From Simple to Complex and Ultra-complex Systems: 
*A Paradigm Shift Towards Non-Abelian Systems Dynamics*

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1. **Introduction**

The development of systems theory has been so far remarkably uneven: phases of tumultuous development arousing vivid expectations have been followed by periods of stagnation if not utter regression. Moreover, within the different sciences, the theory of systems is customarily seen and presented in rather different ways. The differences are often so remarkable that one may ask whether there is in fact anything like “the” theory of systems. Thirdly, it is worth mentioning that more often than not a number of conceptual confusions continue to pester the development of system theory. Remarkably enough, during the past few decades the systems theory has reproduced in its own way the same divide and the same attitude that has characterized recent mainstream philosophy, namely the overwhelming prevalence assigned to the epistemological interpretation to their object as opposed to the ontologically-oriented analysis of their object. According to the epistemological reading, system’s boundaries are in the eye of the observer; it is the observer that literally creates the system by establishing her windowing of attention. On the other hand, the ontological reading claims that the systems under observation are essentially independent from the observer, which eventually discover, or observe, them. Most confusions can be dealt with by distinguishing two aspects of the interactions between observing and observed systems. The thesis that knowing a system, as required e.g. by any scientific development, implies appropriate interactions between an observing and an observed system, does not mean that existence or the nature of the observed system depends on the observing system, notwithstanding the significant perturbations introduced by measurements on microscopic, observed quantum systems.\(^1\)

A measuring device can be taken as one among the simplest types of observing systems (Rosen, 1968; 1994). The resulting *model* depends essentially on the device (e.g., on its sensitivity and discriminatory capacity); on the other hand, the nature of the observed system does not depend on the nature of the measuring device (which obviously shouldn’t exclude the possibility that the very process of measuring may eventually modify the observed system). Furthermore, the ontological interpretation helps in better understanding that some systems essentially depend on other systems in either a *constructive* (Baianu and Marinescu, 1973) or an *intrinsic*, sense (Baianu et al, 2006).

Higher-order systems require first-order systems as their constitutive elements, the basic idea being that higher-order systems result from the couplings among other, lower-order, systems. In this sense, melodies require notes, groups require agents and traffic jams involve cars.

This paper is divided in two main parts. The first part (sections 2-4) serves as an introduction to system theory. Our aim is to present the evolution of system theory from a categorical viewpoint; subsequently we shall study systems from the standpoint of a ‘Universal’ Topos (UT), logico-mathematical, construction that covers both *commutative* and *non-commutative* frameworks. In so

\(^1\) Needless to say, both systems – in the case of a two-systems coupling, can be observing. Furthermore, the observing system can observe itself, or parts (subsystems) of itself.
doing, we shall distinguish three major phases in the development of the theory (two already completed and one in front of us). The three phases will be respectively called “The Age of Equilibrium”, “The Age of Complexity” and “The Age of Super-complexity”. The first two may be taken as lasting from approximately 1850 to 1960, and the third being rapidly developed from the late 1960s. Each phase is characterized by reference to distinct concepts of the ‘general’ system, meant in each case to include all possible cases of specific, actual systems, but clearly unable to do so as the paradigm shifts from simple to ‘complex’, and then again to extremely complex or super-complex (previously called ‘ultra-complex’; Baianu, 2006; Baianu et al., 2006) classes of systems. Furthermore, each subsequent phase generalized the previous one, thus addressing previously neglected, major problems and aspects, as well as involving new paradigms. The second part (sections 5-?) deals with the deeper problems of providing a flexible enough mathematical framework that might be suitable for various classes of systems ranging from simple to super-complex. As we shall see, this is something still in wait as mathematics itself is undergoing development from ‘symmetric’ (commutative, or ‘natural’) categories to dynamic ‘asymmetry’, non-Abelian constructs and theories that are more general and less restrictive than any static modelling.

2. The Age of Equilibrium

The first phase in the evolution of the theory of systems depends heavily upon ideas developed within organic chemistry; ‘homeostasis’ in particular is the guiding idea: A system is a dynamical whole able to maintain its working conditions. The relevant concept of system is spelt out in detail by the following, general definition, D1.

D1. A system is given by a bounded, but not necessarily closed, category or super-category of stable, interacting components with inputs and outputs from the system’s environment.

To define a system we therefore need six items:

(1) components,
(2) mutual interactions or links;
(3) a separation of the selected system by some boundary which distinguishes the system from its environment;
(4) the specification of system’s environment;
(5) the specification of system’s categorical structure and dynamics;
(6) a super-category will be required when either components or subsystems need be themselves considered as represented by a category, i.e. the system is in fact a super-system of (sub) systems, as it is the case of emergent super-complex systems.

Point (5) claims that a system characteristics or ‘structure’ should last for a while: thus, a system that comes into birth and dies off ‘immediately’ has little scientific relevance as a system, although it may have significant effects as in the case of ‘virtual particles’, ‘photons’, etc. in physics (as for example in Quantum Electrodynamics and Quantum Chromodynamics). Note also that there are

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2 The given temporal distinctions are to be taken with a substantial grain of salt: some of the deepest aspects of equilibriums (as for e.g. structural stability) have been discovered much later than 1960; on the other hand, complex phenomena have been widely discussed well before 1960.

