A Polar Concept Argument for the Existence of Abstracta

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ABSTRACT In this paper, I present a polar concept argument for the existence of abstract objects. After recalling the fundamentals concerning the debate about the existence of abstracta, I present in a detailed way the argument for the existence of abstracta. I offer two different variations of the argument: one, deductive and the other, inductive. The argument rests primarily on the fact that our universe is well-balanced. This well-balanced property results from the fact that all instantiable polar dualities are instantiated. Hence, the abstract pole of the abstract/concrete duality must also be exemplified. Lastly, I review several objections that can be raised against this latter argument.

There are several famous problems about abstract entities. One of them is whether there exist any abstract objects. A second issue is concerned with the definition of which sorts of entities are genuinely abstract. A third issue relates to whether the abstract/concrete duality is exhaustive or not. The purpose of this paper is to address the first of these issues and to describe a polar concept argument that entails the existence of abstracta. Before stating the argument in detail and reviewing several objections that can be raised against it, it is worth recalling first the fundamentals of the debate about the existence of abstracta.

1. The debate about the existence of abstracta

Let us recall preliminarily the main lines of the issue of whether there exist abstract objects¹. This latter problem rests primarily on the abstract/concrete distinction. Uncontroversially, the following objects are considered as abstracta: the natural numbers, the cosine function, the Pythagorean theorem. For this reason, I shall only be concerned in what follows with paradigm abstracta, i.e. natural numbers, setting aside other sorts of entities whose status is controversial. On the other hand, an instance of jay or of an oak-tree, the mountain in front of me, the sun, our galaxy are paradigmatically classified as concrete objects. In this context, concreta are considered as existent objects. But at this stage, agreement stops. In effect, by contrast, the mere existence of abstracta is at issue. Do abstract objects truly exist? There are two main philosophical answers to this latter question. On the one hand, some philosophers simply deny the existence of abstracta. According to the corresponding line of thought, only concrete objects exist in our universe, and abstract concepts are a mere product of our brain circuitry. Thus, natural numbers, the sine function, etc. which are considered as paradigm cases of abstracta, exist only in our mind. The view that denies the existence of abstract objets is nominalism.

On the other hand, according to a line of thought originating from Plato, abstract objects do truly exist. According to *platonism*, abstracta have an existence of their own, in the same way as concreta do exist. In addition, abstracta are standardly considered as having no spatiotemporal position, in contrast to concreta which possess a given position in space and time.

¹ E. J. Lowe (1995) distinguishes three different conceptions of abstract objects. In what follows, my concern will be with what he terms abstract₁ objects, i.e. in opposition to concrete objects.

The main argument for abstracta is the Quine-Putnam *indispensability argument*.² Its first premise is that we should be committed to the existence of all entities that are indispensable to our best scientific theories. From the consideration that natural numbers are indispensable to these latter scientific theories, it follows that we should be committed to the existence of natural numbers. The indispensability argument is controversial and has notably led to important objections raised by Hartry Field (1980), Penelope Maddy (1992) and Elliott Sober (1993). Without entering into the details of these criticisms, I will offer here a different line of argument for abstract objects than the indispensability argument.

2. The polar concept argument for the existence of abstracta

In what follows, I borrow the expression 'polar concept argument' from the characterization of Ryle's argument against skepticism³ (1960) provided by Anthony Grayling (1995, pp. 49-50). Grayling describes Ryle's argument in the following terms:

The point can be simply illustrated by a consideration of Gilbert Ryle's attempt to refute the argument from error by a 'polar concept' argument. There cannot be counterfeit coins, Ryle observes, unless there are genuine ones, nor crooked paths unless there are straight paths, nor tall men unless there are short men. Many concepts come in pairs which are polar opposites of one another, and these conceptual polarities are such that one cannot grasp either pole unless one grasps its opposite at the same time. Now error and 'getting it right' are conceptual polarities. If one understands the concept of error, one understands the concept of getting it right. But to understand this latter concept is to be able to apply it. So our very grasp of the concept of error implies that we sometimes get things right.

Grayling characterizes thus as a *polar concept argument* the argument used by Gilbert Ryle to refute the argument from error, which takes place in the debate against skepticism. The argument from error puts in parallel two types of situations. On the one hand, it appears that we often mistakenly have some knowledge that comes from perceptual experience. But these latter situations, argues the skeptic, are indistinguishable from present situations in which we have some knowledge that stem from our current perceptual experiences. Therefore, concludes the skeptic, our present knowledge could also be mistaken. According to Ryle, the argument from error is inconclusive, since 'getting it right' and 'error' are opposites and originate from the same duality. Hence, Ryle pursues, whenever one grasps the concept of 'error', one also grasps the opposite concept of 'getting it right'. A further step states that to understand the corresponding concept is tantamount to being able to apply it in practice. And this finally undermines the conclusion of the argument from error. I shall not discuss here whether Ryle's argument is valid or not. Rather, my concern will be with showing that an analogous polar concept argument can be used in support of the existence of abstracta.

