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ORGANISMIC SUPERCATEGORIES: I. PROPOSALS FOR A GENERAL UNITARY THEORY OF SYSTEMS

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The present paper is an attempt to outline an abstract unitary theory of systems. In the introduction some of the previous abstract representations of systems are discussed. Also a possible connection of abstract representations of systems with a general theory of measure is proposed. Then follow some necessary definitions and authors' proposals for an axiomatic theory of systems. Finally some concrete examples are analyzed in the light of the proposed theory.

Introduction. This paper is an attempt to outline a general unitary theory of systems based on the previous abstract representations as they were developed by Rashevsky (1959, 1961a, 1961b, 1964, 1967a, b, c), Rosen (1958a, b, 1961, 1962, 1963, 1964, 1965, 1966, 1967), and Comorosan (1967).

Familiarity of the reader with these papers is essential for the understanding of the following discussion. Since this paper represents the first step in a new direction, therefore, the authors are well aware of a number of deficiencies in its clarity and possibly in its rigor. The paper is expected to be followed by a series of others where necessary clarification will take place. We shall recall some important results on these lines. In some previous papers (Rashevsky 1959, 1961a, b; Sommerfield, 1963; Mullin, 1962), was shown the importance of relations between sets as a first necessary step in relational biology. Later on these relations proved to represent only a part of biological relations, namely direct relations. So were introduced "Indirect relations" (Rashevsky, 1967b), which are biologically significant. Also it was shown that the distinction between "metric" aspects of systems and "relational" aspects of systems disappear by accepting the mathematical definition of relation, which was called "G-relation" (Rashevsky, 1967c). Another important step was the representation of biological systems from the standpoint of the theory of categories (Rosen 1958b, 1959a). Rosen's categorical representation refers to the "coarse structure" of systems and in some special cases to the "fine structure". For some particular systems an attempt was made to approach the "fine structure" of some particular systems (Comoroşan, 1967). In fact some fine structures are always correlated with a coarse structure. The coarse structure is defined from a functional point of view (the word "functional" has here its biological meaning); fine structures are defined concerning "structures" (in its biological meaning). We can define a coarse structure on the biochemical level or on other levels and this is also true for fine structures.

We get information on fine structures by measurement. Through our experiments we see which characteristics of the studied object allow us to describe the object in an unambiguous way and to predict the behavior of this object. There are a minimum number of characteristics which allow us to do it. We make predictions on the behavior of the object with a certain approximation and probability.

The minimum number of characteristics will define at a given moment the "state" of the object at that moment. We call these characteristics "observables". So a state of an object at a given moment is defined by a number of observables. In a more advanced study we associate mathematical elements to observables in order to get new results which were not at hand before. We operate on mathematical correspondents of observables with some operators. The choice of mathematical tools depends on our understanding of the problem, and the results will depend on our choice. The approximation in which our results are true depends on the precision with which observables are defined.

In the last decade there has been strong evidence for an attempt to unify different abstract branches of mathematics in the frame of the theory of categories.

So we can legitimately ask if the theory of categories is a proper base for a unitary abstract theory of systems.

We shall try to give an answer to this question starting with some necessary definitions then making proposals for some axioms and principles and finally examining some concrete examples in the light of our proposals.

DEFINITIONS

Definition 1. A category C is a class Ob C, together with a class Fl C which is a union of the form:

$$\mathrm{Fl}C = \bigcup_{(A_i,\, A_j) \in \mathrm{Ob}C \times \mathrm{Ob}C} C(Ai, Aj) \quad \mathrm{I} = \text{a class of indexes } i, j \varepsilon \mathrm{I} \tag{1}$$

To avoid logical difficulties we postulate that each $C(Ai, Aj)$ is a set (possibly void).

