The representation of biological systems in terms of organismic supercategories, introduced in previous papers (Bull. Math. Biophysics, 30, 625-636; 31, 59-70) is further discussed. To state more clearly this representation some new definitions are introduced. Also, some necessary changes in axiomatics are made. The conclusion is reached that any organismic supercategory has at least one superpushout, and this expresses the fact that biological systems are multistable. This way a connection between some results of Rashevsky's theory of organismic sets and our results becomes obvious.

1. Introduction. In two recent papers (Băianu and Marinescu, 1968; Comorosan and Băianu, 1969; herein afterwards referred to respectively as I and II), a new abstract representation of biological systems was introduced.

The purpose of the present paper is to make more rigorous and clear some of the points concerning the principle of choice (as introduced in I and II), and to derive some of its indirect consequences.

In I and II biological observables were not explicitly introduced. However, it seems that any biophysical model or representation has to introduce observables explicitly. The study of biological systems by means of "supercategories" can be approached from two distinct and complementary points of view. On the one hand, a biological system can be represented in terms of sequences of states, regarding the states as "primary" concepts. The hope is that the general invariants of these sequences of states will determine the general properties of biological systems. On the other hand, states will be defined in terms of
observables on the basis of a logical analysis of biological knowledge. However, there is a connection between these approaches, and this connection will be discussed here.

A number of unclarities of our representation were pointed out by Professor Rashevsky, by Professor Rosen, and by Professor Arbib. To make matters clear I shall introduce some corrections and precise definitions. In order to do it some basic questions must be emphasized and some clarifications must be made. A crude and simple biological example will be quite sufficient. When we consider the relations among the neurons of the brain, it may be in a general sense said, that these relations are more complex than those among the liver cells. A better idea is given by separating groups of cells which have the same biological activity, and then consider the relations among these (Rashevsky 1967c, 1969). One should like to look for a few essential relations, and then to express all other relations as compositions of the essential ones. This is a very simple way which leads to an algebraic structure, and which may give a simplified insight into the biological activity of organisms. However, other properties which refer to the whole organism may be of interest, such as connectedness (Rosen, 1965). A formalization of these properties leads to a topological structure. The algebraic structure which should be assigned to the brain in order to account for its biological activity will be different from that one which should be assigned to the liver. Even more, there are some relations between the brain and the liver that should be taken into account. Thus, in order to represent the whole organism, one has to consider aggregates of distinct types of structures and connections among structures.

Aggregates of this kind were called “supercategories”. “Supercategories” may appear in many other cases when a mathematical study of complex systems is involved. These superstructures provide a better understanding of differences in structural complexities of systems. Also, any optimality principle which would consider the organism as a structure in space-time (Rashevsky, 1966, p. 293), could not avoid such superstructures. Thus, in growth and differentiation processes the degree of complexity of the organisms increases with time, although one should refer to the organism as being the same system. In order to get a very crude idea of the fact, consider the following intuitive image (Fig. 1). From a geometric point of view one would recognize these three figures as squares; in this case, the shape is a geometric invariant—all the figures are alike. However, these have not the same number of components, and thus, from a topological point of view, they are different. In order to represent both aspects, invariance and change, one should have to adopt both points of view.

A few words must be added about mathematical structures. Any unification of mathematical theories makes explicit or implicit use of the basic notion of a
structure. "The theory of structures over sets admits a more general and axiomatic form within the theory of categories and functors, and this theory of categories seems to be the most characteristic trend in present day mathematics..." (Ehresmann, 1966, p. 5).

Generally speaking, a category is a class together with a partially defined law of composition satisfying some axioms (loc. cit.). An element of a category is called a morphism and this generalizes the old notion of a mapping which was considered by Dedekind, Eilenberg and MacLane as the basic tool of mathematics (Ehresmann, loc. cit.; Eilenberg and MacLane, 1945). However, the elementhood relation $e \in S$, with $e$ denoting an element and $S$ denoting a set, is not essential in modern mathematical constructions and may be avoided in a foundation of mathematics (Lawvere, 1966). Lawvere's foundation makes explicit use of the category of categories. Supercategories of diagrams were independently introduced as general representations of systems (axioms I–IV of I, p. 629). A concrete example will provide an intuitive basis for the understanding of the necessity of Lawvere's axiomatic foundation, and for its utility in defining supercategories. Let us discuss diagram 1 of I (p. 631). Essentially the same notations are used here and are explained in more detail.

Hormonal control is established as a result of certain connections among the components of an organism. Some centers from the diencephalon ($A_1$) produce by neurosecretion the corticotropin-releasing-factor which is then transported to adenohypophyse ($A_2$) where it stimulates the release of ACTH (adrenocorticotropic hormone). ACTH acts on corticosuperrenala ($A_3$), initiating the secretion of corticosteroids, which are then transported by the circulatory flow to the tissues. Through a feedback mechanism ($v_{32}$) the activity of adenohypophyse depends on the level of corticosteroids concentration in the circulatory flow. Also, it affects the level of neurosecretion ($v_{31}$). There is a morphism $\alpha$ in diagram 1 which connects the activity of $A_3$ with both feedback circuits. From this diagram and from equation (12) of I it may be inferred...
that a finer control of this activity could be performed by an upper cycle $A_1 \xrightarrow{u_{21}} A_2$, where $u_{21}$ would represent a neural path which must have only stimulatory synapses with some centers from $A_1$. Let us discuss in more detail the underlying concepts of this diagram. From diagram 1 it may be seen that the main discussion concerns connectedness and ordered connections (arrows). If one goes further, then one has to consider some rules which define the composition of these connections. These rules may be formulated as conditions of coincidence for the vertices of the connections that are to be composed. Such rules are called laws of composition. The most simple of these laws is the categorical law of composition, which requires that the end of a morphism $f$ must coincide with the source of another morphism $g$ if the two morphisms are composable. This law of composition is illustrated by the triangular diagram

$$
\begin{array}{c}
\circ \\
\circ \\
\circ \\
\end{array}
$$

(The morphism denoted by $g \circ f$ is dotted to indicate that it comes last, as a construction based on $g$ and $f$.)

