

# Survival of the flexible: explaining the dominance of meta-heuristics within a rapidly evolving world

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*Abstract*— Although researchers often discuss the rising popularity of meta-heuristics (MH), there has been a paucity of data to directly support the notion that MH are growing in prominence compared to deterministic methods (DM). Here we provide the first evidence that MH usage is not only growing, but indeed appears to have surpassed DM as the algorithm framework of choice for solving optimization problems. Motivated by these findings, this paper aims to review and discuss the origins of meta-heuristic dominance.

Explanations for meta-heuristic success are varied, however their robustness to variations in fitness landscape properties is often cited as an important advantage. In this paper, we review explanations for MH popularity and discuss why some of these arguments remain unsatisfying. We argue that a more compelling and comprehensive explanation would directly account for the manner in which most MH success has actually been achieved, e.g. through hybridization and customization to a particular problem environment.

This paper puts forth the hypothesis that MH derive much of their utility from being flexible. This flexibility is empirically supported by evidence that MH design can adapt to a problem environment and can integrate domain knowledge. We propose what flexibility means from a search algorithm design context and we propose key attributes that should exist in a flexible algorithm framework. Interestingly, a number of these qualities are observed in robust biological systems. In light of these similarities, we consider whether the origins of biological robustness, (e.g. loose coupling, modularity, partial redundancy) could help to inspire the development of more flexible algorithm frameworks. We also discuss current trends in optimization problems and speculate that highly flexible algorithm frameworks will become increasingly popular within our diverse and rapidly changing world.

*Index Terms*— decision theory, genetic algorithms, mathematical programming, meta-heuristics, operations research, optimization

## I. INTRODUCTION

DATA on meta-heuristic usage in public, private, and academic sectors is sparse, however there has been some evidence that their use in computer-based problem solving is growing [1] [2] [3]. On almost a daily basis, there are new nature-inspired algorithms being proposed, new journals and conferences being advertised, as well as a continuous supply of new applications being considered within academic research. In [2], bibliographic data on genetic algorithms is used to show that publications within this field experienced a 40% annual growth from 1978 to 1998. More recently in [1], they present survey data showing that evolutionary computation (EC) usage is growing at a super linear rate in both public and private sectors.

Although these studies clearly indicate a growth in EC usage, it is not clear how these usage trends compare with similar research and development activity in deterministic methods. In particular, it has not been determined whether MH growth is actually outpacing alternative optimization techniques. By analyzing data from a number of publically accessible databases, we

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provide evidence in [Box 1](#) that the usage of meta-heuristics is not only growing, but in many respects meta-heuristics are surpassing deterministic methods as the framework of choice for solving optimization problems.

It is clear from the results in [Box 1](#) that the number of optimization publications, case studies, and patents is growing and that this growth is in many ways irrespective of the search paradigm being considered. There are undoubtedly a number of inter-related factors contributing to this growth including technological innovation, global prosperity, as well as a growth in the number of problems that can be solved through computer-based methods, e.g. due to simulation technology and the growing availability of computing resources. However, it is also apparent from [Box 1](#) that meta-heuristic implementation has been growing at a rate that is not matched by deterministic methods. Our aim in this paper is to try to understand why this is happening.

Before concluding that MH's newly found popularity is a reflection of utility, it is important to consider alternative explanations for this uneven growth. There are different reasons why a component, product or service grows in popularity within a competitive environment and not all of these are based on fitness. For instance, in evolving complex systems, the prominence (e.g. high connectivity) of a component within the system can sometimes be attributed to historical reasons. In particular, historical forces often bias growth in favour of past historical prominence, e.g. the well known "rich get richer" paradigm [4].

Comparing the rise in usage of the two optimization classes in [Box 1](#), it is apparent that historical arguments can not account for the observed trends. Deterministic methods have a well-known rich history and were actively studied for decades prior to the first appearance of meta-heuristics. US patents of deterministic methods for solving linear programming and dynamic programming problems were first granted in 1972, while the first meta-heuristic (simulated annealing) was not patented until 1986. Taking the data from Figure 2a, for the ten years leading up to 1990 there were 2525 DM journal publications compared with only 208 for MH. Over the next ten years the relative size of this gap narrows (DM=15619, MH=8403), however the historical advantage at the turn of the century was still clearly in DM's favour.

Other plausible reasons for biased growth in favour of MH include superficial reasons such as the conceptual appeal of meta-heuristics, e.g. the appeal of "nature-inspired" algorithms. Although this cannot be ruled out as a significant factor and may indeed account for some academic publication trends, conceptual appeal is less likely to explain trends in patents or the usage of MH in industry.

In industry applications, those responsible for deciding which search techniques to implement should be primarily concerned with the anticipated efficacy of the algorithm framework and less concerned with any conceptual or otherwise superficial attachment. As we indicate in [Box 2](#) for the specific case of genetic algorithms (GA) and industrial scheduling problems, there is considerable evidence that MH are being broadly implemented and that these implementations are often successful. Similar arguments might also apply to the patent trends shown in Figure 1. In this case, the costly decision to file for a patent is likely

based on anticipated efficacy and not on superficial appeal.

In short, the available data supports the conclusion that MH are being preferentially selected based on evidence of algorithm utility. However, this naturally raises the question of why MH are more useful for today’s problems. In the next section, we review fitness landscape-based arguments for understanding the utility of MH. We also review past arguments and evidence that MH success is strongly correlated with hybridization and customization efforts. In Section 3, we explore the hypothesis that the flexibility of an algorithm framework is the most important factor in determining the likelihood of algorithm success. We consider what algorithm flexibility means and the conditions that favor flexibility. This section also reviews trends taking place in industry and society and we speculate on important features to expect in future optimization techniques. In Section 4, we propose a theoretical basis for algorithm flexibility and discuss the relationship between these ideas and those developed in the study of complex adaptive systems. A summary of our main findings and arguments is given in Section 5 with experimental methods provided in Section 6.

## II. EARLY EXPLANATIONS OF ALGORITHM UTILITY

Early arguments in favour of MH focused on fitness landscape features or theories related to the operation of genetic algorithms, e.g. schema theory [5] and the building block hypothesis [6]. For instance, genetic algorithms were touted for their ability to deal with discontinuities in the fitness landscape, non-Gaussian noise in objective function measurements, non-stationarity of the fitness landscape, errors in determining objective function gradients, and numerical errors from computer calculations [7] [8] [9]. Their unabated success in multi-objective and multimodal fitness landscapes have also commonly been cited as important advantages. Furthermore, they often benefit from larger and more distributed computing resources; something that is increasingly available in both industry and academia.

However, there is another narrative surrounding the success of MH that should be considered seriously when trying to understand the merits of these algorithms. As many successful algorithm developers repeatedly emphasize in conferences, workshops and lectures, an MH’s success or failure hinges upon the designer’s ability to integrate domain knowledge into the algorithm design and generally customize the algorithm to handle the particular needs of the customer and problem. This customization mantra extends beyond heuristic knowledge. The importance of customization to the success of a GA has been documented repeatedly over the last 15 years within reviews and individual studies [10] [11] [12] [13] [14] [15] [16] [17]. In the case of GA applied to industrial scheduling problems, which we review in [Box 2](#), it is notable that almost all successful case studies involve a custom GA or GA hybrid.

