

MENTALISM AND BEHAVIORISM:  
MERGING?

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# MATHEMATICAL MODELING OF REFLEXIVE PROCESSES

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## MENTALISM AND BEHAVIORISM: MERGING?

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### ABSTRACT

The Reflexive-Intentional Model of the Subject (RIMS) connects the subject's bipolar probabilistic behavior with its mental domain. We demonstrate that the Matching Law is a formal consequence of this tie. RIMS allows us also to deduce theoretically the main patterns of animal behavior in the experiments with two alternatives where the Matching Law reveals itself. This finding inclines us to put forth a hypothesis that this law reflects the process of self-programming of the subject with mental domain. As a result, the subject acquires the ability to choose alternatives with fixed probabilities. With this explanation, the relative frequencies of pressing a pedal or pecking at a key play the role of half-finished-products which after being downloaded into the self turn into the probabilities of choice. The Matching Law can be regarded as an operational indication of the mental domain existence.

### Introduction

Mentalism is a science about subjective matters that gives a living creature a niche for the inner world. Behaviorism is a science about behavior depriving a living creature of it. Both of these sciences have a common feature; in them, an organism appears as an entity. The first one focuses on a subject's relation to the self, while the second one focuses on the relations between the subject and the environment (Tolman, 1932). For the last few decades, the border between mentalism and behaviorism has moved: a formal model of the subject has appeared which includes both its mental domain and its behavior. The model's verification goes through its penetration into various branches of psychology, sociology, and anthropology. Behaviorism represents the most attractive field for such a penetration, because of its strict inner discipline and methodological honesty that allows us to distinguish clearly what is understood and what is not. One of the unsolved problems in the science

of behavior is the *Matching Law* (Herrnstein, 1961). It describes the ability of birds and mammals to regulate the ratio between a sequence of reinforcements and a sequence of responses. This ability looks *strange* from the point of view of the utilitarian common sense (see Williams, 1988). In this work we offer a solution to this problem with the help of Reflexive-Intentional Model of the Subject (RIMS)<sup>1</sup>.

In creating this model we tried to understand a phenomenon of “moral choice” from a purely scientific point of view, rather than from a moralistic one. A great number of specialists from psychiatrists to sociologists studying criminals and terrorists are interested in finding objective laws of moral choice. A human mental domain must be represented in their studies as clearly and unambiguously as behavior is represented in behaviorism.

RIMS is a special mathematical representation of a subject making choice between two alternatives. This model reflects two aspects of the subject’s activity: utilitarian and deontological. The utilitarian aspect relates to the behavior which is advantageous from the practical point of view, for example, obtaining money or food. The deontological aspect relates to the idealistic behavior, for example, choosing between good and evil. It may happen that the “moral” orientation of the alternative does not correspond to the utilitarian one. For example, a deal with an enemy may be more profitable than the deal with a friend. Both these aspects are connected into a single process of behavior generation by the formal model.

RIMS is a probabilistic model. It predicts probabilities with which the subject chooses the alternatives, one playing the role of the positive pole and the other that of the negative pole. The idea that the subject’s choice is probabilistic appeared early in the twentieth century and was used in many theoretical models (Thurstone, 1927; von Neuman & Morgenstern, 1944; Savage, 1951; Mosteller & Nogee, 1951; Bradley & Terry, 1952; Davidson, Suppes & Siegel, 1957; Bower, 1959; Luce, 1959; Audley, 1960; Spence, 1960; Restle, 1961; LaBerge, 1962; Atkinson et al., 1965). This line of investigations changed significantly the view that behavior is a process completely determined by the environment. Although effective methods have been developed to predict the results of probabilistic choice, a problem of its essence remained untouched. We still do not have clear ideas about whether all living creatures are capable of probabilistic choice or only some of them. Also, we do not know how an organism “learns” the probabilities with which it “must” make a choice in a given situation. RIMS connects the subject’s probabilistic behavior

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<sup>1</sup> RIMS is described in great detail in Lefebvre’s *Algebra of Conscience* (2001) and in its supplemented translation into Russian (2003). Various aspects of this model were considered in works by Adams-Webber (1987; 1995; 1997), Baker (1999), Batchelder (1987), Kaiser & Schmidt (2003), Kauffman (1990), Krylov (1994), Lefebvre (1965; 1967; 1972; 1977a,b; 1980; 1987; 1992a,b), Lefebvre & Adams-Webber (2002), Levitin (1987), McClain (1987), Miller & Sulcoski, 1999a,b), Rapoport (1990, 1996), Schreider (1994, 1998), Taran (1999), Townsend (1983, 1990), Wheeler (1987), Zajonc (1987), Popper (1992).

with its mental domain and allows us to formulate a few new hypotheses. In the framework of this model, prior to the act of choice, the subject's state is uncertain and can be characterized by the distribution of probabilities over alternative choices. Using a quantum-mechanical metaphor we can say that immediately before the act of choice, the subject is in a mixed state, and the act of choice is a "collapse" of the mixed state. As a result, the subject moves into one of the pure states. It is worth emphasizing that the ability of the subject to make a choice between the alternatives with *fixed* probabilities indicates a rather high level of the development. The specialists in mathematical modeling know well how difficult it is to construct a technical device which would generate a random sequence of 0's and 1's with a fixed probability of their appearance. It is possible that the organism's ability to give response undetermined by a stimulus raises its chances to find food and not to become another organism's food (Lefebvre, 1999a).