Composition laws may also be formulated by means of some unary function symbols, and Lawvere’s axiomatics proceeds this way. The advantage is that his axiomatic system may be extended in such a way as to define other composition laws, distinct from the categorical composition law. Now, making use of Lawvere’s elementary theory of abstract categories (loc. cit.) I shall give precise definitions instead of definitions 6, 7 and 9 of I. For a better understanding, Lawvere’s axiomatic system is reproduced almost in its original form.

Axioms of the elementary theory of abstract categories (ETAC-Lawvere, loc. cit.).

0. For any letters $x, y, u, A, B$, the following are formulas $\Delta_0(x) = A$, $\Delta_1(x) = B$, $\Gamma(x, y; u)$, $x = y$ ($\Delta_0$ and $\Delta_1$ are unary function symbols). These are to be read respectively, “$A$ is the domain of $x$,” “$B$ is the codomain (range) of $x$,” “$u$ is the composition $x$ followed by $y$” and “$x$ equals $y$.”

1. If $\Phi$ and $\Psi$ are formulas, then “[\Phi] and [\Psi]”, “[\Phi] or [\Psi],” “[\Phi] \Rightarrow [\Psi],” “not [\Phi]” are also formulas.

2. If $\Phi$ is a formula and $x$ is a letter, then “$\forall x[\Phi]”, “\exists x[\Phi]” are also formulas. These are to be read “for every $x, \Phi” and “there is an $x$ such that $\Phi,” respectively.

3. A string of symbols is a formula of ETAC iff it follows from 0, 1, 2, above.
By a *sentence* is meant any formula in which every occurrence of each letter $x$ is within the scope of a quantifier $\forall x$ or $\exists x$. The theorems of ETAC are all those sentences which can be derived by logical inference from the following axioms.

4. $\Delta_i(\Delta_j x) = \Delta_j(\Delta_i x)$ if $i, j = 0, 1$.

5. a) $\Gamma(x, y; u)$ and $\Gamma(x, y; u') \Rightarrow u = u'$
   b) $\exists u[\Gamma(x, y; u)] \Rightarrow \Delta_1(x) = \Delta_0(y)$
   c) $\Gamma(x, y; u) \Rightarrow \Delta_0(u) = \Delta_0(x)$ and $\Delta_1(u) = \Delta_1(y)$.

6. Identity axiom:
   
   $\Gamma(\Delta_0(x), x; x)$ and $\Gamma(x, \Delta_1(x); x)$.

7. Associativity axiom
   
   $\Gamma(x, y; u)$ and $\Gamma(y, z; w)$ and $\Gamma(x, w; f)$ and $\Gamma(u, z; g) \Rightarrow f = g$.

Now, with these axioms in mind, it may be seen that axioms $C_1$, $C_2$, $C_3$, $C_4$ in definition 1 of I comprise abbreviated formulas of ETAC. Thus $A \rightarrow B$ means $\Delta_0(f) = A$ and $\Delta_1(f) = B$; $g \circ f$ means $\Gamma(f, g; h)$;

![Diagram](attachment:image.png)

means

$$\Delta_0(f) = \Delta_0(h) = A \text{ and } \Delta_1(f) = \Delta_0(g) = B \quad (1)$$

and

$$\Delta_1(g) = \Delta_1(h) = C \text{ and } \Gamma(f, g; h).$$

Commutative diagrams are regarded as abbreviated formulas, signifying the associated systems of equations as (1) above. These diagrams have the advantage of a geometric-intuitive image of the underlying equations. For example, the associativity axiom $C_3$ of def. 1 of I becomes clear on contemplat-
ing the following commutative diagram made up of four elementary triangles of the above sort.

This simple example given by Lawvere (loc. cit.) corresponds precisely to the less intuitive diagram of $C_9$ of I. However, axiom $C_4$ of def. 1 of I must be completed; a complete formulation is given by the identity axiom (6) or equivalently by the following commutative diagrams:

\[
\begin{array}{c}
\begin{array}{c}
1_A \text{ such that }
\end{array}
\begin{array}{c}
\begin{array}{c}
1_A \\
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
A
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
f \\
g
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
A
\end{array}
\end{array}
\end{array}
\end{array}
\]

for any $A, f, g$, as in the above diagrams. (1$_A$ is completely defined by these diagrams.)

Obj ($A$) is another abbreviated formula which means:

- a) $A = A_0(A) = A_1(A)$,
- b) $\exists x[A = A_0(x)]$ or $\exists y[A = A_1(y)]$,
- c) $\forall x\forall y[\Gamma(x, A; u) \Rightarrow x = u]$ and $\forall y\forall v[\Gamma(A, y; v) \Rightarrow y = v]$.

These three formulas express a common property of $A$, that of being an object. Ob $C$ of def. 1 of I contains only objects of this type. In these terms a category is understood intuitively as any structure which is an interpretation of ETAC. A functor is understood as a triple consisting of two categories and of a rule $F$ which assigns to each morphism $x$ of the first category, a unique morphism $F(x)$ of the second category in such a way that conditions $F_1, F_2$ of def. 2 of I are fulfilled. These conditions are interpretations of the corresponding conditions from ETAC (see p. 4, loc. cit.). In order to proceed further the category of all functors is considered; the world of all functors is in fact a metacategory, or a large category (supercategory, in the classical terminology). Lawvere has shown which axioms must be added to ETAC in order to get a starting point for further investigations into the structure of categories. According to Lawvere, a set is defined as a discrete category. (That is, every morphism in a set is an object (loc. cit.).) This definition suggests that sets are relatively poor structures. However, it must be noticed here that Rashevsky’s theory of organismic sets deals with more complex structures over sets which are sometimes implicit in the logical formulations. A specific example will be given on pages 550-552. For this reason I shall prefer to introduce explicitly the basic structures. An advantage of this choice, will be the gain in operationality.
Let us reconsider diagram 1 of I. In the terminology of Eilenberg and MacLane (1945), the composition law (p. 632 of I) is defined as a mapping $D(A_3) \times D(A_3) \rightarrow R(A_3)$ with $D(A_3)$ the set of morphisms which have the same range $A_3$, and $R(A_3)$ the set of morphisms which have the same range $A_3$. Correspondingly, there may be introduced two new unary function symbols $\Delta_0$ and $\Delta_1$ which express this composition law by the equations:

$$\Delta_0(u_2^+_{23}) = \Delta_0(u_1^+_{23}) = \Delta_1(u_o),$$

or equivalently $u_1^+_{23}, u_2^+_{23} \in D(A_3)$ and $u_o \in R(A_3)$. (See p. 632 of I.) The symbol $\square$ must be replaced by the categorical composition law inasmuch as we have $\Delta_1(u_o) = \Delta_1(i_3)$ and $\Delta_0(u_o) = \Delta_0(u_{23})$. Also the composition law $\varpi$ may be defined as a mapping $C(A_2, A_3) \times C(A_2, A_3) \rightarrow C(A_2, A_3)$ where $C(A_2, A_3)$ stands for the set of morphisms between $A_2$ and $A_3$ [it is sometimes denoted by $H(A_2, A_3)$ or by $\text{hom}(A_2, A_3)$]. The symbol $\varpi$ may be also defined by equations of the type

$$\Delta_0(u^0_{23}) = \Delta_0(u^+_{23}) = \Delta_0(u_{23})$$

and

$$\Delta_1(u^0_{23}) = \Delta_1(u^+_{23}) = \Delta_1(u_{23}).$$

This simple example suggests that a supercategory must comprise categories, terms which do not contain the elementhood relation and some composition laws to operate on its morphisms. The basic idea is to start with metadiagrams of the type

![Diagram](image)

where the two outer diagrams belong to distinct categories and the dotted morphisms simply connect these diagrams but do not belong to any of these. A specific example is provided by the morphisms which assign to a given topological space its corresponding homological groups.

Now, if we consider the property of being self-reproducing as an essential property of biological organisms or of pairs of biological organisms, then we have to introduce this property axiomatically. However, in an organism there are some highly specialized cells which do not reproduce themselves after the specialization is completed. Consequently, we must allow the construction of new objects which are not self-reproducing, from the self-reproducing ones.
Thus, triples which consist of categories, entities which are self-reproducing and terms which do not contain the elementhood relation, will be introduced first. Then, *supercategories* are constructed from such triples and morphisms among these. Definitions follow.

(D1). a) A "class" (when the word is used in quotation marks) is a triple $K = (C, \Pi, N)$, where

- $C$ is an arbitrary category;
- $\Pi$ is a category of $\pi$-entities;
- $N$ is a non-atomic expression.

Explanations follow.

b) $\pi$-entities were introduced by Löfgren as "complete self-reproducing entities" subject to the negation of the axiom of restriction:

$$\exists S: (S \neq \emptyset) \land \forall u: [u \in S] \Rightarrow \exists v: (v \in u) \land (v \in S)$$

which is known to be independent from ordinary logical-mathematical-biological reasoning. The above axiom says that there exists a nonempty set $S$, such that for any element $u$ of $S$, there is an element $v$ of $u$ which is also an element of $S$. An *atomically self-reproducing entity* is a unit class relation $\pi$ such that:

$$\pi \pi < \pi > \quad (\pi \text{ stands in the relation } \pi \text{ to } \pi),$$

$$\pi \pi < \pi, \pi >, \pi \pi < \pi, \pi, \pi >, \text{ etc.}$$

(Löfgren, 1968). A *symbiotically self-reproducing pair* of distinct entities $\pi_1$ and $\pi_2$ is defined by the equalities

$$\pi_1(\pi_2) = <\alpha, \beta, \ldots, \pi_2>; \quad \pi_2(\pi_1) = <\alpha', \beta', \ldots, \pi_1>.$$  \hspace{1cm} (3)

That is, $\pi_1$ is the behavior function of an automaton that reproduces the entity $\pi_2$. From the input $\pi_2$ the automaton $\pi_1$ thus produces a sequence of constructs $\alpha, \beta, \ldots$, that ends up in the final output $\pi_2$ (Löfgren, 1968). I shall introduce first a few changes in this formalism. Let $\pi_1, \pi_2, \ldots, \pi_n$ be objects in a category $\Pi$. The identity morphisms $1_{\pi_1}, 1_{\pi_2}, \ldots, 1_{\pi_n}$ will be interpreted as *descriptions* of $\pi_1, \pi_2, \ldots, \pi_n$, respectively. A morphism $\pi_1 \rightarrow \pi_2$ will represent the fact that the automaton $\pi_1$ produces finally the output $\pi_2$. A morphism in an epsilon-graph, from an object $X$ to an object $Y$ means $X \in Y$. The composition of morphisms has natural interpretations according to the axiom of restriction.

c) A *connected category* $O^+$ is a category in which any two objects are connected by a chain of morphisms, no matter how the morphisms of the chain are oriented. A connected category in which no morphism is interpreted as an elementhood relation, will be called a *non-atomic expression*. 
Now, suppose that there are some "classes" $K_0 = (C_0, \Pi_0, N_0)$ such that $(C_0, \Pi_0, N_0) = C_0$, for some categories $C_0$. This equality will be taken as an axiom and will be referred to as $K_1$. Def. 6 of I is now replaced by D1 above (see a). Making use of ETAC and D1, an elementary theory of abstract supercategories (ETAS) will be introduced. As a consequence def. 7 of I will have a precise formulation. Let us consider first a biological example.

If $F$ represents a cell which after some transitions leads to a symbiotically self-reproducing pair $\langle \pi_1, \pi_2 \rangle$ and if after $z$ steps it is transformed in a pair which is not self-reproducing, the situation may be represented by a transition of a "class" which comprises the self-reproducing pair into another "class" which comprises a pair which is not self-reproducing. Equivalently, the process may be represented by the following diagram

\[
\begin{array}{ccc}
F & \xrightarrow{T} & \pi_1 \\
\downarrow & & \downarrow z \\
\langle \pi_2, \langle \cdots, \pi_2 \rangle \rangle & \xleftarrow{\pi_1, \langle \cdots, \pi_1 \rangle} & \pi_2 \\
\end{array}
\]

where $(T, Z)$ is the pair which is not self-reproducing. For example, $(T, Z)$ would correspond to higher metazoan brain cells which do not reproduce themselves after the specialization is completed (Rashevsky, personal communication). The symbiotically self-reproducing pair may correspond to the DNA ~ DNA-polymerase system in a cell. Let us denote the above introduced "classes" by $K_+$ and $K^+$. Thus, $K_+ = (F, (\pi_1, \pi_2), N_0)$, and $K^+ = ((T, Z), \Pi_0, N_0)$, with $N_0$ and $\Pi_0$ being void categories, that is, which have no morphisms. This representation introduces a new morphism $K_+ \rightarrow K^+$ (the transition from the "class" $K_+$ to the "class" $K^+$), and consequently a new type of structure. The expression $K_+ \rightarrow K^+$ will be called a supercategory. Now follows the general definition of a supercategory, which will be given in terms of the elementary theory of supercategories.

