On the Nature of Intelligence: The Relevance of Statistical Mechanics

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Abstract

A conundrum that results from the normal distribution of intelligence is explored. The conundrum concerns the chief characteristic of intelligence, the ability to find order in the world (or to know the world) on the one hand, and the random processes that are the foundation of the normal distribution on the other. Statistical mechanics is explored to help in understanding the relation between order and randomness in intelligence. In statistical mechanics, ordered phenomena, like temperature or chemical potential, can be derived from random processes, and empirical data indicate that such a relationship between ordered phenomena and random processes must exist as regards intellect. The apparent incongruity in having both order and randomness characterize intelligence is found to be a feature that allows for intelligence to be known without recourse to underpinnings that are independent of the knowing individual. The contrast of ordered processes and random processes indicates that probabilistic knowledge of the world, stemming from the latter processes, is a basis for knowing the world in a fundamental manner, whether the concern is the physical world or mind. It is likely that physiological concomitants involved in the development, and perhaps current operation, of intellect also demonstrate the same relationship between ordered and random phenomena found on a psychological level. On a microscopic level, it is expected that random neurophysiological processes would give rise to ordered patterns of neurophysiological activity on a macroscopic level.

Text

Measures of human ability in general are frequently nearly normally distributed. Whether or not some ties can be drawn between this fact and the relationship of the normal curve to the probability distribution of groups of binomial trials for chance events is a task we leave gladly to the more philosophically inclined. (Glass & Stanley, 1970, p. 104)

In an earlier paper (Snyder, 1986), an apparent incongruity in the normal distribution of intelligence was discussed. This incongruity contrasted

the fundamental characteristic of intelligence, namely the capacity to order or to find order in one's experience of the world, with the randomness that is at the heart of the derivation of the normal distribution itself. This incongruity, though, reflects a similar one in the physical world, and it will be shown that these incongruities allow for a greater understanding of the physical world and of the person who, using his intellect, attempts to understand it.

More specifically, statistical mechanics will be explored first to show that an uncomplicated derivation of the normal distribution relies on random events and how macroscopic phenomena that act in predictable ways are derived from such events. How this randomness, reflected in the second law of thermodynamics, contrasts with the nature of physical law is then discussed. The normal distribution in psychology is then explored using Galton's description of the nature of this distribution. The reflection of the ordered nature of intelligence in one of the most widely used psychometric instruments for assessing intelligence, the Wechsler Adult Intelligence Scale, Revised (WAIS-R), is discussed. The distribution of test results for the standardization sample on this psychometric instrument is discussed to show that this distribution reflected the U.S. population within the sample's age range and that the sample approximates very closely the theoretical normal curve.

The involvement of both order and randomness in quantum mechanics, part of the bedrock of modern physical theory, is explored because certain features of quantum mechanics make the role of knowledge in this theory and its relationship to probabilistic prediction particularly clear. Further, quantum mechanics specifies a particular relationship between knowledge of the physical world and the physical world itself (Snyder, 1989, 1992). These relationships are shown to apply as well to the physical world in statistical mechanics and the nature of intelligence.

STATISTICAL MECHANICS

In physics, there is an incongruity between physical law on the one hand, characterized by order, and randomness, on the other, that is at the heart of statistical mechanics in what is called the second law of thermodynamics.²

¹ One definition of the second law of thermodynamics is that interacting physical systems that together comprise an isolated physical system tend toward maximum disorder. This law of thermodynamics will be discussed in detail.

² That the second law of thermodynamics is called a law is a bit problematic because it is built on randomness even though on the macroscopic level it describes the physical world with great predictive power.

The foundations of modern physical law are quantum theory and the theory of relativity. Each in their own way supplanted what had been for over two hundred years prior to their development the basis of physical law, namely Newtonian mechanics. Though there are very significant differences between all three theories, they share a very important characteristic. Each theory, in its own way, prescribes some sort of completely ordered development of variables related directly or indirectly to quantities in the physical world. It is this characteristic that entitles these theories to be considered the basis for physical law.

The Conceptual Foundation of the Normal Distribution in Statistical Mechanics

The normal distribution is central to the second law of thermodynamics, and the second law of thermodynamics underlies certain prominent observable phenomena such as thermal equilibrium for interacting physical systems. The development of the normal distribution for various physical quantities is based upon three principles. They are:

- 1) the existence of a closed (or isolated) physical system composed of a number of entities, such as particles;
- 2) the fundamental assumption of statistical mechanics and thermal physics, namely that such an actual isolated system is equally likely to be in any accessible stationary quantum state;³
- 3) an ensemble of physical systems all constructed like the actual closed physical system of interest except that each system is in exactly one of the accessible stationary quantum states for the closed physical system of interest (Kittel, 1969).

An isolated, or closed, physical system is a physical system in which the energy, the number of particles, and the volume remain constant over time. When all observable quantities of a physical system, including the energy, are independent of time, the system is said to be in a stationary quantum state. The fundamental assumption noted above indicates that the state of the system is randomly determined.

³ The term "accessible" refers to statistically accessible, given a particular overall specification of the physical system such as its energy.

There is assumed to be enough leeway in the specification of the system so that, although the concern is with the stationary state of the system, the fundamental assumption remains applicable. The fundamental assumption implies that over the course of long periods of time, the physical system will be in all of the stationary quantum states allowed within its overall specification. In conjunction with the fundamental assumption, the use of a representative ensemble allows for the calculation of average values of observable quantities for this group of systems at a particular time.⁴

That an isolated physical system composed of distinguishable, but similar, particles is equally likely to be in any accessible stationary quantum state may most reasonably be accomplished by assuming that:

- 1) each particle of the system undergoes independent selection with regard to the values of the quantities needed to specify completely the state of the particle;
- 2) the particle is equally likely to have any of the accessible values of these quantities.

The random determination of the system's state is at the heart of the fundamental assumption noted above. It is also at the heart of the assumption that each distinguishable particle in the system is equally likely to have any of the accessible values of the quantities needed to specify that particle's state completely.

Boltzmann and Gibbs made a conceptual advance in the problem of calculating average values of physical quantities. Instead of taking time averages over a single system, they imagined a group of a large number of similar systems, suitably randomized. Averages at a single time are taken over this group of systems. The group of similar systems is called an *ensemble of systems*. The average is called the ensemble average or the thermal average.

An ensemble is an *intellectual construction* [italics added] that represents at one time the properties of the actual system as they develop in the course of time....Our assumption is that this ensemble represents the system in the sense that an average over the ensemble gives correctly the value of the average for the system [over time].

The Gibbs scheme replaces time averages over a single system by ensemble averages, which are averages over all systems in an ensemble. The demonstration of the equivalence of the ensemble and time averages is difficult and has challenged many mathematicians....It is certainly plausible that the two averages might be equivalent, but one does not know how to state the necessary and sufficient conditions that they are exactly equivalent. (pp. 32-33)

⁴ It is to be emphasized that the physicist's imagination is the basis for the representative ensemble of physical systems like the actual system of interest and that the use of the representative ensemble has been verified by experiment. Kittel (1969) wrote:

An Example of the Conceptual Development of the Normal Distribution in Statistical Mechanics

Consider an isolated physical system composed of distinguishable, but similar, particles to which the fundamental assumption is applied. With the use of a representative ensemble, it can be mathematically derived that the normal distribution is generally a very close approximation to the distribution found where the number of accessible stationary quantum states for the isolated physical system is a function of the number of particles in a particular state. Another way of stating this point is that for a physical system composed of a large number of distinguishable particles, with the use of a representative ensemble, the distribution of particles comprising the system as regards certain physical quantities and to which the fundamental assumption applies will closely resemble the normal distribution. This resemblance will be demonstrated through the use of the binomial distribution for a particular quantity. This resemblance will be applicable to Galton's description of the normal distribution that is presented later as well as our discussion of the normal distribution of intelligence.