We may suppose that probabilistic behavior of organisms appears at the same time as their mental domain. Their appearance indicates the moment of an organism's "liberation" from the "necessity" to respond in one only way to an external influence. To choose alternatives with fixed probabilities, the organism must somehow "download" them into the self. We presume that the "secret" of the Matching Law is that it reflects a procedure of forming a mixed state in the subject, during which the subject processes information received from the environment into probabilistic distribution. Let us imagine that an organism, say of a rat, a pigeon, and even a man cannot solve this problem through its brain activity only. Because of that failure the entire organism becomes involved in a computational process. When an animal is running between the two feed hoppers (in the experiments in which the Matching Law is revealed), it is an external demonstration of this process, whose goal is to generate *frequencies* which would later transform into *probabilities*. As a result of such a "downloading" of the probability the subject became capable of making an instant probabilistic choice. But this ability is not "free" for the subject; to obtain it an organism must spend energy.

The experiments with two keys in which human subjects were used (see Ruddle et al., 1979; Wearden & Burgess, 1982) allow us to hypothesize that generation of a mixed state in humans is also connected with their motor activity. This activity may also reveal itself during a process of estimation. For example, when the subject is given a task to mark the intensity of a stimulus on a scale, the subject's pencil oscillates before it makes the final mark. Sometimes it is even difficult to determine which mark is final (see, for example, Poulton & Simmonds, 1985). We may suppose that these oscillations are functionally analogous to rats' running from one food-hopper to another. Let us note that RIMS can explain the process of categorical estimation as well as that of matching (Lefebvre, 1992a). We cannot exclude the possibility that human beings may download the probabilities by eye movements.

The most important difference of RIMS from the models existing previously consists in the introduction of a new special variable which corresponds to the subject's *model of the self* (Lefebvre, 1965; 1977b). We interpret the value of this variable as the subject's *intention* to make a choice. The intentional behavior is given as  $B = I$ , where  $B$  is the value of the variable which describes the subject's behavior, and  $I$  is the value of the variable corresponding to the model of the self. In this case, variable  $I$  can be omitted, and we obtain a behavioristic type model which can be empirically falsified. In the framework of RIMS, the organism of the subject tends to generate a line of behavior such that it reaches and holds equation  $B = I$ . This principle of behavior generation we will call *The Law of Self-Reflexion* (Lefebvre, 2002).

### 1. The Law of Self-Reflexion in a Logical Scheme of Evolution of Behaviorism

In the evolution of the science of behavior, one may see a clear logic which does not depend much on individual preferences of researchers, neither on prohibitions on use introspective concepts (as for example, Pavlov's avoiding such expression as "a dog noticed," "a dog understood," etc.). We single out four stages in the development of behaviorism and can see a move toward the fifth one, today (Fig. 1). Each stage can be described by a "law," which expresses a rule of behavior of an organism in concise form.

At the first stage, which appeared in the Cartesian time, an organism was represented as a black box with an input and output (here we use a metaphor belonging to later times). An organism's life actions, called responses, correspond to the output, and the environment's demands - to the input. A mechanism inside the box *automatically* transforms each demand into a response. This transformation was called *reflex*.

Pavlov's (1927) discovery that reflexes are of two types: inborn and acquired, indicates the appearance of the second stage. The inborn reflexes were called unconditional, and those acquired as the life experience were called conditional. The probabilistic models, as a rule, belong to the first or second stage. Unlike deterministic models, they describe statistical characteristics of stimulus-response rather than functional relation between a set of stimuli and a set of responses.

During the third stage, it was found that the automatic response to a stimulus may change becoming more effective. At the beginning of this stage was Thorndike (1932) who formulated *The Law of Effect*, which shows the ability of a living creature to modify its response to a stimulus depending on its "effect." For example, a cat's organism in Thorndike puzzle box performs selections of successful manipulations with the lock and at the end of successive trials exits cage faster than at the beginning (see also Herrnstein, 1970; Williams, 1988).

The fourth stage is connected with experimental methods developed by Skinner (1938) and his followers. In their experiments, an animal response may influence the generation of stimuli. It turned out that under this condition, an animal

generates a line of behavior which establishes a fixed ratio between a sequence of stimuli and a sequence of responses. A mathematical formula for this ratio has been called the *Matching Law*. Numerous attempt to explain this law in the framework of behaviorism have not given us a convincing explanation.

From the RIMS point of view, the Matching Law displays bipolarity and the law of self-reflexion (Lefebvre, 1999a; 2002). The concepts of the *image of the self* and *intention* lie beyond the vocabulary of behaviorism. Without broadening its vocabulary the science of behavior may not be able to explain the Matching Law. The broadening of behaviorism framework leads us toward the fifth stage which indicates merging of mentalism and behaviorism (Fig. 1).

