Iterative Algorithms for Radar Signal Processing

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Abstract—**In this paper, we are interested in the application and the evaluation of the performances of adaptive recursive subspace-based algorithms of linear complexity for the suppression of interferences in Space Time Adaptive Processing (STAP), namely PAST and OPAST. To highlight their application in the STAP, we present the reduction of the rank by the principal components (PC) and the SINR metric methods. The simulation results will be presented and the performances of the STAP for a reduced rank will be discussed with a comparative study made between the used methods.**

Keywords-PAST and OPAST algorithms; STAP; RADAR

I. INTRODUCTION

A target in a scenario of airborne surveillance is darkened by the clutter of ground and the jammer in multiple dimensions. It was shown in [1] that the target is separated from the clutter and jammer in two-dimensional angle/Doppler. Space-time processing can provide a rejection of such clutter and thus, be able to detect the slow targets. Brennan and Reed [2] first introduced STAP to the radar community in 1973. With the recent advancement of high speed, high performance digital signal processors, STAP is becoming an integral part of airborne or space-borne radars for MTI functions. However, it is a well-known that optimum STAP detection implies a large computational cost, since it utilizes complex matrix operations and often in an iterative fashion. In fact, the methods of the STAP with full rank use all the available degrees of freedom to eliminate the interferences, so requiring a cost of high calculation. For this reason, some reduced-rank STAP algorithms have been developed. In [2-7] and references therein, it was shown that STAP has the unique property of compensating for the Doppler spread induced by the platform motion and thus, making the detection of slows targets possible.

In this paper, we analyze, at first, the STAP with reduced rank by using two methods, namely the method of the principal components (PC) and the SINR metric method. Then, we apply two iterative and adaptive algorithms of subspace tracking to reduce the rank. Finally a comparison is made to justify the use of these algorithms in the radar processing. In Section 2, the mathematical model is given as well as the structure of the matrix of covariance. In Section 3, we give a brief description of STAP with reduced rank and define the PC and SINR metric methods. The iterative algorithms proposed are treated in the Section 4. Results and discussion are presented in Section 5, while the conclusion is presented in Section 6 highlighting the main results presented.

II. MATHEMATICAL MODEL OF DATA

We consider a space time network with N antennas uniformly spaced and M delay elements for any antenna. The data are then processed on one range of interest which corresponds to one slice of the data cube in Figure 1.
The space time covariance matrix is determinated by

$$R = R_c + R_j + R_n \tag{1}$$

where, R_c, R_j and R_n are the covariance matrices of the clutter, jammers and thermal noise, respectively. More details for the computation of these matrices are given in [3].

The performance of the processor can be discussed in terms of the Improvement Factor (IF). IF is defined as the ratio of the SINR of the output to that of the input of the Direct Form Processor (DFP):

$$IF_{opt} = \frac{W^H S.S^H W.tr(R)}{W^H R.W.S^H S} \tag{2}$$

W is the optimum weights of the interference plus noise rejection filter and S is the steering vector.

III. STAP WITH REDUCED RANK

The fully adaptive techniques of signal processing can not be applied for a real-time processing. The methods with reduced rank realize the adaptation on a space of reduced dimension obtained after an adaptive transformation on the

