The Violation of Bell Inequalities in the Macroworld*

Diederik Aerts, Sven Aerts, Jan Broekaert and Liane Gabora

Center Leo Apostel, Brussels Free University
Krijgskundestraat 33, 1160 Brussels, Belgium.
diraerts@vub.ac.be, saerts@vub.ac.be
jbroekae@vub.ac.be, lgabora@vub.ac.be

Abstract

We show that Bell inequalities can be violated in the macroscopic world. The macroworld violation is illustrated using an example involving connected vessels of water. We show that whether the violation of inequalities occurs in the microworld or in the macroworld, it is the identification of nonidentical events that plays a crucial role. Specifically, we prove that if nonidentical events are consistently differentiated, Bell-type Pitowsky inequalities are no longer violated, even for Bohm's example of two entangled spin 1/2 quantum particles. We show how Bell inequalities can be violated in cognition, specifically in the relationship between abstract concepts and specific instances of these concepts. This supports the hypothesis that genuine quantum structure exists in the mind. We introduce a model where the amount of nonlocality and the degree of quantum uncertainty are parameterized, and demonstrate that increasing nonlocality increases the degree of violation, while increasing quantum uncertainty decreases the degree of violation.

Dedication: Marisa always stimulated interdisciplinary research connected to quantum mechanics, and more specifically she is very enthusiastic to the approach that we are developing in CLEA on quantum structure in cognition. It is therefore a pleasure for us to dedicate this paper, and particularly the part on cognition, to her for her 60 th birthday.

1 Introduction

This article investigates the violation of Bell inequalities in macroscopic situations and analyses how this indicates the presence of genuine quantum structure. We explicitly challenge the common belief that quantum structure is present only in micro-physical reality (and macroscopic coherent systems), and present evidence that quantum structure can be present in the macro-physical reality. We also give an example showing the presence of quantum structure in the mind.

Let us begin with a brief account of the most relevant historical results. In the seventies, a sequence of experiments was carried out to test for the presence of nonlocality in the microworld described by quantum mechanics (Clauser 1976; Faraci at al. 1974; Freeman and Clauser 1972; Holt and Pipkin 1973; Kasday, Ullmann and Wu 1970) culminating in decisive experiments by Aspect and his team in Paris (Aspect, Grangier and Roger, 1981, 1982). They were inspired by three important theoretical results: the EPR Paradox (Einstein, Podolsky and Rosen, 1935), Bohm's thought experiment (Bohm, 1951), and Bell's theorem (Bell 1964).

Einstein, Podolsky, and Rosen believed to have shown that quantum mechanics is incomplete, in that there exist elements of reality that cannot be described by it (Einstein, Podolsky and Rosen, 1935; Aerts 1984, 2000). Bohm took their insight further with a simple example: the 'coupled spin-$\frac{1}{2}$ entity' consisting of two particles with spin $\frac{1}{2}$, of which the spins are coupled such that the quantum spin vector is a nonproduct vector representing a singlet spin state (Bohm 1951). It was Bohm's example that inspired Bell to formulate a condition that would test experimentally for incompleteness. The result of his efforts are the infamous Bell inequalities (Bell 1964). The fact that Bell took the EPR result literally is evident from the abstract of his 1964 paper:

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"The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum theory could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which is has interacted in the past, that creates the essential difficulty."

Bell's theorem states that statistical results of experiments performed on a certain physical entity satisfy his inequalities if and only if the reality in which this physical entity is embedded is local. He believed that if experiments were performed to test for the presence of nonlocality as predicted by quantum mechanics, they would show quantum mechanics to be wrong, and locality to hold. Therefore, he believed that he had discovered a way of showing experimentally that quantum mechanics is wrong. The physics community awaited the outcome of these experiments. Today, as we know, all of them agreed with quantum predictions, and as consequence, it is commonly accepted that the micro-physical world is incompatible with local realism.

One of the present authors, studying Bell inequalities from a different perspective, developed a concrete example of a situation involving macroscopic 'classical' entities that violates Bell inequalities (Aerts 1981, 1982, 1985a,b). This example makes it possible to more fully understand the origin of the violation of the inequalities, as well as in what sense this violation indicates the presence of quantum structure.

2 Bell Inequalities and Clauser Horne Inequalities

In this section we review Bell inequalities, as well as Clauser and Horne inequalities. We first consider Bohm's original example that violates these inequalities in the microworld. Finally we we put forth an example that violates them in the macroworld.

2.1 Introduction of the Inequalities

Bell inequalities are defined with the following experimental situation in mind. We consider a physical entity S, and four experiments e_1, e_2, e_3, and e_4 that can be performed on the physical entity S. Each of the experiments $e_i, i \in \{1,2,3,4\}$ has two possible outcomes, respectively denoted $o_i(up)$ and $o_i(down)$. Some of the experiments can be performed together, which in principle leads to 'coincidence' experiments $e_{ij}, i,j \in \{1,2,3,4\}$. For example e_i and e_j together will be denoted e_{ij}. Such a coincidence experiment e_{ij} has four possible outcomes, namely $(o_i(up), o_j(up))$, $(o_i(up), o_j(down))$, $(o_i(down), o_j(up))$ and $(o_i(down), o_j(down))$. Following Bell, we introduce the expectation values $\mathbb{E}_{ij}, i,j \in \{1,2,3,4\}$ for these coincidence experiments, as

$$\begin{aligned} \mathbb{E}_{ij} = \quad & +1.P(o_i(up), o_j(up)) + 1.P(o_i(down), o_j(down)) \\ & -1.P(o_i(up), o_j(down)) - 1.P(o_i(down), o_j(up)) \end{aligned} \tag{1}$$

From the assumption that the outcomes are either +1 or -1, and that the correlation $\mathbb{E}_{ij}$ can be written as an integral over some hidden variable of a product of the two local outcome assignments, one derives Bell inequalities:

$$|\mathbb{E}_{13} - \mathbb{E}_{14}| + |\mathbb{E}_{23} + \mathbb{E}_{24}| \leq 2 \tag{2}$$

When Bell introduced the inequalities, he had in mind the quantum mechanical situation originally introduced by Bohm (Bohm 1951) of correlated spin-$\frac{1}{2}$ particles in the singlet spin state. Here e_1 and e_2 refer to measurements of spin at the left location in spin directions $\mathbf{a_1}$ and $\mathbf{a_2}$, and e_3 and e_4 refer to measurements of spin at the right location in spin directions $\mathbf{a_3}$ and $\mathbf{a_4}$. The quantum theoretical calculation in this situation, for well chosen directions of spin, gives the value $2\sqrt{2}$ for the left member of equation (3), and hence violates the inequalities. Since Bell showed that the inequalities are never violated if locality holds for the considered experimental situation, this indicates that quantum mechanics predicts nonlocal effects to exist (Bell 1964).

We should mention that Clauser and Horne derived other inequalities, and it is the Clauser Horne inequalities that have been tested experimentally (Clauser and Horne 1976). Clauser and Horne consider the same experimental situation as that considered by Bell. Hence we have the coincidence experiments e_{13}, e_{14}, e_{23} and e_{24}, but instead of concentrating on the expectation values they introduce the coincidence probabilities p_{13}, p_{14}, p_{23} and p_{24}, together with the probabilities p_2 and p_4. Concretely, p_{ij} means the probability that the coincidence experiment e_{ij} gives the outcome $(o_i(up), o_j(up))$, while p_i means the probability that the experiment e_i gives the outcome $o_i(up)$. The Clauser Horne inequalities then read:

$$-1 \leq p_{14} - p_{13} + p_{23} + p_{24} - p_2 - p_4 \leq 0 \tag{3}$$

Although the Clauser Horne inequalities are thought to be equivalent to Bell inequalities, they are of a slightly more general theoretical nature, and lend themselves to Pitowsky's generalization, which will play an important role in our theoretical analysis.