3 Claim (4) can be included under (2) by reformulating it as “repeated mutual interactions”.

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many other, different mathematical definitions of ‘systems’ ranging from (systems of) coupled
differential equations to operator formulations, semigroups, monoids, topological groupoids and
categories that are sub-summed by X.

Clearly, the more useful system definitions include algebraic and/or topological structures rather
than simple, structureless sets or their categories (Baianu, 1970; Baianu et al, 2006).
The main intuition behind this first understanding of system is well expressed by the following
passage:

“The most general and fundamental property of a system is the interdependence of parts or
variables. Interdependence consists in the existence of determinate relationships among the parts or
variables as contrasted with randomness of variability. In other words, interdependence is order in
the relationship among the components which enter into a system. This order must have a tendency
to self-maintenance, which is very generally expressed in the concept of equilibrium. It need not,
however, be a static self-maintenance or a stable equilibrium. It may be an ordered process of
change – a process following a determinate pattern rather than random variability relative to the
starting point. This is called a moving equilibrium and is well exemplified by growth” (Parsons
1951, p. 107).

2.1. Boundaries

Boundaries are peculiarly relevant to systems. They serve to distinguish what is internal to the
system from what is external to it. By virtue of possessing boundaries, a system is something on the
basis of which there is an interior and an exterior. The initial datum, therefore, is that of a
difference, of something which enables a difference to be established between system and
environment.

An essential feature of boundaries is that they can be crossed. There are more open boundaries and
less open ones, but they can all be crossed. On the contrary, a horizon is something that we cannot
reach or cross. In other words, a horizon is not a boundary. The difference between horizon and
boundary is useful in distinguishing between system and environment. “Since the environment is
delimited by open horizons, not by boundaries capable of being crossed, it is not a system”
(Luhmann 1984).

As far as systems are concerned, the difference between inside and outside loses its common sense,
or ‘spatial’ understanding. As a matter of fact, ‘inside’ doesn’t anymore mean ‘being placed
within’, but it means ‘being part of’ the system. One of the earlier forerunners of system theory
clarified the situation in the following way:

“Bacteria in the organism ... represent complexes which are, in the organizational sense, not
‘internal’, but external to it, because they do not belong to the system of its organizational
connections. And those parts of the system which go out of its organizational connections, though
spatially located inside it, should also be considered as being ... external.” (Bogdanov 1981,1984).

In other words, internal and external are first and foremost relative to the system, not to its location
within physical space. The situation is, however, less clear-cut in the case of viruses that insert
themselves into the host genome and are expressed by the latter as if the viral genes ‘belonged’ to
the host genome. Even though the host may not recognize the viral genes as ‘foreign’, or ‘external’
to the host, their actions may become incompatible with the host organization as in the case of
certain oncogenic viruses that cause the death of their host. This point is further elaborated in the
next paragraphs. This problem seems to us of a different nature. What one says pertains to a situation of interaction among or between systems, not to the definition of system. The fact that some systems are able to enslave other systems or to exploit them should also be treated further in a subsequent report.

Let us state also that the internal and external aspects can also be taken as features describing the difference between the world of ‘inanimate’ things or machines and the very different world of organisms, which runs against the old Cartesian ideas about the world of living animals without necessarily invoking any so-called ‘vitalism’. In the mechanistic--automaton-like, or ‘linear’ order of things or processes, the world is regarded as being made, or constituted, of entities which are outside of each other, in the sense that they exist independently in different regions of space (and time) and interact through forces, either by contact or at a distance. By contrast, in a living organism, each part grows in the context of the whole, so that it does not exist independently, nor can it be said that it merely ‘interacts’ with the others, without itself being essentially affected in this relationship. The parts of an organism grow and develop together. Nevertheless, one could make a similar argument about various regions of the spacetime of General Relativity in our inflationary Universe. Then, the more appropriate, distinguishing feature that remains for all organisms is their ability to reproduce themselves either as single entities or through sexual reproduction in pairs. Moreover, in spite of physical and chemical changes that take place in a functional, or living, organism, its dynamic organization is maintained for prolonged periods of time, and it is then propagated to future generations through complex reproduction processes that are not however merely making exact, perfect copies of the already existing organism; in the biological reproduction process there is thus an essential “fuzziness” at the molecular, genetic level, that is only partially translated into the phenotype. This characteristic and intrinsic ‘fuzziness’ (Baianu and Marinescu, 1968) is distinct from the quantum mechanical indeterminacy of all quantum systems as determined quantitatively by the Heisenberg Principle, although as pointed out by Schrödinger in his widely read book “What is Life?” the latter may also affect organismal reproduction. Thus, the ‘self-reproduction’ of organisms is quite different in nature from that of self-reproducing automata, or machines, as it is ‘fuzzy’ only to a certain degree permitted by natural selection and also necessary for organismal evolution. Without going further afield into the details of biological self-reproduction, and also how it is indeed entailed, one realizes that such an intrinsic ‘fuzziness’ does lead to a logical heterogeneity of classes of organisms- another characteristic feature of organisms that was pointed out by Elsasser. This realization leads also to necessary logical adjustments to the ‘general’ type of definition D1 of systems if one intends to apply such a general concept to organisms and formulate a Complex Systems Biology (CSB); it leads both to the introduction of an operational logic of an organism which is both many-valued and also probabilistic (Baianu, 1977). Autopoiesis (Várilly, 1997) without an operational many-valued logic is not sufficient to understand the function and stability of an organism, ecosystem or society.