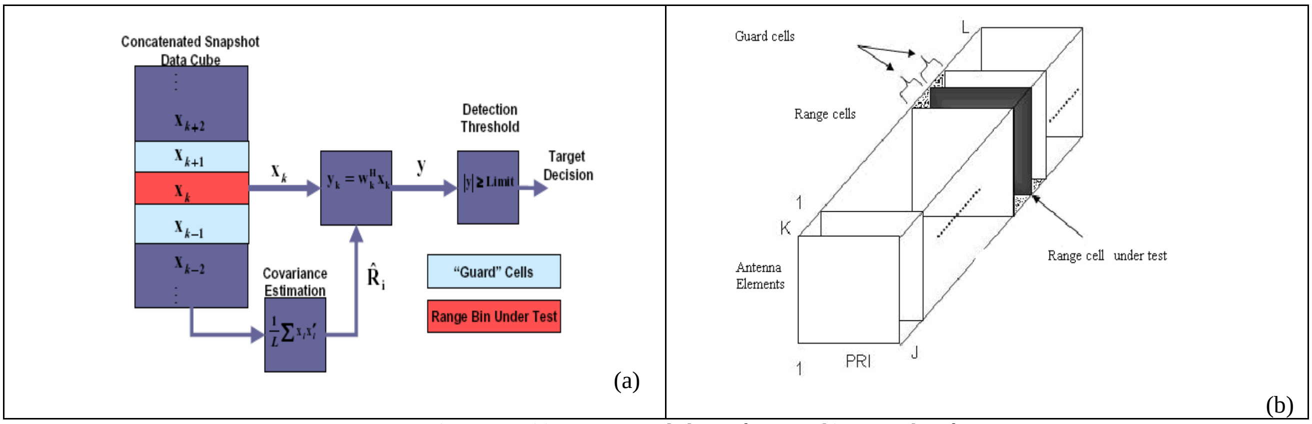

Figure 1. (a) Conventional chain of STAP; (b) Data cube of STAP

data. They also exploit the nature of the low rank of the interferences.

The partially adaptive algorithms of the STAP consists in transforming the data with a matrix $V \in C^{MN \times r}$ where r<<MN. There are several methods of rank reduction, analyzed in [3-7] and references therein, which differ in the shape of the processor and in the selection of the columns of the matrix. The principal component is based on the eigenvectors conservation of the matrix of covariance of interferences corresponding to the dominant eigenvalues [3]. In the SINR metric method, the objective is to choose the r columns of *V* such that the loss in the performances of the SINR will be minimized. Berger and Welsh [5] chose the columns of *V* as being the eigenvectors of R, which minimized the loss in the performance of the SINR. If we assume that the r columns of *V* are a subset of the eigenvectors of R, the improvement factor of the reduced rank can then be written as

$$IF_{RR} = S^H V (V^H R\ V)^{-1} V^H S \frac{tr(V^H R\ V)}{S^H V\ V^H S} \qquad (3)$$

IV. ITERATIVE AND ADAPTIVE ALGORITHMS OF SUBSPACE TRACKING

The iterative/adaptive algorithms of subspace tracking allow to follow the temporal variations of the subspace and to update it in every new observation. One very large number of algorithms were proposed in the literature [8-12] and references therein. They showed their efficiency in several domains of signal processing in particular in antennas processing and in spectrum analysis. They offer interesting perspectives in the STAP.

In this context, we consider only the class of the fastest, most strong and effective algorithms said classify in linear complexity. We propose the application and the evaluation of the performances of the algorithms, PAST as well as its orthogonal version (OPAST).

PAST and OPAST algorithms

The Projection Approximation Subspace Tracking (PAST) [9] is based on the optimization of the following criterion:

$$J(W(t)) = \sum_{i=1}^{t} \mu^{t-i} \left\| x(i) - W(t).y(i) \right\|^2 \qquad (4)$$

where x is the observed data vector and W is the estimated interference subspace basis and μ is a forgetting factor, $0 \le \mu < 1$ and $y(i) = W^H(i-1).x(t)$ this cost function has a global minimum which yields a non orthonormal basis of the interference subspace and which may be attained by a RLS adaptive algorithm given in table 1.

In order to resolve the problem of convergence and orthogonality of the weighty matrix, the Orthonormal PAST (OPAST) was presented [10].

V. RESULTS AND DISCUSSIONS

In this part, we will discuss the influence of some algorithms of reduction of the rank on the detection of a target supposed by low power (SNR= 0 dB) and of low speed. The simulated environment is a linear side looking network of N=8 antennas spaced out by half of the emitted wavelength $d = \lambda / 2$ and M=10 impulses in the coherent processing cube. The dimension of the adaptive process is thus MN = 80. The elevation angle is fixed to 20°. The speed of the airborne radar is V_R=100m/s, and the frequency of transmission is 0.3GHz. The environment of interferences consists of five jammers and the clutter of ground. The jammers have the angles of azimuth respectively: 0°, 180°, 60°, 90°, and 72°, with ratios jammer to noise (JNRs) of 13dB, 12dB, 11dB, 10dB and 9dB respectively. The clutter to noise ratio (CNR) is equal to 8dB. This clutter covers the band [30°, 30°].

Note that a notch, which is a reversed peak of the clutter appears at the frequency in the direction of sight of the radar, while the width of this notch gives a measurement for the detection of slow moving targets.

First of all, we studied the influence of the number of antennas N and M pulses on the number of eigenvalues of the covariance matrix that has given us guidance on environment statistics in which the target is to be tested. To do this, we traced the changes in eigenvalues which focuses the energy of the system according to their numbers.