Let us turn now to the description of the present argument for abstracta. It can be sketched informally as follows. Begin with the fact that our universe is well-balanced. A summary analysis reveals that this well-balanced property is exemplified many times. For consider for example the existence, at a macroscopic level, of very large objects such as stars, supernovae or galaxies. Contrast now with the existence, at a microscopic level, of very small objects such as atoms or molecules. This illustrates how our universe is well-balanced for what concerns the large/small duality. Consider also how both poles of the dark/bright duality are exemplified. It suffices to consider the existence, on the

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² For a survey of the *indispensability argument*, see Colyvan (2001).

³ Cf. Ryle (1960, pp. 94-95): "A country which had no coinage would offer no scope to counterfeiters. There would be nothing for them to manufacture or pass counterfeits of. They could, if they wished, manufacture and give away decorated disks of brass or lead, which the public might be pleased to get. But these would not be false coins. There can be false coins only where there are coins made of the proper materials by the proper authorities. In a country where there is a coinage, false coins can be manufactured and passed; and the counterfeiting might be so efficient that an ordinary citizen, unable to tell which were false and which were genuine coins, might become suspicious of the genuineness of any particular coin that he received. But however general his suspicions might be, there remains one proposition which he cannot entertain, the proposition, namely, that it is possible that all coins are counterfeits. For there must be an answer to the question 'Counterfeits of what?'".

one hand, of dark objects such as black holes, our sky at night and on the other hand, of bright objects such as the sun and the snow over the Himalaya. Now consider also how many dualities such as soft/harsh, hot/cold, static/dynamic, positive/negative, neutral/polar, visible/invisible, one/many, etc. are also instantiated. To be more accurate, for a given object o having a property A, the well-balanced property relative to the A/ \bar{A} duality results from the fact that o also has the opposite property \bar{A} . More generally, being well-balanced relative to our universe results from a generalization of this latter property to all polar dualities. At this step, the necessary well-balance of our universe requires that the abstract/concrete duality be also exemplified. Now it follows that concrete objects exist in our universe. For *concrete* pertains to the abstract/concrete duality. And this entails the necessary existence of abstracta in our universe.

At this step, a more in-depth analysis is needed. Let us then state the present argument more explicitly:

(1) (2)	our universe is well-balanced the well-balanced property relative to the A/Ā duality results from the fact, for each possessed neutral property A, of possessing also the opposite	Premise Definition
(3)	property Ā in our universe the following neutral polar dualities are instantiated: large/small, positive/negative, external/internal, bright/dark, static/dynamic, hard/soft, visible/invisible, one/many, cold/hot, etc.	Evidence
(4)	in our universe all instantiable polar dualities are instantiated	From (1), (2)
(5)	for each simple quality, there exists an opposite quality	Premise
(6)	concrete is a simple quality	Definition
(7)	abstract is the simple opposite quality from concrete	Definition
(8)	there exists in our universe some concrete objects	Evidence
(9)	:. there exists in our universe some abstract objects	From (5), (7),
	·	(8)

Let us highlight some distinctive features of the present argument. It should be pointed out first that the argument is deductive. In effect, it starts from the consideration that our universe is well-balanced and derives the conclusion that abstract objects do exist. The well-balanced property is crucial to the argument and two different types of well-balanced properties are distinguished: (i) well-balanced relative to a given polar duality A/\bar{A} ; (ii) well-balanced relative to our universe.

At this step, it is worth defining accurately the scope of the argument. More generally, the argument postulates that for each pole observed in our universe, there exists an opposite pole. The argument is thus based on the fact that for every object that *exists* and exemplifies a pole, there also exist other objects that instantiate the opposite pole. The argument postulates that there do not exist things in our universe that instantiate a pole of a given duality without also instantiating the opposite pole.

It is also worth emphasizing that the present argument is only concerned with *neutral* polar opposites, i.e. polar concepts that are neither meliorative nor pejorative. Accordingly, *large*, *small*, *bright*, *dark*, *concrete*, *abstract*, etc. are neutral concepts, to the difference of such concepts as *beautiful*, *ugly*, *courageous*, etc which are non-neutral.