The Resemblance of the Binomial and Normal Distributions

Consider Kittel's (1969) case of a quantity for some distinguishable physical existent that has two possible values, such as the direction of the magnetic moment of a small magnet that can be oriented only in one of two directions along some particular axis in space. Allow that the probabilities of the magnetic moment being "up" or "down" along the axis for a particular magnet when measured are equal. Allow further that many of these small magnets are the components of an isolated physical system. The direction of the moment of any magnet is independent of the direction of the moment of any other magnet in this system. For a representative ensemble of the system, the distribution of the magnets as a function of the number of magnets in the physical system having a particular magnetic moment is binomial in nature and given by:

$$(a+b)^{N} = a^{N} + Na^{N-1}b + (N/2)(N-1)a^{N-2}b^{2} + ... + b^{N} = \sum_{0}^{N} [N!/(N-q)!q!][x^{N-q}y^{q})$$
(1)

where N is the number of events for which there are outcomes, q is 0 or a positive integer that increases by 1 in each successive term, ! before a term

(such as in q!) indicates that we are concerned with the term as a factorial (such as q factorial), and a and b are the respective outcomes "up" and "down." The coefficient of each term in the expression represents the number of ways the outcome of "up"s and "down"s can occur.

More specifically, in a representative ensemble with as few as 100 magnets, the resulting distribution of possible states of the overall system of magnets in terms of excess magnetic moment in one of the two possible directions (in our case, the difference between those magnets with magnetic moment "down" subtracted from those magnets with magnetic moment "up") is a binomial distribution that closely resembles a normal distribution (Kittel, 1969). Where there are 10 small magnets in the physical system, the general form of the binomial distribution above becomes:

$$[(up) + (down)]10^{20} = \sum_{0}^{20} [10^{20}!/(10^{20} - q)!q!][(up)10^{20-q}(down)^{q}]$$
(2)

where $N=10^{20}$, the number of magnets of concern, and "up" represents the direction "up" of the magnetic moment for a magnet and "down" represents the direction "down" of this magnetic moment. The coefficient $[10^{20} !/(10^{20} - q)!q!]$ represents the number of possible ways that the state with 10^{20-q} "up"s and q "down"s can be attained. It can be shown that this coefficient is very close to the general expression:

$$[N!/(1/2N)!(1/2N)!][e^{-2m^2/N}]$$
 (3)

or specifically,

$$[10^{20}!/(1/210^{20})!(1/210^{20})!][e^{-2m^2/10^{20}}] (4)$$

where m is a term representing 1/2 the excess magnetic moment for the system. Equations 3 and 4 describe normal distributions. As can be seen from these equations, a general equation for the normal distribution is:

$$y = e^{-x^2}$$
 (5).

Entropy and Thermal Equilibrium

In statistical mechanics, an isolated physical system composed of many similar, but distinguishable, particles in which all accessible stationary states have the same energy does not itself yield a great deal of information (Kittel, 1969). But the conceptual development of the normal distribution for such a

system provides the basis for making very informative predictions concerning various quantities when certain physical systems interact. One of the most important predictions concerns the temperatures of two physical systems that are placed in thermal contact (i.e., they are allowed to exchange energy but not particles) and that together form an isolated physical system.

The second law of thermodynamics allows one to predict that these two component systems will tend toward thermal equilibrium (i.e., toward having the same temperature) when they are brought into thermal contact. How is the second law responsible for this tendency? The answer lies in the fundamental assumption that applies to the overall, isolated physical system. If the overall, isolated physical system has a particular energy, this energy may be divided between the two component systems in many ways. Simply put, the energy of the overall system is divided between the component systems so as to maximize the number of accessible states of the overall system. The normal distribution for each of the component systems considered as isolated physical systems sets up the number of accessible states of each of the component systems at different energy levels for each of the component systems.

Consider our example involving a physical system composed of many small magnets. Here the energy of the system can be specified by introducing a uniform magnetic field to the system. Then there would be a number of different energy levels of the overall system of magnets depending on the excess magnetic moment for the overall system. The number of accessible states for the physical system would be determined by its particular energy level in the presence of the uniform external magnetic field.

Allow that two systems of small magnets are in a uniform magnetic field (and thus have specific energies associated with them) and that these systems are brought into thermal contact, but not diffusive contact (i.e., in which systems can exchange energy but not magnets) to form a large physical system. There would then be a particular energy for the overall physical system. The two systems would tend toward thermal equilibrium, that is they would tend toward a configuration (i.e., a set of states having a particular value for the excess magnetic moment for each of the component systems) that would maximize the number of accessible states for the overall system. This configuration, called the most probable configuration, has a specific excess magnetic moment for each of the component systems and thus has for a particular energy for each of the component systems.

The temperature of each of the component systems is given by:

$$1/T = k_{R}(\partial \sigma/\partial U)_{N} (6)$$

where T is the temperature given in Kelvin, σ is the entropy of the component system, U is the energy of the system, N is the number of particles in the system, and k_B is the Boltzmann constant. The symbol ∂ indicates partial differentiation and thus the term $(\partial \sigma/\partial U)_N$ indicates the partial differentiation of the entropy with respect to the energy, while the number of particles in the system is held constant. The entropy, σ , is the natural logarithm of the number of accessible states for a specified physical system and, because of the fundamental assumption, is thus a measure of the randomness in the system.

The condition of thermal equilibrium, where the entropy of the combined system is at a maximum, is given by:

$$k_{R}(\partial \sigma_{1}/\partial U_{1})_{N1} = k_{R}(\partial \sigma_{2}/\partial U_{2})_{N2}$$
 (7)

or

$$T_{1} = T_{2}$$
 (8)

where the subscript 1 indicates values of quantities in one of the component systems and the subscript 2 indicates values of quantities in the other component system. The entropy of the component systems toward which the systems tend is determined once the energy (for example, as determined by the specific excess magnetic moment), the number of particles, and the volume of the overall physical system is specified. Small fluctuations in the energy of the component systems must meet equations 7 and 8 when these systems are in thermal equilibrium. In thermal equilibrium, any change in the energy of one system results in a corresponding change in the energy of the other system such that the energy of the combined systems is constant. Any change in the entropy of one system associated with a change in the energy of this system results in a corresponding change in the entropy of the other system such that the entropy of the combined systems remains constant.