2.2 The 'Entangled Spins $\frac{1}{2}$' Example

Let us briefly consider Bohm's original example. Our physical entity S is now a pair of quantum particles of spin-$\frac{1}{2}$ that 'fly to the left and the right' along a certain direction v of space respectively, and are prepared in a singlet state Ψ_S for the spin (see Fig. 1). We consider four experiments e_1, e_2, e_3, e_4, that are measurements of the spin of the particles in directions $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \mathbf{a_4}$, that are four directions of space orthogonal to the direction v of flight of the particles. We choose the experiments such that e_1 and e_2 are measurements of the spin of the particle flying to the left and e_3 and e_4 of the particle flying to the right (see Fig. 1).

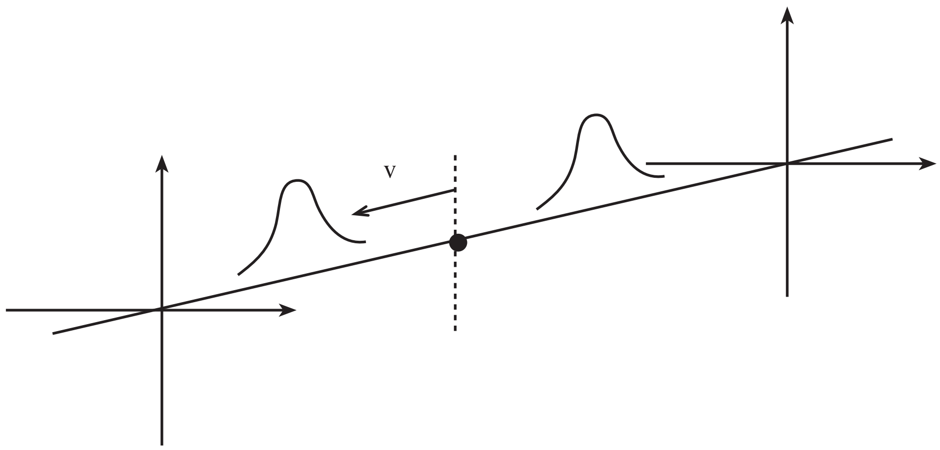

Fig. 1 : The singlet spin state example.

We call p_i the probability that the experiment e_i gives outcome $o_i(up)$. According to quantum mechanical calculation and considering the different experiments, it follows:

$$p_1 = p_2 = p_3 = p_4 = \frac{1}{2} \tag{4}$$

For the Bohm example, the experiment e_1 can be performed together with the experiments e_3 and e_4, which leads to experiment e_{13} and e_{14}, and the experiment e_2 can also be performed together with the experiments e_3 and e_4, which leads to experiments e_{23} and e_{24}.

Quantum mechanically this corresponds to the expectation value $\langle \sigma_1 a, \sigma_2 b \rangle = -a.b$ which gives us the well known predictions:

$$\begin{array}{ll}
\mathbb{E}_{13} = -\mathbf{cos}\angle(\mathbf{a_1}, \mathbf{a_3}) & p_{13} = \frac{1}{2}\ \mathbf{sin}^2 \frac{\angle(\mathbf{a_1}, \mathbf{a_3})}{2} \\
\mathbb{E}_{14} = -\mathbf{cos}\angle(\mathbf{a_1}, \mathbf{a_4}) & p_{14} = \frac{1}{2}\ \mathbf{sin}^2 \frac{\angle(\mathbf{a_1}, \mathbf{a_4})}{2} \\
\mathbb{E}_{23} = -\mathbf{cos}\angle(\mathbf{a_2}, \mathbf{a_3}) & p_{23} = \frac{1}{2}\ \mathbf{sin}^2 \frac{\angle(\mathbf{a_2}, \mathbf{a_3})}{2} \\
\mathbb{E}_{24} = -\mathbf{cos}\angle(\mathbf{a_2}, \mathbf{a_4}) & p_{24} = \frac{1}{2}\ \mathbf{sin}^2 \frac{\angle(\mathbf{a_2}, \mathbf{a_4})}{2}
\end{array} \tag{5}$$

Let us first specify the situation that gives rise to a maximal violation of Bell inequalities. Let $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \mathbf{a_4}$ be coplanar directions such that $\angle(\mathbf{a_1}, \mathbf{a_3}) = \angle(\mathbf{a_3}, \mathbf{a_2}) = \angle(\mathbf{a_2}, \mathbf{a_4}) = 45^o$, and $\angle(\mathbf{a_1}, \mathbf{a_4}) = 135^o$ (see Fig. 2).

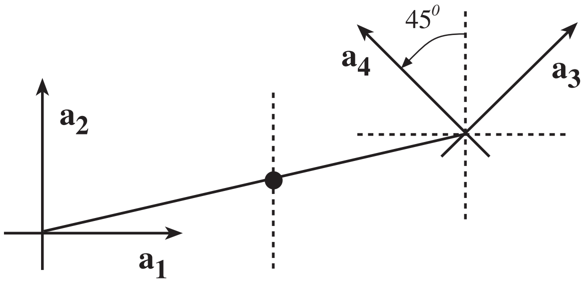

Fig. 2 : The violation of Bell inequalities by the singlet spin state example.

Then we have $\mathbb{E}_{13} = \mathbb{E}_{23} = \mathbb{E}_{24} = \frac{\sqrt{2}}{2}$ and $\mathbb{E}_{14} = -\frac{\sqrt{2}}{2}$. This gives:

$$
\begin{aligned}
|\mathbb{E}_{13} - \mathbb{E}_{14}| + |\mathbb{E}_{23} + \mathbb{E}_{24}| &= |\tfrac{\sqrt{2}}{2} - (-\tfrac{\sqrt{2}}{2})| + |\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}| \\
&= +2\sqrt{2} \\
&> +2
\end{aligned}
\tag{6}
$$

which shows that Bell inequalities are violated. To violate the Clauser Horne inequalities, we need to make another choice for the spin directions. Let us choose $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_4}$ again coplanar with $\angle(\mathbf{a_1}, \mathbf{a_2}) = \angle(\mathbf{a_1}, \mathbf{a_4}) = \angle(\mathbf{a_2}, \mathbf{a_4}) = 120°$ and $\mathbf{a_2} = \mathbf{a_3}$ (see Fig. 3). The set of probabilities that we consider for the Clauser Horne inequalities is then given by $p_1 = \frac{1}{2}, p_2 = \frac{1}{2}, p_3 = \frac{1}{2}, p_4 = \frac{1}{2}, p_{12} = \frac{3}{8}, p_{14} = \frac{3}{8}, p_{23} = 0, p_{24} = \frac{3}{8}$. This gives:

$$
\begin{aligned}
p_{14} - p_{13} + p_{23} + p_{24} - p_2 - p_4 &= +1 - 0 + 1 + 1 - 1 - 1 \\
&= +1 \\
&> 0
\end{aligned}
\tag{7}
$$

which shows that also the Clauser Horne inequalities are violated (see Fig. 3).

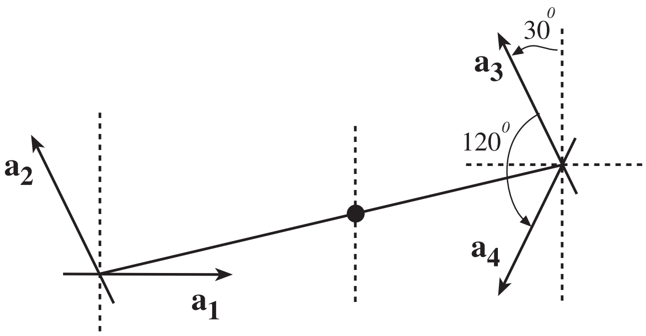

Fig. 3 : The violation of Clauser Horne inequalities by the singlet spin state example.

2.3 The ‘Vessels of Water’ Example

We now review an example of a macroscopic situation where Bell inequalities and Clauser Horne inequalities are violated (Aerts 1981, 1982, 1985a,b). Following this, we analyze some aspects of the example in a new way.