In the literature, it is common to find search operators that are custom-designed for a specific problem and that are effective at finding feasibility regions or more generally for finding useful parameter combinations in solution space. Domain knowledge is

also frequently used to custom design stopping criteria, restart strategies, initialization strategies, constraint representation, and fitness functions, as well as to develop special encoding/decoding processes for solution representation.

Acknowledging the influence that customization has had on the success of this field is important. In our previous statements, we implied that a specific set of MH algorithms are growing in popularity, which is not entirely accurate. A more accurate statement is that an ever diversifying set of algorithms labelled as MH are increasingly being used, and that many of these start off with a common algorithmic origin, e.g. the canonical genetic algorithm.

Evidence of the importance of customization does not contradict previous claims that there are recognizable characteristics of problems that MH are particularly adept at handling. However it does suggest that the power of these problem characteristics to explain MH success is limited. Without accounting for the important role of algorithm design adaptation, fitness landscape arguments for algorithm success can have the unintended effect of implying that an MH is effective as an “off the shelf” or black-box optimization technique, so long as these problem characteristics are present. This broader statement is not at all supported by the available evidence.

#### A. NFL and NFL phenomena

It is by now well recognized that the search bias ingrained in an optimization algorithm will be more or less suited to different problems and that adapting an algorithm to an application domain is important for its effective usage. The No Free Lunch theorems for optimization (NFL) provides some theoretical support to this common knowledge [18]. In its most common (but simplistic) interpretation, NFL states that no search algorithm is better than any other when performance is averaged over all problems. Hence, in theory we know that no optimization algorithm is universally better than any other. This implies that a statement about algorithm utility that does not specify the context does not have any meaning. Fitness landscape characteristics, search algorithm characteristics, and computational resources will all directly play a role in any assessment of algorithm performance.

One can readily find weaknesses in NFL, e.g. because the conditions of the theory may not perfectly overlap with the conditions observed across operations research applications. However, decades of research and development efforts in optimization largely validate the most important implications that are drawn from NFL: that algorithms must be adapted to be effective on different problems. This reality can be obscured by the tendency to label (as we have here) a diverse array of algorithms under a single umbrella term such as *meta-heuristic* or *genetic algorithm*.

### III. SURVIVAL OF THE FLEXIBLE: A NEW PERSPECTIVE ON ALGORITHM UTILITY

If we accept NFL theory or we accept the empirical evidence of NFL-like phenomena in practice, then a compelling theory of

algorithm success should address these realities as opposed to ignoring them. If MH customization has been a key ingredient to success, then it is important to understand why this is true and why this would favor MH over DM. More generally, we need to understand why some algorithms are better positioned to quickly and effectively adapt to new problems.

Although the idea of algorithm flexibility is not new (e.g. [15] [14]), little effort has been devoted to exploring its theoretical basis or its implications for the field. For instance, few studies have explicitly considered why flexibility is important to algorithm utility or the consequences this should have for future algorithm research. Some issues related to flexibility also arise in the study of dynamic optimization, e.g. see [19], however our interests here are considerably broader than what would normally constitute a non-stationary optimization environment. In particular, we are interested in more dramatic problem changes or the emergence of new problems where sufficient algorithm modifications cannot be fully automated and instead require human intervention.

In this section, we consider how algorithm flexibility influences the utility of an algorithm framework, the conditions where flexibility should be favoured, and tradeoffs between the efficacy and efficiency of the algorithm adaptation process. We also discuss plausible explanations for why DM may be generally less flexible than MH. Finally we explore the theoretical underpinnings of algorithm flexibility and consider what insights may be derived from recent developments in the study of complex adaptive systems.

#### *A. Important timescales in algorithm adaptation*

The idea of algorithm flexibility is conceptually simple and is outlined in Figure 5. In short, the utility of an algorithm is evaluated based on its ability to adapt to the needs of a problem and not based on “off the shelf” performance characteristics.

It is common knowledge that any search process will exhibit a trade-off between solution quality and the computational costs expended. Similarly, the flexibility of an algorithm framework is expected to have a trade-off between the solution quality and the amount of time expended on algorithm adaptation. To understand flexibility, it is thus necessary to account for the efficiency and efficacy of the adaptation process (Figure 5b). Efficiency becomes increasingly important when there are pressing deadlines that constrain algorithm development time or when the problem is susceptible to changes in definition (e.g. size, scope) that require quick changes in algorithm search behaviour.

To help understand the trade-off between algorithm adaptation speed and final solution quality, we introduce three timescales: algorithm runtime ( $T_1$ ), algorithm development time ( $T_2$ ), and problem lifespan ( $T_3$ ).  $T_1$  measures the time needed to reach a stopping criteria during the search process,  $T_2$  measures the total time permitted to design an algorithm for a problem, and  $T_3$  measures the amount of time that a problem is relevant, e.g. to a client.

Assuming  $T_1$  is small compared with the time it takes a person to make an algorithm design change, the primary concern in

algorithm development is to quickly discover a sequence of design changes that provide sufficient and reliable solution quality. The performance of the initial algorithm design is not of tremendous importance, so long as it can be modified in the given time ( $T_2$ ). This makes the magnitude of  $T_2$  have influence over how we view sufficiency and the speed of adaptation. If short development times are preferred by a client or necessitated by a short problem lifespan ( $T_3$ ), then preference should be given towards an algorithmic framework that can rapidly adapt to new conditions, e.g. movement to the left in the bottom graph in Figure 5b. We speculate that MH are particularly adept at making rapid (but possibly suboptimal) gains in algorithm performance through design adaptation and should be favoured as  $T_2$  decreases.

The meaning and importance of  $T_3$  depends on whether a problem needs to be solved once (e.g. most design problems) or is solved many times (e.g. in scheduling). For instance, if a problem only needs to be solved once to meet some stated solution goals and if the solution can be reached at any time during the problem's lifespan, then  $T_3$  poses a straightforward constraint on the feasibility of a particular methodology, e.g.  $T_2$  must be less than  $T_3$ .

In the case where a problem is repeatedly being solved, the utility of an algorithm might be naively measured by its improvement over other algorithms multiplied by the amount of time that the algorithm is to be implemented, e.g.  $\Delta\{\text{solution quality}\} \times \{T_3 - T_2\}$ . However when  $T_3$  is small, the importance given to the early stages of implementation can be unexpectedly high (e.g. the importance of being "first to market" or avoiding bottlenecks within a larger project) and the rapid design of sufficient algorithms can trump what would otherwise appear to be a more superior alternative. In short,  $T_2$  has a strong impact on an algorithm's utility, especially when  $T_3$  is small.

