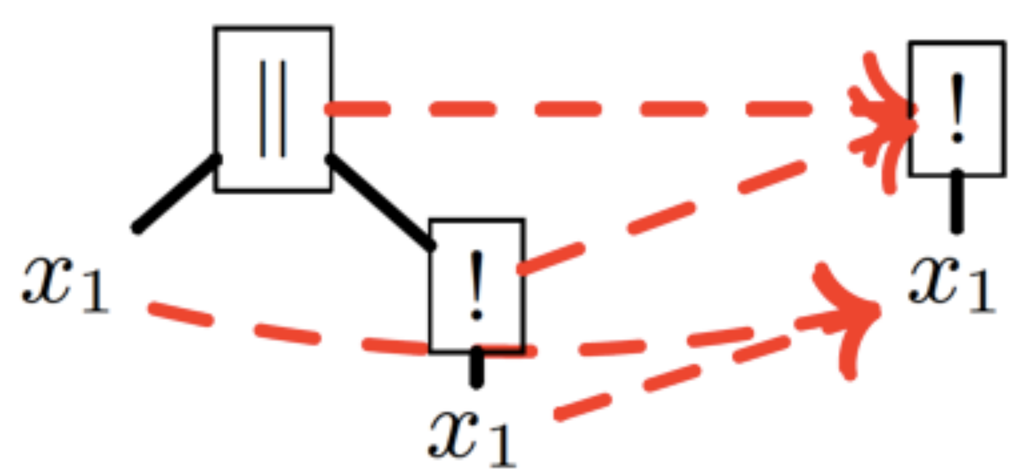
Term Lineage Graph (TLG)

Definition. (Term lineage graph)
Let s, t be Σ-terms. A *term lineage graph (TLG)* ρ from s to t, denoted by $\rho : s \Rightarrow t$, is an unlabeled directed graph whose nodes are nodes of s and t (seen as parse trees), such that:

- any edge is from a node in the *domain term* s to a node in the *codomain term* t;
- each node in s has exactly one outgoing edge;
- the edges are *monotone*: assume that the origin of one edge is a descendant (in the parse tree s) of the origin of another edge. Then the target of the former is also a descendant of (or the same as) that of the latter;
- an edge from an operator symbol σ goes into a (not necessarily the same) operator symbol;
- an edge from a variable x_i in s goes into the same variable x_i in t.

* Bipartite graph, from a term to another
  * Terms as parse trees
  * Example:

$$x_1 \,\|\, !x_1 \quad \overset{\rho(!)}{\Longrightarrow} \quad !x_1$$



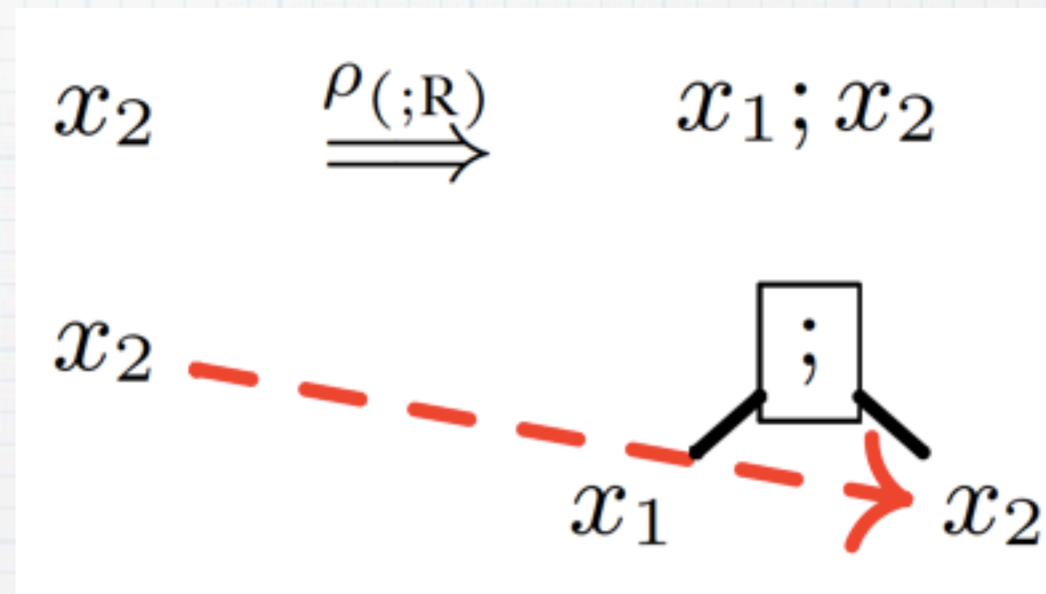
Hasuo (Tokyo)

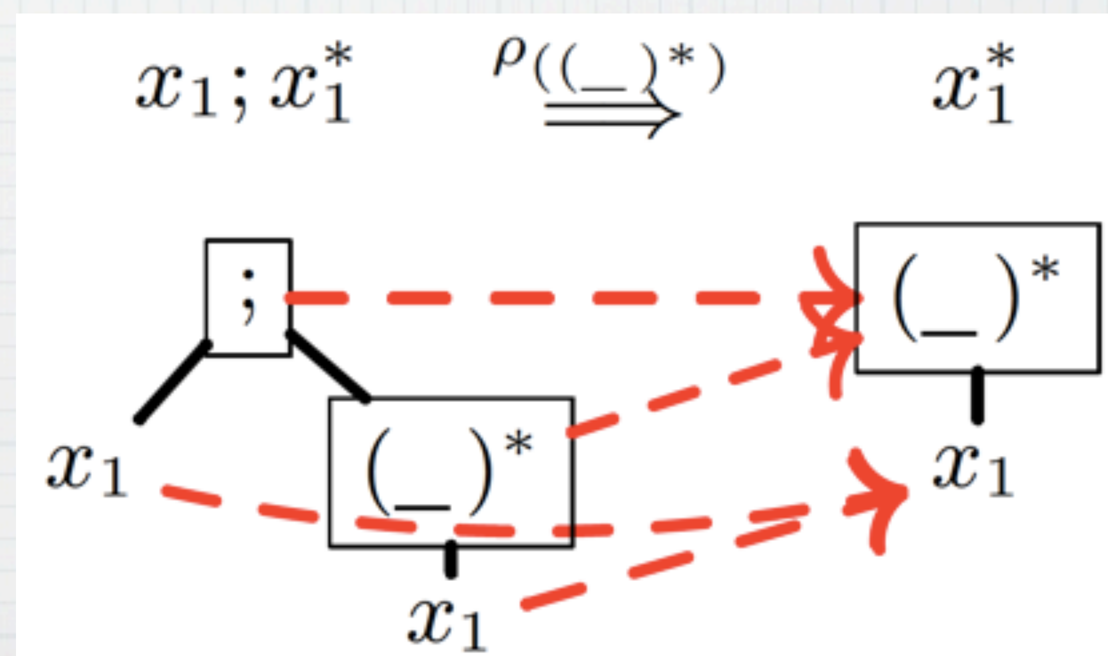
Term Lineage Graph (TLG)

Definition. (Term lineage graph)
Let $\boldsymbol{s}, \boldsymbol{t}$ be $\boldsymbol{\Sigma}$-terms. A *term lineage graph (TLG)* $\boldsymbol{\rho}$ from $\boldsymbol{s}$ to $\boldsymbol{t}$, denoted by $\boldsymbol{\rho} : \boldsymbol{s} \Rightarrow \boldsymbol{t}$, is an unlabeled directed graph whose nodes are nodes of $\boldsymbol{s}$ and $\boldsymbol{t}$ (seen as parse trees), such that:

- any edge is from a node in the *domain term* $\boldsymbol{s}$ to a node in the *codomain term* $\boldsymbol{t}$;
- each node in $\boldsymbol{s}$ has exactly one outgoing edge;
- the edges are *monotone*: assume that the origin of one edge is a descendant (in the parse tree $\boldsymbol{s}$) of the origin of another edge. Then the target of the former is also a descendant of (or the same as) that of the latter;
- an edge from an operator symbol $\boldsymbol{\sigma}$ goes into a (not necessarily the same) operator symbol;
- an edge from a variable $\boldsymbol{x_i}$ in $\boldsymbol{s}$ goes into the same variable $\boldsymbol{x_i}$ in $\boldsymbol{t}$.