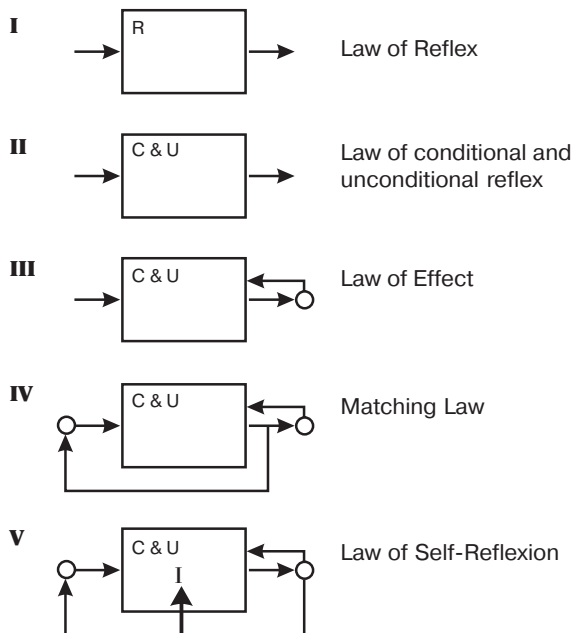


Fig. 1. Logical stages in the development of behaviorism. The scheme does not reflect the time order of the ideas' appearance. For example, Pavlov, Thorndike, Watson and Bechterev worked on their studies at about the same time, but contributions by early Watson and Bechterev belong to the first stage, by Pavlov to the second one, and by Thorndike to the third stage.

## 2. The Matching Law

The ability of an organism to regulate relations between the sequence of responses and the sequence of reinforcements was found by Herrnstein (1961) in the experiments with pigeons. A cage had two keys. When a pigeon pecks a key it may result in the appearance of a grain. Each key has an independent schedule by which rein-

forcement is delivered. The mean interval between the appearance of grains can vary (Variable-Interval Schedules, *VI*). The experiment consisted of a series of sessions with fixed mean intervals for each key in each session. The pair of intervals were chosen in such a way that sometimes the reinforcement appeared more often in one key, and sometimes in the other.

It turned out that the birds choose the line of behavior such that the ratio of the numbers of pecks to the keys ( $B_1$  and  $B_2$ ) is approximately equal to the ratio of the numbers of corresponding reinforcements ( $r_1$  and  $r_2$ ):

$$\frac{B_2}{B_1} = \frac{r_2}{r_1} . \quad (2.1)$$

Equation (2.1) was called the *Matching Law*. There were also experiments with rats and humans. Besides *VI* other schedules were also used. For example, in Variable-Ratio Schedule, *VR*, the mean number of pecks, necessary to receive reinforcement, varied. The results of the experiments led to the formulation of the Generalized Matching Law (Baum, 1974):

$$\frac{B_2}{B_1} = c \left( \frac{r_2}{r_1} \right)^\beta , \quad (2.2)$$

where  $c$  and  $\beta$  are parameters which characterize a subject in a given experiment consisting of a sequence of sessions. Quite recently, Baum et al. (1999) suggested that (2.2) can be reduced to

$$\frac{B_P}{B_N} = c \left( \frac{r_P}{r_N} \right) , \quad (2.3)$$

where  $B_P > B_N$ . Variables with subscript  $P$  relate to the more often chosen alternative, and those with  $N$  relate to less often chosen one. In his recent publication, Baum (2002) substituted condition  $B_P > B_N$ . with condition  $r_P > r_N$ .

Equations (2.1), (2.2) and (2.3) are particular cases of the more general equation:

$$\frac{B_2}{B_1} = c \left( \frac{\varphi(r_2)}{\varphi(r_1)} \right) , \quad (2.4)$$

which represents the subjects' behavior in the experiments described above (Davison & Jones, 1995; Baum & Aparicio, 1999).

### 3. The Attempts to explain the Matching Law within the Framework of the Science of Behavior

Why does equation (2.4) hold? It is natural to assume that it is a by-product of more fundamental processes (Williams, 1988). Baum and Aparicio (1999) gave the following interpretation which expresses the dominant point of view: "Despite

claims to the contrary, all leading theories about operant choice may be seen as models of optimality” (p. 75). The idea of optimality is consonant with the main thesis of behaviorism, in accordance to which an animal is adapting to the environment in a way that it looks rational and goal oriented.

There are numerous general and experimental arguments *pro* and *contra* the principle of optimality as an explanation of the Matching Law (Williams, 1988; Baum et al., 1999). The results of the experiments conducted by Mazur (1981) present a serious argument against the principle of optimality. The conditions of the experiments permitted the researchers to easily discover the pigeons tendency to maximize the amount of food they received. Nevertheless, the birds did not choose the optimal strategy, they chose the strategies to make equation (2.4) hold instead.

Let us consider one more argument against the principle of optimality. Equation (2.2), which describes the Generalized Matching Law, has two *free* parameters  $c$  and  $\beta$ . Their value must be found experimentally for each subject. There were many discussions concerning  $\beta$  (Baum, 1979; Wearden & Burgers, 1982; Aparicio, 2001), but  $c$  was considered just a scale coefficient connecting utility-values of reinforcements from two different sources. A usual explanation of the necessity of its introduction can be clarified with the following example. Let a piece of food from the left food-hopper is 0.75 of that from the right one. If we write (2.1) for the “sum” of utilities, instead of the number of pieces ( $r_1$  and  $r_2$ ), we will obtain the following ratio:

$$\frac{B_2}{B_1} = \frac{0.75r_2}{r_1}. \quad (3.1)$$

Similar argumentation was used for the experiments in which the pieces of food were equal. In these cases, it was said that parameter  $c$  reflects a hidden factor which changes utility-measures of the same product from two different sources. Sometimes for the sake of saving this argumentation, the researchers had to assume that the organism of the subject was capable of finding statistical characteristics of non-simultaneous factors and reflecting them onto  $c$ . How else could they explain the fixed value of  $c$  in the experiments, in which one alternative was connected with schedule *VI*, and the other with schedule *VR*? (see, for example, Baum, 1974, Fig. 5.) The explanation of  $c$  constant value turns into an independent problem similar in its complexity with the explanation of the Matching Law. But if we reject the interpretation of  $c$  as a scale coefficient, all the attempts to reduce the Matching Law to optimality look unconvincing.