### *1) Adaptiveness during and after development*

Optimization problems have so far been described as having a lifespan over which they are practically relevant and a time window when algorithm development must take place. Of course the reality is more varied and more complicated. Once we look closely at the individual components of a problem lifecycle, we find that the need for algorithm adaptation is pervasive.

First, it is common for a problem definition to change during the algorithm development phase. The constraints, the problem definition (e.g. scope, fidelity, representation), and even the objectives are subject to change over the course of an algorithm development project. The reasons that these changes occur are varied. For instance, it is common to learn more about the underlying nature of a problem, and consequently want to change the problem definition, as one develops ways to solve it. Also, a client's true interests are rarely captured entirely by a well defined problem and instead are more likely to involve a network of connected sub-problems and soft objectives that exist as tacit domain knowledge. Early success during algorithm development can also breed a desire for change, e.g. a desire to expand the scope of the problem. However, it is worth stressing that a change in the problem definition does not necessarily reflect poor planning or poor understanding by a client. Instead, these problem changes are often a consequence of intelligent yet boundedly rational individuals attempting to make sense of a dynamic and

complex world (cf [20] [21]). This implies that changes to a problem during algorithm development are not always preventable and hence are likely to persist within future optimization contexts.

Changes to a problem can also occur for reasons that are completely outside the control of the client and may take place after an algorithm is already being implemented. This may be the result of unexpected changes within the market that an organization competes in or other changes in the internal operating conditions of that organization. One example of “after implementation” changes in an industrial production problem is given in Section 1.2.6 in [8]

In summary, problem definitions are subject to change both during and after the span of time allocated to algorithm development (T2). An algorithm must effectively adapt but also do so efficiently to keep up with changing requirements, e.g. of a client during algorithm development or a market during algorithm implementation. Moreover, any algorithmic approach whose success is tightly dependent upon assumptions surrounding the original problem definition are less likely to be able to accommodate new conditions that arise.

#### *B. An evolving purpose for optimization research*

There is no single cause for the current pace of technological, organizational, social, and environmental changes being witnessed in the world today however their presence is unmistakable. Organizations operate within environments that are becoming more volatile and consequently less certain. It is common now for organizations to operate in an environment where customer needs, organizational capabilities, and resources can change frequently and with short notice. A chaotic yet continuous stream of technological innovations provides new opportunities but also creates new problems that demand new solutions. We believe that these changing business conditions have direct bearing on the utility of the future algorithm frameworks. In Figure 6, we summarize some of the major relevant trends that are currently taking place.

First, the number of new optimization problems is growing quickly due to technological and social changes. As this trend continues, one can expect to also see a continued demand for new algorithm designs. Viewing this as an environment of expanding and diversifying resources, we speculate that the most fit algorithms, i.e. those that tend to be utilized, will be those that can most quickly exploit these changing resources. In other words, algorithmic paradigms that are the most flexible and most quickly adapted will be those that are used most often.

The second major trend is one of growing volatility in extant problems; for many industries the ability to predict future conditions (e.g. in organization capabilities, resources, markets, competitors) is becoming increasingly difficult. Hence, the problems that an organization or industry would like to solve tomorrow are becoming more distinct from the problems that an industry is solving today. Again, we argue that the utility of an algorithm will not be dictated by its ability to solve a static problem. Instead it is the ability to adapt to changing conditions that will define success or failure in the optimization algorithms

of tomorrow.

### *C. Why are DM less flexible than MH?*

Whether and how MH are more flexible than DM is not yet known, however the arguments illustrated in Figure 5 are useful for entertaining possible explanations. We consider arguments derived from classical discussions of algorithm utility (static problem argument) as well as those that account for the efficiency and efficacy of algorithm adaptation (dynamic problem argument).

#### *1) Static problem argument (Fitting the algorithm to the problem versus fitting the problem to the algorithm)*

DM research has traditionally been decomposed along lines that are based on broad characteristics of fitness landscapes. Because of this decomposition, some of the effort in DM algorithm development involves determining how a problem can be formulated to meet a set of conditions. The rationale for this decomposition approach is straightforward; the algorithm can be touted as applicable for any problems falling within a particular set of conditions and hence is potentially useful outside the problem being solved. However, the rich diversity of problems and their unique problem-specific attributes may mean that these algorithms are of less practical utility than would otherwise be expected based on scientific studies involving heavily controlled conditions.

By decomposing the world of optimization problems into mathematically tractable contexts and focusing research within particular assumptions, it may breed a culture in which algorithm designers are compelled to fit a problem to a set of conditions instead of fitting an algorithm to a problem. This form of algorithm development bias could also be further exacerbated if (DM research) decomposition constraints make it more difficult to exploit fuzzy domain knowledge. The reason for this difficulty is that domain knowledge is often only approximately true, regardless of the fact it often represents highly relevant information about a fitness landscape. The DM algorithm development approach contrasts sharply with the MH culture where efforts are predominantly given to fitting an algorithm to a problem and where less concern is given to the generality of the final algorithm design.

#### *2) Dynamic problem argument*

These arguments change somewhat once we account for the importance of algorithm flexibility. As already noted, DM rely on a problem meeting certain assumptions that continue to be met throughout the algorithm development phase and throughout its implementation. In short, the DM places constraints on the problem definition and how that definition is allowed to evolve over time.

One difficulty is that the underlying problem that is actually of interest to a client generally has an imperfect overlap with any particular problem definition being used. The simplifying assumptions needed to specify a problem at any point in time are only

approximately true, however DM often will exploit these conditions. A major consequence of this is that future problem definitions (e.g. the problem representation, constraint definitions, objective definitions) are now constrained by the DM; the problem can only be altered in certain ways before the DM no longer can handle the conditions and breaks down. If a client finds the original problem definition is no longer satisfactory, or if the client's needs change over time, this can prove problematic for DM. On the other hand, if the problem can be restructured to still meet the required optimization assumptions, a DM can be highly effective and is often capable of finding higher quality solutions than a MH.

Because MH do not require highly specific fitness landscape conditions, they are not fragile to the loss of these conditions. As noted earlier, they generally do not require gradient information, high accuracy in objective function measurement, stationarity, linearity and continuity in fitness landscapes. This creates tremendous flexibility in the problem representation and the operators used. Yet at the same time, it is straightforward to incorporate local operators that do make such assumptions. Hence, MH can also be designed to exploit landscape features when they are known to exist at some level of scope and resolution within the fitness landscape.

In short, DM's sensitivity to optimization assumptions may have the inadvertent effect of constraining the flexibility of any given DM algorithm and thereby limiting its utility when problem definitions evolve. If this is generally true, it suggests that the robustness of MH to certain problem characteristics may play a direct role in facilitating design flexibility.