**Axioms of the elementary theory of "supercategories" (ETAS).**

S1) For any "classes" $K_+, K^+$ and any letters $k_+, k^+, z$, the following are formulas:

\[
\begin{align*}
\Delta_0(k_+) &= K_+; \\
\Delta_1(k^+) &= K^+; \\
\bar{\Delta}(k_+, k^+, z) &= K^+; \\
\bar{\Delta}(w; x, y, \ldots) &= K_+ \text{ or in } K^+, \quad k_+ = k^+.
\end{align*}
\]
These formulas are to be read as in axiom 0 of ETAC, replacing the letters \( A \) and \( B \) by the “classes” \( K_+ \) and \( K^+ \). \( \bar{I}(w; x, y, \ldots) \) is to be read: “for a given letter \( w \) there is an assignment \( w \rightarrow (x, y, \ldots) \), with \( x, y, \ldots \) being in \( K_+ \) or in \( K^+ \).”

\[ S_2 \] a) If \( \Delta_0(z) = K_+ \), there is a letter \( x \) in \( K_+ \) such that
\[
\Delta_0(x) = \Delta_1(x) = A, \text{ with } A \text{ being an object in } K_+;
\]
b) If \( \Delta_1(z) = K^+ \), there is a letter \( y \) in \( K^+ \) such that
\[
\Delta_1(y) = \Delta_0(y) = B, \text{ with } B \text{ being an object in } K^+.
\]

\[ S_3 \] If \( \Phi \) and \( \overline{\Phi} \) are formulas of ETAS, and \( k \) is a letter, then: “[\( \Phi \)]” and “[\( \overline{\Phi} \)]”, “[\( \Phi \)] or [\( \overline{\Phi} \)]”, “[\( \Phi \)]  \Rightarrow [\( \overline{\Phi} \)]”, “\( \forall k[\Phi] \)”,” “\( \exists k[\Phi] \)” are also formulas.

\[ S_4 \] A string of symbols is a formula of ETAS if and only if this follows from \( S_1, S_2, S_3 \).

The theorems of ETAS are all those sentences which can be derived by logical inference from these axioms and from the following axiom:

\[ S_5 \] a) \( \bar{I}(k_+, k^+, z) \) and \( \bar{I}(k_+, k^+, z') \Rightarrow z = z' \);

b) \( \exists z[\bar{I}(k_+, k^+; z)] \iff \Delta_1(k_+) = \Delta_0(k^+) \);

c) \( \bar{I}(k_+, k^+; z) \Rightarrow \Delta_0(z) = \Delta_0(k_+) \) and \( \Delta_1(z) = \Delta_1(k^+) \).

Axioms \( S_1, S_2, S_5 \) become clear on contemplating the following metadiagram (supercategorical diagram):

(D2). Any structure which is an interpretation of ETAS will be called a supercategory.

In order to obtain a simplified insight into the structure and dynamics of a “complex” system such as a biological or a social system, one has to start with a rather simple structure and then to construct from this simple structure more complicated structures. The next section introduces a formal mean to deal with such problems on the basis of the above definitions.

2. Generators. Besides the basic idea of a structure (Bourbaki, 1958), the concept of a generation of higher type structures and larger classes is very im-

\[ X \rightarrow A \]

\[ k_+ \rightarrow B \]

\[ Y \rightarrow K^+ \]
important. This concept may be used to obtain a simplified insight into the
dynamics of systems. The generation of structures of higher types often
implies the presence of some adjoint functors (Lawvere 1967, 1969). Now, it is
usual in algebra to define theories by *generators and relations* (Eilenberg and
Wright, 1967). A rather simple example is provided by a group. Some
groups may be generated by *multiplication* from a set of their elements. The
elements of this set are called *generators*. Such considerations may lead to
more general concepts concerning generating procedures. One of these is
introduced here.

(D3). A “class” $K$ together with some rules of transformation (which may be
as well functors), will be called a generating class.

If we take $K = K_0 = C_0$, and if $C_0$ is a discrete category then a $T$-algebra $A$
(Eilenberg and Wright, 1967, p. 3) may be defined as a particular generating
class $(A, R)$ with $A = C_0$ and $R$ a rule which assigns to each $\phi: [n] \to [p]$ in the
theory $T$ and to each $p$-tuple $(x_1, \ldots, x_p)$ of elements of $A$, an $n$-tuple $(x'_1,
\ldots, x'_n) = (x_1, \ldots, x_p)\phi$ of elements of $A$, which are also subject to some
axioms. (Here by $[n]$ was denoted the set $\{1, \ldots, n\}$.) Another example is
provided by the representation of analogous systems in terms of observables
(Rosen, 1968a). The one-dimensional harmonic oscillator was shown to be
represented analogously in terms of observables of the system comprising a
free gravitating particle. The equations of motion of the two systems are,
respectively:

$$\begin{cases}
kp = Q \\
Q = P
\end{cases} \quad \text{and} \quad \begin{cases}
\dot{p} = \hbar; \\
\dot{q} = \rho.
\end{cases} \quad (4)$$

Let there be two small categories, the morphisms of which are, respectively,
$S_1 \xrightarrow{Q} R$, $S_1 \xrightarrow{P} R$, and $S_2 \xrightarrow{r} R$, $S_2 \xrightarrow{\rho} R$, with $S_1$, $S_2$ the corresponding state
spaces of the two systems, and $R$ the set of real numbers. The quantities $Q$, $P$
and $q$, $p$, $\hbar$ are observables of the two systems $(S_1, \{f_i\})$ and $(S_2, \{g_i\})$. Two
diagrams may be constructed with these morphisms as objects:

![Diagram D1](image1)