#### 4. RIMS

The Reflexive-Intentional Model of the Subject reflects the inner domain and behavior of a subject in its interactions with two objects which we called “agencies.”



One of them plays the role of the *positive* pole and the other that of the *negative* pole. The subject is represented with the following equation:

$$X_1 - x_1 - (1 - x_1)(1 - x_2)M(x_3) = 0, \quad (4.1)$$

where all variables and function  $M(x_3)$  take on the values from interval  $[0, 1]$  (Lefebvre, 1999b; 2001).

Variable  $X_1$  represents the subject's executive system. Its value is the relative probability with which the subject's executive system is *ready* to influence the positive agency. Variable  $x_3$  corresponds to the subject's model of the self. It represents the intentional domain of the subject. Its value is the relative probability with which the subject *intends* to influence the positive agency. The appearance of *intention* and the appearance of *readiness* are considered to be two independent events. For the sake of language simplicity, we will call  $X_1$  – readiness, and  $x_3$  – intention.

Variable  $x_1$  represents the relation between the subject and the positive agency in a given moment. Its value is interpreted in two ways. First, it is a relative probability of the positive agency *influencing* the subject. Second, it is the subject's *requirement* of the positive agency to influence him with this probability.

Variable  $x_2$  represents the subject's experience. Its value is the integral estimation of the relative probability with which the positive agency influenced him in the *past*.

Function  $M(x_3)$  represents the subject's *prognostic* activity. Its value is the subjective evaluation of the future positivity under condition that intention  $x_3$  turns into reality.

The environment may determine all values of the variables  $X_1, x_1, x_2, x_3$  or only some of them. If the set of values determined by the environment is such that limitation (4.1) does not hold, we consider the subject *disadapted*. In the opposite case, we consider the subject *adapted* to the environment. The variables not determined by the environment may take on any values which do not violate (4.1).

If the value of  $x_1$  is determined by the environment, we interpret it as the influence *received* from the environment. In the opposite case, it is interpreted as the subject's *need* in the environment's influence.

The subject adapted to the environment is called *intentional* if (4.1) is supplemented with the following limitation:

$$X_1 = x_3, \quad (4.2)$$

that is, the intentional subject's readiness is equal to its intention. The intentional subject's choice will be called an *intentional probabilistic choice*.

Under condition (4.2) expression (4.1) turns into the following equation in relation to  $X_1$ :

$$X_1 - x_1 - (1 - x_1)(1 - x_2)M(X_1) = 0. \quad (4.3)$$

The absence of its solution means that the subject is not capable of intentional actions. If this equation does have a solution, that is, there is a function of the type

$$X_1 = f(x_1, x_2), \quad (4.4)$$

satisfying (4.3), we can exclude  $x_3$  from our considerations. RIMS then turns into a behavioristic model, all variables of which can be related to observable values.

If  $X_1$  is not an effective function of  $x_1$  and  $x_2$ , we assume that the subject has the ability to make a choice, even probability of which cannot be predicted by external observers. We call this choice a *free* choice.

While modeling some psychological processes we may introduce more functional limitations on connections between variables  $X_1, x_1, x_2, x_3$  to reflect subjects' specific peculiarities. It follows from (4.1) that the following inequalities hold independently from the type of function  $M(x_3)$ :

$$x_1 \leq X_1 \leq 1 - x_2 + x_1 x_2. \quad (4.5)$$

In the framework of this work, we consider function  $M(x_3)$  to be linear and look as follows:

$$M(x_3) = (1 - d)x_3, \quad (4.6)$$

where  $d \in [0, 1]$ . The value of  $d$  is interpreted as an *index of depression*. This index lowers a degree of the positivity of the future. For example, when this index is maximal,  $d = 1$ , function  $M(x_3) \equiv 0$ , that is, the future looks negative for the subject. When  $d = 0$ ,  $M(x_3) \equiv x_3$ , that is, the degree of positivity of the future is equal to the value of intention. If (4.6) holds, (4.3) turns into equation in relation to  $X_1$ :

$$X_1 = x_1 + (1 - x_1)(1 - x_2)(1 - d) X_1, \quad (4.6a)$$

hence, with  $x_1 + x_2 + d > 0$

$$X_1 = \frac{x_1}{1 - (1 - x_1)(1 - x_2)(1 - d)}. \quad (4.7)$$

When  $x_1 = x_2 = d = 0$ , the value of  $X_1$  is not a function of  $x_1$  and  $x_2$ ; so, in accordance with the definition, the subject is capable of making a free choice.