#### IV. TOWARDS A THEORY OF ALGORITHM FLEXIBILITY

Although the concept of algorithm flexibility is straightforward, the conditions that dictate whether an MH is flexible are much less obvious and need to be more deeply explored. Along these lines, we feel it is important to make progress in answering the following (related) questions:

- **Plasticity to environmental (problem) context:** Are there general conditions that make it easy/difficult to successfully incorporate domain knowledge into an algorithm?
- **Robustness to internal (algorithm) context:** Are there general conditions where the inclusion of a particular operator or a design change has a catastrophic impact on other important search characteristics of the algorithm?
- **Origins of design innovation:** When is it possible to combine algorithm "building blocks" in new ways to achieve a more effective search process for a specific problem?

Below we propose some qualitative attributes that one would expect in an algorithm that is readily adaptable to different optimization contexts.

- **robust yet adaptable behavior:** Particular search characteristics do not demand highly specified algorithm conditions

(robust) but at the same time these search characteristics can be changed and fine-tuned when needed (adaptable).

- **modularity and loose coupling:** Different aspects of the algorithm design can be added or removed while the others can robustly maintain their functionality. More generally, there are little requirements that one feature of the algorithm design places on other design features or on the problem definition.
- **Responsive:** Algorithm changes are easy to make and easy to test. Learning by doing is rapid such that the time needed to adapt the algorithm to a local context is fast enough to make learning by doing a viable approach.
- **Feedback:** Useful feedback information is available to tell us when things are going right and when things need to be changed. Furthermore, feedback information should provide some guidance about what aspects of the algorithm design may need to change.
- **Simple:** Adapting the algorithm design can proceed without expertise in optimization. Similarly, integrating domain knowledge is straightforward to achieve through experimentation and does not require intimate knowledge of the algorithm framework.

#### A. *Lessons from nature*

The qualities of an adaptive algorithm listed above describe several features that are present in naturally evolving biological systems. For instance, in a review by Kirschner and Gerhart [22], they highlight modularity, loose coupling, component versatility, and exploratory behavior as being highly relevant to the adaptability of a system.

The notion of a robust yet flexible algorithmic core that can broadly adapt to different problem conditions provides the basis of our conceptual understanding of algorithm flexibility. This conceptual model shares many similarities with observations of biological evolution [22]. For instance, it has been shown that the vast majority of extant species share a set of conserved core systems and processes [23, 24]. Although individual species are highly sophisticated specialists operating within unique environments, they share many internal similarities. The most obvious and universal of these include the underlying principles governing natural evolution, which clearly constitute an astounding generalist. This view of natural evolution mirrors our coarse illustration of algorithm utility in Figure 5b, where we have robust algorithmic frameworks that can be exploited and modified to fit a broad range of distinct optimization conditions.

Finally, it is worth noting that there have been recent advances in our understanding of the relationship between robustness and adaptation in the context of biological evolution and artificial life [25] [26] [27] [28]. These advances could provide new insights into the design principles that are needed to create more flexible and robust algorithms. For instance, recent studies by this author [27] [28] have provided evidence that a partial overlap in the functionality of components within a system can provide levels of versatility and adaptability that are completely unexpected based on the capabilities of the individual parts.

Evidence has also been given that particular system design principles can reconcile the conflicting needs of adaptability and robustness and can lead to systems with evolvable fitness landscapes [28]. This, along with other theoretical studies, may ultimately lead to a deeper understanding of the principles governing algorithm flexibility.

## V. CONCLUSIONS

Historically optimization problems were not thought of as having an expiration date. However, waning are the days when a problem could be defined and studied for years without the problem changing. More and more in today's world, new problems rapidly come into existence and existing problems unexpectedly change due to new conditions. Solution quality will always be a primary concern, however the algorithm development time and an algorithm's capacity to deal with new information and new conditions is expected to become an increasingly valued asset when addressing optimization problems.

In this paper, we provided evidence that meta-heuristics such as genetic algorithms are becoming increasingly favoured to solve today's optimization problems. We proposed that this growing dominance may be the result of an inherent flexibility that allows these algorithms to be efficiently and effectively modified to fit the characteristics of a problem. In other words, MH popularity may have less to do with the efficacy of a particular set of algorithm designs on a particular set of problems and have more to do with the ability of MH (but also the people and culture surrounding their development) to incorporate domain knowledge and to be advantageously combined with other methods.

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## VI. METHODS

### A. *Box 1 data analysis: selecting keywords*

Keywords for meta-heuristics (MH) and deterministic methods (DM) were selected based on several considerations. A list of keywords was first compiled from active researchers within the respective disciplines. This list was then expanded through concept mapping services such as Google Sets ([labs.google.com/sets](https://labs.google.com/sets)) and Kartoo.com . The additional keywords were obtained by conducting searches with prototypical examples for MH (genetic algorithms, evolutionary computation, metaheuristics), DM (mathematical programming, nonlinear programming, linear programming), and optimization (operations research, optimization, decision theory). This resulted in roughly 20 keywords for MH and 40 keywords for DM. These lists were then culled to 12 keywords per group in order to address the following issues: i) some search engines were not able to handle search strings larger than 256 characters, ii) some keywords were commonly used in both DM and MH research, and iii) some keywords had

significant meaning outside of optimization. Some of the DM keywords refer to classes of optimization problems, however these terms are used almost exclusively within the DM research community and therefore provided effective classifiers for DM data.

MH keywords used: genetic algorithm, evolutionary computation, meta-heuristic, swarm optimization, ant colony optimization, memetic algorithm, genetic programming, simulated annealing, estimation of distribution algorithm, greedy randomized adaptive search, nature inspired algorithm, bioinspired optimization

DM keywords used: mathematical programming, constraint programming, quadratic programming, quasi-Newton method, nonlinear programming, interior-point method, goal programming, Integer programming, simplex method, branch and cut algorithm, linear programming, dynamic programming

keywords that were culled from list: reinforcement learning, artificial neural networks, data mining, game theory, learning classifier systems, evolutionary programming, gene expression programming, evolution strategies, artificial immune systems, polynomial optimization, parametric programming, geometric programming, non convex programming, gradient methods, numerical algorithms, tabu search, deterministic global optimization, Lagrangian relaxation method, KKT condition, branch and bound, transportation method, cutting plane method, line search, Hungarian algorithm, penalty method, Barrier method, upper bounding techniques, combinatorial optimization, convex optimization, robust optimization, non-smooth optimization, stochastic programming, fractional programming, separable programming, linearly constrained optimization, mixed integer linear programming, affine-scaling, duality, global convergence, complementarity problems

### *1) Keyword validation*

Validation of the selected search terms aimed to demonstrate that the two sets of data classifiers are retrieving unique, non-overlapping information. Using Web of Science (Wos) Query 1, we found that the extent of this overlap is negligible (< 3.3 %).

- Independence of groups based on analysis of search strings from WoS Query 1
- Articles returned where topic is contained in search string one but not search string two: 27,693 (R1)
- Articles returned where topic is contained in search string two but not search string one: 38,024 (R2)
- Articles returned where topic is contained in search string one and in search string two: 2,160 (R3)
- The extent that search strings retrieve different sets of articles ( $\text{intersection/union} = R3/(R1+R2-R3) = 0.0328$ )

In interpreting the results that were generated using these keywords, we assume that the large majority of DM and MH research will refer to one of the keywords within the respective classes and hence more specialized and nascent research topics will be captured by these search strings. Although experience suggests that specialized research does refer back to its more well-known origins, this is an important assumption that is being made in our analysis. If this assumption does not hold, then it is

possible that unintended bias is present in the keyword lists that favours one of the two classes of algorithms.