![Diagram D2](image2)

such that equations (4) are obtained from the diagrams taking $Y = d/dt$. Now, if $R$ is organized as a discrete category, then covariant functors $F$, $G$
from $D_1$ and $D_2$ to $R$ may be considered. Thus, natural transformations (or functorial morphisms (p. 628, def. 3 of I) $F_i \xrightarrow{\eta_i} F_j$, $G_i \xrightarrow{\epsilon_i} G_j$ will define in fact the 1-parameter families of natural transformations $\{f_t\}$, $\{g_t\}$ with $f_t$: $S_1 \rightarrow S_1$ and $g_t$: $S_2 \rightarrow S_2$. The couples $(D_1, \eta_t)$, $(D_2, \epsilon_t)$ are particular cases of generating classes. It may be noted that there is a monomorphism $\text{Ob} D_1 \rightarrow \text{Ob} D_2$ and an isomorphism $\text{Fl} D_1 \rightarrow \text{Fl} D_2$. Even more, the type of the operators involved ($Y = d/dt$) is left invariant (is preserved by a functor $D_1 \rightarrow D_2$). These may be equivalent conditions to those of Lemma 2 and Lemma 4 (Rosen, 1968a) which ensure the existence of an analogy between systems $S_1$ and $S_2$. Thus, the generating class $(\rho(\text{Ob} D_1), s^{-1})$, with $\rho(\text{Ob} D_1) \subset \text{Ob} D_2$, and conforming to the above mentioned conditions, allows the generation of $D_1$. This shows that the generating diagram of the first system is, up to an isomorphism, among the diagrams generated by the generating diagram of the second. (Note that $h$ is a constant mapping $S_2 \rightarrow R$ such that $h(s) = h$ for any $s \in S_2$.) The above example suggests a more general definition of an observable: as a morphism. Sometimes, it will be more convenient to consider this morphism in the category of categories, that is, to define an observable as a functor. These observables connected in a diagram, lead to a generating class of the system under consideration. Also, this representation provides a natural formalization of linked observables (Rosen, personal communication). The next example shows that this definition may be applied in a specific form to the study of organismic sets, revealing their basic structure. Let $S_0$ be an organismic set of $N$ elements $e_i$, $i = 1, 2, \ldots, N$. Then according to axiom 1a) of (Rashevsky, 1967a) each $e_i \in S_0$ is characterized by $m_i$ potential activities $a_{i1}$, $\ldots$, $a_{im_i}$. Let $S_i^{(a)}$ be the set of all activities: $S_i^{(a)} = \{a_{i1}, \ldots, a_{im_i}\}$. According to 1b of Rashevsky (1967) the sets $S_i^{(a)}$ may result in a number $r_i$ of products (in the economic sense), $p_{i1}, \ldots, p_{ir_i}$. Let $S_i^{(p)} = \{p_{i1}, \ldots, p_{ir_i}\}$ be the set of all products $p_{ik}$. These sets are subject to a number of conditions (axioms 1c–1f, and postulates 1–6, loc. cit.). I shall construct with these sets two other sets

$$S^{(a)} = \{S_i^{(a)} \mid i = 1, 2, \ldots, N\}$$

and

$$S^{(p)} = \{S_i^{(p)} \mid i = 1, 2, \ldots, N\}.$$
$S^{(p)}$ are actually assigned to the corresponding $e_i$. Thus, we are forced to consider the products

$S_i^{(a)} \times S_i^{(p)}, (S_i^{(a)} \times S_i^{(a)} \times S_i^{(a)}), (S_i^{(a)} \times S_i^{(a)} \times S_i^{(p)}) \ldots, (S_i^{(a)} \times S_i^{(a)} \times \ldots), (S_i^{(p)} \times S_i^{(p)} \ldots),$ 

where $m = \max m_i (i = 1, \ldots N)$ and $r = \max r_i; (i = 1, \ldots, N)$. Denote $S_i^{(a)} \times S_i^{(p)}$ by $S_{ap}$; denote $(S_i^{(a)} \times S_i^{(a)} \times S_i^{(p)} \times S_i^{(p)})$ by $S_{ap}^{(2)}$, and so on. $S_{ap}^{(0)}$ will be denoted by 1 (that is when each $S_i^{(a)} = \emptyset$, and each $S_i^{(p)} = \emptyset$). With these sets an algebraic theory $S$ may be constructed. Its objects are 1, $S_{ap}$, $S_{ap}^{(1)}$, and its morphisms are $S_{ap} \leftarrow 1, \ldots, S_{ap}^{(r)} \rightarrow S_{ap}$, acting separately on each member of the products which is written between parenthesis. Now, if $S_0$ is considered as a discrete category, there are some functors $S_0 \rightarrow S$ which define each $e_i$, at each moment, in terms of activities and products according to 1a–1f (loc. cit.). This, in turn, implies that functors $S_0 \rightarrow S$ may be seen as coarse observables of $S_0$. The category $S_0^S$ of covariant functors $F^{-1}$ (which commute with products), from the algebraic theory $S$ to the discrete category $S_0$, is an algebraic category (Lawvere, 1963), as it may be seen by comparation with the following definition.

(D4). The category of covariant functors (which commute with products) from the algebraic theory $A$ to the category of sets, the morphisms of which are the natural transformations between these functors, is called the associated algebraic category of $A$.

An algebraic category will be denoted by $\overline{A}$ or by $\overline{Ens}$, with $\overline{Ens}$ being the category of all sets and mappings between sets. In order to pass from the coarse observables $F$ to fine observables, like those introduced by Rosen (1968a), we have to consider intensities of the activities $a_i$, that is, mappings $S_i^{(a)} \rightarrow a_i \rightarrow R, S_i^{(p)} \rightarrow R$, with $R$ being the set of real numbers. Thus, the connection between coarse observables and fine observables is given by the commutative diagram

\[
\begin{array}{ccc}
S & \xrightarrow{F} & S \\
\downarrow & & \downarrow \Psi \\
S_0 & \xrightarrow{x} & R \\
\end{array}
\]

with $\overline{R}$ being the algebraic theory whose objects are 1, $R, R^2 = R \times R, \ldots, R^n = R \times R \times \cdots \times R (n \text{ times})$. In the above diagram functors $x = \Psi \circ F$
play the rôle of some generalized observables, and are natural in the sense of Eilenberg and MacLane (1945), that is, $F$ stands as a functor for any category $S_0$ as defined above and for any categories $S, R$, of the type indicated above; even more, this diagram joins objects which did not seem to be related previously. These observables remove partially an asymmetry of the triplet, physics, biology, sociology (Rashevsky, 1967, p. 151). The following theorem gives a characterization of the categories $S^S_0$ which are associated with organismic sets.