* More examples:



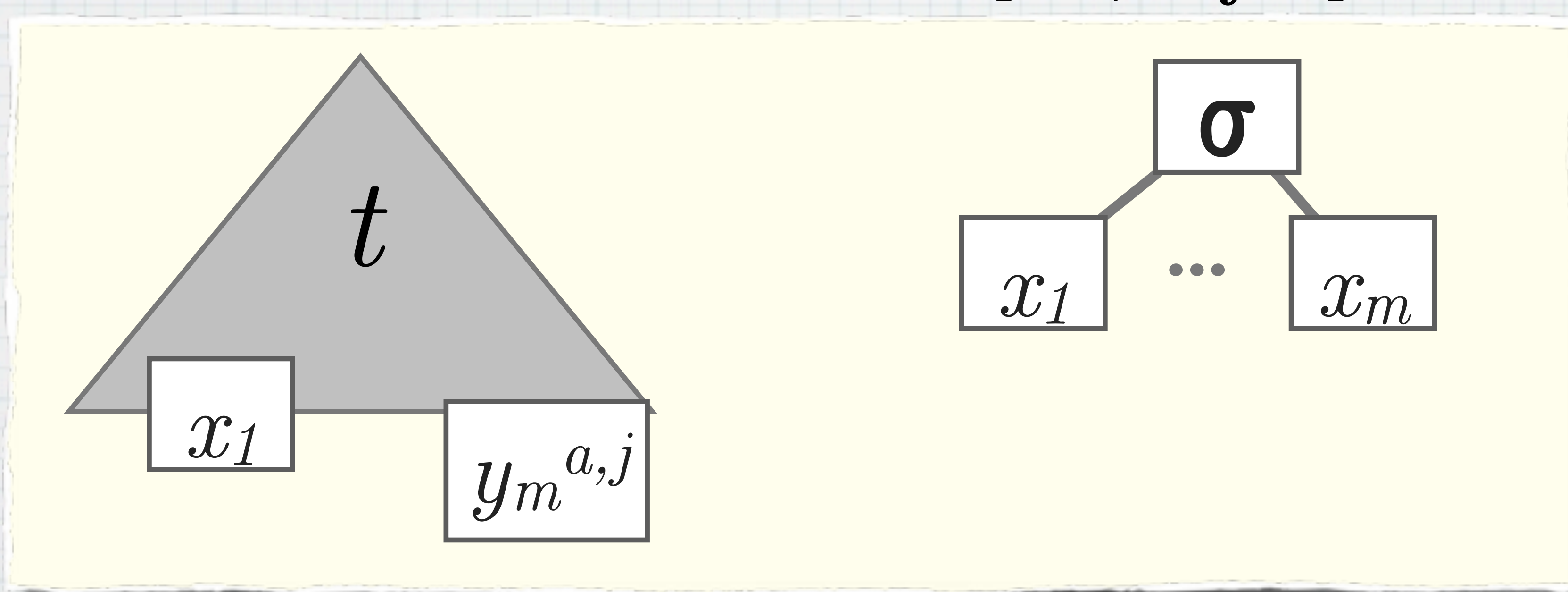


GSOS Rule → TLG

* A GSOS-rule

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}^{a\in A, j\in[1,N_i^a]}_{i\in[1,m]} \quad \{x_i \stackrel{b}{\not\to}\}^{b\in B_i}_{i\in[1,m]}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t} \quad \text{(R)}$$

induces a TLG $\rho_R : t[x_i/y_i^{a,j}] \Rightarrow \sigma(x_1,\dots,x_m)$



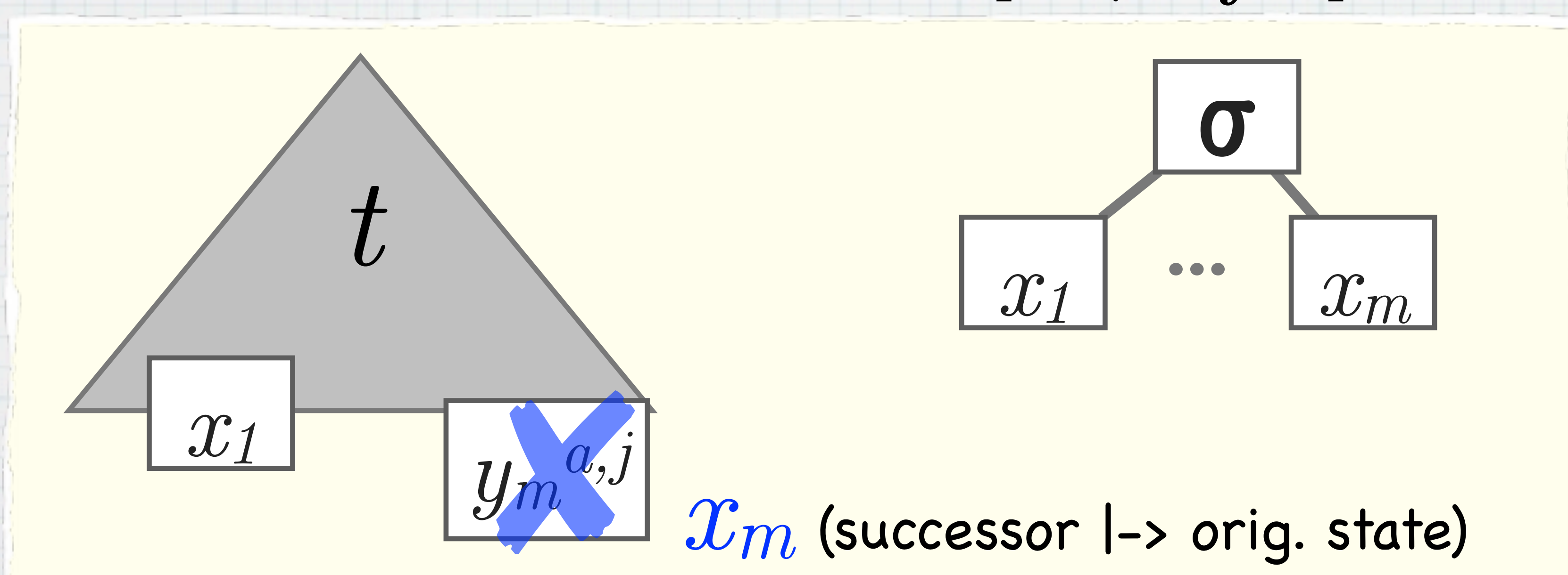
Hasuo (Tokyo)

GSOS Rule → TLG

* A GSOS-rule

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}^{a\in A, j\in[1,N_i^a]}_{i\in[1,m]} \qquad \{x_i \stackrel{b}{\not\to}\}^{b\in B_i}_{i\in[1,m]}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t} \quad \text{(R)}$$