## *B. Box 1 data analysis: search engines*

### *1) Delphion Patent Search*

Patent searches using Delphion (delphion.com) were restricted to granted US patents (“submitted only” patents were excluded). Each of the keywords were searched separately and only those contained in the “front pages” text of ten or more patents were included in the analysis (listed below).

MH: genetic algorithm, evolutionary computation, genetic programming, simulated annealing,

DM: mathematical programming, constraint programming, quadratic programming, nonlinear programming, interior-point method, Integer programming, simplex method, linear programming,

### *2) Google Scholar*

Google Scholar (scholar.google.com) was searched in one year increments to gather time series data on publications containing one or more keywords. Specific categories (listed below) were excluded from the search if they were likely to include publication outlets directly associated with the field of optimization.

#### categories excluded

1. Business, Administration, Finance, and Economics
2. Engineering, Computer Science, and Mathematics

#### categories included:

1. Biology, Life Sciences, and Environmental Science
2. Chemistry and Materials Science
3. Medicine, Pharmacology, and Veterinary Science
4. Physics, Astronomy, and Planetary Science
5. Social Sciences, Arts, and Humanities

#### Search strings

MH “genetic algorithm\*” OR “evolutionary computation” OR “metaheuristic\*” OR “swarm optimization” OR “ant colony optimization” OR “memetic algorithm\*” OR “nature inspired optimization” OR “genetic programming” OR “simulated annealing” OR “estimation of distribution algorithm\*” OR “bioinspired algorithm\*” OR “greedy randomized adaptive search”

DM “mathematical programming” OR “constraint programming” OR “quadratic programming” OR “quasi-Newton method\*” OR “nonlinear programming” OR “interior-point method\*” OR “goal programming” OR “Integer programming” OR “simplex method\*” OR “branch and cut algorithm\*” OR “linear programming” OR “dynamic programming”

### 3) *Web of Science (query one)*

Web of Science (WoS, <http://apps.isiknowledge.com>) was searched for all articles where the topic matched at least one keyword. A publication frequency time series was extracted using the WoS results analysis tool. Citation databases that were searched are listed below (conferences databases were excluded).

#### Databases

- Science Citation Index Expanded (SCI-EXPANDED)--1945-present
- Social Sciences Citation Index (SSCI)--1956-present
- Arts & Humanities Citation Index (A&HCI)--1975-present

#### Search Strings

MH TS=(genetic algorithm OR evolutionary computation OR metaheuristic OR swarm optimization OR ant colony optimization OR memetic algorithm OR nature inspired optimization OR genetic programming OR simulated annealing OR estimation of distribution algorithm OR bioinspired algorithm OR greedy randomized adaptive search) AND Document Type=(Article)

DM TS=(mathematical programming OR constraint programming OR quadratic programming OR quasi-Newton method OR nonlinear programming OR interior-point method OR goal programming OR Integer programming OR simplex method OR branch and cut algorithm OR linear programming OR dynamic programming) AND Document Type=(Article)

### 4) *Web of Science (query two)*

WoS query two used the same conditions as query one with the addition of the phrase “AND TS=(case study)”.

### 5) *Scientific WebPlus*

Scientific WebPlus (<http://scientific.thomsonwebplus.com>) is a search engine that gathers a small selected set of websites that Thompson Scientific claims are most relevant to the search string. The website provides domain statistics associated with the returned results and these statistics were used in the analysis in [Box 1](#).

### 6) *Google Trends*

Google Trends ([www.google.com/trends](http://www.google.com/trends)) provides information on the relative search volume of individual keywords and phrases. More information on the analysis methods that are used is provided at ([www.google.com/intl/en/trends/about.html](http://www.google.com/intl/en/trends/about.html)). The search volume is measured by the Search Volume Index and not by absolute search volume. To calculate the Search Volume Index, Google Trends scales search volume data based on the first term entered so that the first term’s average search volume over the selected time period is 1.0. Subsequent terms are scaled relative to the first term. Google Trends only allows 5 search terms to be compared at one time. To compare data for more than 5 terms required that all analysis be conducted starting with the same starting term (so that the data normalization that Google Scholar conducts is consistent). This was done with *genetic*

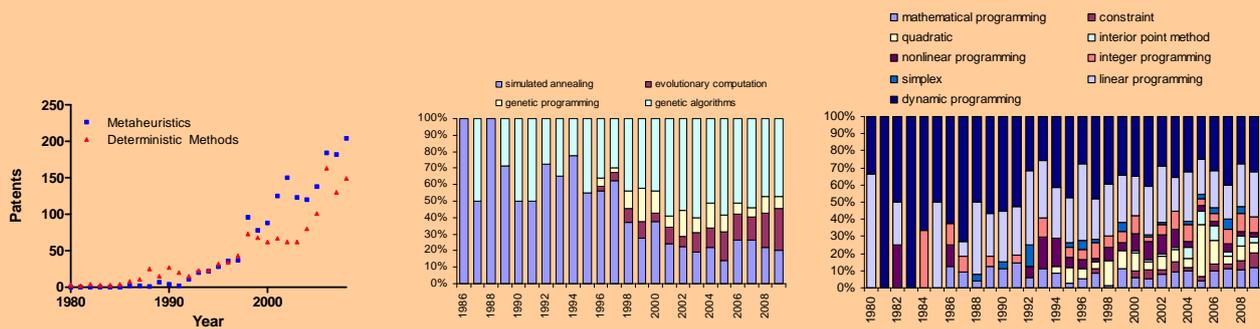
*algorithms* as the first search term, however using this term only affects the scaling of the Search Volume Index and not the relative values reported. Search strings that returned negligible activity included: memetic algorithm, estimation of distribution algorithm, greedy randomized adaptive search, metaheuristic, interior-point method, quasi-Newton method, goal programming, branch and cut algorithm.