If  $x_1 > 0$ , (4.7) can be represented as

$$\frac{1 - X_1}{X_1} = (1 - (1 - x_2)(1 - d)) \left( \frac{1 - x_1}{x_1} \right). \quad (4.8)$$

Let us connect this equation with the number of the subject's influence on the agencies and the number of the agencies influences on the subject as follows:

$$X_1 = \frac{N_1}{N_1 + N_2}, \quad x_1 = \frac{n_1}{n_1 + n_2}, \quad (4.8a)$$

where  $N_1$  and  $N_2$  are the numbers of the subject's influences on the positive and negative agencies, and  $n_1$  and  $n_2$  are the numbers of the positive and negative agencies' influences on the subject. By substituting these values into (4.8), we obtain

$$\frac{N_2}{N_1} = p \left( \frac{n_2}{n_1} \right), \quad (4.9)$$

where

$$p = 1 - (1 - x_2)(1 - d). \quad (4.10)$$

Equation (4.9) represents the *intentional subject* when  $M(x_3) = (1 - d)x_3$ . On the other hand, (4.9) corresponds to the Generalized Matching Law (2.2) for  $\beta = 1$ . If in equation (2.2),  $B_1$  corresponds to the positive pole and  $B_2$  to the negative pole, then the value of  $p$  corresponds to free parameter  $c$ .

The values

$$\frac{n_1}{N_1} = D_1 \quad \text{and} \quad \frac{n_2}{N_2} = D_2 \quad (4.10a)$$

will be called *reinforcement densities*  $D_1$  and  $D_2$ . Now we write (4.9) as follows:

$$\frac{D_1}{D_2} = p. \quad (4.11)$$

It follows from the equivalency of (4.11), (4.9), (4.8) and (4.7) that the subject is intentional if and only if (4.11) holds.

## 5. Modeling the Experiment with Two Keys

We assume that in the experiments with the two keys, the subject's activity aims not only at obtaining food, but also at generating a mixed state (see Introduction). First, an organism stabilizes the relative frequencies of contacts with the agencies by holding  $X_1 = x_3$ . After stabilization, frequency  $N_1/(N_1+N_2)$  "turns" into the *probability* equal to the frequency, which characterizes the subject's mixed state. Further we demonstrate that the experiment with two keys can be modeled with the help of RIMS.

The subject is placed into a cage with two keys each connected with a food-hopper. The subject's pecks onto keys are reinforced by pieces of food but rarely. Each key has its own schedule of reinforcement (type *VI* or *VR*). The experiment consists of a series of sessions. In every session, the mean interval between reinforcements is fixed for each key. Let suppose that

- (1) The preparation of the subject to the experiment and the conditions of the experiment determine the value of the depression index  $d$ , being constant during the entire experiment.
- (2) The following events take place at the beginning of each session:
  - (a) One key acquires the status of the positive agency, and the other one that of the negative agency.
  - (b) Variable  $x_2$  takes on a fixed value equal to or depending on the relative mean frequency of reinforcements in the previous sessions from the key, which in a given session plays the role the positive agency. If a given session is first,  $x_2=1/2$ .

The goal of this process is to form and preserve the state in which an organism has the ability to make an intentional probabilistic choice. RIMS does not indi-

cate the strategy which the subject's organism will choose to reach and hold equation (4.9). It is possible that the subject controls equation (4.11) for local densities in the way similar to the one described by the model of melioration (Rachlin, 1973; Vaughan, 1985).

## 6. The Patterns of Behavior Predicted by RIMS

We will call the alternative (key) richer if, in a given session, the subject behaves in such a way that this alternative is reinforced more often. In the framework of RIMS, the keys are polarized. One of them plays the role of the positive pole, and the other that of the negative pole. Consider three possible relations between polarization and richness along the set of sessions.

- (A) One alternative is the positive pole in all sessions independently from being richer or not.
- (B) In each session, the least rich alternative is the positive pole.
- (C) In each session, the richer alternative is the positive pole.

We will call one key right and the other left. Let  $K_1$  and  $K_2$  be the numbers of pecks to the right and left keys, and  $k_1$  and  $k_2$  the numbers of corresponding reinforcements. Using functions (4.7) and (4.9) we will construct now a graph of  $K_1/(K_1 + K_2)$  depending on  $k_1/(k_1 + k_2)$  and a graph of  $\log(K_2/K_1)$  depending on  $\log(k_2/k_1)$  for cases (A), (B) and (C).  $N_1$  and  $n_1$  relate to the positive pole, and  $N_2$  and  $n_2$  to the negative pole. Each graph corresponds to a set of sessions (Fig. 2).

Graphs A1 and A2 map the case, when the right key plays the role of the positive pole and the left one the role of the negative pole, during the entire set of sessions

Graphs B1 and B2 map the case, in which the right or the left key corresponds to the positive pole only in those sessions in which it is not richer. Because of that, the graphs have a breach. Let us look at B1. For those sessions in which  $k_1 < k_2$ , the right key is the positive pole. At  $k_1 = k_2$ , there is a breach, corresponding to the reorientation of the poles. At  $k_1 > k_2$ , the left key is the positive pole. The logarithmic graph B2 consists of two rays going at the angle of  $45^\circ$  to the horizontal axis. The left ray corresponds to the sessions with the left key being the positive pole, and the right ray to those with the right one.

Graphs C1 and C2 map the case when one of the keys (right or left) is the positive pole only in those sessions in which it is richer than the other. Consider C1. When  $k_1 > k_2$ , the right key is the positive pole. At  $k_1 = k_2$ , as in B1, there is a breach. At  $k_1 < k_2$  the left key is the positive pole. The logarithmic graph C2, as B2, consists of two rays.

