Contextual Geometric Structures: modeling the fundamental components of cultural behavior Bradly Alicea
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Abstract

The structural complexity of culture is inadequately characterized simply by modeling beliefs or the inheritance of discrete ideas. Formal computational modeling and algorithmic models of culture have been focused on the inheritance of discrete cultural units, which can be hard to define and map to practical contexts. In cultural anthropology, research involving structuralist and post-structuralist perspectives have helped us better understand culturallydependent classification systems and oppositional phenomena (e.g. light-dark, hot-cold, goodevil). Research in cognitive neuroscience has placed these oppositions in the context of complementary sets that are represented dynamically in the brain, but no model for the evolution of these sets has of yet been proposed. To fill this void, a method for simulating cultural or other highly symbolic behaviors called contextual geometric structures will be introduced. The contextual geometric structures approach is based on a hybrid model that approximates both individual/group cultural practice and a fluctuating environment. The hybrid model consists of discrete automata (particles) with a soft classificatory structure embedded in a Lagrangianinspired dynamical system that simulates phase space relations and environmental inputs essential to understanding the evolution of cultural patterns. Stochastic and deterministic evolutionary dynamics are approximated by perturbing the flow field, and conditional features provide a top-down means of guiding cultural evolution. These two models are connected by several equations related to diversity, learning, and forgetting. This model can yield important information about multiple structures and social relationships, in addition to phenomena related to sensory function and higher-order cognition observed in neural systems.

Introduction

Why is cultural change so complicated? Intuitively speaking, it seems as though cultural change should be easy to predict. Given the adaptable nature of culture and other symbolic behaviors, changes in the environment should be quickly matched by corresponding changes in cultural representations. Yet there are also many anecdotal examples of cultural inertia, or practices that persist even in the face of negative selection. There are two other features of culture that are puzzling from a functional perspective. Exaptive-like phenomena are cultural practices that emerge for one use and then are eventually used for another purpose. Likewise, reaggregation allows for practices to be assembled from several disparate cultures into the same conceptual framework.

All three of these features (inertia, exaptation, and re-aggregation) suggest that cultural change is highly complex and nonlinear. Yet how can we represent this complexity using a computational framework? The patterns that define cultural behaviors across generations and contexts are most likely synthesized via emergent and evolutionary processes. Unlike goal-directed behaviors such as reaching for a cup of water or following a scent, there is often no clear optimum to achieve or even guiding principles to adhere to. Cultural representations should "make sense" of procedural knowledge in a way that is not only flexible but also constrained by conceptual interlinkage.

Cultural systems have been understood using a number of theoretical perspectives. Structural [1] and post-structural [2] perspectives are based on the notion that cultural life is based on a set of structures orthogonal to human cognition. These structures ostensibly emerge from common patterns of behavior over multiple generations, and represent the outcomes of cultural evolution. One signature of these ephemeral structures is the cognitive representation of oppositional sets, which are bounded by extreme concepts for each category. For example, there may be an unnamed category shared across cultures that are bounded by light and dark. The extremes of this category are bounded by human perception, so that experience of each culture can be contained within.

A "structure" can be defined as a set of relationships between objects in the environment, or experiences that can vary from person to person but are grounded in the same underlying concepts. These structures, which are a critical and implicit component of human cultural practice, have an underappreciated computational potential. This is particularly useful since many of these features lie outside the scope of experience but are essential to understanding the evolution of culture across multiple generations [3]. Even more importantly, these structures might be an essential feature of how cultural practices are represented in a neural architecture. In recent years, brain scientists have applied this idea to a system of oppositional sets called complementary pairs [4]. In this approach, oppositional sets are contingent upon coupling, oscillatory, and heterogeneity in the dynamics of neural circuits. While these approaches hold much promise for the study of culture and symbolic systems, there remains a need to more fully integrate dynamical and structural approaches. We propose that by combining the structural features of cultural practice with a dynamical view of evolution will result in a model of cultural evolution that maps to both social phenomenology and physiological function.