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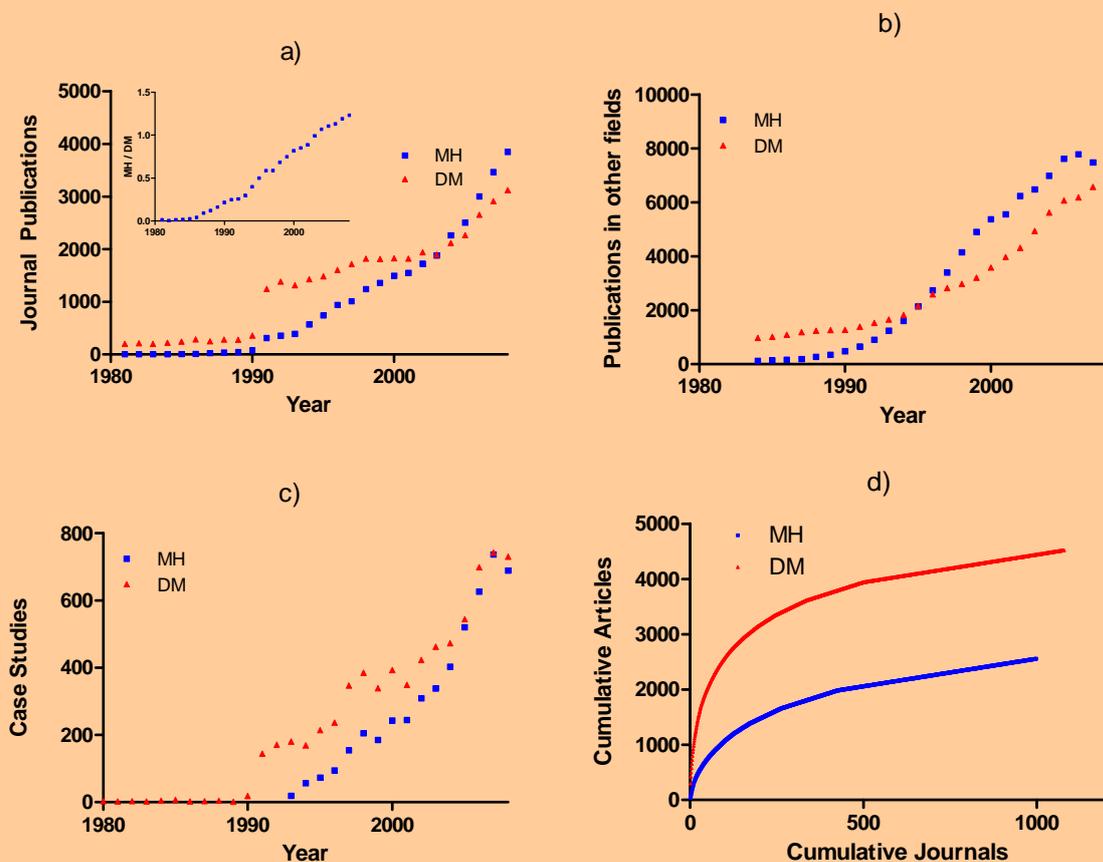
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## Box 1: Analysis of search algorithm usage

The following results evaluate research activity and usage of MH and DM algorithms. The analysis considers patent trends, publication trends, and trends related to internet sites and search traffic. Data for MH and DM is extracted using a set of 12 keywords related to each algorithm class. See the methods section for more information on how keywords were selected and how results were obtained.

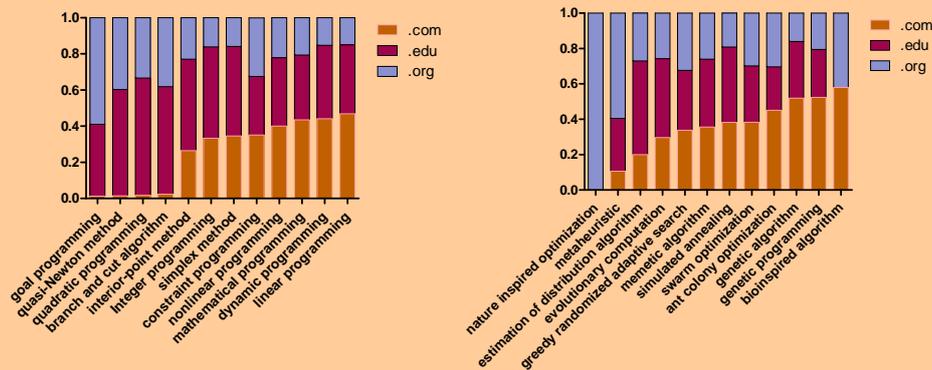


**Figure 1 US patent results: Patents granted in the US are shown for each year from 1980 to the present for all DM and MH related patents (left) with the relative contribution from individual keywords also presented for MH (centre) and DM (right). Only keywords occurring in the “front pages” of 10 or more US granted patents are included in the results shown in the centre and right graphs.**

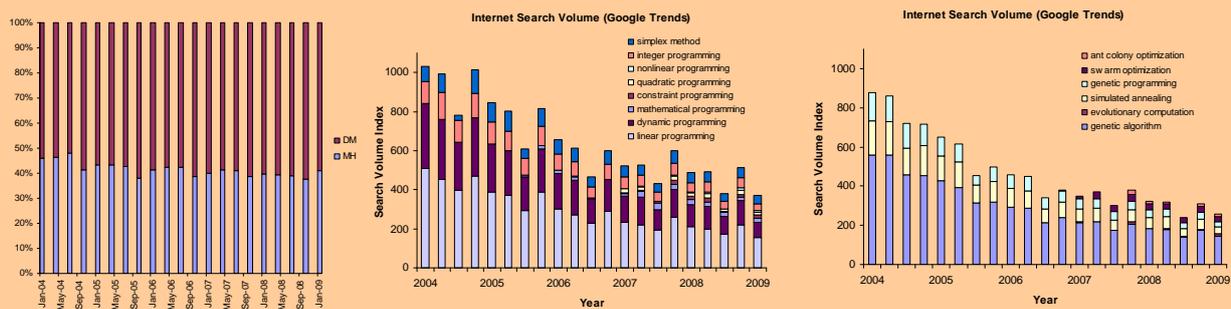


**Figure 2 Publication results: a) Journal publications found using Web of Science® (WoS). This publication time series provides evidence of three distinct trends. Prior to 1990, DM maintained a relatively stable publication frequency (between 200-350) while the MH:DM publication ratio rose steadily from 1980 (0.5%) to 1990 (21.5%). Between 1991 to 2003, DM initially experiences a single year 250% increase but then rises only an additional 47% for the rest of the 12 year period. This is compared with 380% growth in MH publications over the same period of time. Since 2003, MH has surpassed DM in publication frequency and has grown an additional 158% compared with 71% for DM. The inset graph displays the annual MH:DM publication ratio, which exhibits a steady rise (from 1985-present) that does not appear to be influenced by these trends. b) Publication data from Google Scholar is shown for any type of publication outlet (e.g. conferences, journals, books) but eliminates any publication sources that could be related to optimization research. For this data, the publication frequency of MH surpassed DM in the mid 1990s. c) Publication data from Web of Science is shown for any MH or DM publications that also include the topic “case study”. For this data, MH is only now approaching the same publication rate as DM. d) Case study results from “graph c” are shown to indicate how publications are distributed across publication sources. These results demonstrate that per case study, MH articles are spread out over a larger number of journals compared with DM case studies. For instance, 289 of the DM case studies were reported in the European Journal of Operational Research while for MH the most**

case studies reported in a single journal was 36 (also within the same journal). On average, a new MH case study article had a 40% chance of being published in a new publication source compared with only 24% for DM. Finally, 60% of all DM case studies can be found in just 11% of the journals where DM case studies have been published. In contrast, 60% of all MH case studies are found in 22% of the journals where MH case studies have been published.