We shall impose to a category the following axioms of definition:

C_1). For each two distinct couples (A_i, A_j), (A_i', A_j') of objects from ObC we must have:

$$C(A_i, A_j) \cap C(A_i', A_j') = \varnothing. \tag{2}$$

C_2). For each ordered triple of objects A_i, A_j, A_k of C we have defined a mapping of sets:

$$\Theta(A_i, A_j, A_k): C(A_i, A_j) \times C(A_j, A_k) \rightarrow C(A_i, A_k) \tag{3}$$

which we shall call the "composition of morphisms", where morphisms are elements from the sets $C(A_i, A_j)$ of the class FlC. We shall write $g \circ f$ instead of $\Theta(A_i, A_j, A_k)(f, g)$ for each $f \varepsilon C(A_i, A_j)$ and $g \varepsilon C(A_j, A_k)$.

C_3). For each quadruple of objects from ObC the following diagram is commutative:

$$\begin{array}{ccc} C(A_i, A_j) \times C(A_j, A_k) \times C(A_k, A_l) & \xrightarrow{\theta(A_i, A_j, A_k) \times 1C(A_k, A_l)} & C(A_i, A_k) \times C(A_k, A_l) \\ \Big\downarrow {\scriptstyle 1C(A_i, A_j) \times \theta(A_j, A_k, A_l)} & & \Big\downarrow {\scriptstyle \theta(A_i, A_k, A_l)} \\ C(A_i, A_j) \times C(A_j, A_l) & \xrightarrow{\theta(A_i, A_j, A_l)} & C(A_i, A_l) \end{array}$$

that is, for each $f \in C(A_i, A_j)$, $g \in C(A_j, A_k)$ we have:

$$h \circ (g \circ f) = (h \circ g) \circ f, \quad \text{with} \quad h \in C(A_k, A_l). \tag{4}$$

C_4). For any $A \in ObC$, the set $C(A, A)$ has at least an element denoted by 1_A or $i\,dA$ called the identity morphism.

Definition 2. Let C and C' be two categories. We define a covariant (contravariant) functor from C to C' if the following conditions are fulfilled:

F_1). For any $A \in$ ObC we have defined a determinate object, denoted by FX or $F(X)$ from ObC'.

F_2). For any couple $A_i, A_j \in$ ObC and for any $f \in C(A_i, A_j)$ was defined a unique morphism $F(f)$, $F(f) \in C'(FX, FY)$ [$F(f) \in C'(FY, FX)$ for the case of contravariant functors] so that:

i) $F(1_X) = 1_{FX}$
ii) $F(g \circ f) = F(g) \circ F(f)$, [$F(g \circ f) = F(f) \circ F(g)$ for contravariant functors].

Definition 3. Let $F, G: C \to C'$ be two covariant (contravariant) functors. A functorial morphism $\alpha: F \to G$ is defined if we have a morphism $\alpha_X: FX \to GX$ from C' for any $X \in \mathrm{Ob}C$, so that for any $f: X \to Y$, morphism of C, we have the following commutative diagram:

$$\begin{array}{ccc} FX & \xrightarrow{F(f)} & FY \\ \downarrow{\scriptstyle \alpha_X} & & \downarrow{\scriptstyle \alpha_Y} \\ GX & \xrightarrow{G(f)} & GY \end{array} \qquad \left(\begin{array}{ccc} FX & \xrightarrow{F(f)} & FY \\ \uparrow{\scriptstyle \alpha_X} & & \uparrow{\scriptstyle \alpha_Y} \\ GX & \xrightarrow{G(f)} & GY \end{array} \right)$$

Definition 4. A partly ordered set is a system X in which a binary relation $x \geqq y$ is defined, which satisfies:

P_1) For all x, $x \geqq x$.
P_2) If $x \geqq y$, and $y \geqq x$, then $x = y$.
P_3) If $x \geqq y$ and $y \geqq z$, then $x \geqq z$. (Birkhoff, G., 1948.)

Definition 5. Let X be a partly ordered set; we associate to X an oriented system of categories. The inductive limit of the system of categories is a category which classes contain all other classes of the system.

Definition 6. A "class" is a collection of abstract elements; some elements of the "class" are perfectly determinated, that is, we are able to assign them some observables in a clear manner; some other elements do not have this property.