It can be seen clearly from Figure 2, that an increase in NM leads to an increase in powers and in the number of degrees of freedom of the system filter. We also note that the interference-noise space can be separated into two subspaces: the interference subspace and the noise subspace.

As the number of eigenvalues is a measure of the degree of freedom of clutter elimination filter, we can notice that the number of small eigenvalues is high, while the number of those with a low value is significantly large, which increases the degree of freedom.

If we work with all size of the covariance matrix (MNxMN), there will be difficulties in implementing real-time especially since it also requires a very cumbersome equipment. So the use of rank reduction methods reduce the computational complexity and therefore the cost, and this by taking a dimension (rxr) with r the rank which must be very inferior to MN. This choice is motivated by the fact that the observation space is divided into two areas (noise and interference) as it was commented on in Figure 2.

On the other hand, it is important to consider the performances of the SINR for each method partially adaptive according to the rank. We notice from Figure 3 that there is a strong degradation in the performances for a small rank. It is obvious that the method of the PC cannot obtain the optimal SINR of exit until the rank is equal to the dimension of the eigenstructure of noise subspace. The SINR metric is best when the rank is reduced below full dimension.

Figure 4 represents the improvement factor, IF, according to the normalized Doppler frequency (Ft) for the DFP-PC, with different values of PRF (without ambiguities, $PRF = 8.V_R / \lambda$ and the case with ambiguities $PRF = 2.V_R / \lambda$). We notice the appearance of new weak undulations in the bandwidth due to the estimate used for the rank reduction. We notice, also, the appearance of ambiguous notches and the width of the notch does not change. Thus the rank reduction does not eliminate the ambiguities of the clutter during the suppression of the noise, and that this reduction has no effect on the detection of the slow targets.

For the iterative algorithms, we use the improvement factor given by the expression (2) where $W = (I - ww^H)S$, and w is the estimated subspace of interferences. The factor of forgetting of both algorithms is fixed at $\mu = 0.99$. Figure 5 represents the IF according to Ft for the algorithms PAST and OPAST and that of the optimal. We considered the scenario without ambiguities to light the effect of the application of the algorithms on the detection. We notice that the notch due to the PAST follows the same look as that of the optimal processor. It is relatively wide but can indeed allow an acceptable detection of the slow targets. We can thus say that the application of the algorithm PAST allows the suppression of the interferences.

Comparing both considered iterative algorithms, we notice that the detection using the algorithm OPAST is closer to that obtained by the optimal processor.

To compare the effect of the reduction of the rank by applying the algorithm PAST with that of the PC-DFP, we drew the factor of improvement IF according to Ft for the algorithm PAST and PC-DFP for various values of PRF, Figure 6. We notice that the ambiguous notches of the algorithm PAST appears at the same level as those of the method PC-DFP but, they are much wider. Thus these iterative algorithms get ready well for the application in the STAP.

By drawing the same curves as with the OPAST and the PC-DFP, Figure 7, we notice that the results are similar with an improvement at the level of the width of the notch as well as at the level of the secondary undulations. Indeed, we notice that the ambiguous notches of the algorithm OPAST is appreciably wider with regard to those of the method PC-DFP. However, they are less wide than those of the algorithm PAST.

We have to notice that similar results are found with the SINR metric method.

TABLE I. PAST AND OPAST ALGORITHMS

Initialization :

$$W(0) = \begin{bmatrix} I_r \\ O_{(NM-r)\times r} \end{bmatrix}; Z(0) = I_r$$

For *t = 1,2,… do*

PAST Section:

$$y(t) = W(t-1)^H x(t)$$

$$h(t) = Z(t-1)y(t)$$

$$g(t) = \frac{h(t)}{\mu + y(t)^H h(t)}$$

$$Z(t) = \frac{1}{\mu}(Z(t-1) - g(t)y(t)^H Z(t-1))$$

$$e(t) = x(t) - W(t-1)y(t)$$

$$W(t) = W(t-1) + e(t)g(t)^H$$

OPAST Section:

$$\in(t) = \gamma(t)(x(t) - W(t-1)y(t))$$

$$\tau(t) = \frac{1}{\frac{1}{\mu^2}\|h(t)\|^2}\left(\frac{1}{\sqrt{1 + \frac{1}{\mu^2}\|\in(t)\|^2\|h(t)\|^2}}\right)$$