It should be noted that the temperature of a physical system thus depends not only on the energy but the entropy, the randomness, of this system. Two physical systems in thermal equilibrium have the same temperatures when the change in entropy in one of the component systems that

⁵ Feynman, Leighton, and Sands (1963) wrote that entropy is the logarithm of "the number of ways that the insides [of a specified physical system] can be arranged, so that from the outside it looks the same" (p. 46-7).

occurs as a result of change in the energy of this system is equal to the change in the entropy that occurs in the other component system as a result of the change of energy in this other system. As the change in energy in one system must be balanced by an opposite change in the energy of the other system (because the energy of the overall system is constant), so too must a change in the entropy of one system be balanced by an opposite change in the other system. Thus, thermal equilibrium is achieved when the number of accessible states cannot increase. Concerning the importance of entropy to the temperature of a physical system, Kittel (1969) attributed the following quote to Planck:

The general connection between energy and temperature may only be established by probability considerations. [Two systems] are in statistical equilibrium when a transfer of energy does not increase the probability. (p. 37)

The three fundamental principles, and the normal distribution which they lead to, have been shown to provide the basis from which to understand certain aspects of how physical systems interact when they combine to form a large isolated system as well as a particularly important characteristic of such systems, temperature. Also, these principles can explain a phenomenon, the tendency to thermal equilibrium resulting from the probabilistic character of microscopic physical phenomena. A similar analysis holds for the quantity of chemical potential, where particles as well as energy are exchanged.

The Significance of the Fundamental Assumption

In *The Principles of Statistical Mechanics*, Tolman (1938) stated the basis for adopting the fundamental assumption in statistical mechanics, a basis that will be shown to apply equally well to the development of the normal distribution of intelligence. Kittel (1969) provided an excellent quote from Tolman's book:

It has been made clear by the foregoing that statistical methods can be employed in a very natural manner for predicting the properties and behaviour of any given system of interest in a partially specified state, by the procedure of taking averages in an appropriately chosen state, by the procedure of taking averages in an appropriately chosen ensemble of similar systems as giving reasonable estimates for quantities pertaining to the actual system....In the first place, it is to be emphasized, in accordance with the viewpoint here chosen, that the proposed

methods are to be regarded as *really statistical* in character, and that the results which they provide are to be regarded as true on the average for the systems in an appropriately chosen ensemble, rather than as necessarily precisely true in any individual case. In the second place, it is to be emphasized that the representative ensembles [each of which represents one accessible stationary quantum state of the large physical system] chosen as appropriate are to be constructed with the help of an hypothesis, as to equal a priori probabilities, which is introduced at the start, without proof, as a necessary postulate....It is to be appreciated that *some* postulate as to the a priori probabilities...has in any case to be chosen. This again is merely a consequence of our commitment to statistical methods. It is analogous to the necessity of making some preliminary assumption as to the probabilities for heads or tails in order to predict the results to be expected on flipping a coin. In the second place, it is to be emphasized that the actual assumption, of equal a priori probabilities...is the only general hypothesis that can reasonably be chosen....In the absence of any knowledge except that our systems do obey the laws of mechanics, it would be arbitrary to make any assumption other than that of equal a priori probabilities....The procedure may be regarded as roughly [emphasis added] analogous to the assumption of equal probabilities for heads and tails, after a preliminary investigation has shown that the coin has not been loaded. (p. 34)^{6,7}

It is to be emphasized that the fundamental assumption leads to the remarkable property in statistical mechanics that the elements in a physical system tend to occupy the widest variety of states possible. This tendency of the elements to occupy the widest variety of states possible is statistical in nature. This remarkable property of statistical mechanics that the fundamental

⁶ The term "roughly" is emphasized because statistical mechanics is fundamentally probabilistic, whereas the outcome of the flipping of a single true coin can be known using the laws of mechanics if the initial condition of the coin is adequately specified. In the case of the single true coin, probabilistic knowledge is an approximation of the more fundamental state of affairs governed by the laws of mechanics.

⁷ The original quote is from Tolman (1938), pages 64-65.

assumption leads to is illustrated in the following gedankenexperiment (i.e., thought experiment):

A classic irreversible process, and one that helps in defining the concept of entropy a little more precisely, is called free expansion. Suppose a chamber filled with gas is separated by a partition from a vacuum chamber of the same size. If a small hole is made in the partition, gas will escape (that is, it will expand freely) into the formerly empty chamber until both chambers are filled equally.

The reason the molecules spread out to fill both chambers is mathematical rather than physical, if such a distinction can be made. The numbers of molecules on the two sides of the partition tend to equalize not because the molecules repel one another and move as far apart as possible, but rather because their many collisions with the walls of the container and with one another tend to distribute them randomly throughout the available space, until about half of them are one side of the partition and about half are on the other side.

The disordered state—the state in which the gas has spread into both chambers rather than residing compactly in a single chamber—is more probable than the ordered state. That is, there are more configurations of molecules in which the molecules occupy both chambers, just as, when 100 coins are tossed, there are more ways to achieve a total of 50 heads and 50 tails than there are to achieve 100 heads and no tails [emphasis added]. In

saying that the entropy of the universe tends to increase, the second law is simply noting that the universe tends to fall into more probable states as time passes. (Bennett, 1987, p. 110)

In principle, physical law has nothing to do with the tendency of the gas molecules to distribute throughout the enlarged chamber as opposed to their congregating in one of the chambers. One might think that this tendency to distribute throughout the enlarged chamber is only a gradually increasing tendency to disorder that is a reflection of the physical law governing the interaction of the gas molecules at work. This thesis, though, does not allow for the effect of the doubling of the number of paths that the molecules can travel when the chamber is enlarged. The tendency for the molecules to distribute uniformly reflects a relationship between the many gas molecules considered as a system, and this relationship is not determined by physical law. This tendency is not dependent on physical contingencies. Instead, like Bennett noted, the relationship between the many gas molecules is mathematical, or statistical.

PHYSICAL LAW

The preceding section describing statistical mechanics and the importance of randomness to it apparently contradicts what is perhaps the most distinguishing characteristic of physics, the significant degree of order that physics has deciphered in human experience of the physical world. For over two hundred years from the latter 1600's, Newtonian mechanics (Newton, 1686/1962), including the kinematics underlying it, was the dominant theory explaining the functioning of the physical world. It was considered the theory that accurately described the ordered functioning of the physical world. Newton's three laws of motion in conjunction with his law of gravity were able to account for a vast array of physical phenomena. These laws were deterministic, not probabilistic, in nature.

Newtonian mechanics, and the kinematics underlying it, though, could not adequately account for a number of features that characterize electromagnetic phenomena. The development of the classical laws of electromagnetism that were finally set by Maxwell in the 1860's and 1870's was a tremendous achievement. One has only to look around at everyday life and see the degree of control over nature that has been achieved with the application of Maxwell's laws. One consequence of these laws was the prediction of electromagnetic waves and the identification of these waves with

light. An attempt was made by Fitzgerald and then Lorentz in the late 1800's to account for the invariant velocity of light in all inertial reference frames in a manner consistent with Newtonian mechanics and the kinematics underlying it through maintaining the existence of an absolutely stationary ether. Their theory was inelegant and ultimately failed empirical test (Resnick, 1968).