In addition, cultural and symbolic behavioral systems share many features with physical systems that exhibit chaotic behavior. It is this combination of quasi-evolutionary and chaotic dynamics that makes our approach unique. The approach presented here, called **contextual geometric structures**, is a Lagrangian-inspired approach that focuses on the structural complexity of cultural and other symbolic behavioral phenomena. In this paper, I will introduce a **hybrid soft classification/hydrodynamics model** in the context of generalized cultural phenomena. Initially, we will outline basic features of the contextual geometric structure model. Next, we will demonstrate how this model fits into the milieu of cultural diversity and evolution. This includes features that approximate complex and diverse phenomena. Finally, we will consider this model in the context of neuronal processes.

Contextual Geometric Structures

Prior approaches to modeling culture have included forays into population genetics and game theory [5-7], memetic representations [8-9], specialized genetic algorithms [10-11], and conceptual blending models [12-13]. In this paper, a different approach will be used to move towards a computational approach that incorporates some of these prior approaches.

Contextual geometric structures provide advantages that previous models do not. Models inspired by population genetics and game theory are explicitly discrete and focus on inheritance, and so do not produce many of the nonlinear behaviors that culture embodies. While memetic and conceptual blending models may both provide insights into the "churn" (e.g. combinatoric potential) of cultural change, neither are explicitly dynamical. While computationally efficient, specialized genetic algorithms do not express the nonlinear and "fluid" output that cultural

behaviors not explicitly associated with beliefs might exhibit. The ability of contextual geometric structures to map both of these metaphors to a formal, computable structure is perhaps greatest advantage of this technique.

Model Components

Contextual geometric structures consist of a hybrid model: a "soft" computational structure and a dynamical system representing the individual automata and the environment, respectively. Each automaton represents an individual with a brain that houses multiple conceptual spaces we call kernels. The automata then interact in an environment represented by a flow field. The dynamics of this flow field reinforces evolutionary behaviors and ultimately complex structural patterns.

Single automata

The cultural repertoire of each automaton (or particle) uses a soft classification scheme to represent the elements of culture. **Soft classification** [10], a fuzzy logic-inspired methodology, provides the several advantages. One of these advantages involves the capacity to represent different cultural contexts in the same model. Another advantage involves the capacity to represent degrees of specific cultural and symbolic behaviors rather than merely its presence and absence. All natural phenomena classified by any single cultural group has a membership function on a membership kernel (Figure 1), bounded by the capacity of a sensory system. The resulting cultural representation of a phenomenon will sit somewhere on this scale.

n-dimensional "Soft" Kernels

Figure 1 show one- and two-dimensional examples of cultural representation of "hot" to "cold". Figure 2 demonstrates the membership kernel for three different cultures. The logical structure consists of various **membership kernels** which serve to classify the experience of each automaton into a common, objective scale. This graded scale acts to link together related concepts as shown in Figure 1. In this sense, they can be high-dimensional structures. One- and two-dimensional structures tend to represent concepts related to practice, while higher-dimensional structures represent a mapping from neurobiology to the cultural domain (see equations 1-5).

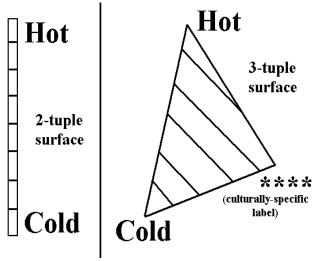


Figure 1. One- and Two-dimensional kernels embedded with *n*-tuple encodings.

In Figure 2, the objective scale for hot and cold stimuli has been mapped to a 2-tuple surface for three cultures (A-C) and their overlap. There will be **variability between individuals and cultures**, which can be evaluated using a common scale. To map physiological function to cultural and symbolic representations, **contextual anchors** will be used (see 3-tuple surface, Figures 1 and 2). In context, contextual anchors provide a means to mediate the membership between hot and cold with procedural knowledge.