Definition 7. A "supercategory" S is a "class" ObS of objects together with a "class" of morphisms FlS which satisfy the following axioms:

S_1). For any two objects A_i and A_j from ObS there is a "class" of morphisms which connect A_i with A_j. We shall denote this "class" by $S(A_i, A_j)$; $i, j \in I$; I = a class of indexes.

S_2). We have always defined a law of composition, denoted by $\circ$, or $\square$, or $*, \ldots$, which associates to an n-tuple of morphisms another m-tuple of morphisms having satisfied a set of rules $\{R_r\}$ for origins extremities and iterated composition.

Distinct sets of rules $\{R_r\}$ define distinct laws of composition which operate on distinct classes (here with its usual meaning), of morphisms. In other words, some restrictions are imposed to diagrams.

Note. In particular we get a category by restrictions which are evident.

Definition 8. A "fuzzy supercategory" is a "supercategory" in which objects and morphisms are present in its "classes" with some probabilities.

Definition 9. An "organismic supercategory" is a "supercategory" in which objects are organismic sets (as they were defined by Rashevsky, 1967a), its morphisms representing some G-relations between organismic sets. We shall denote an "organismic supercategory" by S_σ.

Definition 10. A "system" is a structure defined by a certain "class" of G-relations which acts through a certain finite sequence of operations associating to a "class" of inputs a "class" of outputs.

Axioms, Principles and Laws

A. Axioms of representation

I. A state of a "system" (that is, a state of its components and relations which are defined by a table of its observables) is represented by an appropriate diagram of a "supercategory".

II. Passing from one state to another, that is, from diagram to diagram, we obtain, by means of some rules of transformations for diagrams, a dynamical characterization of a "system".

III. Operating a measure on a "system" we can get with a certain probability a state of the given "system". This probability must result from some statistical considerations and from the rules of transformation.

III′.—An equivalent formulation of III. There is a "fuzzy supercategory" associated to the "system" which predicts the behavior of the given "system" under measurements.

IV. A system is abstractly represented by a "supercategory" built from appropriate diagrams together with a corresponding "fuzzy supercategory"; consequently these supercategories are imposed to some restrictions given by the principles and laws of particular sciences which study particular classes of systems.

The following principles are an attempt to generalize some principles which were discovered by particular sciences.

B. Principles.

I. *The Principle of G-relations.* "The elements of any system, . . ., will aggregate or disperse if this aggregation or dispersal results in certain specified G-relations." (Rashevsky, 1967c.)

II. *The Principle of Choice.* Of all possible sequences of configuration of a "system", that one which occurs (is chosen) is the one which satisfies a certain variational condition (or a finite number of such conditions).

When we want to apply these axioms and principles to concrete "systems" we encounter some difficulties: there are many particular restrictions which are not taken into account by these propositions. So we must know the principles, laws, rules and propositions of particular sciences in order to get concrete results. But sometimes they are insufficient. Then we return to experiments and experimental data.

Biological "Systems". Some explications of the general proposals and some concrete examples in the light of the previous proposals: Rashevsky suggested that we could get a definition of "living systems" from the Principle of G-relations. So a concretion of the Principle of G-relations states the problem of biological "systems". The Principle of Choice infers the Principle of Biological Epimorphism and the Principle of Adequate Design. We may use these principles in the study of concrete biological "systems". These principles will give us the "conditions of interaction" for two objects of an "organismic supercategory", that is, the conditions under which some morphisms between two objects of an "organismic supercategory" S_σ would exist. We could represent the evolution of biological "systems" during their lifetimes, as an oriented system of diagrams of some S_σ. At a given moment the state of the "system" will be represented by the inductive limit of the system of diagrams. The temporal changes in components could be represented by identical morphisms:

$$A_{t_0} \xrightarrow{1A} A_{t_1}$$

where t_0, t_1 are different instants.