The special theory of relativity could account for all electromagnetic phenomena governed by Maxwell's laws, including the invariant velocity of light in vacuum irrespective of the motion of the emitting body. Many results in the special theory of relativity that distinguish the special theory from Newtonian mechanics are not reflected in everyday experience of the physical world. But they are commonly found in the empirical work done by physicists. The invariant velocity of light in inertial reference frames, the slowing of physical processes moving in a uniform translational manner relative to an observer at rest in an inertial reference frame, the special relativistic Doppler effect, and mass-energy equivalence reflected in the creation and disintegration of particles are examples. Perhaps mass-energy equivalence reflected in nuclear fission is the most dramatic evidence of the control over nature achieved with the special theory. It should be emphasized that many of the empirical results that support Newtonian mechanics are part of the empirical support for the special theory. That is, the special theory accounts for the empirical results accounted for by Newtonian mechanics, except for gravitation, as well as many empirical results that Newtonian mechanics cannot account for (French, 1968; Resnick, 1968).

The general theory of relativity accounts for a wider range of phenomena than does the special theory. In particular, it accounts for gravitation in addition to those phenomena explained by the special theory. Empirical results concerning gravitation that distinguish the general theory from other physical theory are far removed from everyday experience. An example is the apparent bending of light rays as they pass near to the sun, which in the general theory is a result of the introduction of spacetime curvature near the sun due to the sun's mass. Thus, one can see the ever increasing scope of what is called classical physical theory over the last four centuries since Galileo (1638/1954).

In addition, empirical data were found in the late 19th and early 20th centuries that could not be explained by classical models based on determinism and the mutually exclusive concepts of particle and wave. Out of these circumstances a new type of physical theory arose, one that was probabilistic in

character and that allowed for a physical existent such as an electron to be at times considered as a particle and at other times as a wave. The theory that accounted for these new data was called quantum mechanics, and more generally quantum theory.

One might ordinarily think that quantum mechanics has little to do with everyday life as the primary area of physical phenomena that distinguishes it from classical physics (i.e., Newtonian mechanics and the special and general theories of relativity) is that of atomic and sub-atomic phenomena. But this is not the case. One description of the Pauli exclusion principle, a basic principle of quantum mechanics, states that two fermions cannot be in the same quantum state. Formally, fermions are sub-atomic particles with half-integral spin. The electron is one type of fermion. It follows from the Pauli exclusion principle that two electrons in an atom cannot occupy the same state, for example the innermost shell of the energy ground state of an atom. If electrons could occupy the same state, the consequences would be, to say the least, dramatic. As Eisberg and Resnick (1985) stated:

To emphasize just how fundamental the problem is...if all the electrons in an atom were in the innermost shell, then the atom would be essentially like a noble gas. The atom would be inert, and it would not combine with other atoms to form molecules. If electrons did not obey the exclusion principle this would be true of all atoms. Then the entire universe would be radically different. For instance, with no molecules there would be no life! (p. 309)

Given the significance of probability in quantum mechanics, how is it that quantum mechanics forms part of the bedrock of modern physical law? The answer is that the wave functions associated with particles evolve in a precise manner. It is these wave functions that provide the basis for making probabilistic predictions concerning observable physical quantities when measurements are made.

THE CONTRAST BETWEEN PHYSICAL LAW AND STATISTICAL MECHANICS

The importance of randomness in statistical mechanics has been discussed previously. Now that the nature of physical law has been briefly presented as well, another example will bring statistical mechanics and physical

law into sharp contrast as concerns the relationship of order and randomness in physical theory.

Contrast individual interactions between two moving particles with a physical system composed of many moving and distinguishable particles, such as that found in a gas. In statistical mechanics, the motion of these particles is considered anything but ordered. Their motion is considered random in nature with the components of this motion along three orthogonal spatial axes considered random as well.

At the same time, it has not been doubted that individual components of a large physical system composed of many distinguishable particles, such as a gas, each follow the laws of nature. But it is not the lawful behavior of individual physical existents that leads to the second law of thermodynamics. Rather, it is the complexity attendant on the in principle random behavior of a large collection of such existents. Brewer and Hahn (1984) reviewed the contrast between physical law and the fundamental assumption of statistical mechanics in its historical context.

In 1872 Ludwig Boltzmann, a founder of modern thermodynamics, gave a lecture in which he said that the entropy, or disorder, of isolated systems increases irreversibly as time passes. On hearing this the physicist Joseph Loschmidt rose in protest. He argued that the laws governing the motions of all particles are symmetric with respect to time. Thus any system that had decayed from order to chaos could be made orderly once again simply by reversing the momentum of each particle, without affecting the total kinetic energy of the system. In defiance Boltzmann pointed his finger at Loschmidt and said, "You reverse the momenta."

This scholarly conflict illustrates the paradoxical nature of the second law of thermodynamics, which states that systems tend toward maximum entropy. Yet Loschmidt's argument remains cogent. If one were able to film the motions of any small group of particles and show the film to a physicist, he or she would have no way of telling in principle whether the projector was running forward or backward. Consequently, according to Loschmidt's criticism (which has come to be called the Loschmidt paradox), any law that governs the behavior of large

collections of particles should be symmetric with respect to time. (p. 50)

It is to be emphasized that physical law does not prescribe the behavior of large collections of physical existents when they are considered in terms of their aggregate behavior. That these physical existents should operate in a random manner is not due to physical law; rather it is due to chance. The problem with considering the large system as simply the sum of the individual components for which physical law governing the individual components governs the behavior of the components considered as an aggregate, is that for the individual components the possible ways in which the same interactions between the components can occur is not at issue. Rather, it is the lawful character of the interactions between the individual components that is of concern. But for the large system considered as a whole, the lawful character of the particular interactions is not the major concern. Rather, it is the possible ways in which the same interactions between constituents can occur that is of concern. And the possible ways in which each of the interactions between specified constituents occur are equally likely. That the individual components of the large system each follow the laws of nature is the basis for Tolman's statement quoted above:

In the absence of any knowledge except that our systems do obey the laws of mechanics, it would be arbitrary to make any assumption other than that of equal *a priori* probabilities. (Tolman, 1938, p. 64)

It is this assumption that allows Tolman (1938) to call statistical mechanics, "really statistical in character" (p. 34). According to Tolman, it is as if the order found in the laws of mechanics is necessary to support the thesis that only the fundamental assumption can reasonably be considered to account for the distributions that are the basis of statistical mechanics. Basically, we need to know what physical law is so as to know what it is not. Thus, the Loschmidt paradox, though relevant to the description of the individual particles comprising a physical system, does not affect the validity of the fundamental assumption for isolated systems.

THE NORMAL DISTRIBUTION IN PSYCHOLOGY

Now that the distinction between ordered and random processes in the physical world has been discussed in some detail, the same general analysis concerning ordered and random processes can be applied to the nature of

intelligence. The previous analysis of normal distribution in statistical mechanics will serve as a model for developing the normal distribution of intelligence. The same theoretical integrity found in developing the normal distribution in statistical mechanics from the three assumptions noted will be found to characterize the development of the normal distribution of intelligence.

The normal distribution of psychological characteristics can be derived from assumptions modeled after those presented earlier for statistical mechanics. Three principles appear to be the basis for the derivation of the normal distribution of psychological characteristics. These principles were modeled after those found in physics for the description of a physical system in statistical mechanics. These principles in psychology are:

- an isolated mental system composed of certain elements underlying a psychological characteristic or its development;
- 2) an assumption analogous to the fundamental assumption in physics that applies to the state underlying this psychological characteristic or its development;
- 3) a representative ensemble of these mental systems.