Although boundaries allow the distinction between organisms to be made, and also simplify the logical heterogeneity problem to the extent of becoming a solvable one, they are neither clear-cut nor sharp, nor absolute and rigid/ixed, nor totally impermeable, because otherwise an organism whose boundaries are completely closed will die in a short time. As soon as a boundary is established, both separating and connecting the system to its environment, a second type of boundary may arise, namely the one distinguishing the centre of the system from its periphery (the former boundary will be termed ‘external’ boundary, and the latter ‘internal’). The centre, once

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4 However, note the interesting and intriguing question: ‘Is a virus dead or alive?’ which has not yet been answered satisfactorily through the molecular biology approach in spite of major advances being made in this field.
established assumes control over the system’s external boundary and can modify the boundary’s behaviour. Multi-modal systems may require a multiplicity of centres, for each of the relevant modalities. When different centres are active, a secondary induced dynamic arises among them. Boundaries may be clear-cut, precise, somewhat rigid, or they may be vague, blurred, mobile, or again they may be intermediate between these two typical cases, according to how the differentiation is structured.

The usual dynamics is as follows. It begins with vague, random oscillations. These introduce differences among the diverse areas of a region. The formation of borderline phenomena (such as surface tension, pressure, competition) only occurs later, provided that the differences prove to be sufficiently significant. Even later there arises a centre, or a node, whose function is primarily to maintain the boundaries. Generally speaking, a closed boundary generates an internal situation characterized by limited differentiation. The interior is highly homogeneous and it is distinct from whatever lies outside. Hence, it follows that whatever lies externally is inevitably viewed as different, inferior, inimical; in short, as something to be kept at a distance. A second consequence of closed boundaries is the polarization of the internal space of the system into a centre and a periphery. The extent of this problem was already noted by Spencer, who accounted for it with his ‘law of the concentration of matter-energy’. Open boundaries allow instead, and indeed encourage, greater internal differentiation, and therefore, a greater degree development of the system than would occur in the presence of closed boundaries. In its turn, a population with marked internal differentiation, that is, with a higher degree of development, in addition to having numerous internal boundaries is also surrounded by a nebula of functional and non-coincident boundaries. This non-coincidence is precisely one of the principal reasons for the dynamics of the system. Efforts to harmonize, coordinate or integrate boundaries, whether political, administrative, military, economic, touristic, or otherwise, generate a dynamic which constantly re-equilibrates the boundary situation. The border area becomes highly active, and it is in this sense that we may interpret the remark by Ludwig von Bertalanffy that “ultimately, all boundaries are dynamic rather than spatial” (Bertalanffy 1972, p. 37). However, note that in certain, ‘chaotic’ systems organized patterns of spatial boundaries do indeed occur, albeit established as a direct consequence of their ‘chaotic’ dynamics. This non-coincidence is precisely one of the principal reasons for the dynamics of the system. Efforts to harmonize, coordinate or integrate boundaries, whether political, administrative, military, economic, touristic, or otherwise, generate a dynamic that constantly re-equilibrates the boundary situation. In certain, ‘chaotic’ systems organized patterns of spatial boundaries occur as a direct consequence of their ‘chaotic’ dynamics. In such cases, the border area becomes highly active, and it is in this sense that we may interpret the remark by Ludwig von Bertalanffy that “ultimately, all boundaries are dynamic rather than spatial”, or merely spatial (Bertalanffy 1972, p. 37).

Corresponding to the ‘logic of boundaries’ is a more or less correlative ‘logic of centers’. If to every boundary there is an associated centre which is responsible for its maintenance, the dynamics of boundaries reverberates in a corresponding dynamics of centers. The multiplicity of boundaries, and the dynamics that derive from it, generate interesting phenomena. Campbell was the first to point out that boundaries tend to reinforce one another (Campbell 1958). We quote Platt on this matter:

“...The boundary-surface for one property ... will tend to coincide with the boundary surfaces for many other properties ... because the surfaces are mutually-reinforcing. I think that this somewhat astonishing regularity of nature has not been sufficiently emphasized in perception-philosophy. It is this that makes it useful and possible for us to identify sharply-defined regions of space as ‘objects‘.
This is what makes a collection of properties a ‘thing’ rather than a smear of overlapping images. ...any violation of boundary-coincidence has an upsetting fascination for us, as in tales of ghosts, which can be seen but not touched” (Platt 1969, p. 203).