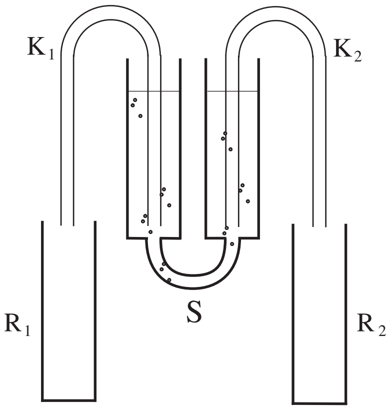

Fig. 4 : The vessels of water example violating Bell inequalities. The entity S consists of two vessels containing 20 liters of water that are connected by a tube. Experiments are performed on both sides of the entity S by introducing syphons K_1 and K_2 in the respective vessels and pouring out the water and collecting it in reference vessels R_1 and R_2. Carefully chosen experiments reveal that Bell inequalities are violated by this entity S.

Consider an entity S which is a container with 20 liters of transparent water (see Fig. 4), in a state s such that the container is placed in the gravitational field of the earth, with its bottom horizontal. We introduce the experiment e_1 that consists of putting a siphon K_1 in the container of water at the left, taking out water using the siphon, and collecting this water in a reference vessel R_1 placed to the left of the container. If we collect more than 10 liters of water, we call the outcome $o_1(up)$, and if we collect less or equal to 10 liters, we call the outcome $o_1(down)$. We introduce another experiment e_2 that consists of taking with a little spoon, from the left, a bit of the water, and determining whether it is transparent. We call the outcome $o_2(up)$ when the water is transparent and the outcome $o_2(down)$ when it is not. We introduce the experiment e_3 that consists of putting a siphon K_3 in the container of water at the right, taking out water using the siphon, and collecting this water in a reference vessel R_3 to the right of the container. If we collect more or equal to 10 liters of water, we call the outcome $o_3(up)$, and if we collect less than 10 liters, we call the outcome $o_3(down)$. We also introduce the experiment e_4 which is analogous to experiment e_2, except that we perform it to the right of the container.

Clearly, for the container of water being in state s, experiments e_1 and e_3 give with certainty the outcome $o_1(up)$ and $o_3(up)$, which shows that $p_1 = p_3 = 1$. Experiments e_2 and e_4 give with certainty the outcome $o_2(up)$ and $o_4(up)$, which shows that $p_2 = p_4 = 1$.

The experiment e_1 can be performed together with experiments e_3 and e_4, and we denote the coincidence experiments e_{13} and e_{14}. Also, experiment e_2 can be performed together with experiments e_3 and e_4, and we denote the coincidence experiments e_{23} and e_{24}. For the container in state s, the coincidence experiment e_{13} always gives one of the outcomes $(o_1(up), o_3(down))$ or $(o_1(down), o_3(up))$, since more than 10 liters of water can never come out of the vessel at both sides. This shows that $\mathbb{E}_{13} = -1$ and $p_{13} = 0$. The coincidence experiment e_{14} always gives the outcome $(o_1(up), o_4(up))$ which shows that $\mathbb{E}_{14} = +1$ and $p_{14} = +1$, and the coincidence experiment e_{23} always gives the outcome $(o_2(up), o_3(up))$ which shows that $\mathbb{E}_{23} = +1$ and $p_{23} = +1$. Clearly experiment e_{24} always gives the outcome $(o_2(up), o_4(up))$ which shows that $\mathbb{E}_{24} = +1$ and $p_{24} = +1$. Let us now calculate the terms of Bell inequalities,

$$\begin{aligned} |\mathbb{E}_{13} - \mathbb{E}_{14}| + |\mathbb{E}_{23} + \mathbb{E}_{24}| &= |-1-1| + |+1+1| \\ &= +2+2 \\ &= +4 \end{aligned} \tag{8}$$

and of the Clauser Horne inequalities,

$$\begin{aligned} p_{14} - p_{13} + p_{23} + p_{24} - p_2 - p_4 &= +1 - 0 + 1 + 1 - 1 - 1 \\ &= +1 \end{aligned} \tag{9}$$

This shows that Bell inequalities and Clauser Horne inequalities can be violated in macroscopic reality. It is even so that the example violates the inequalities more than the original quantum example of the two coupled spin-$\frac{1}{2}$ entities. In section 5 we analyze why this is the case, and show that this sheds new light on the underlying mechanism that leads to the violation of the inequalities.

3 The Inequalities and Distinguishing Events

In the macroscopic example of the vessels of water connected by a tube, we can see and understand why the inequalities are violated. This is not the case for the micro-physical Bohm example of coupled spins. Szabo (pers. com.) suggested that the macroscopic violation of Bell inequalities by the vessels of water example does not have the same 'status' as the microscopic violation in the Bohm example of entangled spins, because events are identified that are not identical. This idea was first considered in Aerts and Szabo, 1993. Here we investigate it more carefully and we will find that it leads to a deeper insight into the meaning of the violation of the inequalities.

Let us state more clearly what we mean by reconsidering the vessels of water example from section 2.3, where we let the experiments e_1, e_2, e_3 and e_4 correspond with possible events $A_1(up)$ and $A_1(down)$, $A_2(up)$ and $A_2(down)$, $A_3(up)$ and $A_3(down)$ and $A_4(up)$ and $A_4(down)$. This means that event $A_1(up)$ is the physical event that happens when experiment e_1 is carried out, and outcome $o_1(up)$ occurs. The same for the other events. When event $A_1(up)$ occurs together with event $A_3(down)$, hence during the performance of the experiments e_{12}, then it is definitely a different event from event $A_1(up)$ that occurs together with event $A_4(up)$, hence during the performance of the experiment e_{14}. Szabo's idea was that this 'fact' would be at the origin of the violation of Bell inequalities for the macroscopic vessel of water example. In this sense the macroscopic violation would not be a genuine violation as compared to the microscopic.

Of course, one is tempted to ask the same question in the quantum case: is the violation in the microscopic world perhaps due to a lack of distinguishing events that are in fact not identical? Perhaps what is true for the vessels of water example is also true of the Bohm example? Let us find out by systematically distinguishing between events at the left (of the vessels of water or of the entangled spins) that are made together with different events at the right. In this way, we get more than four events, and unfortunately the original Bell inequalities are out of their domain of applicability. However Pitowsky has developed a generalization of Bell inequalities where any number of experiments and events can be taken into account, and as a consequence we can check whether the new situation violates Pitowsky inequalities. If Pitowsky inequalities would not be violated in the vessels of water model, while for the microscopic Bohm example they would, then this would 'prove' the different status of the two examples, the macroscopic being 'false', due to lack of correctly distinguishing between events, and the microscopic being genuine. Let us first introduce Pitowsky inequalities to see how the problem can be reformulated.

3.1 Pitowsky Inequalities

Pitowsky proved (see theorem 1) that the situation where Bell-type inequalities are satisfied is equivalent to the situation where, for a set of probabilities connected to the outcomes of the considered experiments, there exists a Kolmogorovian probability model. Or, as some may want to paraphrase it, the probabilities are classical (Pitowsky 1989). To put forward Pitowsky inequalities, we have to introduce some mathematical concepts. Let S be a set of pairs of integers from $\{1,2,...,n\}$ that is,

$$S \subseteq \{\{i,j\} \mid 1 \leq i < j \leq n\} \tag{10}$$

Let $R(n,S)$ denote the real space of all functions $f : \{1,2,...,n\} \cup S \mapsto \mathbb{R}$. We shall denote vectors in $R(n,S)$ by $\mathbf{f} = (f_1, f_2, ...f_n, ...f_{ij}, ...)$, where the f_{ij} appear in a lexicographic order on the $i, j's$. Let $\{0,1\}^n$ be the set of all n-tuples of zeroes and one's. We shall denote elements of $\{0,1\}^n$ by $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$ where $\varepsilon_j \in \{0,1\}$. For each $\varepsilon \in \{0,1\}^n$ let $\mathbf{u}^\varepsilon$ be the following vector in $R(n,S)$:

$$u_j^\varepsilon = \varepsilon_j \qquad 1 \leq j \leq n \tag{11}$$

$$u_{ij}^\varepsilon = \varepsilon_i \varepsilon_j \qquad \{i,j\} \in S \tag{12}$$

The classical correlation polytope $C(n,S)$ is the closed convex hull in $R(n,S)$ of all 2^n possible vectors $\mathbf{u}^\varepsilon$, $\varepsilon \in \{0,1\}^n$:

Theorem 1 (Pitowsky, 1989) *Let $\mathbf{p} = (p_1, ..., p_n, ..., p_{ij}, ...\}$ be a vector in $R(n,S)$. Then $\mathbf{p} \in C(n,S)$ if there is a Kolmogorovian probability space $(X, \mathcal{M}, \mu)$ and (not necessarily distinct) events $A_1, A_2, ..., A_n \in \mathcal{M}$ such that:*

$$p_i = \mu(A_i) \quad 1 \leq i \leq n \qquad p_{ij} = \mu(A_i \cap A_j) \quad \{i,j\} \in S \tag{13}$$

Where X is ..., $\mathcal{M}$ is the space of events and μ the probability measure

To illustrate the theorem and at the same time the connection with Bell inequalities and the Clauser Horne inequalities, let us consider some specific examples of Pitowsky's theorem.