To review, the fundamental assumption in statistical mechanics is that an actual isolated system is equally likely to be in any accessible stationary quantum state (Kittel, 1969). Randomness is at the core of the fundamental assumption. Further, the outcome events in statistical mechanics, specifically the values of the quantity of concern, for each of the elements composing the system are random and independent of one another. In psychology, the analogous fundamental assumption is that the possible developmental paths of the outcome events that results in some psychological characteristic (and possibly underlying its current operation), such as intelligence, are equally likely. This assumption presupposes that the outcome of each of the events that are part of the fashioning of the intellect of a particular individual are randomly and independently determined of one another. It also presupposes that the stages in the development of intellect are similar.

In *Natural Inheritance*, Galton (1889/1973) provided a simplified, but accurate, demonstration of a generalized version of these first principles in action in the derivation of the normal distribution that he applied to various human characteristics, including intellect. Essentially, Galton presented a version of the random, or drunkard's, walk (Reif, 1965). Galton described an

apparatus, a certain quantity of shot, the manner of operation of the apparatus on the shot, and the result obtained as follows:

[Consider] a frame glazed in front, leaving a depth of about a quarter of an inch behind the glass. Strips are placed in the upper part to act as a funnel. Below the outlet of the funnel stand a succession of rows of pins stuck squarely into the backboard, and below these again are a series of vertical compartments. A charge of small shot is inclosed. When the frame is held topsy-turvy, all the shot runs to the upper end; then, when it is turned back into its working position, the desired action commences. Lateral strips...have the effect of directing all the shot that had collected at the upper end of the frame to run into the wide mouth of the funnel. The shot passes through the funnel and issuing from its narrow end, scampers deviously down through the pins in a curious and interesting way; each of them darting a step to the right or left, as the case may be, every time it strikes a pin. The pins are disposed in a quincunx fashion, so that every descending shot strikes against a pin in each successive row. The cascade issuing from the funnel broadens as it descends, and, at length, every shot finds itself caught in a compartment immediately after freeing itself from the last row of pins. The outline of the columns of shot that accumulate in the successive compartments approximates to Curve Frequency the of [the normal curvel. (pp. 63-64)

Galton then described the principle underlying this result:

The principle on which the action of the apparatus depends is, [sic] that a number of small and independent accidents befall each shot in its career. In rare cases, a long run of luck continues to favour the course of a particular shot towards either outside place, but in the large majority of instances the number of accidents that cause Deviation to the right, balance in a greater or less degree those that cause Deviation to the left. Therefore most of the shot finds its way into the compartments that are situated near to a perpendicular line drawn from the outlet of the funnel, and the Frequency with which shots stray to different distances to the right or left of that line diminishes in a much

faster ratio than those distances increase. This illustrates and explains the reason why mediocrity is so common. (pp. 64-65)

The result of the operation of Galton's apparatus is the normal distribution of the shot with regard to its net displacement from "the perpendicular line drawn from the outlet of the funnel" (Galton, 1889/1973, p. 65) along the base of the apparatus. By mediocrity, Galton referred to the tendency of the distribution of some characteristic to cluster around the mean value of the distribution. Applied to human intelligence, the approximately normal distribution of this psychological characteristic is due to successive independent random events affecting the individuals making up a population with regard to the formation of their respective intellects.

Galton and Randomness in the Normal Distribution

Many have missed the difficulty in relying on randomness as the basis for explaining the distribution of intelligence, knowing all the while that the essence of intelligence is the concern with finding order in the world and that the greater an individual's intelligence, the greater the ability to find order in the world. It appears that Galton himself missed this point. As noted, Galton (1889/1979) wrote in *Natural Inheritance* that the "number of small and independent accidents" (pp. 64-65) that befall some developing entity is the basis of the principle that "explains the reason why mediocrity is so common" (p. 65). This applies whether the concern is the position of the shot or the distribution of intelligence. These "accidents" are the basis of the assumption that helps to explain the clustering of intelligence around its mean value in a human population.⁸

Indeed, Galton (1869/1972) specifically considered mental ability in detail in *Hereditary Genius*, and he found that mental ability in his study was normally distributed. In *Hereditary Genius*, Galton discussed how he came to employ the normal distribution. He noted that the normal distribution was first found to be applicable to the distribution of data from astronomical observations and that Quetelet, the Astronomer-Royal of Belgium, found that it also

⁸ In a lecture on statistical mechanics, I once heard the professor express amazement with regard to the apparent incongruity that order, found in physical law, and randomness, found in statistical mechanics, both appear to characterize the functioning of the physical world. The amazement should be no less, and perhaps should be greater, when it appears that randomness is a factor in intelligence, that psychological characteristic the very nature of which is the capacity to find order in the world.

described proportions of the human body. Then, in the "Prefatory Chapter to the Edition of 1892" (the second edition) of *Hereditary Genius*, Galton described his next step. "After some provisional verification, I [Galton] applied this same law [the law of frequency of error or the normal distribution] to mental faculties, working it backwards to obtain a scale of [mental] ability, and to be enabled thereby to give precision to the epithets employed [such as the term 'eminent']" (Galton, 1869/1972, p. 29).

In his preface to the second edition, Galton did not provide any other information concerning the nature of the normal distribution. Specifically, he did not question how his conception of mental ability could be concerned with order in the world and yet be normally distributed. As we have seen, in the intervening years between the first and second editions of *Hereditary Genius*, Galton wrote specifically on the central role of randomness to the theoretical derivation of the normal distribution.

Perhaps Galton and others, such as Glass and Stanley (1970), have not addressed this point because it is so difficult to conceive in actuality that the development of intelligence is a sequence of similar developmental stages in which the likelihood of any sequence of outcome events at each stage is equally likely. But the conceptual development of the normal distribution from an assumption that fundamentally incorporates randomness is not only applicable to statistical mechanics. As seen in Galton's own description of the normal curve, it is the basis for what he proposed as well regarding the applicability of the normal curve to the distribution of human intelligence.

THE NORMAL DISTRIBUTION OF INTELLIGENCE

Because the conceptual development of the normal distribution depends on randomness, and because intelligence intuitively seems to be exclusively concerned with order, it is important to establish conclusively that intelligence, as measured by a major psychometric instrument, is very close to being normally distributed. Any significant deviation would invalidate the thesis proposed here, namely that probabilistic considerations are at the heart of intelligence which is indisputably characterized by order.

The presence of ordered and random phenomena in intelligence is represented in features of the tests themselves that are used to assess intelligence. That the distribution of intelligence in a large population is very close to being normally distributed is generally acknowledged in psychology. Primary assessment instruments of intelligence, such as the Wechsler Adult

Intelligence Scale and the Wechsler Intelligence Scale for Children have all found this to be so. Yet, in the various tasks used to determine intelligence in these tests, the underlying criterion in them is the degree of order the subject can find in the task presented.

The Wechsler Adult Intelligence Scale, Revised (the WAIS-R) will be discussed in some detail in order to demonstrate the inclusion of ordered and random processes in a major psychometric test of intelligence. The WAIS-R is very likely the most widely used psychometric instrument for assessing intelligence of adults in the United States. It yields an overall score for intelligence called a Full Scale IQ. In addition, it yields a Verbal IQ, a Performance IQ, and a standard score for each of the six Verbal subtests and the five Performance subtests for nine age groups spanning the ages 16 years to one month short of 75 years.