The following synopsis summarizes the main structural features of system boundaries:

- cardinality (as a single boundary or as a border area)
- nature (external, internal)
- form (open/closed, mobile/static)
- structure (gates, filters and overlaps)
- dynamic (changes in location, boundary exchange)
- maintenance (integrity, growth, destruction) (Poli 2001)

To these features, the so-called law of transposition should be added: a characteristic of all structures with an emerging structure at a higher level is that its boundaries can be revealed by transposition. In conclusion, we wish to add a few ontological remarks about boundaries. The analysis set out in this section has been conducted with the deliberate omission of any reference to levels of reality (Poli 2001, 2006a, b; 2007). Boundaries are, however, associated to different levels of an object, or a system. The inclusion of explicit consideration of the problem of stratification into levels will be shown to significantly increase complexity thus leading to ‘super-complexity’ (see Section 4).

3. The Age of Complexity, Computers and Chaotic Dynamics

After the Second World War, cybernetics, game theory, information theory, computer science, general systems theory, and other related fields flourished. Subsequently, there was also the first report of a categorical approach to complex biological systems which is Robert Rosen’s seminal paper on the metabolic-replication, (M,R)-systems and the general, abstract representation of biological systems in the category of sets (Rosen,1958a,b), in a purely functional, or organizational sense. The main result achieved by the first phase of development of system theory has been the proof that the system as a whole is defined by properties not pertaining to any of its parts – a patently non-reductionist view. Equilibrium (stability, etc) are properties of the systems, not of their parts. However, much more than this is required to understand system dynamics. The simplest way to see what is lacking runs as follows. According to equilibrium theories, a system is the whole resulting from the interactions among its elements. There are at least three hidden assumptions embedded in this definition. The first assumption is that all the elements, or components, are given in advance, before the constitution of the system. We shall discuss this problem under the heading of the system’s constitution. The second assumption becomes apparent as soon as one asks what happens when the set of elements changes: What happen when an element goes out of the system? What happen when a new element enters the system? What happen when elements die out? This group of questions can be summarized as the problem of the reproduction of the system, i.e. as the problem of the continuity of the system through time, as distinguished and opposed to the continuity of its elements. The third hidden assumption is that all the changes are placed on the side of the environment. What about systems that are able to learn and to develop new strategies for better dealing with survival or other problems they may run into? Systems with this property will be called adaptive.
3.1 The Constitution of Simple and Complex Systems

Two forms of composition should be distinguished: the bottom-up type of composition from elements to the system (already seen) and the top-down form from (a previous stage of) the system to its elements. This latter form of composition comes in two guises: (1) as constraints on initial conditions and the phase space of elements, and (2) as creation of new elements, i.e. as development of a new organizational layer of the system. A more elaborate treatment of composition and constitution of complex systems of lower-level systems and simple system components in terms of meta-level theories is currently under development. Non-decomposability issues for most meta-level systems may provide also an answer as to the question of the limitations of reductionist schemes in studies of the types of complex and super-complex systems considered in this article.

3.2. Adaptive and Autopoietic Systems

3.2.1. Adaptive systems

Adaptive systems require at least two layers of organization: the layer of the rules governing the interactions of the system with its environment and with other systems and the higher-order layer that can change the rules of interaction. These changes may be purely casual, or may follow both pre-established and acquired patterns. In this regards, the hypothesis can be advanced claiming that the main difference between non-living natural systems from one side and living natural systems, psychological systems and social systems on the other side is that the former systems present only one single organizational layer of interactions, while the latter present at least two layers of organization (the one governing interactions and the one capable of modifying the rules of interaction).

3.2.2. Reproducing Systems, Regeneration, Metabolic Repair and Their Logic Entailments

Autopoiesis (or regeneration) was previously defined as the capacity of a complex system to generate the elements of which it is composed. The simplest mathematical models exhibiting such ‘biological’ capabilities are arguably Robert Rosen's (M,R)-systems (Rosen, 1958a,b). Their categorical construction using natural transformations utilizing the fundamental Yoneda Lemma elicited their implicit algebraic structures (Baianu and Marinescu, 1973), and also showed their possible extension to more general categories than Rosen’s (M,R)-system categories of sets, such as, the cartesian categories of generalized, algebraic (M,R)-systems (Baianu, 1974). Complex, molecular dynamic representations of both (M,R)-systems (Rosen, 1970, 1971, 2004) and organismic sets have also been constructed in terms of natural transformations of molecular biology systems (Baianu, 1982, 1984, 1987, 1992; Baianu et al, 2005, 2006). Such categorical, multi-level constructs of generalized, algebraic metabolic-repair/regenerating systems (Baianu, 1984, 1987; Baianu et al., 2005) illustrate the hierarchical strata occurring in super-complex biological systems and will be further discussed here in Section 8.