The upper ray corresponds to the left key being the positive pole, and the lower ray to the right one. Let us look at the differences between C2 and B2. At  $k_1 > k_2$ , the ray lies above the diagonal in B2 and below it in C2. At  $k_1 < k_2$ , the ray lies below the diagonal in B2 and above it in C2.

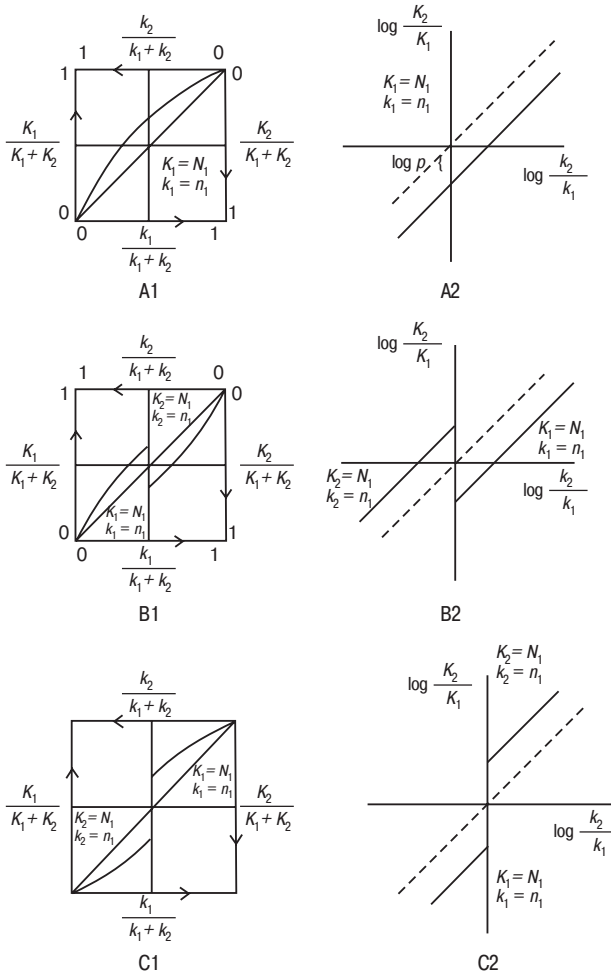


Fig. 2. The patterns of behavior predicted by RIMS

A ray shift up or down on logarithmic graphs A2, B2 and C2 is predetermined by

$$p = 1 - (1 - x_2)(1 - d). \tag{6.1}$$

It is easy to see that  $p=1$  only under condition that at least one of the values,  $x_2$  or  $d$ , is equal to 1. The value  $x_2=1$  means that the subject received all the preceding reinforcements from the key which is positive in the given session. In real experiments, the subjects always have some experience in receiving reinforcements from the negative key, as well. Thus, we have to consider  $x_2 < 1$ .

Therefore, the ideal correspondence

$$\frac{N_2}{N_1} = \frac{n_2}{n_1} \tag{6.2}$$

may appear only at  $d = 1$ , that is, if the subject has been led to the state with the maximal index of depression. When  $d = 0$ , that is, the index of depression is minimal, the following equation is realized:

$$\frac{N_2}{N_1} = x_2 \frac{n_2}{n_1} . \tag{6.3}$$

### 7. The Patterns Observed

Pattern A is well known. It is usually described as a case of  $\beta = 1$  in (2.2). This pattern appears under condition that left and right alternatives differ essentially. For example, the left key is under schedule  $VI$ , and the right one under  $VR$  (see Baum, 1974; Williams, 1988). In this case, a set of sessions can be described by (7.1), where  $c \leq 1$ :

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + cr_2} . \tag{7.1}$$

A type of experimental curves in Fig. 3 corresponds to this case.

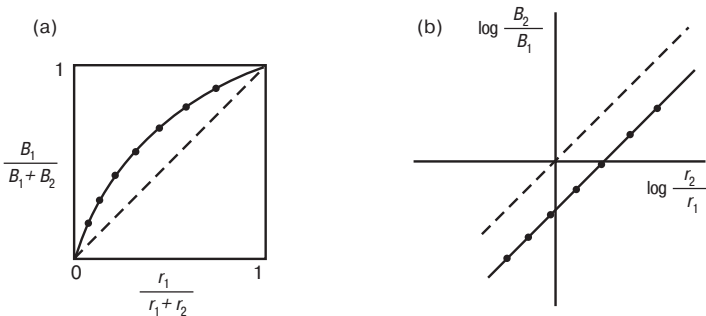


Fig. 3. A type of experimental graphs corresponding to pattern A

In the framework of RIMS we interpret this pattern as follows: the key corresponding to  $B_1$  is the positive pole and  $c = p$ . Therefore (7.1) can be written as

$$\frac{N_1}{N_1 + N_2} = \frac{n_1}{n_1 + pn_2} . \tag{7.2}$$

An analysis of the experiments, in which one key (say the left one) is controlled by schedule *VI*, and the other (right) by *VR*, demonstrates that *VR* key is the positive pole, and *VI* one is the negative pole.

Pattern B is also known well. It is observed when alternatives do not differ but by the ratios of schedules. This observation made Baum et al. (1999, see also Baum, 2002).