When different cultural categories overlap, it is indicative of cultural diversity in the form of **intermingling**. However, separation between categories is also indicative of cultural diversity in the form of **distinction**. Cultural distinction is a common feature of cultural evolution which can sometimes be imposed by its practitioners. In our context, we will assume that cultural distinction is an emergent feature, and is specified by the segregation factor (see Equation 6). Segregation or distinction is characterized by the non-overlapping region between B and C in Figure 2.

Environment

The environmental component of contextual geometric structures involves a second-order Lagrangian system with dynamics that produce solutions analogous to **Lagrangian Coherent Structures** (LCS) [14]. LCS structures are defined as "ridges" of particles aggregates in different partitions of the flow field and obtained using either the **Finite Time Lyapunov Exponent** (FTLE - solved with regard to temporal divergence) or the **Finite Space Lyapunov Exponent** (FSLE - solved with regard to spatial divergence) [15-16]. Characterization of these features can be encapsulated in a measure called the **iterated temporal divergence** (see Equation 7). This methodology has previously been applied as a generalized analogy for evolvability in biological evolution [17]. This work is an extension of this application, the schematic of which is shown in Figure 3.

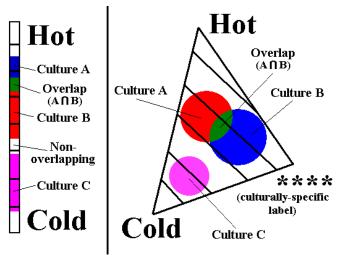


Figure 2. A soft classification kernel populated with the space for three different cultures. In this case, the same automaton is a carrier for three sets of cultural knowledge simultaneously.

As can be seen in Figure 3, the automata are initialized in the same location and then get diffused by the force field environment. The automata also have the properties of **replicator vehicles** that reproduce according to specified parameters. While the selective component of the

model has yet to be specified completely, we believe that LCS-like models produce default effects dominated by evolutionary neutrality [18]. In addition, our goal is to observe cultural diversity, which involves far-from-equilibrium and sub-optimal behaviors obscured by strong selective pressures.

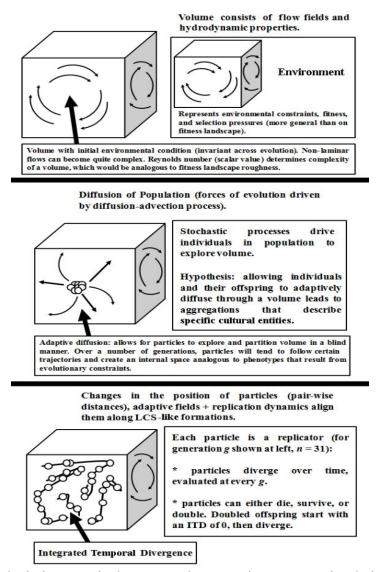


Figure 3. Cartoon depicting a typical contextual geometric structure simulation over the course of cultural evolution. TOP: initial condition, MIDDLE: active diffusion of the automata population, BOTTOM: final volume features contextual geometric structures.

When applied to cultural systems, the LCS approach [19] typically involves **observing the diffusion of particles** in a **hydrodynamic force field** and **tracking the structures** that result (Figure 3). These structures are observed to collide, pull apart, and intermingle over time. Yet external forces introduced by the flow field can influence diffusion, and so the particles will still aggregate into recognizable and orderly structures. In the case of contextual geometric structures, structures form as a consequence of both interactions between agents over time and evolutionary constraints.

Structures, Diversity, and Evolution

In order to better understand the role of evolution in the emergence of contextual geometric structures, it is important to take a closer look at the outcome of interactions between three distinct automata populations. Figure 4 shows an example run using automata from three distinct cultures (red, blue, and black). This 2-D LCS volume features 165 automata present at the following frequencies: black (0.35), blue (0.35), and red = (0.30). This allows us to observe a number of purely physical outcomes after the evolution of an initial population. The first of these are loosely-organized vortices, which can either be homogeneous (all automata of the same color) or heterogeneous (automata of multiple colors). The second physical feature is a cluster, often found along edges of the volume. These aggregates can be either homogeneous or heterogeneous, and can be considered products of pure diffusion. The third physical feature is a ridge, which can be either homogeneous or heterogeneous and often lead to the formation of vortices. The fourth physical feature is a vortex, which are tightly packed aggregations of automata which are usually homogeneous.