3.3. Complexity as an Intrinsic Organizational Property

The overall outcome of constitution, reproduction and autonomy is complexity. The guiding connection changes from the system-environment connection to the connection between the system and its complexity. Not by chance, self-referential phenomena and systems have started to receive substantial attention. Summing up, complex systems are adaptive systems capable of regeneration
making ‘copies’ of themselves by reproducing the elements they are made of. As far as self-referential systems are concerned, the guiding relation is not more the “system-environment” opposition, but the “system-system” one. Ultimately, the difference between openness and closure acquire a different meaning: now openness means exchange with the environment, whereas closure generates structure, an aspect of the system which provides its identity that organizes the system as an integral whole, or holon.

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**Figure 1. Towards Life: The Emergence of Super-Complex Dynamics.**
Single, light arrows indicate the occurrence of a transition; thick arrows are indicating either a logical or sequential implication --“leading to” the concept towards which the thick arrow points. Thick, double arrows signify either conceptual representation or equivalence; for example, systems with high global dynamic symmetry admit canonical (so-called ‘symmetry’) group representations. Further details of these processes and mathematical structures/concepts are provided in Baianu (2007a, b).

4. The Age of Super-complexity

Living systems (including among living systems not only biological systems but psychological and social systems as well) present features remarkably different from those characterizing non-living systems. We propose that super-complexity requires at least four different categorical frameworks, namely those provided by the theories of levels of reality, chronotopoids, (generalized) interactions, and anticipation.

4.1. Levels of Reality

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Note, however, that in the case of bi-sexual organisms the system self-copying, or reproduction, is always not exact, but partial, thus leaving the door widely open to further evolution. Furthermore, inbreeding usually leads to either infertility or very low survival rates of the inbred offspring.
Atoms, molecules, organisms distinguish *layers of reality* because of the causal links that govern their behavior, both horizontally (atom-atom, molecule-molecule, organism-organism) and vertically (atom-molecule-organism). This is the first intuition of the theory of levels. Even if the further development of the theory will require imposing a number of qualifications to this initial intuition, the idea of a series of entities organized on different levels of complexity will prove correct. Briefly, the difference between levels of reality and levels of interpretation requires acknowledging that the *items* composing levels of reality are endowed with their own form of *agency* (Poli, 2006b, 2007). Most details of the links connecting together the various levels of reality are still unknown, because the various sciences had mainly been working on causal links *internal* to their regional phenomena. The lack of a theory of levels of reality has been the major obstruction to the development of the needed theories. Proposal concerning the architecture of levels and their links will improve our understanding of the world and its inter-dependences.

The deepest access to levels of reality is achieved by adopting a categorical viewpoint. In short, a level of reality is represented by a group of (ontological) categories. The next step is to distinguish *universal* categories, those that pertain to the whole of reality, from *level* categories, those that pertain to one or more levels, but not to all of them. The distinction is widespread among four basic strata of reality. Even if the boundaries between them are differently placed, the distinction among the four strata of inanimate, biological, mental and social phenomena is essentially accepted by most scholars. The question now arises as to how the biological, psychological and social strata are connected together. We shall defend the option according to which biological phenomena act as bearers of both psychological and social phenomena. In their turn, psychological and social phenomena determine each other reciprocally. Psychological and social systems are formed through co-evolution, meaning that the one is the environmental prerequisite for the other. Both present a double existential dependence: firstly they both depend on their material bearer; secondly, each depends of the twin stratum: psyches require societies and societies require psyches. The next step is to articulate the internal organization of each stratum. Each stratum of reality has its specific structure. Compared to the lower strata, the psychological and social ones are characterized by an interruption in the categorical series and by the onset of new ones (relative to the psychological and social items).

### 4.2. Chronotopoids

The theory of levels paves the way towards the claim that there could be *different* families of times and spaces, each with its own structure. We shall argue that there are numerous types of real times and spaces endowed with structures that may differ greatly from each other. The qualifier “real” is mandatory, since the problem is not the trivial one that different abstract theories of space and time can eventually be and have been constructed (Poli 2007a, b). Drawing on Brentano, we shall treat the general problem of space and time as a problem of *chrono-topoids* (understood jointly, or separated into *chronoids* and *topoids*). The guiding intuition is that each stratum of reality comes equipped with its own family of *chrono-topoids* (see Poli 2006 for further details).

### 4.3. Interactions

The theory of levels of reality provides as well the natural framework for developing a full-fledged theory of causal dependences (interactions). As for the case of chrono-topoids, levels support the
hypothesis that any level has its own form of causality/interaction (or family of forms of causality/interaction). Material, psychological and social forms of causality/interaction could therefore be distinguished (and compared) in a principled way. Beside the usual kinds of basic causality between phenomena of the same nature, the theory of levels enables us to distinguish upward and downward forms of causality/interaction (from the lower level to the upper one and vice versa). This acknowledgement provides the needed context for distinguishing material, psychological and social types of interactions.