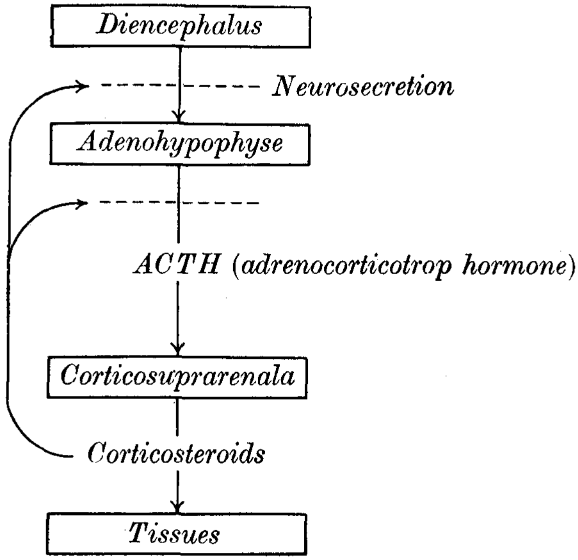
We could approach the problem of "fine structure" in a "structural S_σ" that is, all objects and morphisms of this "structural S_σ" will be defined with reference to some structural observables.

Similarly we could approach the problem of "course structure" in a "functional S_σ", and we could define a functor between a "structural S_σ" and its corresponding "functional S_σ". This functor shall represent the relations between structure and function.

But a structure could perform many functions; so, we have to compare different "functional S_σ" corresponding to the same "structural S_σ". These "functional S_σ" could be "compared" by functorial morphisms (see Def. 3). We could approach these problems in a "unit S_σ" where any object and morphism is defined with reference to structural and functional observables.

Also a hierarchy on components could be established by means of some "relations of order" (see Def. 4), in some S_σ. Each time the question is to get new results. This is equivalent to the explication of some indirect relations which are not evident. We could do it taking into account some new morphisms in our S_σ. We shall be guided by experimental data, axioms and principles.

A. Hormonal Control. Experiments have brought evidence for a hierarchy of endocrine glands (Karlson, 1966). The following scheme represents this hierarchy and hormonal control. Its corresponding abstract diagram is a "unit S_σ".



Scheme 1. Hormonal Control (from Karlson, 1966) (Indications in text)

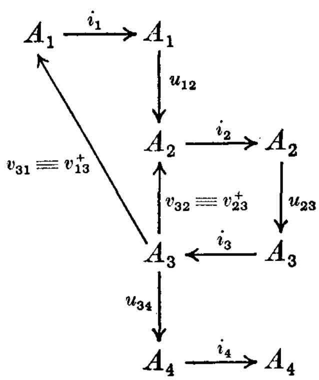


Diagram 1. An abstract representation of Hormonal Control (Indications in text)

The hierarchy on components is established naturally by a relation of order:

$$A_1 > A_2 > A_3 > A_4$$

A_1 is fixed by a number of observables which allow us to recognize a functional diencephalus. Similarly are introduced A_2, A_3, A_4 corresponding respectively to adenohypophyse, corticosuperrenala, and tissues. u_{12} represents the qualitative relation established between A_1, A_2 by means of the corticotropin-releasing-factor. We assign to this morphism an observable f_{u12} which represents the biochemical concentration of corticotropin-releasing-factor; u_{12} and f_{u12} establish a G-relation between A_1 and A_2.

We shall sometimes denote v_{31} and v_{32} by v_{13}^+ and v_{23}^+ respectively. u_{23}, u_{34}, v_{13}^+, v_{23}^+ are defined in a similar way. $A_1 \xrightarrow{i_1} A_1$ represents a chain of morphisms in the structure of A which composed by "$\square$" with v_{13}^+ gives u_{12}, that is, u_{12}—the relation between A_1 and A_2 is expressed as a composition of v_{13}^+ with a chain of morphisms from the structure of A_1. We have:

$$u_{12} = i_1 \square v_{13}^+ \tag{5}$$

where "$\square$" is defined as a law of composition which associates to the couple (i_1, v_{13}^+) another morphism u_{12} with some evident restrictions for origins and extremities of these morphisms, that is, the extremity of v_{13}^+ must be the origin of i_1 and the extremity of $i_1$must be the origin of u_{12}. This association is not ambiguous because u_{12} has a clear definition. In a similar way are defined i_2, i'_2, i_3. We have also:

$$u_{23}^0 = i_2 \square u_{12}, \quad u'_{23} = i'_2 \square v_{23}^+ \tag{6}$$

$$u_{23}^0 \bigtriangledown u'_{23} = u_{23} \tag{7}$$

where "$\bigtriangledown$" is the law of composition which associates to u_{23}^0 and u'_{23} (as they were defined above) another morphism u_{23} from A_2 to A_3; u_{23} expresses the relation between A_2 and A_3 as a result of two distinct relations between A_2 and A_3, namely u_{23}^0 and u'_{23}.

We shall use another law of composition "$\triangle$" which associates to v_{23}^+ and v_{13}^+ another morphism—their composition, that is, u_{34}, v_{23}^+, v_{13}^+ composed by "$\triangle$" result in a morphism

$$u_v = i_3 \square u_{23} \tag{8}$$

with an evident restriction—the extremity of i must be the origin for v_{13}^+, v_{23}^+, u_{34}.

Then we have:

$$u_v = u_{34} \triangle (v_{23}^+ \triangle v_{13}^+). \tag{9}$$

Also from (6), (7), (8), (9) we get:

$$u_{34} \triangle (v_{23}^+ \triangle v_{13}^+) = i_3 \square (u_{23}^0 \bigtriangledown u_{23}) \tag{10}$$

or,

$$u_{34} \triangle (v_{23}^{+} \triangle v_{13}^{+}) = i_3 \square \{[i_2 \square (i_1 \square v_{13}^{+})] \nabla (i_2' \square v_{23}^{+})\} \tag{10'}$$

We have some biochemical concentrations assigned respectively to u_{34}, u_{12}^{+}, v_{13}^{+}, and v_{23}^{+}: $f_{u_{34}}, f_{u_{12}}, f_{v_{13}^{+}}, f_{v_{23}^{+}}$ and some operators assigned respectively to i_1, i_2, i_2', i_3 which are Y_1, Y_2, Y_2', Y_3; they could be obtained explicitly from considerations concerning the functions and structures of A_1, A_2, A_3.

From their definitions we have:

$$Y_1 f_{v_{13}^{+}} = f_{u_{12}}; \quad Y_2 f_{u_{12}} = f_{u_{23}^{0}}; \quad Y_2' f_{v_{23}^{+}} = f_{u_{23}'}; \quad Y_3 f_{u_{23}} = f_{u_{34}} + f_{v_{23}^{+}} + f_{v_{13}^{+}} \tag{11}$$

or,

$$Y_3[Y_2 Y_1 f_{u_{13}} + Y_2' f_{v_{23}^{+}}] = f_{u_{34}} + f_{v_{23}^{+}} + f_{v_{13}^{+}}. \tag{11'}$$

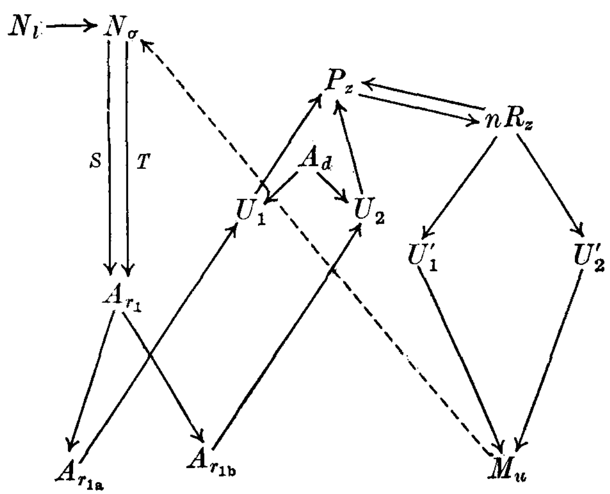
And finally:

$$\frac{df_{u_{34}}}{dt} = Y_3 \frac{d}{dt}[Y_2' Y_1 f_{u_{13}} + Y_2' f_{v_{23}^{+}}] + [Y_2 Y_1 f_{u_{13}} + Y_2' f_{v_{23}^{+}}] \frac{dY_3}{dt} - \frac{df_{v_{23}^{+}}}{dt} - \frac{df_{v_{13}^{+}}}{dt}. \tag{12}$$