Yet how exactly do these formations map to the evolution of culture? Using a mixed initial population can lead to competition, selection, and other quasi-evolutionary dynamics. Yet to allow the diffusion of automata within a flow field to exhibit behaviors relevant to cultural structures and practice, the soft classifications inherent to each automaton must be coordinated using a series of features based on principles of attraction and repulsion. Three features are expected to produce a broad range of highly-complex and realistic cultural scenarios.

Initial condition of model

Our choice of a hybrid soft classificatory/hydrodynamics model allows us to observe evolution enforced by self-organization. The tracking of particle populations allows for complex dynamics to emerge out of interactions particles make between automata and the environment. Given this predicted result, both wildly fluctuating (intermittent) and short-lived (transient) solutions should be possible.

In our model, the automata dynamics will tend to exhibit deterministic behaviors at low Reynolds numbers (R_e). In hydrodynamic contexts, the Reynolds number (R_e) is a dimensionless tuning parameter which provides a means to organize flow fields. This allows for the approximation of complex dynamic patterns given controlled external perturbation, for heterogeneities can arise even at low R_e values.

We have exploited this complex behavior embodied by the R_e value to approximate neutral processes and other evolutionary phenomena. For example, neutral processes might be approximated by adding Gaussian noise into the flow field. This can affect transient portions of the flow, which can mimic the uniform diffusive properties of neutral evolution [20]. Likewise, we can approximate natural selection by adding 1/f noise to the flow field. This and other forms of asymmetric perturbation can mimic the directional properties of selection [21].

Depending on force parameters that constrain the simulation environment, the simulation can yield vastly different behaviors. Yet the relational structure between concepts can remain quite similar across contexts. One feature of evolutionary systems is that they are often constrained to a particular evolutionary trajectory by past trajectories and current features [22].

These constraints combined with environmental fluctuations simulated by the addition of systematic noise produce quasi-evolutionary dynamics.

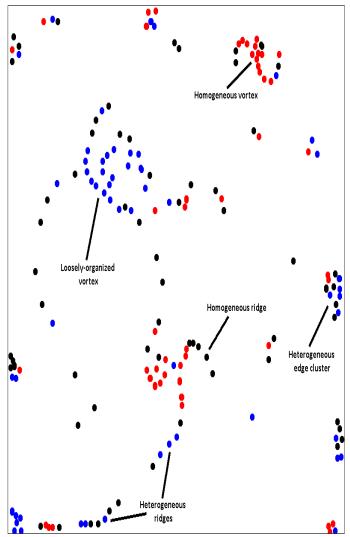


Figure 4. A 2-dimensional space representing an evolved population of automata representing three distinct cultures (Black, 58 automata; Blue, 58 automata; Red, 49 automata). Each subpopulation has a multifaceted set of relationships with regard to the other two.

Features that shape evolution

As previously mentioned, systematic noise can be used to perturb the flow field. This perturbation can approximate different evolutionary dynamics. In a like manner, **conditional features** are top-down, deterministic perturbations of the flow field that act like selective mechanisms. Three conditional features are proposed: purity, associativity, and syncretism. These features are predicted to produce a wide range of contextual geometric structures that may be identified as complex cultural dynamics (see Figure 5). Each conditional feature operates on the *n*-dimensional kernels of each automaton. While a lack of selection can produce evolutionary dynamics, higher-level organizational features can also increase the adaptive capacity of an evolutionary system [23, 24]. In our system, this is realized via simple interaction rules which lead to complex and highly-ordered outcomes.