**Theorem.** Any category $S^S_0$ has generators, kernels, cokernels, products, coproducts, pullbacks, limits and colimits.

**Proof.** The proof is immediate by taking into account the fact that the category $S^S_0$ conforms to definition (D4) and thus is an algebraic category. Any algebraic category has the above mentioned properties (Georgescu and Popescu, 1968: Corollary 2, Theorem 1, Corollary 1 and Theorem 3, respectively), and consequently, $S^S_0$ has the aforesaid properties. Q.E.D.

This characterization gives the general algebraic properties of the coarse observables of an organismic set. Their particular interpretations may reveal interesting properties of organismic sets. In a subsequent paper, Rashevsky (1968a) introduced an organismic set $S_0$ as the union of three disjoint subsets $S_{01}, S_{02}$ and $S_{03}$. The functioning of the core $S_{01}$ is essential for the functioning of $S_0$. The discrete category $S_{01}$ together with three functors $G_1, G_2, G_3$ may "generate" a complete organismic set $S_0$. Thus, if we take $G_1(S_{01}) = S_{02}$, $G_2(S_{02}) = S_{03}$ and $G_3$ as the classical union "$\cup$" of sets, then by sequential application of $G_1$, $G_2$ and $G_3$ a complete $S_0$ is obtained. Consequently, $[S_{01}; G_1, G_2, G_3]$ may be considered a generating class of $S_0$.

(D5). The generating classes of an organismic set will be called organismic classes.

Now, consider the case of an organismic set which is generated by an aggregate $(\tilde{G}_1, \tilde{G}_2, \ldots, \tilde{G}_k)$ of organismic classes. If the members of this aggregate are in fact distinct categories together with functors, and if there exist some connections among these, then a supercategory may be constructed with these organismic classes. This supercategory will be called a generating organismic supercategory and will be denoted by $S_\sigma$. For example, the structural genes of a cell could be represented by sequential machines or by categories associated with semigroups of states of the sequential machines. The regulatory genes could be adequately represented by topological spaces (control spaces) or by subcategories of the category "Top" of topological spaces. The totality of structural and regulatory genes would be then represented by a generating organismic supercategory of semigroups and topological spaces.
Generating classes could have wide applications in developmental biology. Two examples will be given to illustrate this idea. The development of an organism from the ovum may be viewed as the generation of a supercategory from its generating classes. In doing this the term II of the "class" will play a major role. The process of generation will involve the enlargement of the corresponding "state space" of the organism. A rather simple but important question is the following. Could we realize a mathematical construction such that this construction enlarges the state space on the one hand, and continuously maintains stability on the other hand?

It must be noted first that the enlargement of the state space may take place in different ways, some of which are equivalent, that is, lead to the same final result. Thus, if $A$ is a fertilized ovum, while $P$ is a stage of the organism developed from it, then diagrams of the type

$$
\begin{array}{c}
\begin{array}{c}
A_1 \\
A
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
P \quad \quad \quad A_2 \\
A_1 \quad \quad A
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
A_1 \\
A
\end{array}
\end{array}
$$

will correspond to a polymorphism depending on the route which the development of the ovum takes after a stage $P$ (as in the case when from the same ova laid by a queen bee are obtained the other queens, drones and asexual workers; this example was suggested by Professor Rashevsky). In order to realize it $A$ must be a fixed object, while $P$ must have some additional properties. This example corresponds exactly to the categorical concept of a pushout.

(D6). Given two morphisms $A \overset{a_1}{\longrightarrow} A_1$ and $A \overset{a_2}{\longrightarrow} A_2$ with a common domain, a commutative diagram

$$
\begin{array}{c}
\begin{array}{c}
P \\
A_1 \\
A
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
A_2 \\
A
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
A_1 \\
A
\end{array}
\end{array}
$$

is called a pushout $\bar{P}$ for $a_1$ and $a_2$, if for every pair of morphisms $A_1 \overset{\beta'_1}{\longrightarrow} P'$ and $A_2 \overset{\beta'_2}{\longrightarrow} P'$ such that $\beta'_1 \circ a_1 = \beta'_2 \circ a_2$, there exists a unique morphism $P \overset{\gamma}{\longrightarrow} P'$ such that $\beta'_1 = \gamma \circ a_2$ and $\beta'_2 = \gamma \circ \beta_2$. If $P'$ is also a pushout for $a_1$ and $a_2$, there is a morphism $P' \overset{\gamma'}{\longrightarrow} P$ such that $\beta'_1 = \gamma' \circ \beta_0$ and $\beta'_2 = \gamma' \circ \beta_2$. 

7—B.M.B.
Now if we return to the previous example, then it may be seen that $P$ corresponds to the stage when the routes separate in such a manner that $P', P'', \ldots$ correspond to the final organisms developed from the same ova. If also $\bar{P}'$ is a pushout then the stage $P'$ is isomorphic to the stage $P$, and this corresponds to the fact that in this case the ways which lead to $P$ and $P'$ are equivalent. In the above diagrams, morphisms represent transitions from one stage to the other.

The next example shows that pushouts may be used as well to represent stable systems. However, a precise statement has to make explicit the meaning of stability, and this concept implies the idea of recovering of some states for certain intervals of time. A precise definition of stability will be given in the next section, after this example. A virus in an unfavourable environment may survive a very long time. This suggests that all transitions of states lead finally to a stable field $\{P', P'', \ldots\}$ as in the following diagram

\[ A \rightarrow A_1 \rightarrow P \rightarrow A_2 \rightarrow P' \rightarrow P'' \rightarrow \ldots \]