The case $n = 4$ and $S = \{\{1,3\},\{1,4\},\{2,3\},\{2,4\}\}$. The condition $\mathbf{p} \in C(n,S)$ is then equivalent to the Clauser-Horne inequalities (see (3)):

$$\begin{array}{ll}
0 \leq p_{ij} \leq p_i \leq 1 & \\
0 \leq p_{ij} \leq p_j \leq 1 & i = 1,2 \quad j = 3,4 \\
p_i + p_j - p_{ij} \leq 1 & \\
-1 \leq p_{13} + p_{14} + p_{24} - p_{23} - p_1 - p_4 \leq 0 & \\
-1 \leq p_{23} + p_{24} + p_{14} - p_{13} - p_2 - p_4 \leq 0 & \\
-1 \leq p_{14} + p_{13} + p_{23} - p_{24} - p_1 - p_3 \leq 0 & \\
-1 \leq p_{24} + p_{23} + p_{13} - p_{14} - p_2 - p_3 \leq 0 &
\end{array} \tag{14}$$

The case $n = 3$ and $S = \{\{1,2\},\{1,3\},\{2,3\}\}$. We find then the following inequalities equivalent to the condition $\mathbf{p} \in C(n,S)$:

$$\begin{array}{cc} 0 \leq p_{ij} \leq p_i \leq 1 & \\ 0 \leq p_{ij} \leq p_j \leq 1 & 1 \leq i < j \leq 3 \\ p_i + p_j - p_{ij} \leq 1 & \\ p_1 + p_2 + p_3 - p_{12} - p_{13} - p_{23} \leq 1 \leq 0 & \\ p_1 - p_{12} - p_{13} + p_{23} \leq 0 & \\ p_2 - p_{12} - p_{23} + p_{13} \leq 0 & \\ p_3 - p_{13} - p_{23} + p_{12} \leq 0 & \end{array} \tag{15}$$

It can be shown that these inequalities are equivalent to the original Bell inequalities (Pitowsky 1989).

3.2 The Genuine Quantum Mechanical Nature of the Macroscopic Violations

Let us now introduce a new situation wherein the events are systematically distinguished. For the vessels of water example, we introduce the following events: Event E_1 corresponds to the physical process of experiment e_1, leading to outcome $o_1(up)$, performed together with experiment e_3 leading to outcome $o_3(up)$. Event E_2 corresponds to the physical process of experiment e_1 leading to outcome $o_1(up)$, performed together with experiment e_4, leading to outcome $o_4(up)$. In order to introduce the other events and avoid repetition, we abbreviate event E_2 as follows:

$$E_2 = [O(e_1) = o_1(up) \;\&\; O(e_4) = o_4(up)]$$

The other events can then be written analogously as:

$$\begin{array}{ll} E_3 = [O(e_2) = o_2(up) \;\&\; O(e_3) = o_3(up)] & E_4 = [O(e_2) = o_2(up) \;\&\; O(e_4) = o_4(up)] \\ E_5 = [O(e_3) = o_3(up) \;\&\; O(e_1) = o_1(up)] & E_6 = [O(e_3) = o_3(up) \;\&\; O(e_2) = o_2(up)] \\ E_7 = [O(e_4) = o_4(up) \;\&\; O(e_1) = o_1(up)] & E_8 = [O(e_4) = o_4(up) \;\&\; O(e_2) = o_2(up)] \end{array} \tag{16}$$

The physical process of the joint experiment e_{13} corresponds then to the joint event $E_1 \wedge E_5$, the physical process of the joint experiment e_{14} to the joint event $E_2 \wedge E_7$, the physical process of the joint experiment e_{23} to the joint event $E_3 \wedge E_6$, and the physical process of the joint experiment e_{24} to the joint event $E_4 \wedge E_8$.

Having distinguished the events in this way, we are certain the different joint experiments give rise to real joint events. We can now apply Pitowsky's theorem to the set of events $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_1 \wedge E_5, E_2 \wedge E_7, E_3 \wedge E_6, E_4 \wedge E_8$.

Suppose that there is an equal probability of experiment e_1 being performed with e_3 or e_4, and similarly for the joint performance of e_2 with e_3 or e_4. According to this assumption, the observed probabilities are:

$$\begin{array}{c} p(E_1) = p(E_5) = 0 \\ p(E_2) = p(E_3) = p(E_4) = p(E_6) = p(E_7) = p(E_8) = \frac{1}{2} \\ p(E_1 \wedge E_5) = 0 \\ p(E_2 \wedge E_7) = p(E_3 \wedge E_6) = p(E_4 \wedge E_8) = \frac{1}{4} \end{array} \tag{17}$$

The obtained probability vector is then $\mathbf{p} = (0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. Applying Pitowsky's approach, we could directly calculate that this probability vector is contained in the convex hull of the corresponding space, and hence as a consequence of Pitowsky's theorem it allows a Kolmogorovian probability representation (Aerts and Szabo 1993). This means that after the distinction between events has been made, the vessels of water example no longer violates Pitowsky inequalities. An important question remains: would the violation of the inequalities similarly vanish for the microscopic spin example? Let us, exactly as we have done in the vessels of water example, distinguish the events we are not certain we can identify, for the case of the correlated spin situation. Again we find 8 events: Event E_1 corresponds to the physical process of experiment e_1 leading to outcome $o_1(up)$, performed together with experiment e_3, leading to outcome $o_3(up)$. Analogously, events E_2, E_3, E_3, E_4, E_5, E_6, E_7, E_8 are introduced. Again, the physical process of the joint experiment e_{13} corresponds then to the joint event $E_1 \wedge E_5$, the physical process of the joint experiment e_{14} to the joint event $E_2 \wedge E_7$, the physical process of the joint experiment e_{23} to the joint event $E_3 \wedge E_6$, and the physical process of the joint experiment e_{24} to the joint event $E_4 \wedge E_8$.

Suppose that directions $\mathbf{a_1}$ or $\mathbf{a_2}$, as well as $\mathbf{a_3}$ or $\mathbf{a_4}$, are chosen with the same probability at both sides. According to this assumption the observed probabilities are:

$$\begin{array}{c} p_i = p(E_i) = \frac{1}{4} \quad 1 \leq i \leq 8 \\ p_{15} = p(E_1 \wedge E_5) = \frac{1}{4}\mathbf{sin^2}\angle(\mathbf{a_1}, \mathbf{a_3}) = \frac{3}{16} \\ p_{27} = p(E_2 \wedge E_7) = \frac{1}{4}\mathbf{sin^2}\angle(\mathbf{a_1}, \mathbf{a_4}) = \frac{3}{16} \\ p36 = p(E_3 \wedge E_6) = \frac{1}{4}\mathbf{sin^2}\angle(\mathbf{a_2}, \mathbf{a_3}) = 0 \\ p_{48} = p(E_4 \wedge E_8) = \frac{1}{4}\mathbf{sin^2}\angle(\mathbf{a_2}, \mathbf{a_4}) = \frac{3}{16} \end{array} \tag{18}$$

The question is whether the correlation vector $\mathbf{p} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{3}{16}, \frac{3}{16}, 0, \frac{3}{16})$ admits a Kolmogorovian representation. To answer this question, we have to check whether it is inside the corresponding classical correlation polytope $C(8, S)$. There are no derived inequalities for $n = 8$, expressing the condition $\mathbf{p} \in C(n, S)$. Lacking such inequalities we must directly check the geometric condition $\mathbf{p} \in C(n, S)$. We were able to do this for the vessels of water example because of the simplicity of the correlation vector, but we had no general way to do this. It is however possible to prove the existence of a Kolmogorovian representation in a general way:

Theorem 2 *Let events $E_1, E_2, ..., E_n$ and a set of indices S be given such that non of the indices appears in two different elements of S. Assume that for each pair $\{i, j\} \in S$ the restricted correlation vector $p_{\{i,j\}} = (p(E_i), p_(E_j), p(E_i \wedge E_j))$ has an $(X_{\{i,j\}}, \mu_{\{i,j\}})$ Kolmogorovian representation. Then the product space $(X_{\{i_1,j_1\}} \times X_{\{i_2,j_2\}} \times ... \times X_{\{i_{|S|},j_{|S|}\}}, \mu_{\{i_1,j_1\}} \times \mu_{\{i_2,j_2\}} \times ... \times \mu_{\{i_{|S|},j_{|S|}\}})$ provides a Kolmogorovian representation for the whole correlation vector* $\mathbf{p}$.