induces a TLG $\rho_R : t[x_i/y_i^{a,j}] \Rightarrow \sigma(x_1,\dots,x_m)$

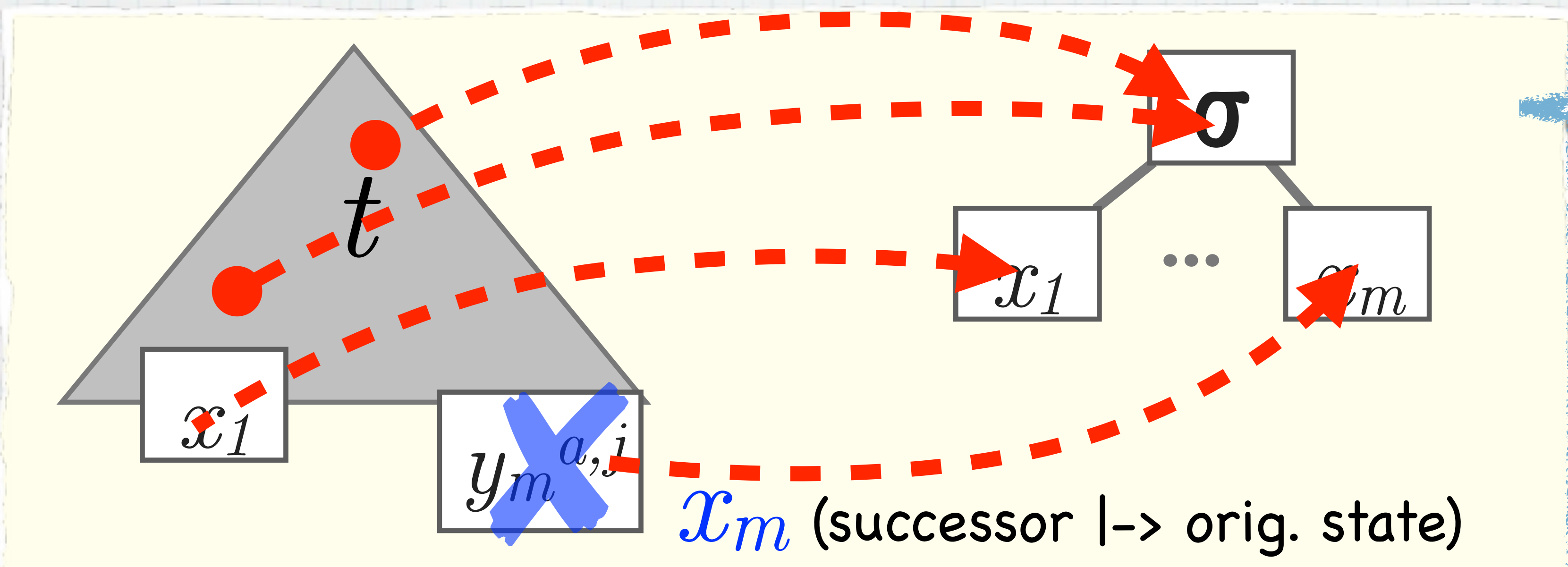


GSOS Rule → TLG

* A GSOS-rule

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \stackrel{b}{\not\rightarrow}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t} \quad \text{(R)}$$

induces a TLG $\rho_R : t[x_i/y_i^{a,j}] \Rightarrow \sigma(x_1,\ldots,x_m)$



* Opr. symbols --> σ
* Variables --> the same var.

More TLGs

- Identity TLG

$$\mathbf{id}_t \ : \ t \Rightarrow t$$

- Composition:

$$\frac{\rho \ : \ s \Rightarrow t \qquad \pi \ : \ t \Rightarrow u}{\pi \cdot \rho \ : \ s \Rightarrow u}$$

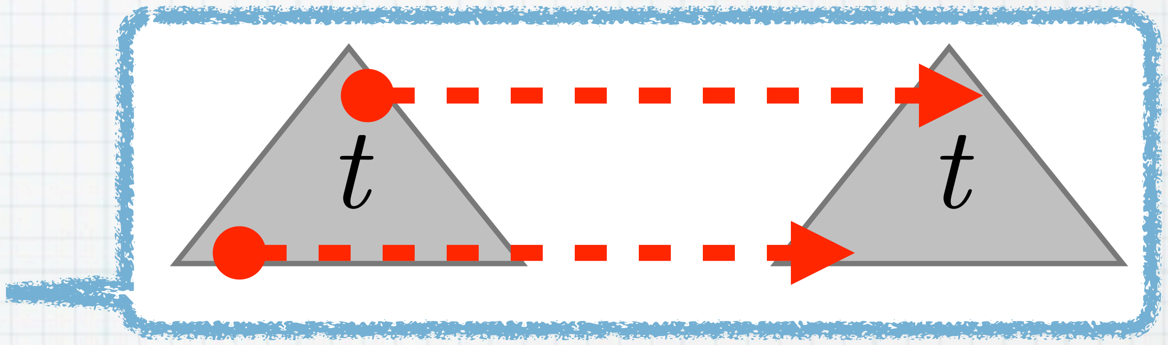
- Substitution:

$$\frac{\rho \ : \ t \Rightarrow u \qquad \rho_1 \ : \ t_1 \Rightarrow u_1 \qquad \cdots \qquad \rho_m \ : \ t_m \Rightarrow u_m}{\rho[\rho_i/x_i] \ : \ t[t_i/x_i] \Rightarrow u[u_i/x_i]}$$

More TLGs

* Identity TLG

$$\mathbf{id}_t \ : \ t \Rightarrow t$$



* Composition:

$$\frac{\rho \ : \ s \Rightarrow t \qquad \pi \ : \ t \Rightarrow u}{\pi \cdot \rho \ : \ s \Rightarrow u}$$

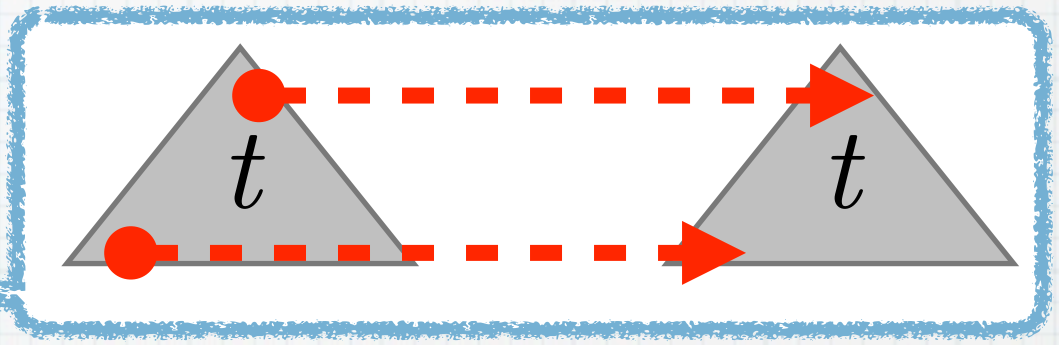
* Substitution:

$$\frac{\rho \ : \ t \Rightarrow u \qquad \rho_1 \ : \ t_1 \Rightarrow u_1 \qquad \cdots \qquad \rho_m \ : \ t_m \Rightarrow u_m}{\rho[\rho_i/x_i] \ : \ t[t_i/x_i] \Rightarrow u[u_i/x_i]}$$

More TLGs

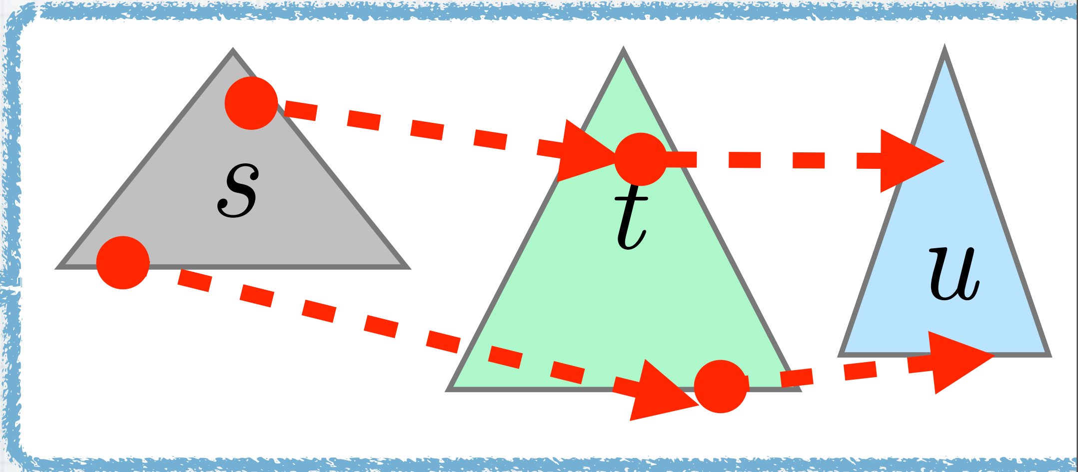
- Identity TLG

$$\mathbf{id}_t \ : \ t \Rightarrow t$$



- Composition:

$$\frac{\rho \ : \ s \Rightarrow t \qquad \pi \ : \ t \Rightarrow u}{\pi \cdot \rho \ : \ s \Rightarrow u}$$