4.4. Anticipation

Finally, as far as the distinction is concerned between living and non-living systems, the claim is defended that anticipation is the single most relevant feature distinguishing them. Intuitively, the choice of the action to perform depends from the system’s anticipations of the evolution of itself and/or the environment in which it is placed (Rosen 1985). Non-living systems, on the contrary, are reactive systems where subsequent states depends entirely from preceding states (usually, according to some law or rule). Anticipation comes in different guises: the simplest distinction is between strong and weak types of anticipation, where the former (the strong one) is meant as coupling between the system and its environment, while the latter (the weak one) is understood in the form of a (cognitive) model developed by the anticipatory system itself. As a straightforward consequence, evolutionary survival implies that all living systems are characterized by some form of another of strong anticipation, while some among the most evolved species may enjoy weak types of anticipation as well. Anticipation can therefore lie low and work below the threshold of consciousness or it may emerge into conscious purpose. In the latter form it constitutes the distinctive quality of causation within the psychological and the social realms. On the other hand, biological systems are better characterized by non-representative (model-based) types of anticipation.

Complexity, as usually—and most likely incorrectly understood—is entirely past-governed and apparently does seem to be unable to include anticipatory behaviour. In order to distinguish anticipatory systems from entirely past-governed systems, the concept of super-complexity has been recently introduced (see Baianu, 2006: *Axiomathes*, 16, 1-2, special issue dedicated to Robert Rosen). A different but not opposite way to understand anticipation is to see the theory of anticipatory systems as providing a phenomenological, or first-person, type of description, while most of complexity theory is usually based on third-person descriptions. The theory of anticipatory systems can therefore be seen as comprising both first- and third-person information. The interactions between the two types of descriptions may result in a substantial reduction of the state space characterizing the dynamics of anticipatory systems.

Besides anticipation, living systems require the capacity of coordinating (again, intentionally or “automatically”) the rhythm of the system with those of its parts. In this respect, the anticipation of the system as a whole may diverge from those of its parts. Furthermore, functional organisms are *multi-strata, multi-stable ‘variable systems’*, composed by different types of components interacting at different levels of organization. Analysis is required of both their material and functional components.
The interaction among the three mentioned aspects: anticipation, part--whole structure relationships and levels of organization provides a basis for a better understanding of organisms regarded as highly-complex systems.

5. Ontological and Mathematical Categories

Systems analysis or theory also requires framing ontology in the form of a theory of categories (Poli 2007). We shall therefore adopt here a categorical viewpoint, meaning that we are looking for “what is universal” (in some domain or in general); this viewpoint is quite different from the philosophical, or categorical meaning of classical, such as Aristotelian categories- for example- in Ontology, even though some encyclopedias blur the distinction between categorical and categorical. In the categorical view, the most universal feature of reality is that it is temporal, i.e. it changes, and it is subject to countless transformations, movements, alterations; mathematically this corresponds to true variable forms, rather than fixed ones. From the point of view of mathematical modelling, the mathematical theory of categories models the dynamical nature of reality by resorting to variable categories (i.e., toposes). The claim advanced by this paper is that mainstream topos theory suits perfectly the needs of complex systems. However, as soon as one passes from complex systems to super-complex systems, the theory of toposes requires suitable generalizations. Our proposal is then to adopt the framework provided by (a suitably generalized) topos theory for modelling super-complex systems.

We have seen that the difference between complex and super-complex systems in based on at least four main issues: levels of reality, chronotopoids, (generalized) interactions, and anticipation. So far, none of them has been adequately formalized. However, considering that chronotopoids and interactions require (and therefore depend on) the theory of levels, and that the issue of anticipation has been advanced by Rosen 1985, subsequent sections will make reference to the issue of levels only. Furthermore, it is apparent that any step towards a proper formalization of the theory of levels (or any other of the mentioned theories) seems to require the development of a non-Abelian framework. Whichever further mathematical property will be required, the first mandatory move is therefore to pass from an Abelian or commutative or natural framework to a non-Abelian one.

6. Organisms Represented as Variable Categories:

Many-Valued LM-logic Algebraic Categories of Functional Biosystems

One of the major road blocks to any successful dynamical theory of complex systems, and also of developing organisms, has been the lack of a flexible enough mathematical structure which could represent the immensely variable and heterogeneous classes of biological and social organisms. In the following subsection we propose to re-examine the representation of organisms in terms of such flexible mathematical structures that can vary in time and/or space, thus providing a natural framework for relational/ theoretical biology, psychology, sociology or global theoretical constructs addressing environmental problems.
We have already mentioned that the problem of time, i.e. the problem of the dynamical nature of reality, is the main problem underlying the philosophical theory of categories as reviewed by Poli (2007). This same problem has also been at the center of the mathematical theory of categories over the last six decades, and found a first outcome in the idea of variable category (be it in the form of variable sets, variable classes, etc), albeit in a formal and precise setting. Furthermore, dealing with varying, or variable, objects such as those formalized by the concepts variable sets, variable classes, etc. leads to a further generalization of this categorical approach which is founded in Logic, be it Boolean (as in the Category of Sets), Heyting-intuitionistic (as in “standard” Topos theory; Moerdijk and MacLane, 1994, 2004), or Many-Valued (MV or Łukasiewicz-Moisil (see Georgescu, 2006 for a review of N-valued logics), as in the new theory of Generalized Topoi (Baianu, Brown, Glazebrook and Georgescu, 2005, 2006, 2011). It is worth mentioning that some of the methodological pitfalls of the categorical approach in specific, logical or mathematical, contexts, as for in example in certain areas of Algebraic Topology or Algebraic Logic, were recognized early by topologists, who also branded this approach as “abstract nonsense”, even though it continues to facilitate and be widely employed in the proof of general theorems. Such an objection lies in the fact that the “universal” may, and does, have specific, subtle exceptions and counter-examples as one might, of course, expect it. Things that may appear to be globally “the same”, or “categorically equivalent”, may still differ quite significantly in their specific, local contexts. Such problems with defining different kinds of equivalence arise not only in the modern theories of Algebraic Topology and Groupoids, Abelian categories, Algebraic Geometry, and so on, but also when attempting to define in precise terms the similarity or analogies between systems that appear physically distinguishable but mathematically equivalent in some selected, specific sense. We are considering in the next section this problem in further detail.