This theorem shows that if the distinctions that we have explained are made, the inequalities corresponding to the situation will no longer be violated. This also means that we can state that the macroscopic violation, certainly with respect to the distinction or identification of events, is as genuine as the microscopic violation of the inequalities.

4 The Violation of Bell Inequalities in Cognition

In this section we show how Bell inequalities are violated in the mind in virtue of the relationship between abstract concepts and specific instances of them. We start with a thought experiment that outlines a possible scenario wherein this sort of violation of Bell inequalities reveals itself. This example was first presented in Aerts and Gabora 1999. We then briefly discuss implications for cognition.

4.1 How Concepts Violate Bell inequalities

To make things more concrete we begin with an example. Keynote players in this example are the two cats, Glimmer and Inkling, that live at our research center (Fig. 5). The experimental situation has been set up by one of the authors (Diederik) to show that the mind of another of the authors (Liane) violates Bell inequalities. The situation is as follows. On the table where Liane prepares the food for the cats is a little note that says: ‘Think of one of the cats now’.

Fig 5: Inkling (left) and Glimmer (right). This picture was taken before Glimmer decided that the quantum cat superstar life was not for him and started to remove his bell.

To show that Bell inequalities are violated we must introduce four experiments e_1, e_2, e_3 and e_4. Experiment e_1 consists of Glimmer showing up at the instant Liane reads the note. If, as a result of the appearance of Glimmer and Liane reading the note, the state of her mind is changed from the more general concept ‘cat’ to the instance ‘Glimmer’, we call the outcome $o_1(up)$, and if it is changed to the instance ‘Inkling’, we call the outcome $o_1(down)$. Experiment e_3 consists of Inkling showing up at the instant that Liane reads the note. We call the outcome $o_3(up)$ if the state of her mind is changed to the instance ‘Inkling’, and $o_3(down)$ if it is changed to the instance ‘Glimmer’, as a result of the appearance of Inkling and Liane reading the note. The coincidence experiment e_{13} consists of Glimmer and Inkling both showing up when Liane reads the note. The outcome is $(o_1(up), o_3(down))$ if the state of her mind is changed to the instance ‘Glimmer’, and $(o_1(down), o_3(up))$ if it changes to the instance ‘Inkling’ as a consequence of their appearance and the reading of the note.

Now it is necessary to know that occasionally the secretary puts bells on the cats’ necks, and occasionally she takes the bells off. Thus, when Liane comes to work, she does not know whether or not the cats will be wearing bells, and she is always curious to know. Whenever she sees one of the cats, she eagerly both looks and listens for the bell. Experiment e_2 consists of Liane seeing Inkling and noticing that she hears a bell ring or doesn’t. We give the outcome $o_2(up)$ to the experiment e_2 when Liane hears the bell, and $o_2(down)$ when she does not. Experiment e_4 is identical to experiment e_2 except that Inkling is interchanged with Glimmer. The coincidence experiment e_{14} consists of Liane reading the note, and Glimmer showing up, and her listening to whether a bell is ringing or not. It has four possible outcomes: $(o_1(up), o_4(up))$ when the state of Liane’s mind is changed to the instance ‘Glimmer’ and she hears a bell; $(o_1(up), o_4(down))$ when the state of her mind is changed to the instance ‘Glimmer’ and she does not hear a bell; $(o_1(down), o_4(up))$ when the state of her mind is changed to the instance ‘Inkling’ and she hears a bell and $(o_1(down), o_4(down))$ when the state of her mind is changed to the instance ‘Inkling’ and she does not hear a bell. The coincidence experiment e_{23} is defined analogously. It consists of Liane reading the note and Inkling showing up and her listening to whether a bell is ringing or not. It too has four possible outcomes: $(o_2(up), o_3(up))$ when she hears a bell and the state of her mind is changed to the instance ‘Inkling’; $(o_2(up), o_3(down))$ when she hears a bell and the state of her mind is changed to the instance ‘Glimmer’; $(o_1(down), o_3(up))$ when she does not hear a bell and the state of her mind is changed to the instance ‘Inkling’ and $(o_1(down), o_3(down))$ when she does not hear a bell and the state of her mind is changed to the instance ‘Glimmer’. The coincidence experiment e_{24} is the experiment where Glimmer and Inkling show up and Liane listens to see whether she hears the ringing of bells. It has outcome $(o_2(up), o_4(up))$ when both cats wear bells, $(o_2(up), o_4(down))$ when only Inkling wears a bell, $(o_2(down), o_4(up))$ when only Glimmer wears a bell and $(o_2(down), o_4(down))$ when neither cat wears a bell.

We now formulate the necessary conditions such that Bell inequalities are violated in this experiment:

(1) The categorical concept ‘cat’ is activated in Liane’s mind.

(2) She does what is written on the note.

(3) When she sees Glimmer, there is a change of state, and the categorical concept ‘cat’ changes to the instance ’Glimmer’, and when she sees Inkling it changes to the instance ’Inkling’.

(4) Both cats are wearing bells around their necks.

The coincidence experiment e_{13} gives outcome $(o_1(up), o_3(down))$ or $(o_1(down), o_3(up))$ because indeed from (2) it follows that Liane will think of Glimmer or Inkling. This means that $\mathbb{E}_{13} = -1$. The coincidence experiment e_{14} gives outcome $(o_1(up), o_4(up))$, because from (3) and (4) it follows that she thinks of Glimmer and hears the bell. Hence $\mathbb{E}_{14} = +1$. The coincidence experiment e_{23} also gives outcome $(o_2(up), o_3(up))$, because from (3) and (4) it follows that she thinks of Inkling and hears the bell. Hence $\mathbb{E}_{23} = +1$. The coincidence experiment e_{24} gives $(o_2(up), o_4(up))$, because from (4) it follows that she hears two bells. Hence $\mathbb{E}_{24} = +1$. As a consequence we have:

$$|\mathbb{E}_{13} - \mathbb{E}_{14}| + |\mathbb{E}_{23} + \mathbb{E}_{24}| = +4 \tag{19}$$

The reason that Bell inequalities are violated is that Liane’s state of mind changes from activation of the abstract categorical concept ‘cat’, to activation of either ‘Glimmer’ or ‘Inkling’. We can thus view the state ‘cat’ as an entangled state of these two instances of it.

We end this section by saying that we apologize for the pun on Bell’s name, but it seemed like a good way to ring in these new ideas.

4.2 The Nonlocality of Concepts

Our example shows that concepts in the mind violate Bell inequalities, and hence entail nonlocality in the sense that physicists use the concept. This violation of Bell inequalities takes place within the associative network of concepts and episodic memories constituting an internal model of reality, or worldview. We now briefly investigate how this cognitive source of nonlocality arises, and its implications for cognition and our understanding of reality.