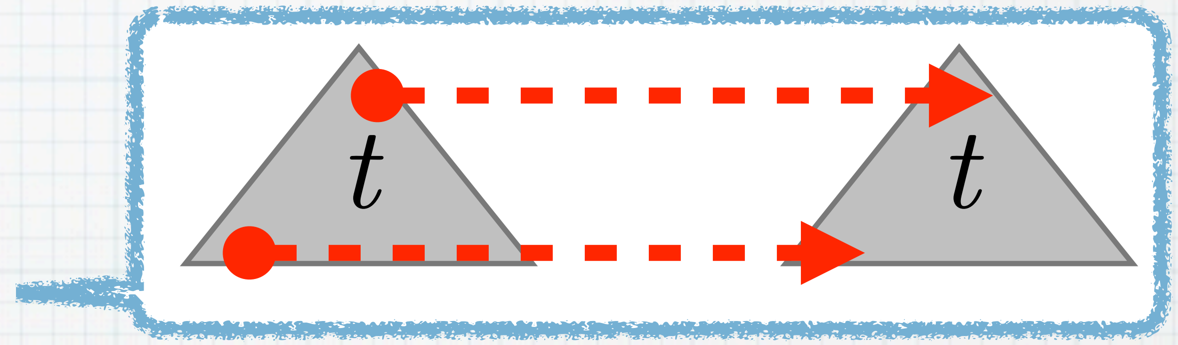
- Substitution:

$$\frac{\rho \ : \ t \Rightarrow u \qquad \rho_1 \ : \ t_1 \Rightarrow u_1 \qquad \cdots \qquad \rho_m \ : \ t_m \Rightarrow u_m}{\rho[\rho_i/x_i] \ : \ t[t_i/x_i] \Rightarrow u[u_i/x_i]}$$

More TLGs

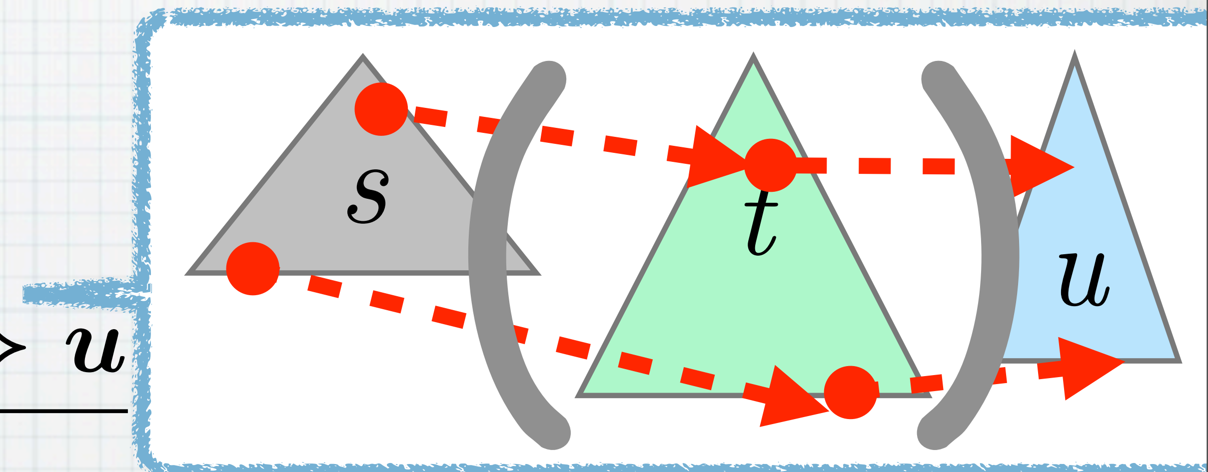
- Identity TLG

$$\mathbf{id}_t \ : \ t \Rightarrow t$$



- Composition:

$$\frac{\rho \ : \ s \Rightarrow t \qquad \pi \ : \ t \Rightarrow u}{\pi \cdot \rho \ : \ s \Rightarrow u}$$



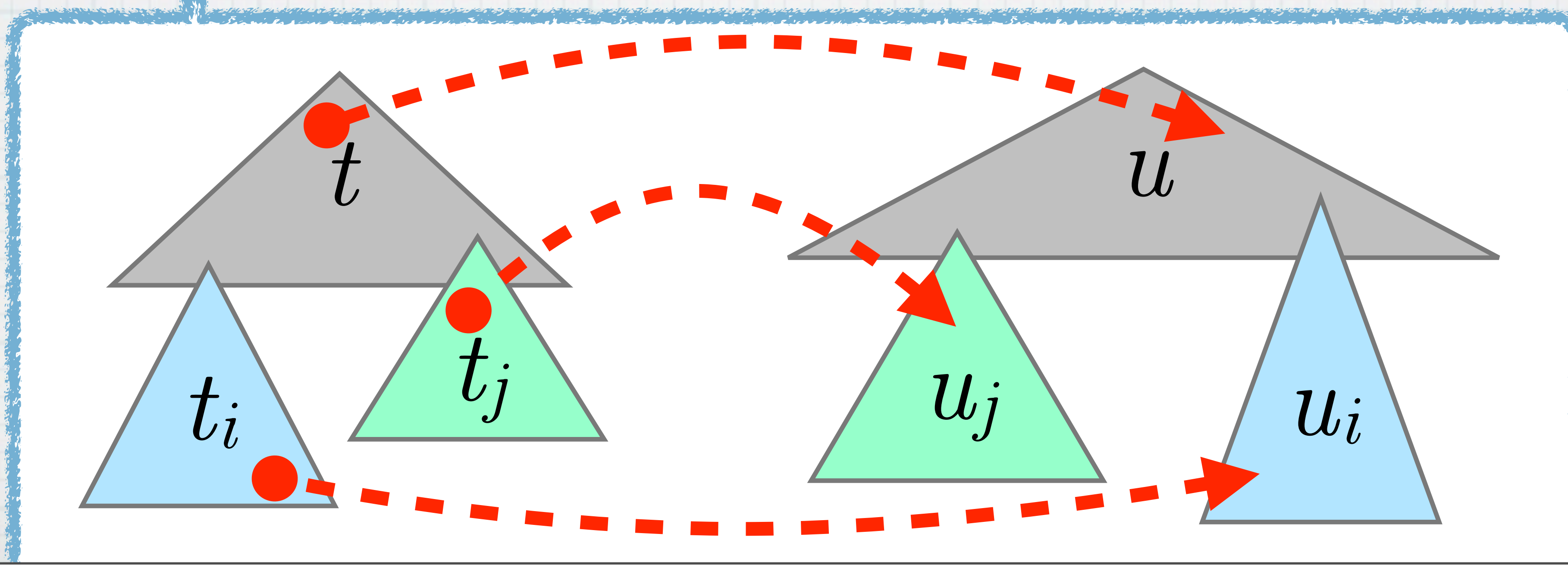
- Substitution:

$$\frac{\rho \ : \ t \Rightarrow u \qquad \rho_1 \ : \ t_1 \Rightarrow u_1 \qquad \cdots \qquad \rho_m \ : \ t_m \Rightarrow u_m}{\rho[\rho_i/x_i] \ : \ t[t_i/x_i] \Rightarrow u[u_i/x_i]}$$

More TLGs

* Identity TLG
* Composition
* Substitution:

$$\frac{\rho : t \Rightarrow u \qquad \rho_1 : t_1 \Rightarrow u_1 \qquad \cdots \qquad \rho_m : t_m \Rightarrow u_m}{\rho[\rho_i/x_i] : t[t_i/x_i] \Rightarrow u[u_i/x_i]}$$



GSOS-Compatible State Space

* $\mathcal{R}$: a set of GSOS rules
* { $\mathcal{R}$-TLG } :=

  { ρ_R | $R \in \mathcal{R}$ }

  closed under id, comp., subst.