Lastly, it should be pointed out that the above argument can be stated alternatively under the form of an inductive argument. It could then be recast as follows:

(1i)in our universe the following neutral polar dualities are instantiated: Evidence large/small. positive/negative, external/internal. bright/dark. static/dynamic, hard/soft, visible/invisible, one/many, cold/hot, etc. (2i) the well-balanced property relative to the A/Ā duality results from the Definition fact, for each possessed neutral property A, of possessing also the opposite property Ā (3i) : in our universe all instantiable polar dualities are instantiated From (1i),generalization (4i)for each simple quality, there exists an opposite quality Premise (5i)concrete is a simple quality Definition (6i) abstract is the simple opposite quality from concrete Definition

(7i) there exists in our universe some concrete objects
(8i) ∴ there exists in our universe some abstract objects
From (3i), (7i), induction

(9i) ∴our universe is well-balanced From (8i)

Note that the inductive form of the argument proceeds by enumerating all instantiable polar dualities and then generalizing to all polar dualities. It follows then straightforwardly by induction that the abstract/concrete duality is also instantiated.

3. Response to objections

At this stage, it is worth considering a set of objections that can be pressed against the present argument for abstracta. Let us review, first, a line of objection that stems from the issue of whether Ryle's argument is a sound one. Grayling (1995, p. 50) mentions in effect that a skeptic critic could object to Ryle's polar concept argument that a same line of reasoning applied to other dualities such as 'perfect/imperfect', 'mortal/immortal', 'finite/infinite' would entail the existence of perfect, immortal or infinite entities. An objection along the same lines could then be raised against the present argument for abstracta. However, in the present context, these three dualities do not deserve the same type of response. For that reason, I shall consider them in turn. Let us begin with the perfect/imperfect duality. From the above, it should be apparent that the perfect/imperfect duality does not fall under the scope of the present argument. For perfect is not a simple quality. In effect, perfect can be defined as the sum of all simple positive qualities. Thus, perfect can be characterized as a complex or composite quality. But the scope of the present argument is restricted to simple qualities. For that reason, the existence of perfect entities is not entailed by the current argument.

Let us turn now to the mortal/immortal duality. At this step, it should be pointed out that it is not clear whether mortal can be considered as a simple quality, in the sense defined above. For that reason, I shall replace it by the temporal/atemporal duality. This latter pair is made up of two conceptual polarities that can be regarded unambiguously as simple qualities. Now it should be acknowledged that the temporal/atemporal duality also falls under the scope of the above argument. And a same line of reasoning yields the existence of atemporal entities. I shall endorse such a consequence here. In effect, the present argument is also for the existence of atemporal entities. But is there something counter-intuitive here? It seems that some objects such as numbers are obvious candidates for this definition. In effect natural numbers can be considered as both abstract and atemporal entities.

Now the same goes for the application of the present argument to the infinite/finite duality. For it should be acknowledged that infinite and finite are simple qualities in the sense defined above. Thus the argument also applies to these latter concepts and yields the existence of infinite objects. But such inference should not be very disturbing, I think, for it is much in line with our current intuitions. Just as in the previous case of the temporal/atemporal duality, there exist natural candidates for the definition of infinite entities. Natural numbers, for example, straightforwardly instantiate the property of being infinite.

Let us consider, second, another line of objection. For it could be opposed to the present argument that a similar line of reasoning postulates the existence of *impossible* objects. Few would doubt, in

effect, that we currently have a large body of evidence in favor of the existence of *possible* objects. Our universe contains many instances of possible objects. Hence, according to the above argument, from the possible/impossible duality, we can infer the existence of impossible objects. But as this latter notion is self-contradictory, the objection goes, the whole enterprise is vowed to inconsistency. However, this line of objection does not undermine the force of the argument, I think. For the present argument is only concerned with existent objects. It begins with the observation that many objects exemplifying both poles of a given duality do exist. It pursues by inferring the existence of objects that instantiate a pole of the abstract-concrete duality. But in all cases, the present argument is only concerned with pairs of polar contraries that are compatible with existence. Perhaps, some would agree that certain objects do possess the property of being impossible, contradictory or imaginary. But such inferences do not fall under the scope of the present argument. For the dualities that fall under the scope of the present argument need at least to be instantiable. As a consequence, predicates such as impossible, inexistent, imaginary, contradictory, inconceivable, etc. should be discarded from the beginning. And all non-instantiable dualities (i.e. dualities which contain at least one non-instantiable pole) such as possible/impossible, existent/inexistent, coherent/incoherent, etc. are not concerned with the current argument. In addition, the same response prevails for a similar line of objection that would respectively infer the existence of inexistent, inconceivable, imaginary objects, from the existent/inexistent, conceivable/inconceivable, real/imaginary dualities.