7. Analogous Systems and Dynamic Equivalence

A scientific and/or engineering strategy for dealing with complex systems has long been the analysis of simpler, more readily accessible ‘models’ of a complex system. One often attempts to arrive at computable models with similar dynamic behavior(s) to that of the original, complex system. Computability of such simple models may often involve the use of a super-fast digital computer, and the models can be made indefinitely more and more complicated through iterated attempts at improved computer simulation.

A formal, categorical approach to analogous systems and dynamic equivalence of systems was first reported by Rosen (1968) from a classical standpoint that is, excluding quantum dynamics; subsequently his approach was extended to the development of biological systems and embryology (Baianu and Scripcariu, 1973: BMB, “On Adjoint Dynamical Systems”).

Returning now to the issue of computer simulation, one finds upon careful consideration that there is no recursively computable (either simple or complicated) model of both super-complex biological systems and simpler ‘chaotic’ systems (see for example, Baianu, 1987 and the relevant references cited therein). Therefore, Complex Systems Biology (CSB) cannot be reduced to any finite number of simple(r) mechanistic models that are recursively computable, or accessible to digital computation or numerical simulation. This basic result does not seem, however, to deter the computationally-oriented scientists from publishing a rapidly increasing number of reports on computer simulation of complex biological systems. There is surprising enthusiasm and optimism, not to mention popularity, funding, etc., for computer simulations in both biology and medicine.
Such ‘mechanistic’ approaches to understanding how parts or subsystems of a complex organism work are necessary but insufficient steps towards developing a CSB theory that takes into account what the over-simplified ‘mechanistic’ models have left out—those irreducible interactions that pertain to the essence of an organism’s existence and its integrated physiological functions. Heuristic results are both attractive and stimulating, culminating with the aroused expectation of ‘final answers’ to either biological or medical problems by means of digital super-computers. It seems, however, that even the fastest and best-programmed super-computers are no match for the super-complexity of organisms, or even for the simpler, ‘chaotic’ dynamics, a result which is also widely recognized by many chaotic dynamic theorists.

Fundamentally, the limitations of digital computers that rely upon finite/recursive computations are traced back to the Boolean (or Chrysippian) logic underlying the design of all existing digital computers, and also to the Axiom of Choice upon which set theory is based (Moerdijk and MacLane, 2004). On the other hand, biological, super-complex system dynamics is governed by a many-valued (MV) logic characteristic of biological processes including genetic (Baianu, 1977) and neural ones Baianu et al, 2006, 2011. Such an MV-logic is both non-commutative (unlike the Boolean or the Heyting- intuitionistic logic of standard toposes) and irreducible to Boolean and/or intuitionistic logic (of course, with the exception of the special cases of the category of centred Łukasiewicz-Moisil logic algebras that can be mapped isomorphically onto Boolean Logic algebras (Georgescu, 1974, 2006). Unlike the well known result of von Neumann's for the Universal Automaton, super-complex biological ‘systems’ are not recursively, or numerically computable. Although this limits severely the usefulness of all digital computers in Complex Systems Biology and Mathematical Medicine, it does not render them completely useless for experimentation, data collection and analysis or graphics and graphical presentation/representation of numerical results. Both the limitations and the advantages of using computers become evident in the final analysis where computer simulations of super-complex ‘system’ dynamics cannot claim a full, or complete, dynamical modelling of organisms as such a result has been formally proven to be unobtainable, in general, through recursive computation with algorithms, universal Turing machines, etc. (Baianu, 1986; Rosen, 1987; Penrose, 1994).

Algebraic computation is still possible, of course, for living systems and their essential subsystems, such as genetic networks, by employing instead non-commutative, irreducible MV-logics, either in a general context (Georgescu,1974,2006) or in more specific contexts, such as the controlled dynamics of genetic networks in biological organisms (Baianu,1977, 2004a,b, 2005a,b; Baianu et al, 2006). Non-commutative super-complex dynamic modelling has just begun in Biology and Medicine, including diagnostics. A Biostatistics formulation based on LM-logic algebras, but independent of current probability theory, has also become a strong possibility (Georgescu, 2006). Such recent developments also suggest a paradigm shift occurring now in system theories – from Abelian to non-Abelian/ non-commutative theories. This new paradigm has perhaps already began with the earlier introduction of noncommutative geometric spaces obtained through deformation as models of quantum spaces in attempts by A. Connes et al. (1992, 2004, 2006, and references cited therein) at formalizing a Noncommutative Geometry theory of Quantum Gravity 8. Non-Abelian Systems Theory.