* $\mathcal{R}$-state space:

$$(\!|\sigma|\!)(X_1, \ldots, X_m) \;:=\; \coprod_{\rho : s \Rightarrow \sigma,\ \mathcal{R}\text{-TLG}} |s|(X_1, \ldots, X_m)$$

GSOS-Compatible State Space

* $\mathcal{R}$: a set of GSOS rules
* { $\mathcal{R}$-TLG } :=

  { ρ_R | $R \in \mathcal{R}$ }

  closed under id, comp., subst.

* $\mathcal{R}$-state space:

$$(\!|\sigma|\!)(X_1, \ldots, X_m) \;:=\; \coprod_{\rho\,:\, s \Rightarrow \sigma,\ \mathcal{R}\text{-TLG}} |s|(X_1, \ldots, X_m)$$

for each "offspring" s

Hasuo (Tokyo)

GSOS-Compatible State Space

* $\mathcal{R}$: a set of GSOS rules
* { $\mathcal{R}$-TLG } := $\{ \rho_R \mid R \in \mathcal{R} \}$ closed under id, comp., subst.
* $\mathcal{R}$-state space:

$$(\!|\sigma|\!)(X_1, \ldots, X_m) \; := \coprod_{\rho : s \Rightarrow \sigma,\ \mathcal{R}\text{-TLG}} |s|(X_1, \ldots, X_m)$$

for each "offspring" s

$|s|$: "plain" state space, e.g.

$$|\sigma|(X_1, \ldots, X_m) \; := \; X_1 \times \cdots \times X_m$$

GSOS-Compatible State Space

- Examples:

$$(\!|\, \| \,|\!)(X_1, X_2) = X_1 \times X_2$$

$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$

$$\begin{aligned}(\!|\, ! \,|\!)(X) &= X^+ \\ &= X + X^2 + X^3 + \cdots\end{aligned}$$

GSOS-Compatible State Space

* Examples:

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\parallel\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\overline{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync})$$

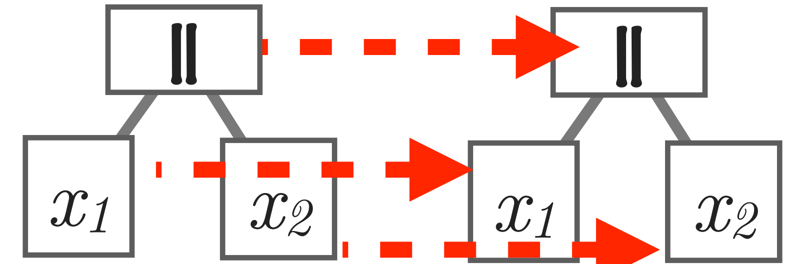
$$(\!|\, \| \,|\!)(X_1, X_2) = X_1 \times X_2$$

$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$

$$(\!|\, ! \,|\!)(X) = X^+ = X + X^2 + X^3 + \cdots$$

Hasuo (Tokyo)

GSOS-Compatible State Spac



* Examples:

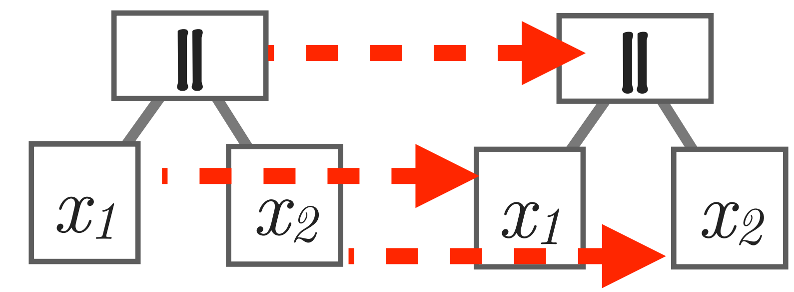
$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\parallel\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\overline{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync})$$

$$(\!|\, \parallel \,|\!)(X_1, X_2) = X_1 \times X_2$$

$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$

$$\begin{aligned}(\!|\, ! \,|\!)(X) &= X^+ \\ &= X + X^2 + X^3 + \cdots\end{aligned}$$

GSOS-Compatible State Spac



* Examples:

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\parallel\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\overline{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync})$$

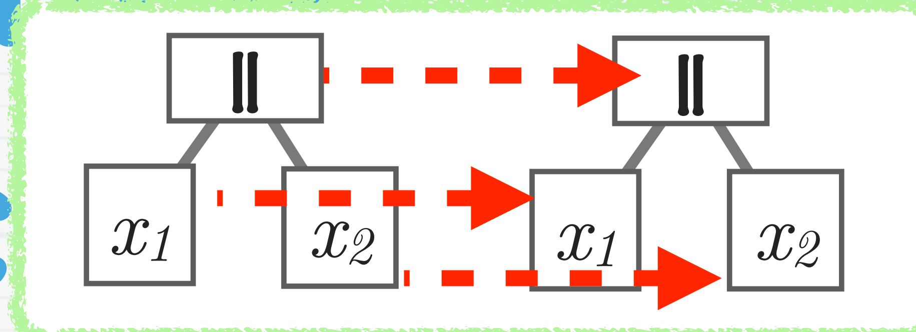
$$(\!|\, \parallel \,|\!)(X_1, X_2) = X_1 \ldots$$

$$\frac{x \xrightarrow{a} x'}{x ; y \xrightarrow{a} x' ; y}\ (;\mathrm{L}) \qquad \frac{x \not\xrightarrow{a} \ (\forall a \in A) \quad y \xrightarrow{b} y'}{x ; y \xrightarrow{b} y'}\ (;\mathrm{R})$$

$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$

$$\begin{aligned}(\!|\, ! \,|\!)(X) &= X^{+} \\ &= X + X^2 + X^3 + \cdots\end{aligned}$$

GSOS-Compatible State Spac



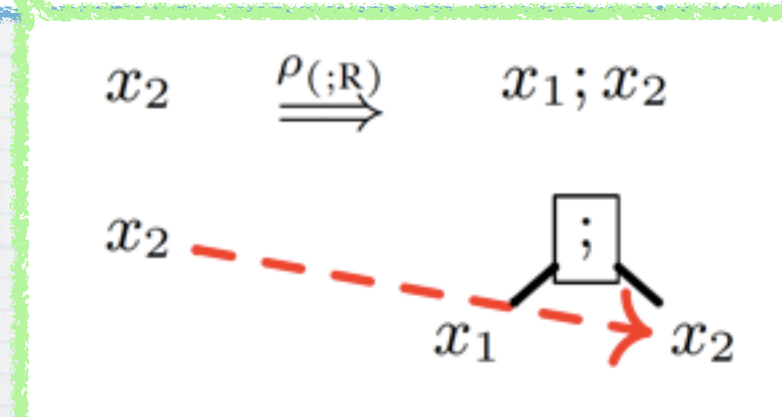
* Examples:

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\parallel\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\bar{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync})$$

$$(\!|\, \parallel \,|\!)(X_1, X_2) = X_1 \ldots$$

$$\frac{x \xrightarrow{a} x'}{x ; y \xrightarrow{a} x' ; y}\ (;\mathrm{L}) \qquad \frac{x \overset{a}{\not\to}\ (\forall a \in A) \quad y \xrightarrow{b} y'}{x ; y \xrightarrow{b} y'}\ (;\mathrm{R})$$