Let us examine, third, a different line of objection. It runs as follows. The present argument rests on the necessity of instantiating both poles of all dualities, the objection goes. But it could be retorted that certain poles of some dualities need not being instantiated. And such is the case, the objection runs, for the abstract pole of the abstract/concrete duality. It should be apparent that this latter objection challenges premise (4), according to which all instantiable polar dualities are instantiated in our universe. But this latter objection is not very promising, I think. For the current argument is concerned with our universe's well-balanced requirement. And this well-balance results precisely from the instantiation of both poles of each duality. Consider the case of the bright/dark duality. Imagine our universe containing only dark objects, with all bright objects missing. Would we expect to find ourselves in such an universe? No. For the emergence of carbon-based life would be impossible in such one-sided (from the viewpoint of the dark/bright duality) universe. Or consider alternatively the situation if all objects in our universe were static and no objects were dynamic. Or else imagine if our present universe only contained cold objects, and were entirely devoid of hot ones. Consider alternately an universe with only one object, thus exemplifying only one pole of the one/many duality. All such universes would be very unfriendly, to say the least. Now the same goes for the abstract pole of the abstract/concrete duality. Perhaps it could be helpful to recall, at this step, one major premise of the fine-tuning argument. This latter argument derives from the fact that many cosmological constants are fine-tuned for the emergence of carbon-based life, the conclusion that this latter feature of our universe is non-random and due to the intention of its Creator. Now the premise according to which the cosmological constants are fine-tuned for further emergence of carbon-based life can be replaced by the three following steps:

- (1f) the cosmological constants of our universe are fine-tuned for the emergence of carbon-based life
- (2f) the instantiation of many dualities in our universe permits the emergence of carbon-based life
- (3f) ∴ the cosmological constants of our universe are fine-tuned for the instantiation of the following dualities: large/small, positive/negative, external/internal, bright/dark, static/dynamic, hard/soft, visible/invisible, one/many, cold/hot, etc.

At this step, it should be apparent that challenging (3f) also implies being committed to doubt (1f), while on the other hand it is a widely accepted premise of the fine-tuning argument.

The above argument could be attacked, fourth, on the grounds that is not deductive, but rather inductive. According to this line of objection, the present argument is a disguised inductive argument.

In effect, if the argument were inductive instead of deductive, it would be probabilistic and as such, its impact would be weaker than in its original presentation. As mentioned above, it should be acknowledged that the argument can be presented alternatively as an inductive argument. The inductive form of the argument begins with an enumeration of all exemplified polar dualities. From this, it derives a generalization to all polar dualities. The conclusion that the abstract/concrete duality and in particular its abstract pole is also instantiated ensues. If the above argument is to be considered as essentially inductive, this has the effect of weakening the argument by making it inductive rather than deductive. However, the present argument is not intended to count as a proof yielding an absolute certainty. Then choose whatever variation of the argument you prefer - the deductive or the inductive form. In either case, the essence of the argument remains in force.

It would also be tempting to challenge, fifth, premise (1), namely the fact that our universe is well-balanced. But such an objection is not very promising, I think. In effect, our universe is about 15 billion years old. How could our universe have lasted so long if it hadn't been well-balanced? Considering now its spatial extension, a question of the same nature arises. For our universe extends billions of light years in any directions. How could our universe have occupied such huge spatial region without being well-balanced? And again: How could our universe contain so much objects such as neutrons, monkeys, stars, galaxies, etc. and a total number of atoms amounting to 10^{80} , without exemplifying a well-balanced property?

Lastly, another line of objection that could be pressed against the present argument is that it is simply a generalization of Ryle's argument. However, I would stress that Ryle's argument is slightly differently motivated. In effect, the key concept in my argument is the well-balanced property. A key premise is in effect that being well-balanced is a prominent feature of our universe. And this last premise is reinforced by the additional premise based on the empirical fact that some properties of our universe currently instantiate this well-balanced property. Thus, the present argument is not entirely a priori as could be characterized Ryle's argument. The present argument incorporates in effect some empirical features of our universe. In addition, I should stress that Ryle's argument contrasts 'error' and 'getting it right'. But it should be apparent that 'error' has a pejorative connotation and 'getting it right' reveals a meliorative nuance. Hence, 'error' is a negative concept and 'getting it right' is a positive one. By contrast, the current argument is only concerned with neutral concepts and pairs of neutral opposites. Consequently, it is worth pointing out that the present argument would be inapplicable to the 'error/getting it right' pair of concepts.

In conclusion, it is worth mentioning the scope of the above argument. It is not designed in effect to serve as a proof of the existence of abstracta. For given our current high standards, it should be acknowledged that it does not meet our present criteria of proof. Rather, the present argument aims at reinforcing an initial credence that abstract objects could exist. The above argument is simply designed to increase our a priori belief about the existence of abstracta. As such, it is consistent with the Quine-Putnam indispensability argument. It is also consistent with recent advances in cosmology and in particular with the level IV of the kind of multiverse described in Tegmark (2003). In this context, the purpose of the present argument for abstracta is to provide some additional grounds in support of the hypothesis that abstract objects do exist.

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