If this diagram is considered as a graph, then it may be seen that the graph has two cycles. Such graphs were called “kinematic” (Ashby, 1965), and give only a topological insight. Now, if a virus with a monostrand nucleic acid enters a cell, then some dynamical constraints are eliminated (the protein coat of the virus), thus generating a new stable field $\{P'_+, P''_+, \ldots\}$. Then, both $\bar{P}$ and $\bar{P}_+$ are pushouts, and transitions may take place between them: $P \Rightarrow P_+$. Indeed, after the infection the virus is able to recover its stable field $\{P', P'', \ldots\}$. Apparently, the two systems (the virus and the cell), do not have to be considered as two systems in interaction, but as a single larger system. However, as far as the generating organismic supercategories of the two systems do not change as a result of their interaction, the virus and the cell may be considered as distinct systems although their first order structure (state space) is changed. Thus, the associated generating organismic supercategories are in fact second order structures which are invariant with respect to systems interactions. Only when these generating organismic supercategories are changed I shall speak of the change of the system as a whole. The same procedure could work in the case of evolutionary systems. This time, a third order structure, the generating organismic supercategory $\bar{S}$ of a large class (in the biological sense) of organisms has to be considered. Then $\bar{S}$ will be invariant for all
organisms in the considered class, generating them one by one. Thus the idea of the fundamental biogenetic law or (exclusive) the principle of biological epimorphism (relational invariance), (Rashevsky 1967c, 1968c) is naturally obtained. Also, from the definition of a generating class, it may be seen that the number of relations among the elements of an organismic set increases with its development (generation). Realizations of these situations are left to the reader as an exercise. The above discussed fact was suggested by Rashevsky as a new biological principle (Rashevsky, 1968b). In a general model of biological systems generating diagrams of generating organismic supercategories would be considered and these would have to be connected in metadiagrams; morphisms in these metadiagrams would represent the connection between structures of distinct orders:

\[
\begin{align*}
S^0_\sigma & \longrightarrow S \\
\text{or} & \\
\text{or} & \\
\vdots & \\
S^0_\sigma & \longrightarrow
\end{align*}
\]

In the above metadiagram by \( S^0_\sigma \) denotes the generating organismic supercategory of zeroth order which generates an organismic set of zeroth order, and \( S^{(n)}_\sigma \) denotes a generating organismic supercategory of \( n \)th order.

3. Multistable Systems. The previous discussion suggests a more general definition of dynamic systems in categorical terms. On the basis of such a definition, stability may be also defined in terms of observables rather than in terms of "equilibria of forces." Thus, a biological system may remain "stable" although its first order structure or configuration is changed. What remains invariant in this case is the associated supercategory \( S_\sigma \). A specific example is provided by an organism which develops from an ovum. New cells may appear from the old ones, new relations arise among the cells and thus, new configurations appear. However, the whole organism is considered stable, regardless of the stage of its development. A change of configuration should be represented by changes in the associated type of algebraic and topological structures. Similarly, it may be said that the configurations are generated by an \( S_\sigma \). Basically, a dynamic system has to be represented in terms of states and transitions among states. In doing this, observables must be also present to account for quantities which can be or are determined by experiments.
A dynamic system $D$ is a commutative diagram of the type

$$
\begin{array}{ccc}
\mathcal{S} & \xrightarrow{L} & \mathbb{R}^n \\
\downarrow & & \downarrow \\
\mathcal{X} & \xrightarrow{} & \mathcal{T}
\end{array}
$$

with: $\mathcal{X}$—the state space of $D$ (that is, a supercategory the objects of which are states of $D$ and whose morphisms are transitions among these); $\mathbb{R}^n$—the category the objects of which are elements from $R$, $R \times R$, ..., $R \times R \times \cdots \times R = \mathbb{R}^n$, and whose morphisms are operators on real numbers; $\mathcal{S}$—a supercategory of generating classes and morphisms among these, and $\mathcal{T}$ a supercategory such that the structure of $\mathcal{S}$ depends primarily on the structure of $\mathcal{T}$ by setting a one-to-one correspondence $(\eta_{ij} : F_{ij} \rightarrow F_{ij}, L(s_i)) \rightarrow \ell$ ($\ell$ is an object in $\mathcal{T}$, $s_i$ is an object in $\mathcal{X}$ and $F_{ij} : \mathcal{S} \rightarrow \mathbb{R}$). The object $\mathcal{T}$ will be called temporal for intuitive reasons. Consider $\mathcal{X}$ and $\mathbb{R}$ as fixed objects in $\mathbb{R}$, and let $\lambda > 0$. Let us consider a functor $G : \mathcal{X} \rightarrow \mathbb{R}$ such that:

a) $G(s) = V(\ell)$ for any $V : \mathcal{T} \rightarrow \mathbb{R}$, $s$ in $\mathcal{X}$, and $\ell$ in $\mathcal{T}$;

b) $G(s') \geq \lambda$ and $G(s') - G(s'') < \lambda$ for some states $s'$, $s''$, ... in $\mathcal{X}$.

c) $G(s^+) < \mu$ and $\mu \leq \lambda$ for some states $s^+$ in $\mathcal{X}$. Then it will be noticed that an apparatus which measures time intervals with a precision $\delta \geq \lambda$ will observe only states $s'$, $s''$, ....

(D8) A category $\mathcal{X}$ which comprises only states $s'$, $s''$, ... of $\mathcal{X}$ will be subject to the foregoing conditions a), b) and will be called stable in $\mathcal{X}$.

After intervals of time which are longer than the transitions which necessarily lead from a stable state $s'$ to other stable states $s''$, $s'''$, ... according to condition b), $\mathcal{X}$ is stable in the algebraic sense, because $s'$ and $s''$, $s'''$, ... are together in $\mathcal{X}$. If we take $\mathcal{S} = \mathcal{S}_\sigma$ in (D7), then the corresponding $\mathcal{X}$ will be called organismic. It must be emphasized that a generating organismic supercategory $\mathcal{S}_\sigma$ is distinct from the corresponding organismic supercategory $\mathcal{X}_\sigma$, although they are closely related. In the following some properties of $\mathcal{X}_\sigma$ will be presented.

(D9) If there are some distinct categories $\mathcal{X}_a$, $\mathcal{X}_b$, ..., $\mathcal{X}_z$ which are stable in $\mathcal{X}$, and if they form a supercategory, then $\mathcal{X}$ is called multistable of order $z$.

Returning to the example given on page 555, it should be noted that a virus may be called ultrastable because its state changes only when some observables take values which exceed some given limits, showing a succession of transient fields concluded by a terminal field which is always stable (Ashby, 1952). Then, a multistable system consists of "many ultrastable systems joined main variable to main variable" (Ashby, 1952), and this is adequately represented by
a multistable supercategory together with its corresponding $S_\gamma$. As a consequence of this construction a multistable system gains "plasticity." The example suggests also that in the case of multistable systems there are present some distinct pushouts in $X$ which are joined by morphisms, thus forming a superpushout in $X$. A geometric image of this situation is given below

In constructing a mathematical model of a differentiating cell, it is useful to introduce "metastable" states $s^*$ (Rosen, 1968), that is, states for which $G(s^*) > \varepsilon, \mu < \varepsilon < \lambda$. If $A$ is a metastable field, $B$ a stable state and $F(X)$, $F(Y)$ unstable fields, then a colimit may be constructed with these.