One can formalize the hierarchy of multiple-level relations and structures that are present in super-complex ‘systems’ and meta-systems in terms of the mathematical Theory of Categories, Functors and Natural Transformations (TC-FNT). On the first level of such a hierarchy are the links between the system components represented as morphisms of a structured category which are
subject to the axioms/restrictions of Category Theory. Then, on the next, second level of the hierarchy one considers functors or links between such first level categories that compare categories without 'looking inside' their objects/system components. On the third level, one compares, or links, functors using natural transformations in a 2-category (meta-category) of categories. At this level, natural transformations not only compare functors but also look inside the first level objects (system components) thus 'closing' the structure and establishing the universal links' between items as an integration of both first and second level links between items. The advantages of this constructive approach in the mathematical theory of categories, functors and natural transformations have been recognized since the beginnings of this mathematical theory in the seminal paper of Mac Lane and Eilenberg (1945). A relevant example from the natural sciences, e.g., neurosciences, would be the higher-dimensional algebra of processes of cognitive processes of still more, linked sub processes (Brown, 2004). Yet another example would be that of groups of groups of item subgroups, 2-groupoids, or double groupoids of groups of items. The hierarchy constructed above, up to level 3, can be further extended to higher, n-levels, always in a consistent, natural manner.

This type of global, natural hierarchy of items inspired by the mathematical TC-FNT has a kind of internal symmetry because at all levels, the link compositions are natural, that is the all link compositions that exist are transitive, i.e., \( x < y \) and \( y < z \Longrightarrow x < z \), or \( f: x \rightarrow y \) and \( g: y \rightarrow z \Longrightarrow h: x \rightarrow z \), and also \( h = g \circ f \). The general property of such link composition chains or diagrams involving any number of sequential links is called commutativity, or the naturality condition. This key mathematical property also includes the mirror-like symmetry \( x \ast y = y \ast x \); when \( x \) and \( y \) are operators and the \( \ast \) represents the operator multiplication. Then, the equality of \( x \ast y \) with \( y \ast x \) implies that the \( x \) and \( y \) operators 'commute'; in the case of an eigenvalue problem involving such commuting operators, the two operators would share the same system of eigenvalues, thus leading to 'equivalent' numerical results. This is very convenient for both mathematical and physical applications (such as those encountered in quantum mechanics). Unfortunately, not all operators 'commute', and not all mathematical structures are commutative. The more general case is the non-commutative one. An example of a non-commutative structure relevant to Quantum Theory is that of the Clifford algebra of quantum observable operators (Dirac, 1962; see also the Appendix); yet another, more recent and popular, example is that of \( C^* \)-algebras of (quantum) Hilbert spaces. Last but-not least, are the interesting mathematical constructions of non-commutative 'geometric spaces' obtained by 'deformation' introduced by Allan Connes (1990) as possible models for the physical, quantum space-time. Thus, the microscopic, or quantum, 'first' level of physical reality does not appear to be subject to the categorical naturality conditions of Abelian TC-FNT—the 'standard' mathematical theory of categories (functors and natural transformations). It would seem therefore that the commutative hierarchy discussed above is not sufficient for the purpose of a General, Categorical Ontology which considers all items, at all levels of reality, including those on the 'first', quantum level, which is not commutative. On the other hand, the mathematical, Non-Abelian Algebraic Topology (Brown and Sáfiro, 2005, 2006), the Non-Abelian Quantum Algebraic Topology (NA-QAT; Baianu et al., 2005-2006), and the physical, non-Abelian gauge theories may provide the ingredients for a proper foundation for non-Abelian, hierarchical multi-level theories of super-complex system dynamics. Furthermore, it was recently pointed out (Baianu et al, 2005, 2006) that the current and future development of both NA-QAT and of Complex Systems Biology theories involve a fortiori non-commutative, many-valued logics of quantum events, such as the Lukasiewicz-Moisil (LM) logic algebra (Georgescu, 2006a), complete with a fully-developed, novel probability/measure theory grounded in the LM-logic algebra (Georgescu, 2006a,b). The latter paves the way to a new projection operator theory founded upon the (non-commutative) quantum logic of events, or dynamic processes, thus opening the possibility of a complete,
Non-Abelian Quantum Theory. Furthermore, such recent theoretical developments point towards a paradigm shift in systems theory and to its extension to more general, non-Abelian theories that lie well beyond the bounds of commutative structures/spaces. Non-Abelian theories are also free from the restrictions or basic limitations imposed by the Axiom of Choice and the elementhood, or parthood/subordination relation in Set Theory or Russell’s theory of classes.

References


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