If we impose some initial conditions and if we make some approximations, we could get an analytical solution to the problem, that is, $f_{u_{34}}$ expressed explicitly as a function of $f_{u_{12}}$, $f_{v_{23}^{+}}$, and $f_{v_{13}^{+}}$.

These results are true if we do not take into account the relations between diencephalus and telencephalus.

B. Biogenesis of ribosomes. We shall discuss the following oversimplified scheme of the biogenesis of ribosomes.



Scheme 2. A biochemical representation of Biogenesis of Ribosomes (Indications in text)

N_l = nucleolus; N_σ = nuclear organizer; A = r-RNA; U_1, U_2 = subunits of ribosomes; A_d = DNA; P_z = polisome; R_z = ribosomes; U_1', U_2' = subunits of ribosomes formed by decomposition of ribosomes; M_u = molecular components of ribosomes formed by ribosome decomposition.
S = synthesis of r-RNA; T = transport of r-RNA.

We suppose the hypothesis of a nuclear organizer to be well known; consequently further explication of Scheme 2 would be useless.

We could get some concrete results by means of a similar algorithm with the previous one for the following S_σ-diagram.

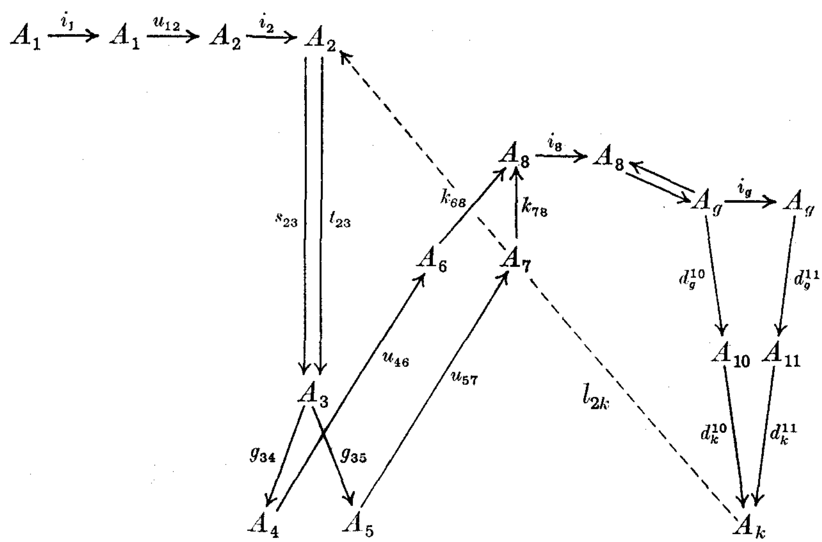


Diagram 2. An abstract representation of Biogenesis of Ribosomes (Indications in text)

This similitude in algorithm allows some generalizations; the previous diagrams are "supercategories S_σ with feedback", that is, morphisms which result from the composition of an oriented chain of morphisms act on some previous steps of the chain. This way, we get a new composition which allows the chain to "adequate" its actions. Now we are able to state this situation more generally: any normal biological "system" is an "organismic supercategory with feedback." This proposition is a general consequence of the Principle of Adequate Design. Some interesting connections could be made among different degrees of "adequation", the number of morphisms which perform a feedback and the Principle of Biological Epimorphism.

Further sophistications of the proposed approach could be achieved by some extensions of the theorems from the theory of categories or in a categorical theory of logics. Some connections with systems analysis and automata theory are not precluded.

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