As a first approximation, we can say that the nonlocality of stored experiences and concepts arises from their distributed nature. Each concept is stored in many memory locations; likewise, each location participates in the storage of many concepts. In order for the mind to be capable of generating a stream of meaningfully-related yet potentially creative remindings, the degree of this distribution must fall within an intermediate range. Thus, a given experience activates not just one location in memory, nor does it activate every memory location to an equal degree, but activation is distributed across many memory locations, with degree of activation falling with distance from the most activated one. Fig. 6 shows schematically how this feature of memory is sometimes modeled in neural networks using a radial basis function (RBF) (Hancock et al., 1991; Holden and Niranjan, 1997; Lu et al. 1997; Willshaw and Dayan, 1990).

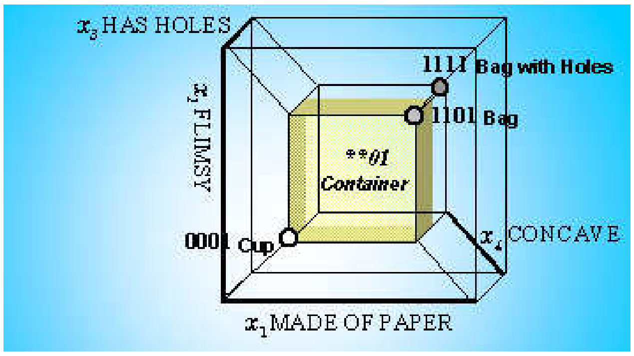

Fig 6: Highly schematized diagram of a stimulus input activating two dimensions of a memory or conceptual space. Each vertex represents a possible memory location, and black dots represent actual location in memory. Activation is maximal at the center k of the RBF, and tapers off in all directions according to a Gaussian distribution of width s.

Memory is also content addressable, meaning that there is a systematic relationship between the content of an experience, and the place in memory where it gets stored (and from which material for the next instant of experience is sometimes evoked). Thus not only is it is not localized as an episodic memory or conceptual entity in conceptual space, but it is also not localized with respect to its physical storage location in the brain.

4.3 The Relationship between Nonlocality and Degree of Abstraction

The more abstract a concept, the greater the number of other concepts that are expected to fall within a given distance of it in conceptual space, and therefore be potentially evoked by it. For example, Fig. 7 shows how the concept of 'container' is less localized than the concept of 'bag'. The concept of 'container' does not just activate concepts like 'cup', it derives its very existence from them. Similarly, once 'cup' has been identified as an instance of 'container', it is forever after affected by it. To activate 'bag' is to instantaneously affect 'container', which is to instantaneously affects the concept 'thing', from which 'container' derives it's identity, and so forth.

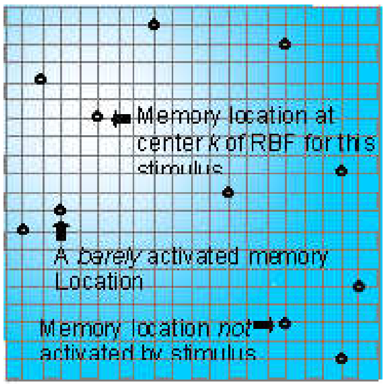

Fig 7: A four-dimensional hypercube that schematically represents a segment of memory space. The stimulus dimensions 'MADE OF PAPER', 'FLIMSY, 'HAS HOLES' and 'CONCAVE' lie on the x_1, x_2, x_3, and x_4 axes respectively. Three concepts are stored here: 'cup', 'bag', and 'bag with holes'. Black-ringed dots represent centers of distributed regions where they are stored. Fuzzy white ball shows region activated by 'cup'. Emergence of the abstract concept 'container' implicit in the memory space (central yellow region) is made possible by the constrained distribution of activation.

An extremely general concept such as ‘depth’ is probably even more nonlocalized. It is latent in mental representations as dissimilar as ‘deep swimming pool’, ‘deep-fried zucchini’, and ‘deeply moving book’; it is deeply woven throughout the matrix of concepts that constitute one’s worldview. In (Gabora 1998, 1999, 2000) one author, inspired by Kauffman’s (1993) autocatalytic scenario for the origin of life, outlines a scenario for how episodic memories could become collectively entangled through the emergence of concepts to form a hierarchically structured worldview. The basic idea goes as follows. Much as catalysis increases the number of different polymers, which in turn increases the frequency of catalysis, reminding events increase concept density by triggering abstraction-the formation of abstract concepts or categories such as ‘tree’ or ‘big’-which in turn increases the frequency of remindings. And just as catalytic polymers reach a critical density where some subset of them undergoes a phase transition to a state where there is a catalytic pathway to each polymer present, concepts reach a critical density where some subset of them undergoes a phase transition to a state where each one is retrievable through a pathway of remindings events or associations. Finally, much as autocatalytic closure transforms a set of molecules into an interconnected and unified living system, conceptual closure transforms a set of memories into an interconnected and unified worldview. Episodic memories are now related to one another through a hierarchical network of increasingly abstract – and what for our purposes is more relevant – increasingly nonlocalized, concepts.

4.4 Quantum Structure and the Mind

Over the past several decades, numerous attempts have been made to forge a connection between quantum mechanics and the mind. In these approaches, it is generally assumed that the only way the two could be connected is through micro-level quantum events in the brain exerting macro-level effects on the judgements, decisions, interpretations of stimuli, and other cognitive functions of the conscious mind. From the preceding arguments, it should now be clear that this is not the only possibility. If quantum structure can exist at the macro-level, then the process by which the mind arrives at judgements, decisions, and stimulus interpretations could itself be quantum in nature.

We should point out that we are not suggesting that the mind is entirely quantum. Clearly not all concepts and instances in the mind are entangled or violate Bell inequalities. Our claim is simply that the mind contains some degree of quantum structure. In fact, it has been suggested that quantum and classical be viewed as the extreme ends of a continuum, and that most of reality may turn out to lie midway in this continuum, and consist of both quantum and classical aspects in varying proportions (Aerts 1992; Aerts and Durt 1994).

5 The Presence of Quantum Probability and Bell Inequalities

We have seen that quantum and macroscopic systems can violate Bell inequalities. A natural question that arises is the following: is it possible to construct a macroscopical system that violates Bell inequalities in exactly the same way as a photon singlet state will? Aerts constructed a very simple model that does exactly this (Aerts 1991). This model represents the photon singlet state before measurement by means of two points that live in the center of two separate unit spheres, each one following its own space-time trajectory (in accordance with the conservation of linear and angular momentum), but the two points in the center remain connected by means of a rigid but extendable rod (Fig. 8). Next the two spheres reach the measurement apparatuses. When one side is measured, the measurement apparatus draws one of the entities to one of the two possible outcomes with probability one half. However, because the rod is between the two entities, the other entity at the center of the other sphere is drawn toward the opposite side of the sphere as compared with the first entity. Only then this second entity is measured. This is done by attaching a piece of elastic between the two opposite points of the sphere that are parallel with the measurement direction chosen by the experimenter for this side. The entity falls onto the elastic following the shortest path (i.e.. orthogonal) and sticks there. Next the elastic breaks somewhere and drags the entity towards one of the end points (Fig. 9). To calculate the probability of the occurrence of one of the two possible outcomes of the second measurement apparatus, we assume there is a uniform probability of breaking on the elastic. Next we calculate the frequency of the coincidence counts and these turn out to be in exact accordance with the quantum mechanical prediction.

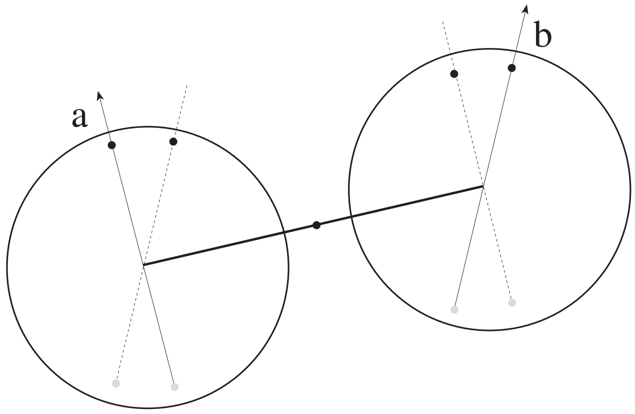

Fig 8: Symbolic representation of the singlet state in the model as two dots in the centers of their respective spheres. Also shown is the connecting rod and the measurement directions chosen at each location a and b.