Each of the subtests on the WAIS-R is concerned with finding order in one's experience and the higher one's standard score on a subtest, the higher is one's ability to find order in the realm of experience gauged by the subtest. A brief description of each of the subtests will be provided to demonstrate the point that each subtest is indeed concerned with finding order in one's experience. Comments provided by the The Psychological Corporation (1985), which publishes the WAIS-R, are noted for most of the subtests to indicate that the test publisher itself considers that these subtests measure a person's ability to find order in experience. Anastasi's (1954/1988) descriptions of the subtests have also been used in developing the present descriptions.

Consider the Verbal subtests in which questions from the examiner and answers from the subject are spoken:

Verbal Subtests -

- Information: Questions assessing general knowledge that people would ordinarily be able to gain in society.
 "Individuals who do well on this subtest usually are alert to the environment and have good long-term memory for facts" (The Psychological Corporation, 1985).
- 2. Comprehension: Questions that assess practical knowledge, including basic reasoning and social judgment, and involvement in mainstream culture.

- 3. Arithmetic: Basic numerical problems that do not require sophisticated mathematical skill. Problems are like those given in elementary school.
- 4. Similarities: Questions asking how two things are alike. This subtest "calls for the ability to see relationships between things and ideas, and to categorize them into logical groups" (The Psychological Corporation, 1985).
- 5. Digit Span: Lists composed of three to nine digits that the subject is first asked to reproduce in the order first stated. Then lists of two to eight digits are read and the subject is asked to reproduce the digits in each list backwards. Besides assessing a form of short-term memory, this subtest "reflects the individual's attention span and ability to concentrate" (The Psychological Corporation, 1985).

On the Performance subtests, questions from the examiner are generally presented pictorially, and the answer from the subject generally requires some manual performance.

Performance Subtests -

- 1. Digit Symbol: Nine symbols are paired with nine numbers. Given a set of numbers, the subject is asked to write down the corresponding symbols. Besides measuring visual-motor speed, this subtest "may be affected by visual memory...and the ability to learn nonverbal material" (The Psychological Corporation, 1985).
- 2. Picture Completion: The subject is presented with a set of pictures and asked to find the particular item missing from each picture. This subtest "measures the individual's alertness to visual detail and the ability to grasp the meaning of details within a complete picture" (The Psychological Corporation, 1985).
- 3. Block Design: The subject is presented with shapes containing areas of red and white and the subject is asked to assemble the objects using a set of blocks the sides of which are red, white, or red and white. This subtest "is

- essentially a measure of the ability to handle spatial relations" (The Psychological Corporation, 1985).
- 4. Picture Arrangement: Stories are told through series of pictures. For each story, the subject is asked to assemble the pictures in the correct order. In this subtest, the subject is required to "evaluate the social relevance of pictured situations, to anticipate the consequences of actions, and to distinguish essential from irrelevant details" (The Psychological Corporation, 1985).
- 5. Object Assembly: Subjects are presented with pieces of flat figures (similar to large pieces of a jig saw puzzle without the tight connections) jumbled together and asked to assemble the various figures. The pieces of each figure are presented separately. "A sense of space relations, visual-motor coordination, and persistence are among the qualities measured by this subtest" (The Psychological Corporation, 1985).

Does the Standardization Sample Reflect the Population from which It was Drawn?

In order to determine whether the distribution of some characteristic in a population is normal, one either has to measure this characteristic on the entire population or use some sampling technique that will create a sample that reflects the population from which the sample is drawn. If the latter alternative is used, it is important to show how closely the sample resembles the population from which it was drawn. In this way, one can be confident that the normal distribution of the sample accurately reflects that the population itself is normally distributed.

This point is particularly important because of the unusual nature of the result presented concerning the relation of randomness to the normal distribution of intelligence. If either the sample distribution of intelligence is not very close to the theoretical normal distribution, or if the sample does not accurately reflect the population from which it is drawn, the basis for the arguments presented in this paper are subject to serious doubt.

In developing the WAIS-R, subjects for the standardization sample were chosen so as to reflect various characteristics of the adult population of the United States. These seven characteristics are: age; sex; race (white or

nonwhite), urban-rural residence; education; occupation; and geographic region. The WAIS-R manual states, "Individual cases [in the standardization sample] were described in terms of age, sex, race, geographic region, and occupational group" (The Psychological Corporation, 1981, p. 18). This appears to indicate that individual subjects were selected so that the subjects in a particular age range, of a particular sex, of a particular race, and from a particular geographic region, and in a particular occupational group would reflect similar groupings of characteristics in individuals in the population according to the representation of these groupings in the population. Because educational attainment and urban-rural residence were not included in the above quote, it appears that educational attainment and urban-rural residence were not included in these sample groupings.

Consider the age group from 25 to 34 years of age. This age group in the standardization sample on the matching variables (except age) is compared to individuals 25 to 34 years of age in the population from which this sample was selected. The data indicate that the standardization sample in the age range 25 to 34 years of age was very close on the seven characteristics noted above to the individuals in the population 25 to 34 years of age. Following are some sample comparisons for this age range. Males with 12 years of education comprised 34.0% of the WAIS-R sample and 35.0% of the U.S. population. Individuals residing in urban areas comprised 74.3% of the WAIS-R sample and 74.6% of the U.S. population. White males who were craftsmen and foremen comprised 18.7% of the WAIS-R sample and 18.8% of the U.S. population. The data for the other age range groups of the standardization sample were likewise very close on the seven characteristics to the corresponding age range groups of the population from which the various parts of the standardization sample were drawn.

The Distribution of Test Results for the Standardization Sample

The Full Scale, or overall, IQ has a mean of 100 and a standard deviation of 15 for each of the age groups. Similarly, the performance and verbal IQ scales also have means of 100 and standard deviations of 15. Each of the standard score scales for the subtests for each age range has a mean of 10

⁹ The data for the comparisons between the WAIS-R sample and the U.S. population for the age range 24-35 are found in Tables 1 through 5 of the *WAIS-R Manual* (The Psychological Corporation, 1981). For a complete comparison of the standardization sample to the adult population of the United States, consult the *WAIS-R Manual*.

and a standard deviation of 3. From the Full Scale IQ for the entire sample population down to the individual subtest scales for each age group, the manual for the WAIS-R indicates that the scale is approximately normally distributed. With regard to the Full, Verbal, and Performance Scales, the manual states:

On any of the scales [Full Scale, Verbal, or Performance] for a given age, an IQ of 100 defines the performance of the average adult at that age. About two-thirds of all adults obtain IQs between 85 and 115 (1 standard deviation below and above the mean, respectively). About 95 percent score between 70 and 130 (2 standard deviations on either side of the mean). More than 99 percent obtain scores between 55 and 145 (3 standard deviations from the mean). (The Psychological Corporation, 1981, p. 27)

These are the percentages that are expected for a normal distribution. In addition, there is in the WAIS-R Manual a table (i.e., Table 22) in which percentile ranks and standard deviations from the mean are given for different scaled scores for any specific age group on any single subtest. The percentile rankings for the standard scores and the standard deviations from the mean are those expected for a normal distribution.