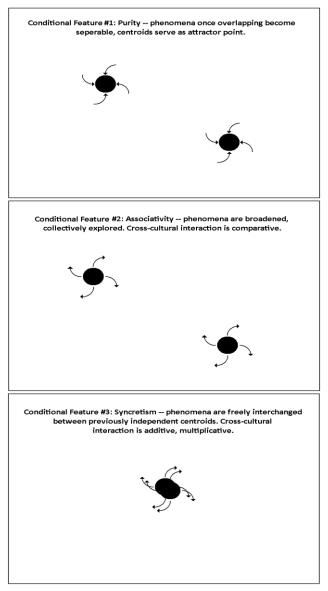


Figure 5. Three types of enforcing selection (conditional features) for the evolution of contextual geometric structures. Cartoon illustrates the general shape and mode of action characterized by each flow field modification.

Purity is successfully enforced when two or more distinct structures are formed. These structures are distinct in that all automata flow inward towards discrete vortices (Figure 5, Scenario #1). Over time, automata of different subpopulations exhibit total separation from one another. Associativity is successfully enforced when automata flow outward from established vortices along several trajectories towards one another (Figure 5, Scenario #2). Associativity often results in heterogeneous structures, and may lead to interactions between subpopulations.

The effectiveness of the purity and associativity sorting mechanisms can be detected using the **conditional diversity measure**, shown in Equation 8. This measure provides a profile of all automata within a certain level of Lagrangian divergence in the flow field by using a single

parameter D. When the value converges upon 0.5, the collection of automata that compose a loosely-associated structure or ridge is highly homogeneous. When the value approaches 0.0, the collection of automata is highly heterogeneous.

Syncretism involves the dispersion of automata towards automata of a competing population. This generally involves automata that are aggregated around two or more vortices. Based on this conditional feature, automata spiral outward from these aggregation centers towards each other in overlapping patterns (Figure 5, Scenario #3). The particles (automata) are freely interchanged in the resulting vortex and trailing flow (Figure 5, Scenario #3).

The predicted features shown in Figure 4 are approximations of what could be referred to as cultural practice space. In this sense, structures represent the aggregation of different cultures, which are distinct from individual automata holding representations for multiple cultures. This may allow us to make complex cross-cultural comparisons.

Intermittent and transient dynamics

One of our main assumptions is that variation in in a flow field of variable turbulence might contribute to local changes in the rate of evolution. Indeed, actively manipulating the flow parameters is another way to observe the "churn" of cultural evolution. Yet the relationship between the two model components might also allow us to observe selective conservation across cultural structures and practices.

What is the evolutionary relationship between the kernel values housed by individual automata and the Lagrangian unfolding in environmental space? To address this, we constructed a rate measure for learning and forgetting (see Equation 9). This measure bridges the gap between model components by tying kernel value segregation between populations to their distance in the Lagrangian flow field. These distances between concepts of practice and the evolutionary trajectory of individual automata (respectively) can be thought of as gaps that are translated between the two models. Learning occurs in cases where the gap between kernel values for different populations of automata is transferred to the evolutionary space (e.g. where the ITD value becomes larger over time). Forgetting occurs when the gap between kernel values for different populations of automata is transferred to the evolutionary space (e.g. where the ITD value becomes smaller over time).

Applying this measure when comparing subpopulations refines the model's ability to simulate the navigation of culturally-specific structures, which result in more coherent structures and life-like behavior. When very large $r_{\rm LF}$ values occur, learning predominates. When very small $r_{\rm LF}$ values occur, forgetting predominates. As in real culture, we expect representations of practice to fluctuate between extremes when the environment is unpredictable. In our model, this could be accomplished when the flow parameters produce a turbulent regime.

Conclusions

In this paper, we have proposed both an architecture and set of testable predictions for a model of cultural evolution focused on approximating the structures of practice. There are also several conclusions regarding the applicability of this model to real-world settings. The ultimate

goal is to model the diversity and evolutionary dynamics of context. The common features and shortcomings of this model can tell us something about the cultural structures related to practice.