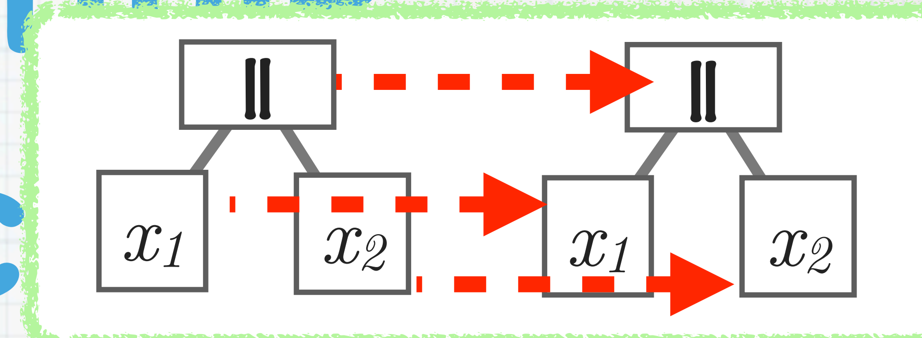
$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$



$$(\!|\, ! \,|\!)(X) = X^{+} = X + X^2 + X^3 + \cdots$$

Hasuo (Tokyo)

GSOS-Compatible State Spac

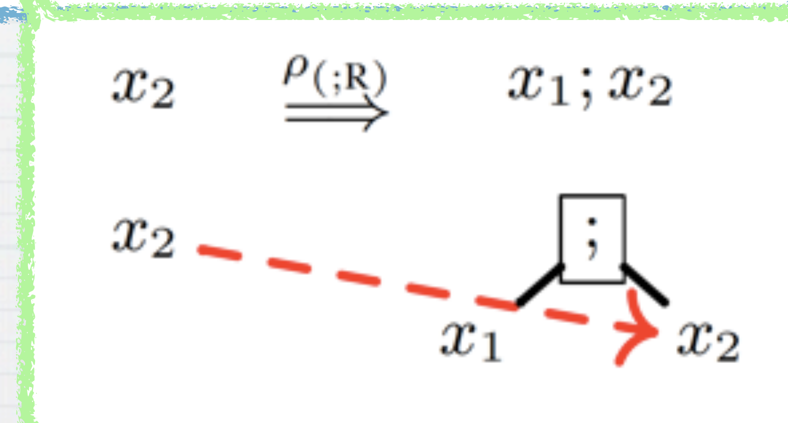


* Examples:

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\|\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\|\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\bar{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\|\textsc{Sync})$$

$$(\!|\, \| \,|\!)(X_1, X_2) = X_1$$

$$\frac{x \xrightarrow{a} x'}{x; y \xrightarrow{a} x'; y}\ (;\mathrm{L}) \qquad \frac{x \not\xrightarrow{a}\ (\forall a \in A) \quad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'}\ (;\mathrm{R})$$



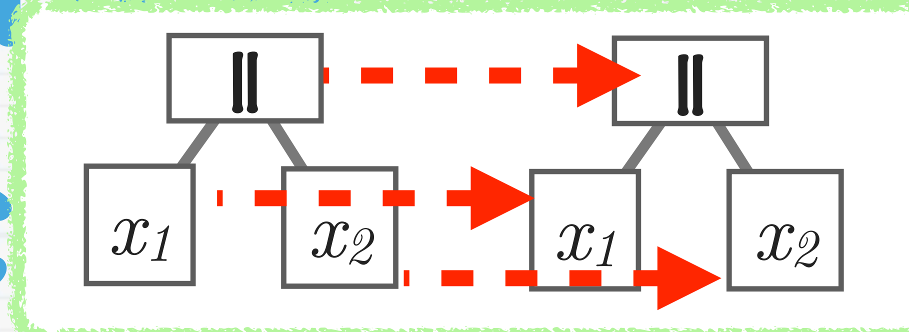
$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$

$$(\!|\, ! \,|\!)(X) = X^+ = X + X^2 + X^3 + \cdots$$

$$\frac{x \xrightarrow{a} x'}{!x \xrightarrow{a} x' \parallel !x}\ (!)$$

Hasuo (Tokyo)

GSOS-Compatible State Spac



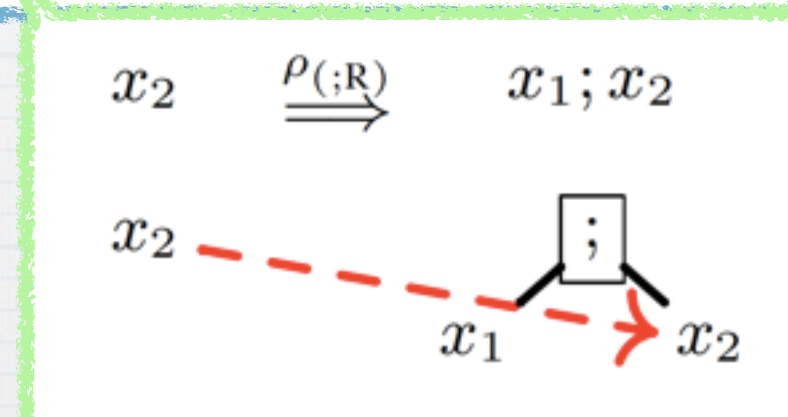
* Examples:

$$\frac{x \xrightarrow{a} x'}{x \parallel y \xrightarrow{a} x' \parallel y}\ (\parallel\mathrm{L}) \qquad \frac{y \xrightarrow{a} y'}{x \parallel y \xrightarrow{a} x \parallel y'}\ (\parallel\mathrm{R}) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{\overline{a}} y'}{x \parallel y \xrightarrow{\tau} x' \parallel y'}\ (\parallel\textsc{Sync})$$

$$(\!|\, \parallel \,|\!)(X_1, X_2) = X_1 \ldots X$$

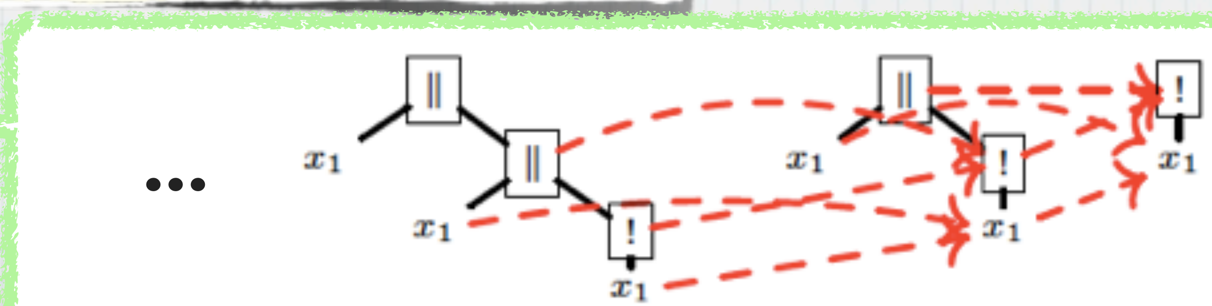
$$\frac{x \xrightarrow{a} x'}{x; y \xrightarrow{a} x'; y}\ (;\mathrm{L}) \qquad \frac{x \not\xrightarrow{a}\ (\forall a \in A) \quad y \xrightarrow{b} y'}{x; y \xrightarrow{b} y'}\ (;\mathrm{R})$$

$$(\!|\, ; \,|\!)(X_1, X_2) = X_1 \times X_2 + X_2$$



$$(\!|\, ! \,|\!)(X) = X^+$$
$$= X + X^2 + X^3 + \cdots$$

$$\frac{x \xrightarrow{a} x'}{!x \xrightarrow{a} x' \parallel !x}\ (!)$$



Microcosm SOS for full GSOS

$F = \mathcal{P}_{\mathrm{fin}}(A \times \_\,)$

* **Thm.** Given $(\Sigma, \mathcal{R})$, GSOS-specification

1. Induces **categorical GSOS rule**

$$\left[\begin{array}{l} (FX_1 \times X_1) \times \cdots \times (FX_m \times X_m) \\ \to F\big((\!|\sigma|\!)(X_1, \ldots, X_m) \big) \end{array} \right]_{X_1, \ldots, X_m}$$