$$e(t) = \frac{\tau(t)}{\mu}W(t-1)h(t) + (1 + \frac{\tau(t)}{\mu^2}\|h(t)\|^2 \in(t)$$

$$Z(t) = \frac{1}{\mu}(Z(t-1) - \frac{1}{\mu}g(t)y(t)^H Z(t-1))$$

$$W(t) = W(t-1) + e(t)g^H(t)$$

End for

VI. CONCLUSION

In this paper, we considered the application and the evaluation of the performances of two adaptive recursive subspace-based algorithms of linear complexity and a comparison with reduced rank methods and the optimal filter. The results of simulation showed that the algorithm OPAST is better than the algorithm PAST in the sense that it gets closer to performances of the optimal processor to full rank. A comparative study was made, proving that the iterative algorithms get ready well for the reduction of the rank for the STAP because they allow similar performances those given by the methods of rank reduction.

Furthermore, they present a very low computational complexity. In fact it can be viewed on table 1, that the complexity burden is O(MN) instead of $O((MN)^3)$ for the eigencanceller processor. That's why these algorithms can be considered as an economical approach in comparison with the other techniques.

REFERENCES

[1] R. Klemm, *Space Time Adaptive Processing Principles and applications*, The Institution of Electrical Engineers, London, 1998.

[2] L.E. Brennan and I.S. Reed "Theory of radar", *IEEE transactions on Aerospace and Electronics*, vol. AES-9, no2, pp. 237-252, 1973.

[3] H. Nguyen, *Robust Steering Vector Mismatch Techniques for reduced Rank Adaptive Array Signal Processing*, PhD dissertation in Electrical and Computer Engineering, Virginia, USA, 2002.

[4] J.S. Goldstein and I.S. Reed, "Theory of partially adaptive radar", *IEEE Transactions on Aerospace and Electronics Systems In Proceedings of the IEEE National Radar Conference*, vol. 33, No.4, pp 1309-1325, 1997.

[5] S.D. Berger, and B.M. Welsh, "Selecting a reduced-rank transformation for STAP, a direct form perspective", *IEEE Transactions on Aerospace and Electronics Systems*, vol. 35, N°2, pp. 722-729, April 1999.

[6] J.R. Guerci et al., "Optimal and adaptive reduced-rank STAP", *IEEE Transactions on Aerospace and Electronics Systems*, Vol. 36, No. 2, pp 647-663, 2000.

[7] S. Dib et al., "A Reduced Rank STAP with Change of PRF", *Eusipco2007*, Poznan, Poland, pp 95-122, 2007.

[8] P. Comon and G.H. Golub, "Tracking a few extreme singular values and vectors in signal processing", *Proc. Of the IEEE*, vol. 78, No. 8, pp 1327-1343, 1990.

[9] B. Yang, "Projection Approximation Subspace Tracking", *IEEE-T-SP*, vol. 44, N°1, pp 95-107, 1996.

[10] K. Abed-Meraim A. Chkeif and Y. Hua, "Fast Orthonormal PAST Algorithm,"*IEEE Signal Processing Letters*, vol. 7, pp. 60–62, Mar. 2000.

[11] R. Badeau et al., "Fast approximated power iteration subspace tracking", *IEEE Transactions on Signal Processing*, vol. 53, pp. 2931–2941, Aug. 2005.

[12] A. Valizadeh and M. Karimi, "Fast Subspace tracking algorithm based on the constrained projection approximation," *EURASIP Journal on Advances in Signal Processing*, 2009.

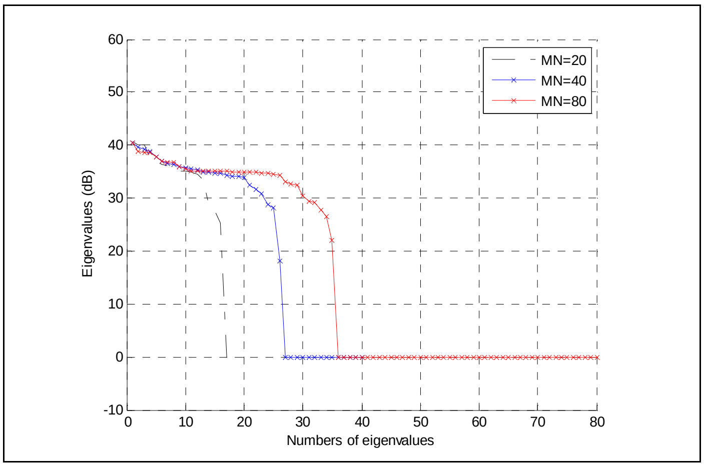

Figure 2. Eigenvalues of the space-time covariance matrix R: with, JNR=35 dB, CNR = 30 dB