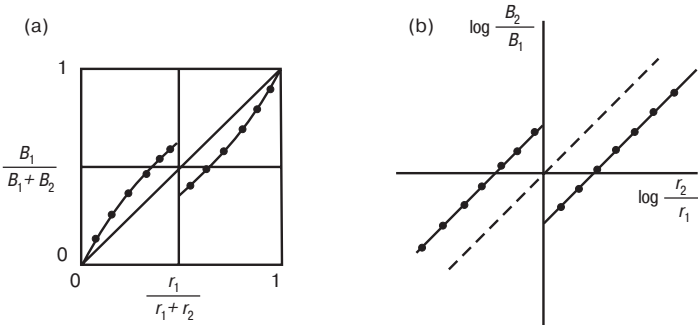


Fig. 4. A type of experimental graphs corresponding to pattern B

This pattern, as Baum et al. noted, corresponds to cases with  $\beta < 1$ , if we describe it with the Generalized Matching Law. The appearance of a twist inherent to this law may be explained as an approximation of broken graph 4(a) by a continuous power function (Fig. 5).

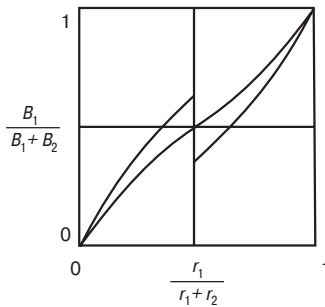


Fig. 5. An approximation of a broken graph (Fig. 4a) by a continuous power function

By analyzing graphs in Fig. 4 with RIMS we see that the positive pole corresponds to the alternative which is *less* reinforced. This peculiar fact, as we will show later, is a key fact for understanding the difference between utilitarian and deontological aspects in animal behavior.

Pattern C can be seen rarely and only recently has been singled out as a special (Baum, 2002, Fig. 1). We can see an example of it in the experiment by Baum and Aparicio (1999) where one alternative was worked on *VR*-schedule with constant

mean ratio, and the other one – on *VI*-schedule and interval changing session by session. The data of this experiments (rats 102, 111, 120, 213) can be represented in the graph in Fig. 6:

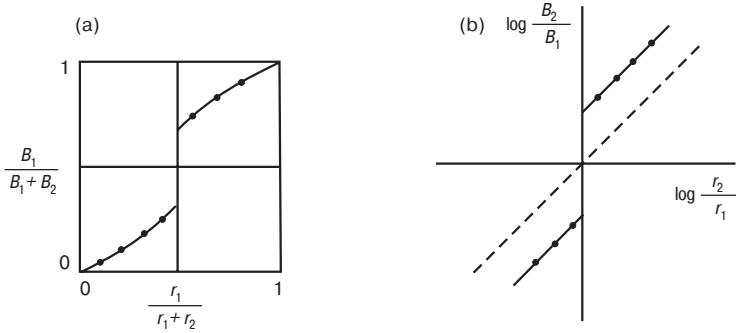


Fig.6. A type of experimental graphs corresponding to pattern C

Following Baum (2002), we may suppose that this pattern reveals itself in the experiments whose description with the Generalized Matching Law requires introduction of  $\beta > 1$ . As in pattern B, the appearance of a curve breach can be explained by approximation of graph 6(a) with a power function (Fig. 7).

The experiment by Baum and Aparicio (1999) and their analysis demonstrates that pattern C can be reduced to pattern A, if instead of location (right/left) alternatives are coordinated by schedules (VR/VI). Then graph 6(a) looks like Fig. 8.

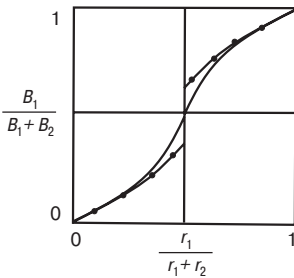


Fig.7. Approximation of broken graph Fig. 6(a) – with a continuous power function

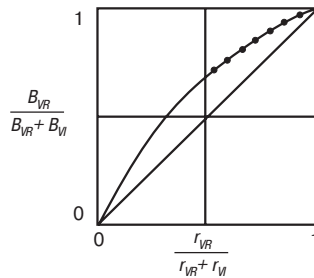


Fig.8. Pattern (A), corresponding to pattern C in Fig. 6(a)

Pattern C may appear in the experiments when a factor which determines a positive-negative polarization of the alternatives in some sessions is connected with the left alternative and in the others with the right one.

What are the experimental values for  $c$  in (7.1)? It is easy to find them for patterns A. We have to find an intersection point of a logarithmic graph with a



vertical axis in Fig. 3(b). According to Williams (1988), when one key works under  $VI$ -schedule, and the other under  $VR$ -schedule,  $c = 0.59$ . Most pattern B data were treated under assumption that the Generalized Matching Law holds, because of this it is close to impossible to extract the value of  $c$  from them. To find the value of  $c$  for pattern B using logarithmic data, we need to perform a linear approximation separately for the dots lying above the horizontal axis and those lying below it. Such a procedure was performed by Baum et al. (1999). We used their data and found the mean value of  $c = 0.58$ . In RIMS  $c$  corresponds to  $p$  found from (4.10).

Let us find now the mean value of the depression index  $d$  for this experiment assuming  $x_2 = 0.5$ . Then (4.10) looks as follows:

$$0.59 = 1 - (1 - 1/2)(1 - d), \quad (7.3)$$

from where  $d = 0.18$ . Therefore, RIMS predicts that in this experiment, a pigeon's prognostic model of the future can be represented as function  $m(x) = 0.82x$ .