2. Induces $\mathbf{Coalg}_F^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$

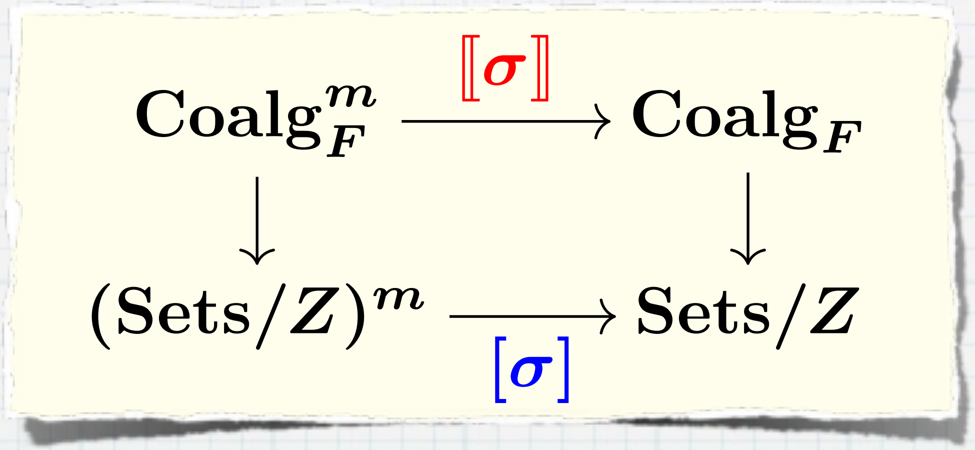
3. Induces $Z^m \xrightarrow{[\sigma]} Z$ (coinduction)

$$\begin{array}{ccc} \mathbf{Coalg}_F^m & \xrightarrow{[\![\sigma]\!]} & \mathbf{Coalg}_F \\ \downarrow & & \downarrow \\ \mathbf{Sets}^m & \xrightarrow[(\!|\_|\!)]{} & \mathbf{Sets} \end{array}$$

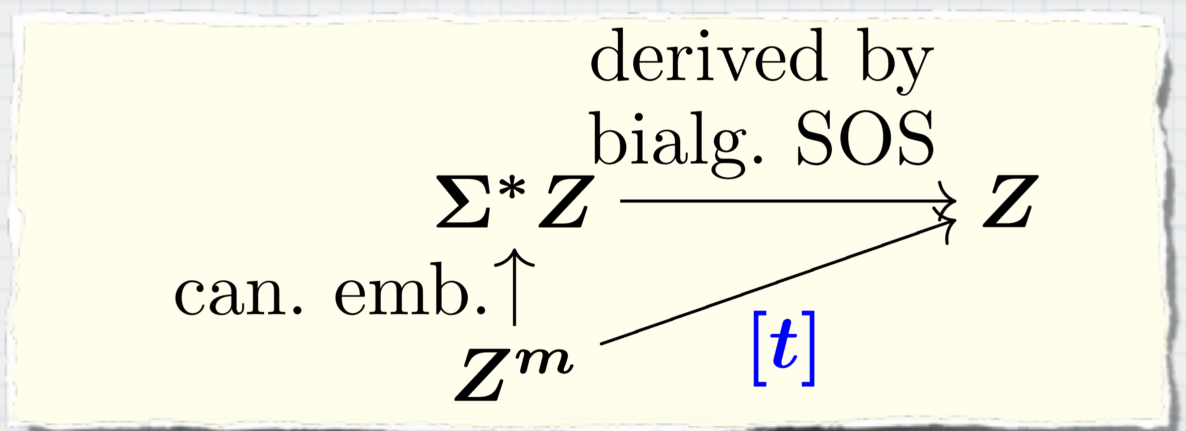
Microcosm SOS for full GSOS

* **Thm. (ctn'd)** Given $(\Sigma, \mathcal{R})$, GSOS-specification

4. Compositionality:



5. Extends Bialgebraic SOS. t: a Σ-term.



Conclusion

- SOS for component calculi:

$$\mathbf{Coalg}^m_F \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

  - from

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \overset{b}{\not\to} \quad \text{in } \mathcal{S}_i\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t \quad \text{in } \sigma(\mathcal{S}_1,\dots,\mathcal{S}_m)}$$

  - i.e.

$$\left[\begin{array}{l}(FX_1 \times X_1) \times \cdots \times (FX_m \times X_m) \\ \to F\big(\,(\!|\sigma|\!)(X_1,\dots,X_m)\,\big)\end{array}\right]_{X_1,\dots,X_m}$$

- Future work:
  - Verification framework (FOL repr., thm. prv.)
  - Combination with duality-based logics

Thank you for your attention!
Ichiro Hasuo (Dept. CS, U Tokyo)
http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro/

Conclusion

* SOS for component calculi:

$$\mathbf{Coalg}_F^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

  * from

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j} \text{ in } \mathcal{S}_i\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \not\xrightarrow{b} \text{ in } \mathcal{S}_i\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t \quad \text{in } \sigma(\mathcal{S}_1,\dots,\mathcal{S}_m)}$$

  * i.e.

$$\Big[\, (FX_1 \times X_1) \times \cdots \times (FX_m \times X_m) \to F\big(\, (\!|\sigma|\!)(X_1,\dots,X_m)\,\big) \,\Big]_{X_1,\dots,X_m}$$

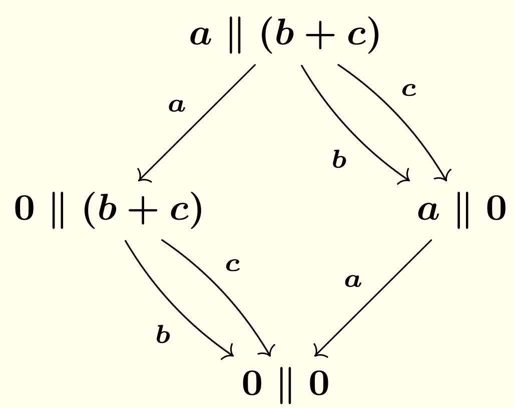
* Future work:
  * Verification framework (FOL repr., thm. prv.)
  * Combination with duality-based logics

Summary

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥



Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

➥ $$\begin{array}{c} \Sigma I \\ \downarrow \text{initial} \\ I \\ \downarrow ! \\ FI \end{array}$$

categorically

Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\|\textsc{Sync})$$

➥ parallel composition of LTSs

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ

In Other Words...

(Conventional) Process SOS

- **Syntactic format:**
  - One of the simplest:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} \sigma'(y_{i_1}^{a_1,j_1},\dots,y_{i_n}^{a_n,j_n})}$$

  - The GSOS format

[Bloom,Istrail&Meyer,JACM'95]

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\dots,x_m) \xrightarrow{e} t}$$

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

- **Categorical format:**
  - Natural transformation

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

  - Categorical GSOS format [Turi&Plotkin, LICS'97]

$$\Sigma(F \times \mathrm{id}) \stackrel{\lambda}{\Rightarrow} F\Sigma^*$$

Microcosm SOS

- **Categorical format:**
  - Natural transformation

$$\Big(FX \times FY \xrightarrow{\mathsf{sync}} F(X\times Y)\Big)_{X,Y\in\mathbf{Sets}}$$

  - Categorical GSOS format

??

"Formats"

(Conventional) Process SOS

* **Syntactic format:**
  * One of the simplest:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} \sigma'(y_{i_1}^{a_1,j_1},\ldots,y_{i_n}^{a_n,j_n})}$$

  * The GSOS format

[Bloom,Istrail&Meyer,JACM'95]

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t}$$

Bialgebraic SOS

[Turi&Plotkin, LICS'97]

* **Categorical format:**
  * Natural transformation

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

  * Categorical GSOS format [Turi&Plotkin, LICS'97]

$$\Sigma(F \times \mathrm{id}) \overset{\lambda}{\Rightarrow} F\Sigma^*$$

Microcosm SOS

* **Categorical format:**
  * Natural transformation

$$\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y)\right)_{X,Y\in\mathbf{Sets}}$$

  * Categorical GSOS format

??