The WAIS-R manual provides a comparison of data for the distribution of Full Scale IQ on the standardization sample with what would be expected in a normal distribution. Just how close do the empirical data with regard to intelligence, specifically Full Scale IQ, resemble the normal distribution? The empirical data are very close to those that would be expected for a normal distribution of intelligence.

Following are the comparisons for Full Scale IQs.¹⁰ For the range of IQs 69 and below, 2.3% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 2.2%. For the range of IQs 70 to 79, 6.4% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 6.7%. For the range of IQs 80 to 89, 16.1% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 16.1%. For the range of IQs 90 to 109, 49.1% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 50.0%. For the range of IQs 110 to 119, 16.6% of the WAIS-R sample fell

¹⁰ Table 6 in the WAIS-R manual contains the data comparing Full Scales IQ distribution of the WAIS-R sample to the expected distribution for a theoretical normal curve.

is this range, while the expected value in a theoretical normal curve is 16.1%. For the range of IQs 120 to 129, 6.9% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 6.7%. For the range of IQs 130 and above, 2.6% of the WAIS-R sample fell is this range, while the expected value in a theoretical normal curve is 2.2%. Thus, one can be confident that intelligence as measured by the WAIS-R is very close to being normally distributed in the adult population of the United States.

It appears that random processes, that are at the core of the normal distribution, play a significant role in the distribution of intelligence as measured by the WAIS-R. If this thesis is true, and ordered and random processes are both characteristic of intelligence, what are its implications? A brief exploration of quantum mechanics will help in determining these implications.

QUANTUM MECHANICS, RANDOMNESS, AND ORDER

How is it that randomness, that is the basis of statistical mechanics, and order, as embodied in physical law, both accurately portray the functioning of the physical world? Prior to quantum mechanics, this circumstance concerning the physical world was particularly troubling as the behavior of discrete physical existents, such as particles, was thoroughly deterministic in nature (as, for example, was discussed regarding Newtonian mechanics) when these discrete existents were considered individually or in terms of interactions among a small number of them.

With the development of quantum mechanics, probabilistic notions extended to the realm of individual existents traditionally known as particles. In quantum mechanics, the wave function associated with a physical existent, which forms the foundation for whatever can be known about this existent, is found to develop in a precise manner, a lawful manner, in accord with the Schrödinger wave equation. The wave function serves as the foundation for this knowledge by providing the basis for developing a probabilistic prediction concerning the result of a measurement of some observable quantity of the physical existent. (The probabilistic interpretation of the wave function was originally suggested by Born in 1926 [Born, 1926/1983]). Further, the wave function associated with the existent, in general, changes throughout space instantaneously upon measurement, and the probabilistic predictions concerning physical quantities of this existent as a result also, in general, change instantaneously upon measurement.

The importance of randomness to the description of individual physical existents through the introduction of probabilistic prediction in quantum mechanics can aid in understanding the role of random processes in statistical mechanics as well as in the distribution of intelligence. It does so in part by indicating that probability can be concerned with knowledge of the world without recourse to other more fundamental underpinnings, specifically a world that exists independently of the knowing individual.

In quantum mechanics, probabilistic prediction is not an approximation due to lack of specific knowledge about some circumstance in the physical world. That probabilistic prediction in quantum mechanics is not an approximation due to lack of specific knowledge is supported by features of quantum mechanics briefly noted above. These features will now be explained in more detail, and the nature of the theoretical structure supporting this probabilistic prediction will be explored. The features noted were:

- Whatever can be known about a physical entity can be derived from the wave function associated with that entity; For all physical existents described within quantum mechanics, and more generally quantum theory, whatever can be known about the world composed of these existents can be derived from the wave functions associated with them.
- Quantum mechanics provides probabilistic predictions about the results of measurements when they are made and does not describe with arbitrary precision the deterministic functioning of the physical world;
- 3) The wave function that underlies the probabilistic predictions of some observable quantity of the physical existent in general changes instantaneously throughout space when the existent is observed in a measurement.

In addition there is a fourth point concerning the basic theoretical structure of quantum mechanics that supports points two and three and provides more theoretical support for the general thesis that the quantum mechanical wave function is concerned with more than the classically considered physical world. This point is:

4) The quantum mechanical wave function associated with a physical existent traditionally considered a particle is

mathematically complex; it has both mathematically real and imaginary components.

The immediate change in the quantum mechanical wave function throughout space that generally occurs in measurement supports the thesis that the probabilistic knowledge derived using the wave function is directly linked to the physical world. It does so because this instantaneous change cannot be explained physically unless the velocity limitation of the special theory of relativity (i.e., the invariant velocity of light in inertial reference frames) is violated. As the knowledge derived using the wave function is probabilistic in nature (one feature of this nature being that the wave function is concerned with the future of the physical world when a measurement is taken), there is no mistaking this knowledge as simply a reflection of an independent physical world functioning in a deterministic manner. The knowledge derived using the wave function is primary, and it can be ascribed to the functioning of the physical world itself because there is no world posited to exist independently of one's knowledge of it. Unless one concludes the special theory is incorrect, the immediate change in the wave function indicates that it has some feature that is non-physical. It is thus natural to consider the wave function to be in part cognitive because the wave function itself is the basis for the probabilistic knowledge in quantum mechanics.

How is point four, the complex nature of the wave function, important to both the probabilistic predictions of quantum mechanics and the in general immediate change in the wave function when an observation is made in the course of a measurement? A complex wave function can change throughout space instantaneously without violating the velocity limitation of the special theory. This is unlike a mathematically real wave function that is traditionally considered to reflect the "real" physical world, that is the measurable physical world (Eisberg and Resnick, 1985).

It would be problematic if the quantum mechanical wave function associated with a physical entity *were* mathematically real, and thus presumably measurable in the physical world. In this case, the act of measurement, including the observation that ultimately determines in quantum mechanics when a measurement is made, could be described only in terms of physical events because the mathematics underlying the measurement result would consist of a mathematically real function. As the measurement result is unavoidably tied to the act of measurement in quantum mechanics, the act of observation central to the measurement result would be subject to physical law

alone. Without violating the velocity limitation of the special theory, for example, there would not be a physical basis for explaining how an observation in general results in an immediate change in the wave function throughout space.

Instead of a fundamentally physical explanation underlying the probabilistic predictions that are derived from the wave function, the use of a complex wave function emphasizes the fundamentally probabilistic character of quantum mechanics, that is its foundation in predictive knowledge of the physical world. A complex wave function is particularly well suited for developing probabilistic predictions of results of arbitrarily precise measurements because the product of complex values of the wave function multiplied by their complex conjugates result in mathematically real values, values that represent the probabilities of some measurement outcome. Eisberg and Resnick (1985) stated, "It is a desirable feature [that the wave function is complex] because it makes it immediately apparent that we should not attempt to give to wave functions a physical existence in the same sense that water waves have a physical existence. The reason is that a complex quantity cannot be measured by any actual physical instrument" (p. 147).^{11,12}

For describing our mental activity [which includes perceptions of the physical world], we require, on one hand, an objectively given content to be placed in opposition to a perceiving subject, while, on the other hand, as is already implied in such an assertion, no sharp separation between object and subject can be maintained, since the perceiving subject also belongs to our mental content. From these circumstances follows...that a complete elucidation of one and the same object may require diverse points of view which defy a unique description....This domain [i.e., perception], as already mentioned, is distinguished by reciprocal relationships which

Resnick (1985) went on to write, "the *wave functions are computational devices* which have a significance only in the context of the Schrödinger theory of which they are a part" (p. 147). Though wave functions clearly have a critical role in the theoretical structure of quantum mechanics, Eisberg and Resnick relegate their status to "computational devices." The wave function is the basis for deriving whatever can be known about the physical existent with which it is associated. As has been shown, this knowledge is integrally linked to the physical world. To consider the wave function simply a "computational device" deprives it of any form of existence that would allow it to provide the basis for probabilistic predictions concerning the physical world, predictions that are empirically verifiable. The only reasonable nature that can be ascribed to the wave function is that it has both cognitive and physical components.