**Figure 3 Web domain statistics:** Web domain statistics are shown for selected websites that have been determined by Scientific WebPlus<sup>®</sup> to have the most pertinent information related to the searched keyword. The goal with this analysis is to determine the extent that important content is located on academic versus non-academic websites. From this data, it was determined that the stated proportionalities for .edu domains are significantly different between MH and DM keywords, with DM keywords more likely to have a higher proportion of .edu domain sites (t-test, unequal variances,  $n=12$ ,  $p < 0.005$ ).



**Figure 4: Internet search volume results:** Google Trends<sup>®</sup> was used to evaluate internet search volume for each of the keywords from 2004 (the oldest recorded data) to the present. In presenting this data, the goal was to provide an indicator of overall interest in individual keywords related to DM and MH. The proportion of search volume related to DM has remained approximately constant between 55% and 60% relative volume (left), however the search volume time series data for DM (centre) and MH (right) indicate a steady decline in search activity. In contrast with data from earlier figures, this could indicate that general interest in both DM and MH keywords is shrinking.

## Box 2: GA in industrial scheduling problems

Here we review recent evidence that GA's and GA hybrids are being successfully applied to industrial scheduling problems. We consider evidence based on: Table 1) case studies where government or industry is directly involved, Table 2) surveys of GA's applied to "industrial strength" test problems, and Table 3) scheduling optimization companies that utilize GAs or hybrids in their software.

**Table 1 GA and GA hybrids applied to scheduling problem case studies. The studies were selected from a WoS search for recent articles including the topics "case study" AND "industry" AND "genetic algorithm" AND "scheduling". Only studies with government or industry participation were included.**

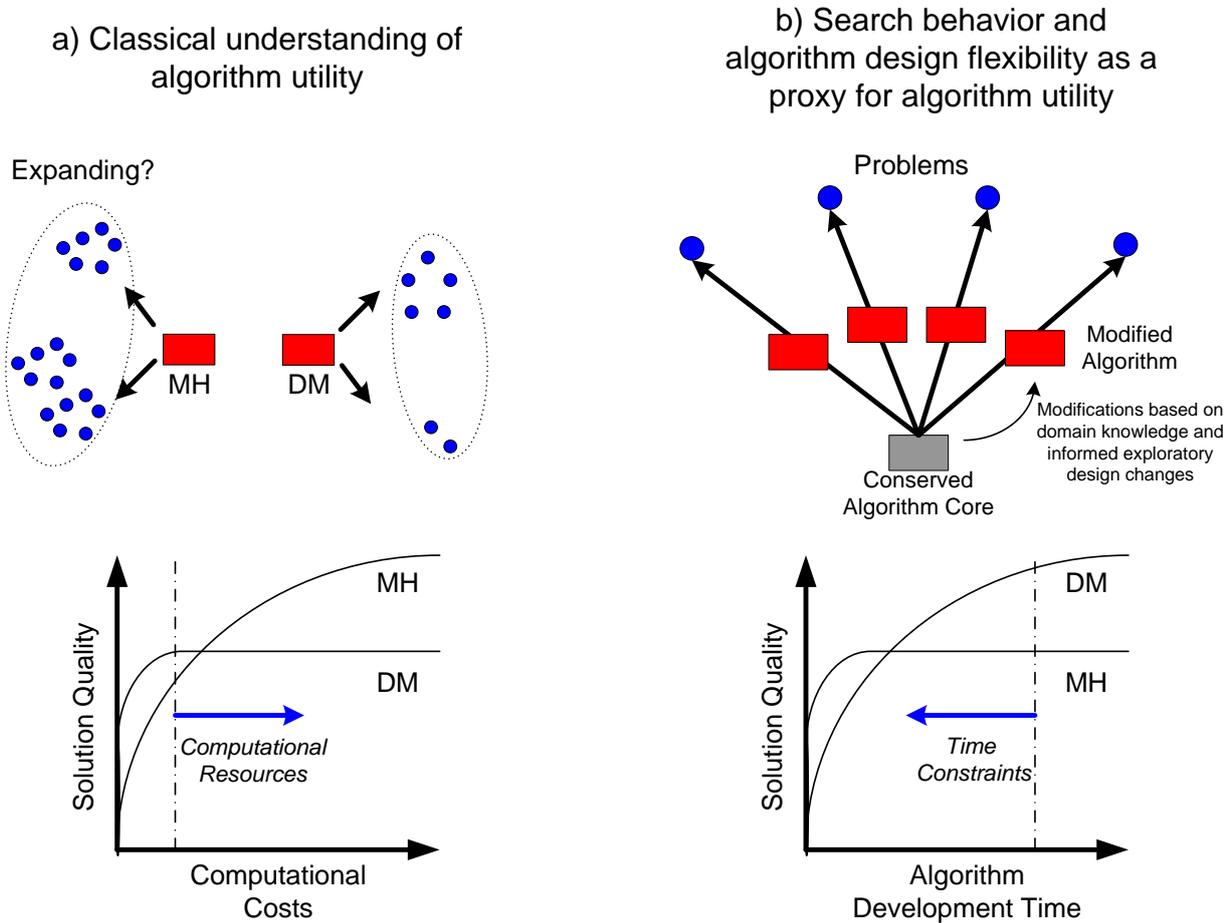
Company	Problem	Algorithm	Results	Year, Ref
General Electric	Maintenance scheduling for low earth orbit satellites	Custom GA	p.21, col.2: Found solution that was within 0.002% of the global optima in about 10 minutes of computation.	2006, (15)
Chilean Foundry: Name Not Provided	Task scheduling in small Chilean foundry.	GA	p.4: Solutions better than an expert found (the production chief) for 50% of cases.	2005, (16)
Dow AgroSciences	Scheduling of new products to market	Custom GA	p.7, tab.2: 18 projects in portfolio. None are solvable by DM. No MH beat all others on all projects, although GA was 10% better on average compared to second best MH. Authors estimate 10s of millions of dollars saved.	2004, (17)
Far East Electro-Mechanical Manufacturing Plant: Name Not Provided	Scheduling design tasks in circuit board	Custom GA	p.8: Claim that good solutions are obtained, however benefits from GA are inconclusive since alternate methods are not compared.	2004, (18)
Chinese company providing parts for internal combustion engines: Name Not Provided	Production scheduling	GA, Custom GA	p.18: 38% improvement in resource utilization. 31% improvement in completion time of jobs (compared to current operations).	2005, (19)
PRECON S.A. (Spanish concrete company)	Scheduling production of prefabricated concrete parts.	Custom GA	p.14: 12% improvement in costs compared to current operations.	2007, (20)