The take-home message from this work is twofold. One part of the message is that that the inability of culture to adapt to rapidly-changing environments is not simply inertia. The other part of this message is to suggest that the ability of culture to adapt rapidly to environmental challenges is not free of constraints. Given these conclusions, this method is not meant to be a general-purpose model for understanding every cultural phenomenon. Our focus is on cultural practices and the structures that underlie descriptive structures best characterized by folk classification. While we have not revealed specific instances of cultural inertia, exaptation, and re-aggregation, we have established a model through which their cultural and neural substrate can be better understood.

To better understand the adaptive capacity of cultural systems, our ultimate goal is to characterize the labyrinthine features of a practice or ritual. This might explain why some practices are resistant to change (such as religious rites), while others can be highly improvisational (like a jazz score). Notably, this model does not account for hierarchical and ecological relationships between cultural and social groups. Our focus is more on the origins of cultural complexity and the spontaneous nature of cross-cultural interplay.

Idiosyncrasies observed in the adaptive capacity of culture can be seen in behaviors unique to our approach. For example, automata and even entire structures can exhibit recursive behaviors such as local cycling. In certain cases, groups of automata can get "stuck" in a local cycle. Automata that cluster into the tightest aggregates in the flow field are those that exhibit not only the same values for their kernels, but also those with shared referential structure. For kernels in a single automaton, these shared structures will tend to be recursive and circular logics. This is an essential ingredient for determining cultural context, but needs further development.

One key advantage of this model over previous approaches to modeling culture is relevance to neurobiological processes, particularly in terms of the information-processing essential to neural and cognitive system function. Thus, cultural-based classification can be placed explicitly in the context of neuronal function. Similar to a typical model of brain function, the fine-grained biological details are implicit in our soft classification model. Yet unlike a typical model of brain function, the evolution of collective behavior and shared cultural information over time are simulated using a physics-based model.

One example of dynamic, nonlinear neuronal processing related to cultural and symbolic behavior is multisensory integration. Multisensory integration involves the integration of visual, auditory, and somatosensory information at selective sites in the brain [25]. In mammals, the superior colliculus integrates visual and auditory sensory information for further processing relevant to the orienting function of attention [26]. This combination of senses is not linear, and the coincidence of stimuli in space and time results in a superadditive electrophysiological response [27].

Several of our particle structures, the classifications which form the basis of practice structures, are based on sensory interactions with the environment. However, we propose that such non-additive relationships form the basis of interactions between cognitive neurobiology and cultural structures and diversity. This may be particularly important when attempting to approximate diverse contextual responses to common stimuli often observed between individuals.

Future work should focus on several common phenomena in cultural systems. One of these is practices where a single dimension is isolated and treated as the entire practice. This often occurs in fundamentalist religions and other "cultish" behaviors. Another target for future research involves understanding seemingly illogical behaviors, such as reinforced ritualized behaviors despite the need for cultural change. Placing the evolution and information processing of these phenomena within a logical framework may lead to further advances in understanding behavior and ultimately human nature.

Methods and Equations

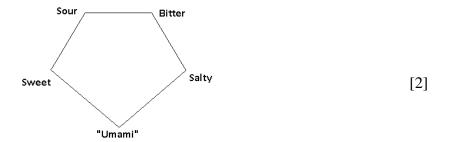
Particle structures. The number of potential structures that can interface with cognitive and neural processes can be quite large. We constructed five distinct particle structures, which can be defined as combination of dimensions representing both the fundamental limits of a neural subsystem (e.g. vision, touch, auditory, gustatory) and the centroid of a contextual variable (e.g. fluctuation, umami, modulation). The contextual variable has cultural meaning, and site in relation to these perceptual limits.

Soft classification allows for an *n*-tuple representational scheme which is not mutually exclusive. Phenomena can belong to two or more categories simultaneously, differing only in terms of degree. For example, changes in "light" do not result in corresponding changes to the "dark" classification. The use of contextual anchors (which also employ soft classification schemes) concurrent with the neural mechanism dimensions allows for non-additive cultural representations that approximate the sub- and super-additivity common in neural mechanisms of sensory integration.