¹² In a related vein, Bohr (1934/1961) implied that the need for complementary description of physical phenomena in quantum mechanics follows from the general nature of perception. (It should be noted that this position is not the one most often associated with Bohr.) In the following quote, Bohr considered the nature of quantum mechanics in the larger context of perception.

KNOWLEDGE, INTELLIGENCE, AND THE PHYSICAL WORLD

Thus, in quantum mechanics, probability is concerned with knowledge of the physical world, and this knowledge of the world is tied directly to the physical world. This knowledge is not an approximation of a physical world that functions independently of the individual considering it. Ouantum mechanics indicates that probability, and the randomness that is at the core of probability, need not be opposed to knowledge or intellect. In fact, the probabilistic character of quantum mechanics points toward the primacy of knowledge and the intrinsic significance of the intellect that possesses it. It points toward the significance of the individual's direct relation, and indeed link, to the physical world. The probabilistic knowledge derived in quantum mechanics can be ascribed to the functioning of the physical world itself. These are the lessons of quantum mechanics that are relevant to understanding the nature of intelligence. In classical mechanics, especially Newtonian mechanics, knowledge derived from physical theory can be seen as secondary to the deterministic functioning of the physical world which does not rely on the observing and thinking individual in any essential way.

Quantum mechanics thus provides a window through which to see that the probabilistic basis for the normal distribution of intelligence as measured by various psychometric instruments is not problematic. Quantum mechanics does so by indicating that the individual applying his intellect to some set of phenomena, in this case the physical world, is linked directly to these phenomena. This result is supported by the normal distribution of intelligence, indicating that probabilistic considerations are at the heart of intellect. There is no independent world, physical or mental, in which intellect "really" resides

depend upon the unity of our consciousness and which exhibit a striking similarity with the physical consequences of the quantum of action (Bohr, 1934/1961, pp. 96, 99).

Bohr arrived at this position because the roots of the "reciprocal relationships" in quantum mechanics, which follow the uncertainty principle, are found in the wave function. It has been noted that the wave function generally changes upon measurement of the existent with which it is associated immediately throughout space (unrestricted by the velocity limitation of the special theory). These "reciprocal relationships" in quantum mechanics then are also generally affected by the change in the wave function that occurs in measurement. To the evidence cited earlier of the link between cognition and the physical world in quantum mechanics, it should be added that in quantum mechanics a measurement is not complete until its result is observed or known by a person (Snyder, 1989, 1992). Thus, the observer's perception of the measurement result is naturally included in the "mental activity" Bohr noted.

and which the knowing individual can only approximate. The knowing individual is directly linked to intelligence as the known phenomena: it is the normal distribution of intelligence, with its fundamentally probabilistic nature, that provides the basis for this direct link.

Instead of the phenomena that are the object of the individual's intellect providing only an ambiguous basis for their being known, a phenomenon which has fundamentally a probabilistic character can fundamentally *only* be known. The presumably lawful character of the building blocks of intellect points only to the genuinely probabilistic nature of the normal distribution of intelligence in the same way that physical law for Tolman carries with it its own limitations. What appear to be difficulties in having both random and ordered processes as features of the same phenomenon, intelligence, instead only points toward the unification of the way intelligence itself is known and how an individual knows the world.

Extending the Thesis from Intelligence to Statistical Mechanics

It should be noted that the previous discussion concerning the nature of probability in quantum mechanics indicates that cognition is tied directly to the physical world in statistical mechanics. Quantum mechanics points toward how physical theory need not depend on an assumption that the physical world functions in a deterministic manner independent of the experiencing person. In statistical mechanics, we see that:

- 1. There is no more fundamental reality behind the probabilities found in statistical mechanics that are developed from the fundamental assumption;
- 2. The probabilities are concerned with knowledge;
- 3. Order as expressed in physical law does not explain the tendency of a large system to occupy that configuration allowing for the most accessible stationary quantum states of the system.

These points allow for the conclusion that knowledge in statistical mechanics is not simply a reflection of the functioning of a physical world independent of the knowing individual. These points indicate further that knowledge of processes in the physical world with which statistical mechanics is concerned itself characterizes the functioning of the physical world that supports these processes. These results are reinforced by the fact that probabilistic predictions

characterize not only the behavior of large groups of physical entities in statistical mechanics, but in quantum mechanics such predictions characterize the behavior of the existents considered individually.

A cognitive component in the physical world in quantum mechanics and statistical mechanics eliminates the artificial separation between the object as perceived and its "real" existence in a world essentially independent of the perceiver and which somehow supports the perception. This thesis is not solipsistic because both quantum mechanics and statistical mechanics provide avenues for empirical verification. It is not surprising that empirical test has provided a great deal of support for both of these theories because they provide support for a natural and straightforward thesis, namely that the observer and observed phenomenon in the physical world are directly linked.

CONCLUSION

The well-known, but nonetheless, problematic empirical evidence that intelligence is normally distributed has been explored. It appears problematic because the chief characteristic of intelligence is the ability to find order in the world, to know the world, and the basis for the normal distribution of intelligence relies on random processes. Exploring statistical mechanics was an aid to understanding the relation between order and randomness in intelligence. Just as phenomena that demonstrate order on a macroscopic level, such as temperature and chemical potential, arise from random processes in statistical mechanics, so the ordered character of intellect arises upon the basis of random processes. Random processes essential to the normal distribution of intelligence provide for a direct link between the knowing individual and intelligence as a known object, where intelligence does not exist as some entity radically separated from the knowing individual.

In that there are similar circumstances in statistical mechanics to those concerning the nature of intelligence, the same general analysis concerning the relation of ordered to random processes in intelligence applies to the nature of the physical world and the relation of mind to it in statistical mechanics. The presence of fundamentally random processes in the physical world indicates that the physical world may be known directly without recourse to more fundamental underpinnings (i.e., a world that exists independently of the individual considering it). There is nothing behind the known physical world that operates in some independent and deterministic manner. There is only that which is known. Lawful processes play a major role in that the parameters of

their applicability indicate their own limitations as the basis to explain characteristics that are naturally explained by the introduction of random processes. It is likely that physiological concomitants involved in the development, and perhaps current operation, of intellect also demonstrate the same relationship between ordered and random phenomena found on a psychological level. On a microscopic level, it is expected that random neurophysiological processes would give rise to ordered patterns of neurophysiological activity on a macroscopic level.

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