(D10) Let $D, C$ be two categories and $F: D \rightarrow C$ a covariant functor. A colimit of $F$ is a pair $(A, \{u_X\})$ with $A$ being an object of $C$ and $u_X: A \rightarrow F(X)$ morphisms which are defined for each object $X$ of $D$, such that for any morphisms $\alpha: X \rightarrow Y$ in $D$, any object $B$ in $C$ and morphisms $v_X: B \rightarrow F(X)$ (defined for each object $X$ in $D$), there exists a unique morphism $v: B \rightarrow A$ which makes commutative the following diagram

In other words we have the following equations:

$$u_Y = F(\alpha) \circ u_X, \quad F(\alpha) \circ v_X = v_Y, \quad u_X \circ v = v_X. \quad (5)$$

(If the category $D$ is small, i.e. $\text{Ob} \ D$ is a set, it is also called a scheme and a functor $F: D \rightarrow C$ is called a diagram in $C$ of scheme $D$.)

Now, if a differentiating cell is considered as a multistable system, supercolimits must be introduced to represent distinct fields of stable, unstable and metastable states. According to this, the general contouring of the state space $X_\gamma$ of a differentiating cell may be visualized as below
A supercolimit may be defined as a metadiagram constructed from aggregates of colimits and the connecting morphisms among these (as morphism \( F(Y) \to G(U) \)). It is not difficult to see the connection between these definitions and the principle of choice. Previously, in II, we suggested that the mathematical form of the principle of choice would be

\[
\left\{ M_k(d_k) \right\}_{k \in K} = \cdots = \text{Lim}_{d_i \to d_j} \left\{ M_1(d_i) \right\}_{1 \leq L}\]

and

\[
\frac{\partial \phi}{\partial \theta} = (m, p_m)
\]

where \( m \) is a certain numerical matrix, \( p_m \) is the corresponding matrix of probabilities and \( \phi, \theta \) are some essential observables of a dynamic system. Then some matrices \( M \) are assigned to each diagram and (6) is a condition of extremum. Qualitatively, the principle of choice asserts that each superdiagram in \( X \) has a superlimit or a supercolimit. From these considerations the following proposition is easily derived.

**Proposition 1.** Any organismic supercategory \( X_\sigma \) has at least one superpushout.

**Proof.** The principle of choice implies that any superdiagram in \( X \) has a supercolimit. Then, from the construction of a supercolimit, there results that each diagram in \( X \) has a colimit. Categories which have a finite number of colimits for all functors over a scheme \( \Sigma \) are called finitely \( \Sigma \)-cocomplete. If these categories are \( \Sigma \)-cocomplete for all diagram schemes \( \Sigma \) then these categories are called finitely cocomplete. I shall consider only the finite case. Generally, a category which is finitely cocomplete has pushouts. Now, it results from the construction of a superpushout that \( X_\sigma \) has at least one superpushout (as far as a superpushout is made up of interconnected pushouts). Q.E.D.

When interpreted, this assertion says that any biological system comprises at least one ultrastable subsystem. Systems with “suspended” life as protozoa...
ORGANISMIC SUPERCATEGORIES: II

(Rosen, 1958) are reduced to the ultrastable subsystem. However, an \(X_a\) is generally multistable. The generation of "undifferentiated" cells may be represented by the degeneration of a supercolimit into a superpushout. The superpushout of an ultrastable system would keep indefinitely and its associated stable field \(\{P', P'', \ldots\}\) will be frozen such that the ultrastable system is "immortal" or almost "immortal." By contrast, a "differentiated" \(X_{a}\) has more "plasticity," and is "mortal" because there is an increased probability for the system to stop between two consecutive stable fields or "steady states," when the system is in an unstable state. This unstable state would be reached when the system is placed in extremely difficult conditions. This is also one of the results of Rashevsky's theory of organismic sets (1968a, Theorem 3). Another connection between this representation and the above quoted theory is established by the following statement.

**Proposition 2.** If \(T: A \to B\) is a full functor, then without any conditions on the supercategories \(A\) and \(B\), \(T\) reflects superlimits and supercolimits.

**Proof.** This is only an extension of a theorem of Freyd (quoted from Mitchell, 1965, p. 56). The reasoning is similar to that one followed in the above quotation. The fact that \(T\) has to be full, shows that any transformation which preserves the stability properties of a biological system must also have this property. This may appear as a condition which must be added to the principle of relational invariance.

**Proposition 3.** A supercategory \(X\) is \(\Sigma\)-cocomplete if and only if the functor \(I: X \to [\Sigma, X]\) has a coadjoint \(L: [\Sigma, \times] \to X\). (Here \([\Sigma, X]\) denotes the class of all superdiagrams in \(X\) over the superscheme \(\Sigma\) and this class can be made into a supercategory.)

**Proof.** Extending the concept of a scheme and the concept of an adjoint functor for the supercategorical case this proposition is a mere generalization of Mitchell's Proposition 12.1 and Corollary 12.1 (Mitchell, 1965, p. 67). An adjoint functor may be intuitively understood as a "close" comparation of two categories which preserves some important properties of the source category. This condition is necessary and sufficient for a biological system in order to reach effectively its steady-states.

**4. Discussion.** It is not difficult to see that the principle of choice as investigated here has functional aspects. One could ask if only these aspects are taken into account by this principle. Rosen (1959) suggested that in a convenient context the principle of adequate design could have structural or morphological implications. We have emphasized in II (Section D) such a
structural implication of the principle of choice. However, when structural aspects are involved, there are some implicit or explicit assertions concerning the principle of $G$-relations, which make convenient the context. Here only limited assertions concerning this principle were made. As a consequence Rashevsky's theorems 1 and 2 (1968a) cannot be proved only on the basis of the principle of choice. Those theorems seem to be related to the principle of $G$-relations. Thus, a mathematical convenient expression of the principle of $G$-relations is necessary if one wants to study the connections between these principles and their possible consequences.

The full consequences of the principle of choice may be rich in concrete cases, and a detailed mathematical study of some particular systems would provide a mean to construct on this basis multistable systems.

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