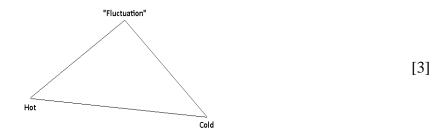
2-tuple without a contextual anchor. The first (and simplest) kernel design is the 2-tuple without a contextual anchor based on light sensing and visual perception. The example in [5] shows a binary opposition representing the transition between light and dark, an exemplar of which can be stated as [0.6, 0.2].



5-tuple with a contextual anchor. The second kernel design is a 5-tuple with a contextual anchor, and maps to the human gustatory system. The example in [6] shows a discrete set of tastes, an exemplar of which can be stated as [0.2, 0.2, 0.4, 0.8, 0.2].



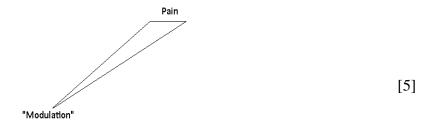
3-tuple with contextual anchor. The third kernel design is a 3-tuple with a contextual anchor, and maps to the function of thermoreceptors in the haptic system. The example in [7] shows a discrete set of tastes, an exemplar of which can be stated as [0.6, 0.2, 0.9].



3-tuple without contextual anchor. The fourth kernel design is a 3-tuple without a contextual anchor, and maps to the functions of arousal and emotion. The example in [8] shows a discrete set of emotional states, an exemplar of which can be stated as [0.1, 0.5, 0.3].



2-tuple with contextual anchor. The fifth kernel design is a 2-tuple with a contextual anchor, and maps to the function of nociceptors in human tissues. The example in [9] shows the degrees between the pain state and modulation of pain (a highly parallel process but represented here as a point), an exemplar of which can be stated as [0.6, 0.8].



Iterated Temporal Divergence (ITD). Iterated Temporal Divergence is defined using the following equation

$$L_{t}(X_{0}) = \int_{t}^{t+1} (\nabla - \nu) |F_{t}^{s}(X_{0}) ds$$
 [6]

where the divergence between two particles subject to the same flow field is integrated over a finite time period, $t: \rightarrow t+1$.

Segregation Factor. The segregation factor is used to understand changes in the distribution of values for a particular soft classification kernel. Sets that define the structure of a certain cultural feature can become segregated over time, resulting from interactions with other particles in the flow field. This can be defined as

$$S = \left| \sum I_{ij} \right|, \left| \sum I_{ij} \right| > 0$$
 [7]

where a value of S \rightarrow 1.0 results in a maximization of movement towards discrete positions on the particle.

Conditional Diversity. To measure the distribution of automata within a given ridge or vortex, we can use a measure of conditional diversity. This measure provides us with a distribution of automata in the flow field for all automata within a certain value of the ITD measure (see equ. [1]). This measure can be stated as

$$D = \sigma(p_1, p_2, \dots, p_n)$$

$$p_i = \frac{A_i}{A_{tot}}$$

$$A_i = \underset{X_0}{\operatorname{argmax}} L_t X_0 \le L_t X_0 \ge 0$$
 [8]

where σ equals the variance of set p_n , A_i equals all automata for a specific subpopulation below the threshold value for the ITD measure, p_i is the number of automata in a specific subpopulation, A_{tot} is the total number of automata, and p_n is the number of subpopulations in the simulation.

Rate of learning and forgetting. To measure the relationship between the kernel representing the structure of practice and the Lagrangian model representing evolution, a rate can be used to characterize a cultural distance between populations based on distinctions in practice. The can be expressed as

$$r_{LF} = \frac{L_t(X_0)}{S_{p_i} - S_{p_j}} \tag{9}$$

where $L_t(X_0)$ is the iterated temporal divergence, and S_{pi} and S_{pj} are segregation factors for different automata populations housing a particular kernel.

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