There are two ingredients of this model that seem particularly important. First we have the rigid rod, which shows the non-separable wholeness of the singlet coincidence measurement (i.e. the role of the connecting tube between the vessels of water, or the associative pathways between concepts). Second, we have the elastic that breaks which gives rise to the probabilistic nature (the role of the siphon in the vessels of water, or the role of stimulus input in the mind) of the outcomes. These two features seem more or less in accordance with the various opinions researchers have about the meaning of the violation of Bell inequalities. Indeed, some have claimed that the violation of Bell inequalities is due to the non-local character of the theory, and hence in our model to the 'rigid but extendable rod', while others have attributed the violation not to any form of non-locality, but rather to the theory being not 'realistic' or to the intrinsic indeterministic character of quantum theory, and hence to the role of the elastic in our model. As a consequence, researchers working in this field now carefully refer to the meaning of the violation as the "non-existence of local realism". Because of this dichotomy in interpretation we were curious what our model had to say on this issue. To explore this we extended the above model with the addition of two parameterizations, each parameter allowing us to minimize or maximize one of the two aforementioned features (Aerts D., Aerts S., Coecke B., Valckenborgh F., 1995). The question was of course, how Bell inequalities would respond to the respective parameterizations. We will briefly introduce the model and the results.

5.1 The Model

The way the parameterized model works is exactly analogous to the measurement procedure described above, with two alterations. First, we impose the restriction that the maximum distance the rigid rod can 'pull' the second photon out of the center is equal to some parameter called $\rho \in [0,1]$. Hence setting ρ equal to 1 gives us the old situation, while putting ρ equal to zero means we no longer have a correlation between the two measurements. Second, we allow the piece of elastic only to break inside a symmetrical interval $[-\epsilon, \epsilon]$. Setting ϵ equal to 1 means we restore the model to the state it was in before parameterization. Setting ϵ equal to zero means we have a classical 'deterministic' situation, since the elastic can only break in the center (there remains the indeterminism of the classical unstable equilibrium, because indeed the rod can still move in two ways, up or down). In fact, to be a bit more precise, we have as a set of states of the entity the set of couples

$$q \in Q = \{(s_1, s_2) | s_1, s_2, c \in R^3, ||s_1 - c|| \leq \rho, ||s_2 + c|| \leq \rho, \rho \in [0,1]\} \tag{20}$$

Each element of the couple belongs to a different sphere with center c (resp $-c$ due to linear momentum conservation) and whose radius is parameterized by the correlation parameter ρ. At each side we have a set of measurements

$$e_1, e_2 \in M = \{\gamma \mathbf{n} | \mathbf{n} \in R^3, ||\mathbf{n}|| = 1, \gamma \in [-\epsilon, +\epsilon], \epsilon \in [0,1]\} \tag{21}$$

The direction $\mathbf{n}$ can be chosen arbitrarily by the experimenter at each side and denotes the direction of the polarizer. (Of course, for the sake of demonstrating the violation of Bell inequalities, the experimenter will choose at random between the specific angles that maximize the value the inequality takes). The value that the parameter γ takes represents the point of rupture of the elastic and is unknown in so far that it can take any value inside the interval $[-\epsilon, +\epsilon]$, and it will do so with a probability that is uniform over the entire interval $[-\epsilon, +\epsilon]$. Hence the probability density function related to γ is a constant: $pdf(\gamma) = 1/2\epsilon$. This represents our lack of knowledge concerning the specific measurement that occurred, and is the sole source of indeterminism. As we will show later, if we take γ to vary within $[-1, +1]$ we have a maximal lack of knowledge about the measurement and we recover the exact quantum predictions. As before, the state q of the entity before measurement is given by the centers of the two spheres. The first measurement e_1 projects one center, say s_1 orthogonally onto the elastic that is placed in the direction **a** chosen by the experimenter at location A and that can only break in the interval $[-\epsilon, +\epsilon]$. Because it will always fall in the middle of the elastic, and because we assume a uniform probability of breaking within the breakable part of the elastic, the chance that the elastic pulls s_1 up is equal to 1/2 (as is the probability of it going down). Which side s_1 is to go depends on the specific point of rupture. Be that as it may, the experimenter records the outcome on his side of the experiment as 'up' or 'down'. At the other side of the experiment, call it location B, s_2 is pulled towards the opposite side of s_1 because of the rigid rod connecting the s_1 and s_2. The maximum distance it can be pulled away from the center is equal to ρ, and hence the old s_2 transforms to $s_2 = -\rho\ \mathbf{a}$. Next, experiment e_2 is performed in exactly the same way as e_1. At location B, the experimenter chooses a direction, say **b**, and attaches the elastic in this direction. Again s_2 is projected orthogonally onto the elastic and the elastic breaks. The main difference between the first and the second measurement is that s_2 is no longer at the center, but rather at $-\rho\ \mathbf{a}.\mathbf{b}$.

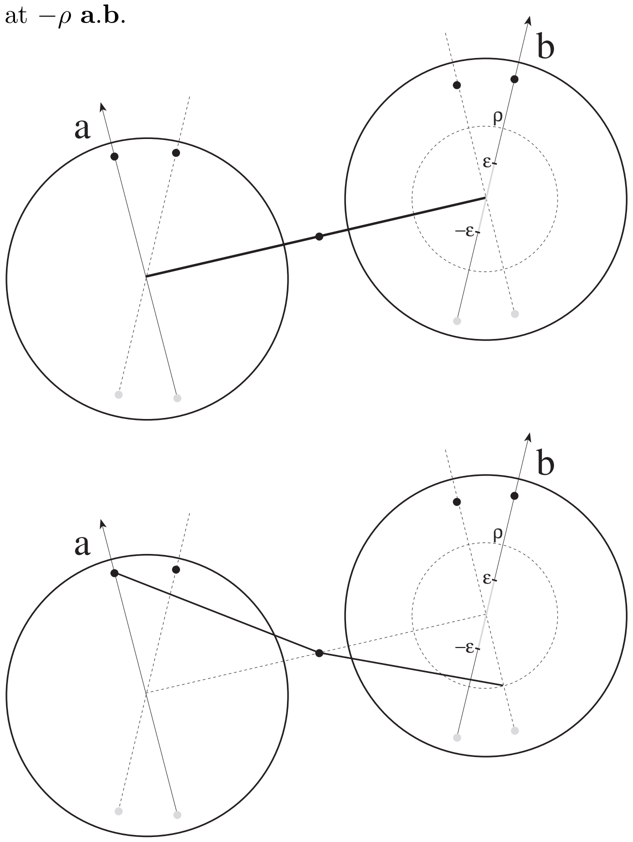

Fig 9: The situation immediately before and after the measurement at one side (left in this case) has taken place. In the first picture the breakable part of the elastic is shown (i.e. the interval $[-\epsilon, +\epsilon]$ on the elastic) and the maximum radius ρ. In the second picture, we see how the measurement at location a has altered the state at location b because of the connecting rod.

5.2 Calculating the Probabilities and Coincidence Counts

There are three qualitatively different parts of the elastic where s_2 can end up after projection: the unbreakable part between $[-\rho, -\epsilon]$, the breakable part between $[-\epsilon, +\epsilon]$ and the unbreakable part between $[+\epsilon, +\rho]$. If s_2 is in $[-\rho, -\epsilon]$, then it does not matter where the elastic breaks: s_2 will always be dragged 'up' and likewise, if s_2 is in $[+\epsilon, +\rho]$, the outcome will always be 'down'. If, however, s_2 is in $[-\epsilon, +\epsilon]$, then the probability of s_2 being dragged up is equal to the probability that the elastic breaks somewhere in $[-\rho\ \mathbf{a}.\mathbf{b}, -\epsilon]$. This is the Lebesgue measure of the interval divided by the total Lebesgue measure of the elastic:

$$P(e_2 = up | s_2 = -\rho\ \mathbf{a}.\mathbf{b}) = \frac{\epsilon - \rho\ \mathbf{a}.\mathbf{b}}{2\epsilon} \tag{22}$$

This settles the probabilities related to s_2. As before, we define the coincidence experiment e_{ij} as having four possible outcomes, namely $(o_i(up), o_j(up))$, $(o_i(up), o_j(down))$, $(o_i(down), o_j(up))$ and $(o_i(down), o_j(down))$ (See section 2). Following Bell, we introduce the expectation values $\mathbb{E}_{ij}, i, j \in \{1, 2, 3, 4\}$ for these coincidence experiments, as

$$
\begin{aligned}
\mathbb{E}_{ij} = \quad & +1.P(o_i(up), o_j(up)) + 1.P(o_i(down), o_j(down)) \\
& -1.P(o_i(up), o_j(down)) - 1.P(o_i(down), o_j(up))
\end{aligned} \tag{23}
$$

One easily sees that the expectation value related to the coincidence counts also splits up in three parts.