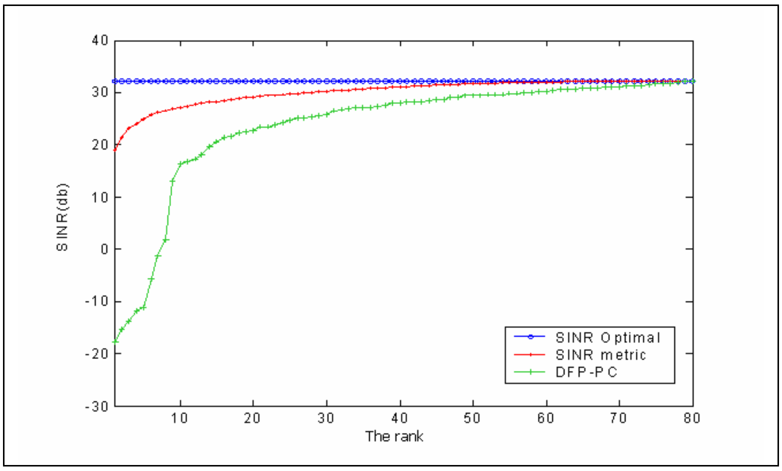

Figure 3. SINR performance versus the rank

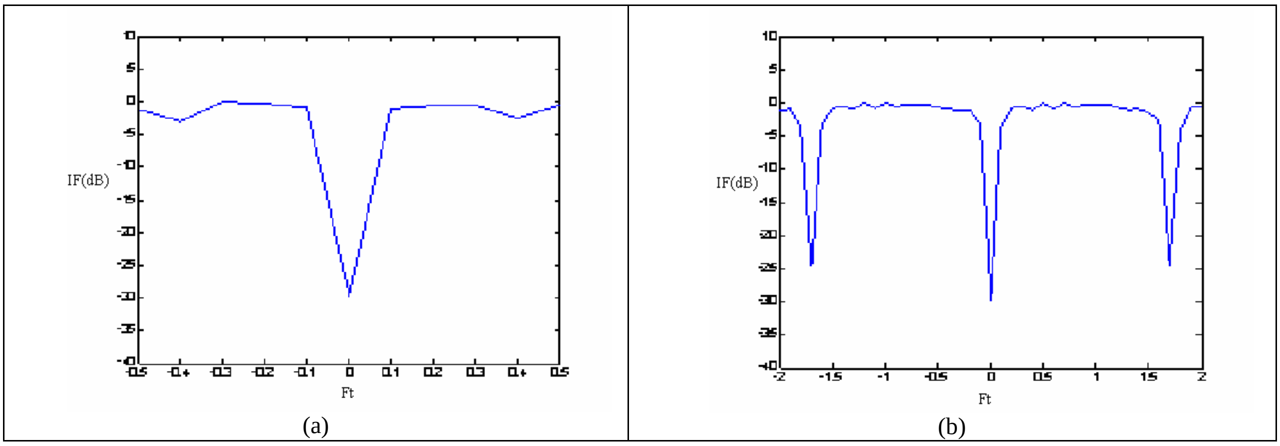

Figure 4. Improvement Factor of the DFP-PC with: (a) $PRF = 8.V_R / \lambda$, (b) $PRF = 2.V_R / \lambda$

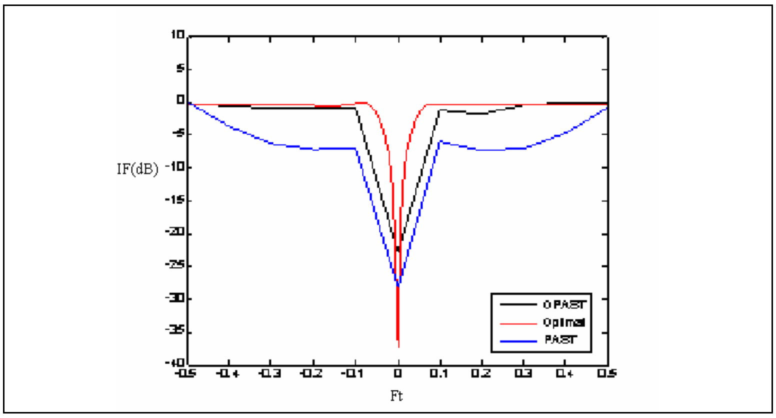

Figure 5. Improvement factor for the optimal processor and the two algorithms: OPAST, PAST