## 8. Animals Deontological Evaluations

We would like to put forth a hypothesis that animals have ability to make bipolar, positive vs. negative evaluations analogous to (and perhaps preceding evolution of) the human moral evaluation good vs. bad. To find out how moral-like evaluations in animals are connected with the utilitarian preferences, let us consider ourselves first. Here is a specific case. In 1918, during the Civil War in Russia, the head of one family ends up in China, while his wife is left in Russia with six children; she is executed. The father finds a wealthy American, who with the risk to his life goes to Russia, finds children and brings all six of them to their father to China. Ten years later, with great difficulties, the same American helps them to move to the USA. The oldest of the saved children feels disappointed there.

Why America was not good for twenty-two-year old Mulia? Nostalgia? No. The thing was that the most important value for her - that of self-sacrifice - contradicted to the American style of life. Mulia was confused. An idea of "living for others", natural to her, somehow could not be applied to America. Of course, Mr. Crane "sacrificed" a lot of time and money to charity (he was helping 50 more families to stand on their feet!). But his activity did not carry that special beauty of total self-sacrifice, admiration of which Mulia got with her mother's milk? (Pann, 2003).

Let us suppress our surprise (and perhaps, indignation) of the lack of psychological grace in this young woman. We will try to analyze this example from a naturalist's point of view. What feature in Mr. Crane behavior was unacceptable for Mulia? This highly noble man helped fifty one families. It is clear that he had to plan his activity and count money spent on each family. In other words, his good deals were connected with money, this is why his activity lacked "that special beauty of self-sacrifice." It looks as if this young woman has an automatic mechanism which forbids combining utilitarian and deontological evaluations

Let us make the next step and suppose that this mechanism has a deep biological nature, while it only looks culturally conditioned. In developing this idea we can hypothesize that birds and mammals have two systems of evaluations. The first system, utilitarian, reflects evaluations of the agencies related to the animal close biological needs. The second one, deontological, is connected with bipolar evaluations positive-negative on a larger time scale.

Consider, for example, a hungry animal choosing between two feeders. The first one contains more food, but the second one is more safe (say, hidden from view). In this case, the agency richer with food gets evaluation “negative” while the poorer one receives “positive” evaluation. Thus, animals’ “idealism” while being related to specific life-important evaluations, is nevertheless separated from this-minute preferences. We cannot exclude the possibility that this distinction is supported by a special mental mechanism, whose work in humans reveals itself in a dramatic contrast between material and ideal values.

From this point of view, let us look at the patterns of behavior described in the previous sections. Pattern B appears when left and right food-hoppers differ only in their frequency of food delivering. In this case, as we found, the alternative which is less reinforced plays the role of the positive pole. We may hypothesize that this phenomenon reflects the work of the same mechanism which counterposes “dirty” money and “pure” intentions in humans. The alternative polarization opposite to their utilitarian preferences as food sources is analogous to the human act of purification, that is, of separation good from practical profit. Let us emphasize that this happens if the alternatives differ only in the frequency of food delivery. If some “non-utilitarian” factor exists, which predetermines polarization of the alternatives, then the polarization remains the same during the entire session independently from reinforcement of the positive alternative. This conclusion results from the interpretation of patterns A and C.

## 9. Sacred Shift

Altruism is understood as spending one’s own resources in order to help others. But this is not the only form of sacrificial behavior. Voluntary expenditure of one’s finances and energy related to creation and support of religious and moral symbols is another form of sacrificial behavior. Everyone may cite an example of people who agree to perform work connected with higher values (say, building a cathedral) for smaller pay than they would require for similar work not connected with these values. RIMS offers explanation for this phenomenon. If we interpret  $N_1$  and  $N_2$  in (4.9) as the subject’s *expenditures* of appeals to the positive and negative agencies, and  $n_1$  and  $n_2$  as his revenues, then the ratios

$$\frac{n_1}{N_1} \text{ and } \frac{n_2}{N_2} \quad (9.0)$$

express the mean payments the subject requires from the agencies for one appeal. It follows from (4.11) that

$$\frac{n_1}{N_1} \leq \frac{n_2}{N_2} . \quad (9.1)$$

Therefore, it turns out that at average, the subject never requires more goods for one appeal from the positive agency than from the negative one. This conclusion is true not only for humans but for the rats and pigeons, as well.

### Conclusion

We are convinced that we have a mental domain. This belief is based only on our own subjective experience. We do not have an *operational criterion* which would allow us to find out whether a particular organism or a technical device has an inner world. Or, on the other hand, are we observing an “inanimate” physical process. This problem extends beyond the framework of science and touches upon the core of our morality. We believe that a rat and a pigeon are able to suffer. But what about a fish or a bee, can they suffer? In this work, we put forth a hypothesis that the mental domain in living creatures appears at the same time as their ability to make a probabilistic choice. We have also substantiated an assumption that the creatures with mental a domain can “self-program.” That is, they can program themselves by downloading the values of probabilities with which they will make their choice. Furthermore, we have shown that the Matching Law is an external demonstration of this self-programming. If this hypothesis proves to be correct, then the Matching Law will turn into an operational criterion. We will thus have grounds to consider organisms, for which it holds, to have a mental domain.

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