- $-\epsilon < \rho\ \mathbf{a}.\mathbf{b} < +\epsilon$:
  In this case we have $P(o_i(up), o_j(up)) = P(o_i(down), o_j(down)) = \frac{\epsilon - \rho\ \mathbf{a}.\mathbf{b}}{4\epsilon}$ and $P(o_i(up), o_j(down)) = P(o_i(down), o_j(up)) = \frac{\epsilon + \rho\ \mathbf{a}.\mathbf{b}}{4\epsilon}$. Hence the expectation value for the coincidence counts becomes

$$
\mathbb{E}(\mathbf{a}, \mathbf{b}) = -\frac{\rho\ \mathbf{a}.\mathbf{b}}{\epsilon} \tag{24}
$$

  We see that putting $\rho = \epsilon = 1$ we get $\mathbb{E}(\mathbf{a}, \mathbf{b}) = -\mathbf{a}.\mathbf{b}$, which is precisely the quantum prediction.

- $\rho\ \mathbf{a}.\mathbf{b} \geq +\epsilon$:
  In this case we have $P(o_i(up), o_j(up)) = P(o_i(down), o_j(down)) = 0$ and $P(o_i(up), o_j(down)) = P(o_i(down), o_j(up)) = 1/2$. Hence the expectation value for the coincidence counts becomes

$$
\mathbb{E}(\mathbf{a}, \mathbf{b}) = -1 \tag{25}
$$

- $\rho\ \mathbf{a}.\mathbf{b} \leq -\epsilon$: In this case we have $P(o_i(up), o_j(up)) = P(o_i(down), o_j(down)) = 1/2$ and $P(o_i(up), o_j(down)) = P(o_i(down), o_j(up)) = 0$. Hence the expectation value for the coincidence counts becomes

$$
\mathbb{E}(\mathbf{a}, \mathbf{b}) = +1 \tag{26}
$$

5.3 The Violation of Bell Inequalities

Let us now see what value the left hand side of the Bell inequality takes for our model for the specific angles that maximize the inequality. For these angles $\mathbf{a}.\mathbf{b} = \pi/4$ and $\mathbf{a}.\mathbf{b} = 3\pi/4$, the condition $-\epsilon < \rho\ \mathbf{a}.\mathbf{b} < \epsilon$ is satisfied only if $\frac{\sqrt{2}}{2} < \frac{\epsilon}{\rho}$. In this case, we obtain :

$$
\mathbb{E}(\mathbf{a}, \mathbf{b}) = -\mathbb{E}(\mathbf{a}, \mathbf{b}') = \mathbb{E}(\mathbf{a}', \mathbf{b}) = \mathbb{E}(\mathbf{a}', \mathbf{b}') = -\frac{\rho}{\epsilon}\ \frac{\sqrt{2}}{2} \tag{27}
$$

If, on the other hand we would have chosen ϵ and ρ such that $\frac{\sqrt{2}}{2} \geq \frac{\epsilon}{\rho}$ we find:

$$
\mathbb{E}(\mathbf{a}, \mathbf{b}) = -\mathbb{E}(\mathbf{a}, \mathbf{b}') = \mathbb{E}(\mathbf{a}', \mathbf{b}) = \mathbb{E}(\mathbf{a}', \mathbf{b}') = -1 \tag{28}
$$

We can summarize the results of all foregoing calculations in the following equation:

$$
|\mathbb{E}(\mathbf{a}, \mathbf{b}) - \mathbb{E}(\mathbf{a}, \mathbf{b}')| + |\mathbb{E}(\mathbf{a}', \mathbf{b}) + \mathbb{E}(\mathbf{a}', \mathbf{b}')| = \left\{ \begin{array}{cc} \frac{2\sqrt{2}\rho}{\epsilon} & \frac{\epsilon}{\rho} > \frac{\sqrt{2}}{2} \\ 4 & \frac{\epsilon}{\rho} \leq \frac{\sqrt{2}}{2} \end{array} \right\} \tag{29}
$$

For $\epsilon = 0$, we have two limiting cases that can easily be derived: for $\rho = 0$ the left side of the inequality takes the value 0, while for $\rho \neq 0$ it becomes 4.

For what couples ρ, ϵ do we violate the inequality? Clearly, we need only consider the case $\frac{\epsilon}{\rho} > \frac{\sqrt{2}}{2}$. Demanding that the inequality be satisfied, we can summarize our findings in the following simple condition:

$$
\epsilon \leq \sqrt{2}\ \rho \tag{30}
$$

This result indicates that the model leaves no room for interpretation as to the source of the violation: for any $\rho < \frac{1}{\sqrt{2}}$, we can restore the inequality by increasing the amount of lack of knowledge on the interaction between the measured and the measuring device, that is by increasing ϵ. The only way to respect Bell inequalities for all values of ϵ is by putting $\rho = 0$. Likewise, for any $\rho > \frac{1}{\sqrt{2}}$ it becomes impossible to restore the validity of the Bell inequalities. The inevitable conclusion is that the correlation is the source of the violation. The violation itself should come as no surprise, because we have identified ρ as the *correlation* between the two measurements, which is precisely the non-local aspect. This is also obvious from the fact that it is this correlation that makes $\mathbb{E}(\mathbf{a}, \mathbf{b})$ not representable as an integral of the form $\int A(a, \lambda)B(b, \lambda)\, d\lambda$ as Bell requires for the derivation of the inequality. What may appear surprising however, is the fact that increasing the indeterminism (increasing ϵ), decreases the value the inequality takes! For example, if we take $\epsilon = 0$ and $\rho = 1$, we see that the value of the inequality is 4, which is the largest value the inequality possibly can have, just as in the case of the vessels of water model.

6 Conclusion

We have presented several arguments to show that Bell inequalities can be genuinely violated in situations that do not pertain to the microworld. Of course, this does not decrease the peculiarity of the quantum mechanical violation in the EPRB experiment. What it does, is shed light on the possible underlying mechanisms and provide evidence that the phenomenon is much more general than has been assumed.

The examples that we have worked out – the 'vessels of water', the 'concepts in the mind', and the 'spheres connected by a rigid rod' – each shed new light on the origin of the violation of Bell inequalities. The vessels of water and the spheres connected by a rigid rod examples, show that 'non-local connectedness' plays an essential role in bringing the violation about. The spheres connected by a rigid rod example shows that the presence of quantum uncertainty does not contribute to the violation of the inequality; on the contrary, increasing quantum uncertainty decreases the violation.

All three examples also reveal another aspect of reality that plays an important role in the violation of Bell inequalities: the potential for different actualizations that generate the violation. The state of the 20 liters of water as present in the connected vessels is potentially, but not actually, equal to '5 liters' plus '15 liters' of water, or '11 liters' plus '9 liters' of water, *etc.* Similarly, we can say that the concept 'cat' is potentially equal to instances such as our cats, 'Glimmer' and 'Inkling'. It is this potentiality that is the 'quantum aspect' in our nonmicroscopic examples, and that allows for a violation of Bell-type inequalities. Indeed, as we know, this potentiality is the fundamental characteristic of the superposition state as it appears in quantum mechanics. This means that the aspect of quantum mechanics that generates the violation of Bell inequalities, as identified in our examples, is the potential of the considered state. In the connected vessels example, it is the potential ways of dividing up 20 liters of water. In the concepts in the mind example, it is the potential instances evoked by the abstract concept 'cat'. In the rigid rod example, it is the possible ways in which the rod can move around its center.

7 References

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