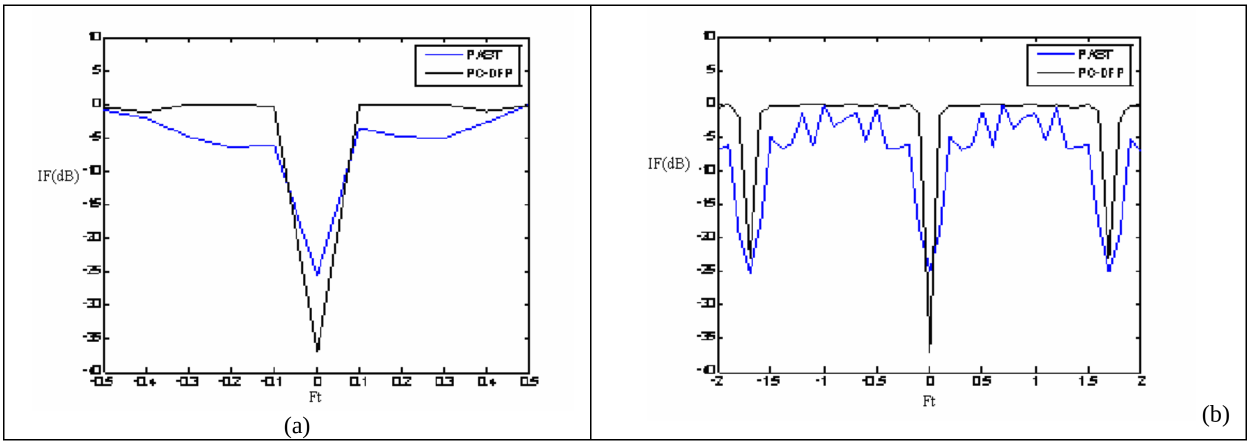

Figure 6. Improvement factor for the PAST and PC-DFP with: (a) $PRF = 8.V_R / \lambda$, (b) $PRF = 2.V_R / \lambda$

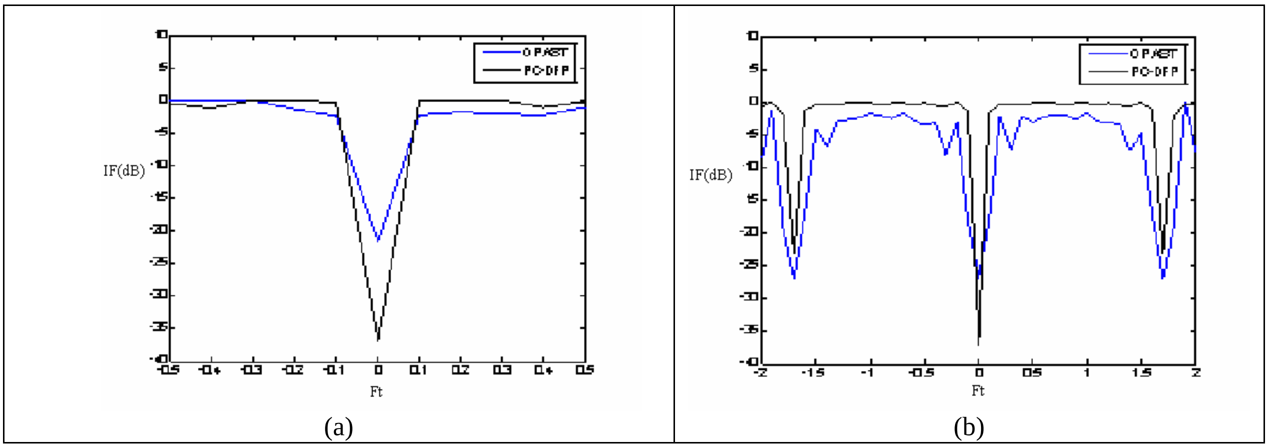

Figure 7. Improvement factor for the OPAST and PC-DFP with: (a) $PRF = 8.V_R / \lambda$, (b) $PRF = 2.V_R / \lambda$