"Formats"

(Conventional) Process SOS

* Syntactic format:
  * One of the simplest:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} \sigma'(y_{i_1}^{a_1,j_1},\ldots,y_{i_n}^{a_n,j_n})}$$

  * The GSOS format [Bloom,Istrail&Meyer,JACM'95]

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \quad \{x_i \stackrel{b}{\not\to}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t}$$

Bialgebraic SOS [Turi&Plotkin, LICS'97]

* Categorical format:
  * Natural transformation

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

  * Categorical GSOS format [Turi&Plotkin, LICS'97]

$$\Sigma(F \times \mathrm{id}) \stackrel{\lambda}{\Rightarrow} F\Sigma^*$$

Microcosm SOS

* Categorical format:
  * Natural transformation

$$\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y)\right)_{X,Y\in\mathbf{Sets}}$$

  * Categorical GSOS format

"Formats"

(Conventional) Process SOS

* **Syntactic format:**
  * One of the simplest:

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} \sigma'(y_{i_1}^{a_1,j_1},\ldots,y_{i_n}^{a_n,j_n})}$$

  * The GSOS format
[Bloom,Istrail&Meyer,JACM'95]

$$\frac{\{x_i \xrightarrow{a} y_i^{a,j}\}_{i\in[1,m]}^{a\in A, j\in[1,N_i^a]} \qquad \{x_i \not\xrightarrow{b}\}_{i\in[1,m]}^{b\in B_i}}{\sigma(x_1,\ldots,x_m) \xrightarrow{e} t}$$

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

* **Categorical format:**
  * Natural transformation

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

  * Categorical GSOS format [Turi&Plotkin, LICS'97]

$$\Sigma(F \times \mathrm{id}) \stackrel{\lambda}{\Rightarrow} F\Sigma^*$$

Microcosm SOS

* **Categorical format:**
  * Natural transformation

$$\left(FX \times FY \xrightarrow{\text{sync}} F(X \times Y)\right)_{X,Y\in\mathbf{Sets}}$$

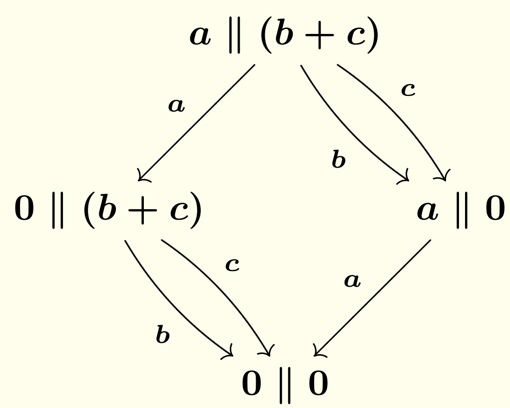
  * Categorical GSOS format

$$\begin{bmatrix} (FX_1 \times X_1) \times \cdots \\ \times (FX_m \times X_m) \\ \to F\big(\,(\!|\sigma|\!)(X_1,\ldots,X_m)\,\big) \end{bmatrix}_{X_1,\ldots,X_m}$$

SOS: Variations

(Conventional) Process SOS

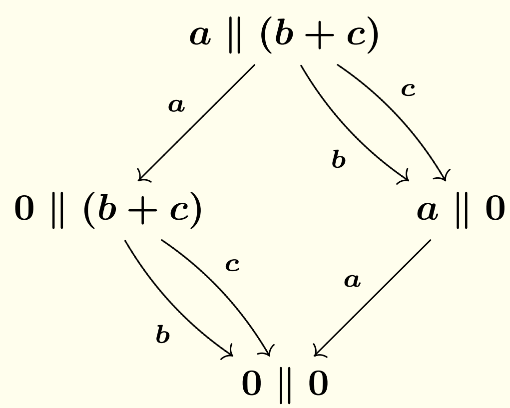
$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$



SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$

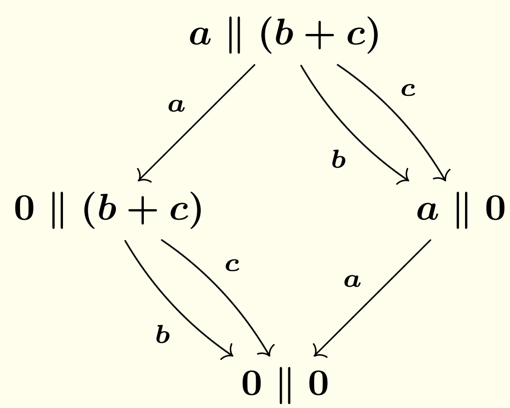


Categorically

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\textsc{Sync})$$



categorically

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \overset{\lambda}{\Rightarrow} F\Sigma$$

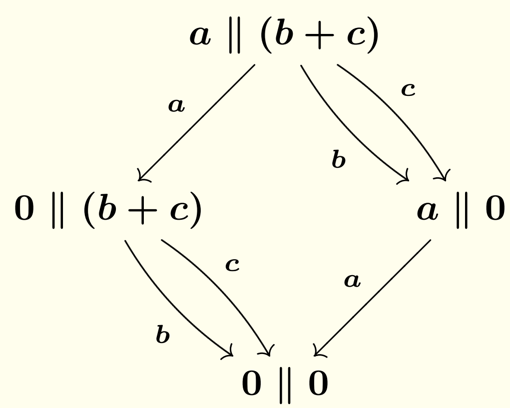
$$\begin{array}{c} \Sigma I \\ \big\downarrow \text{initial} \\ I \\ \big\downarrow ! \\ FI \end{array}$$

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\text{SYNC})$$

➡



Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \ (\parallel\text{SYNC})$$

➡ parallel composition of LTSs

categorically

Bialgebraic SOS
[Turi&Plotkin, LICS'97]

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

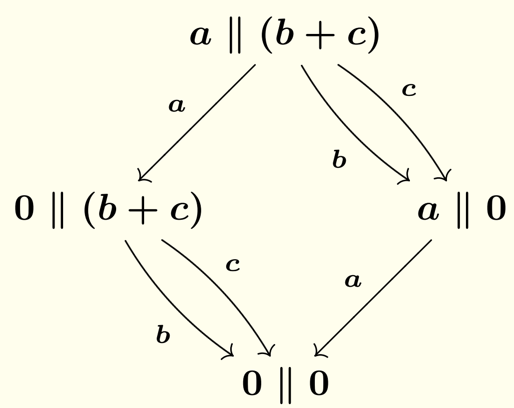
➡

$$\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$$

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'}\ (\|\textsc{Sync})$$



Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'}\ (\|\textsc{Sync})$$

➡ parallel composition of LTSs

categorically

Bialgebraic SOS

[Turi&Plotkin, LICS'97]

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

➡

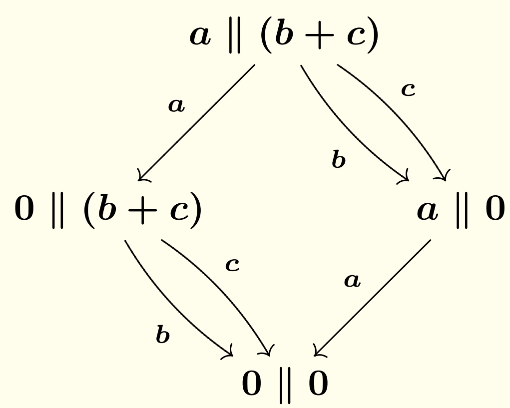
$$\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$$

SOS: Variations

(Conventional) Process SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\|\textsc{Sync})$$

➥



Component SOS

$$\frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y'}{x \parallel y \xrightarrow{a|b} x' \parallel y'} \;(\|\textsc{Sync})$$

➥ parallel composition of LTSs

Bialgebraic SOS

[Turi&Plotkin, LICS'97]

$$\Sigma F \stackrel{\lambda}{\Rightarrow} F\Sigma$$

➥

$$\Sigma I \xrightarrow{\text{initial}} I \xrightarrow{!} FI$$

categorically

Microcosm SOS

[Hasuo,Jacobs&Sokolova, FoSSaCS08]

$$(\mathbf{Coalg}_F)^2 \xrightarrow{\parallel} \mathbf{Coalg}_F$$

synchronous parallel composition

Current work

$$(\mathbf{Coalg}_F)^m \xrightarrow{[\![\sigma]\!]} \mathbf{Coalg}_F$$

for any GSOS-specified σ