**Table 2 Reviews and large scheduling problem studies.**

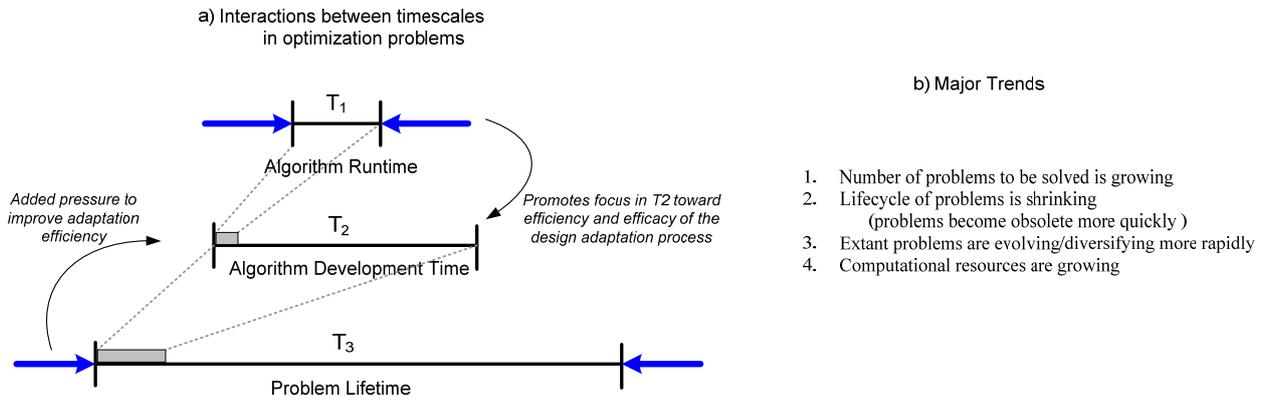
Problem	Alg	Overview	Year, Ref
Resource-Constrained Project Scheduling	Custom GAs	This review provides evidence that GA hybrids are dominating scheduling optimization research. Evidence is based on artificial scheduling problems.	2006, (21)
Airline Crew Scheduling	Custom GA	This paper considers airline crew scheduling with 28 real datasets taken from an airline. Problem sizes ranged from small to large however problem definitions appear to be simplified (e.g. less constraints) relative to other real world problems. GA reaches within 4% of global optimal solution on average.	2004, (22)
Workforce Constrained Preventative Maintenance Scheduling	ES, SA	This paper looks first at Evolution Strategies (ES) on 60 small scale scheduling problems and shows ES can quickly reach optimal solutions. On 852 large scale problems (where optimal solutions are not known), ES was shown to find what they call "near optimal" solutions 12 times faster than Simulated Annealing (SA).	2000, (23)

**Table 3 Private companies that provide schedule optimization services using a GA or GA-hybrid. Most service providers also explicitly point out that algorithms are customized to fit particular client needs. The table is not exhaustive, e.g. companies such as OptTek Systems, Inc, AntOptima SA, and known to incorporate other MHs into their algorithm designs.**

<b>Company</b>	<b>Online info</b>	<b>Description</b>
NuTech Solutions	<a href="http://nutechsolutions.com/index.asp">http://nutechsolutions.com/index.asp</a> <a href="http://nutechsolutions.com/lit_featured.asp">http://nutechsolutions.com/lit_featured.asp</a>	NuTech provides a range of nature-inspired optimization and computer modeling services. The second link describes their success with solving Air Liquide's scheduling distribution problems.
XpertRule	<a href="http://www.xpertrule.com/pages/case_ud.htm">http://www.xpertrule.com/pages/case_ud.htm</a>	XpertRule is a GA optimization tool. The provided link describes a case study where XpertRule provided substantial improvements in distribution solutions for the company United Vinters and Distillers
Bio-Comp	<a href="http://www.bio-comp.com/industrial/maximizeproduction.htm">www.bio-comp.com/industrial/maximizeproduction.htm</a>	Bio-Comp provides evolution-based solutions, some of which deal with scheduling related problems.
Advanced Computational Technologies	<a href="http://www.ad-comtech.co.uk/application.shtml">www.ad-comtech.co.uk/application.shtml</a> <a href="http://www.ad-comtech.co.uk/study7.htm">www.ad-comtech.co.uk/study7.htm</a>	ACT uses a GA to provide solutions to a number of problems including roster scheduling problems
Esteco	<a href="http://www.esteco.com/schedulers.jsp">www.esteco.com/schedulers.jsp</a>	Esteco is a provider of optimization software. Their optimization package includes two types of GA.
IcoSystem	<a href="http://icosystem.com/technology.htm">http://icosystem.com/technology.htm</a> <a href="http://icosystem.com/apps_personnel.htm">http://icosystem.com/apps_personnel.htm</a>	IcoSystem uses a GA for personnel management and scheduling problems
Bright Rivers	<a href="http://www.brightivers.com/page.asp?id=home">www.brightivers.com/page.asp?id=home</a>	Bright Rivers has developed a hybrid GA to solve scheduling optimization problems.
Blue Kaizen	<a href="http://www.bluekaizen.com/">www.bluekaizen.com/</a>	Blue Kaizen provides scheduling, planning, and routing solutions using a GA and other MH
SolveIt	<a href="http://www.solveitsoftware.com/">www.solveitsoftware.com/</a>	SolveIt provides custom scheduling and planning solutions using a broad range of MH techniques including GA hybrids



**Figure 5** Different perspectives for assessing algorithm utility. a) At small spatio-temporal scales (where NFL applies), we only consider the utility of an algorithm from the perspective of a static algorithm design. Under these conditions, classical explanations of algorithm utility can be broken down into fitness landscape (top) and computational resource (bottom) arguments. *Top*: MH are effective for problems with fitness landscape characteristics that are common, or are becoming increasingly so, for so called “real world” problems. See text for more details on these problem characteristics. In the diagram, the distance between an algorithm (box) and problem (circle) indicates the suitability of the pairing. *Bottom*: DM and MH commonly have different cost-benefit profiles, such that the preferred technique will often depend on the amount of computational resources available. Although the amount and type of computational resources varies from problem to problem, it is often argued that MH are increasingly favoured as computational resources become cheaper and more readily available. The blue arrow indicates current trends in the availability of computational resources. b) From a broader perspective, we propose to assess utility based on an algorithm framework’s adaptability towards different problems. *Top*: Illustration of how to assess the utility of an algorithm framework. *Bottom*: Algorithm adaptation profiles which show hypothetical values of solution quality as a function of the amount of time given to algorithm development. Given a fixed amount of available computational resources, here we speculate that problems that can be studied for long periods of time may favour a DM approach while shorter algorithm development times would favour an MH approach. The blue arrow indicates current trends in the available algorithm development times (see Section 3.2).



**Figure 6 a) Trends and interactions between optimization timescales. The amount of time needed for an algorithm to search for a solution ( $T_1$ ) is rapidly decreasing due to technological improvements in computational resources. As a result, a smaller proportion of algorithm development time ( $T_2$ ) is spent with the algorithm running and more is spent on algorithm design changes. Because a problem's lifespan ( $T_3$ ) is decreasing and problem definition volatility is increasing, the available time to make algorithm design changes is becoming more and more constrained. b) Major trends that have a direct bearing on optimization research.**