Elements of dialectical contextualism

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Abstract In what follows, I strive to present the elements of a philosophical doctrine, which can be defined as dialectical contextualism. I proceed first to define the elements of this doctrine: dualities and polar contraries, the principle of dialectical indifference and the one-sidedness bias. I emphasize then the special importance of this doctrine in one specific field of meta-philosophy: the methodology for solving philosophical paradoxes. Finally, I describe several applications of this methodology on the following paradoxes: Hempel's paradox, the surprise examination paradox and the Doomsday Argument.

Keywords: dialectical contextualism, contextualism, dialectics, one-sidedness bias, cognitive distortion, meta-philosophy, polar contraries

In what follows, I will endeavour to present the elements of a specific philosophical doctrine, which can be defined as dialectical contextualism. I will try first to clarify the elements that characterise this doctrine, especially the dualities and dual poles, the principle of dialectical indifference and the one-sidedness bias. I will proceed then to describe its interest at a meta-philosophical level, especially as a methodology to assist in the resolution of philosophical paradoxes. Finally, I will describe an application of this methodology to the analysis of the following philosophical paradoxes: Hempel's paradox, the surprise examination paradox and the Doomsday Argument.

The dialectical contextualism described here is based on a number of constitutive elements which have a specific nature. Among these are: the dualities and dual poles, the principle of dialectical indifference and the one-sidedness bias. It is worth analysing in turn each of these elements.

1. Dualities and dual poles

To begin with, we shall focus on defining the concept of dual poles (polar opposites). Although intuitive, this concept needs to be clarified. Examples of dual poles are static/dynamic, internal/external, qualitative/quantitative, etc. We can define the dual poles as concepts (which we shall denote by A and Ā), which come in pairs, and are such that each of them is defined as the opposite of the other. For example, internal can be defined as the opposite of external and symmetrically, external can be defined as the contrary of internal. In a sense, there is no primitive notion here and neither A nor Ā of the dual poles can be regarded as the primitive notion. Consider first a given duality, that we can denote by A/Ā, where A and Ā are dual concepts. This duality is shown in the figure below:

The dual poles A and Ā

At this point, we can also provide a list (which proves to be necessarily partial) of dualities:

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1 Such notion is central to the concept of matrices of concepts introduced in Franceschi (2002), of which we can consider that it constitutes the core, or a simplified form. In this paper that bears more specifically on the elements of dialectical contextualism and their application for solving philosophical paradoxes, merely presenting the dual poles proves to be sufficient.

In order to characterize more accurately the dual poles, it is worth distinguishing them from other concepts. We shall stress then several properties of the dual poles, which allow to differentiate them from other related concepts. The dual poles are neutral concepts, as well as simple qualities; in addition, they differ from vague notions. To begin with, two dual poles $A$ and $\bar{A}$ constitute neutral concepts. They can thus be denoted by $A^0$ and $\bar{A}^0$. This leads to represent both concepts $A^0$ and $\bar{A}^0$ as follows:

![Diagram of dual neutral poles $A^0$ and $\bar{A}^0$]

The dual poles are neutral concepts, i.e. concepts that present no ameliorative or pejorative nuance. In this sense, external, internal, concrete, abstract, etc., are dual poles, unlike concepts such as beautiful, ugly, brave, which present either a ameliorative or pejorative shade, and are therefore non-neutral. The fact that the dual poles are neutral has its importance because it allows to distinguish them from concepts that have a positive or negative connotation. Thus, the pair of concepts beautiful/ugly is not a duality and therefore beautiful and ugly do not constitute dual poles in the sense of the present construction. Indeed, beautiful has a positive connotation and ugly has a pejorative connotation. In this context, we can denote them by beautiful' and ugly'.

It should be emphasised, second, that the two poles of a given dual duality correspond to simple qualities, as opposed to composite qualities. The distinction between single and composite qualities can be made in the following manner. Let $A_1$ and $A_2$ be simple qualities. In this case, $A_1 \land A_2$, and $A_1 \lor A_2$ are composite qualities. To take an example, static, qualitative, external are simple qualities, while static and qualitative, static and external, qualitative and external are composite qualities. A more general definition is as follows: let $B_1$ and $B_2$ be single or composite qualities, then $B_1 \land B_2$ and $B_1 \lor B_2$ are composite qualities. Incidentally, this also highlights why the pairs of concepts red/non-red, blue/non-blue concepts can not be considered as dual poles. Indeed, non-red can thus be defined as follows as a composite quality: violet $\lor$ indigo $\lor$ blue $\lor$ green $\lor$ yellow $\lor$ orange $\lor$ white $\lor$ black. In this context, one can assimilate non-blue to the negation-complement of blue, such complement negation being defined with the help of composite qualities.

Given the above definition, we are also in a position to distinguish the dual poles from vague objects. We can first note that dual poles and vague objects have certain properties in common. Indeed, vague objects come in pairs in the same way as dual poles. Moreover, vague concepts are classically considered as having an extension and an anti-extension, which are mutually exclusive. Such a feature is also shared by the dual poles. For example, qualitative and quantitative can be assimilated respectively to an extension and an anti-extension, which also have the property of being mutually exclusive, and the same goes for static and dynamic, etc.. However, it is worth noting the differences between the two types of concepts. A first difference (i) lies in the fact that the union of the extension and the anti-extension of vague concepts is not exhaustive in the sense that they admit of borderline cases (and also borderline cases of borderline cases, etc., giving rise to a hierarchy of higher-order vagueness of order $n$), which is a penumbra zone. Conversely, the dual poles do not necessarily have such a characteristic. Indeed, the union of the dual poles can be either exhaustive or non-exhaustive. For example, the abstract/concrete duality is then intuitively exhaustive, since there does not seem to exist any objects that are neither abstract nor concrete. The same goes for the vague/precise duality: intuitively, there does no exist indeed objects that are neither vague nor precise, and that would belong to an intermediate category. Hence, there are dual poles whose extension and anti-extension turns out to be exhaustive, unlike vague concepts, such as the two poles of the abstract/concrete duality. It is worth mentioning, second, another difference (ii) between dual poles and vague objects. In effect, dual poles are simple qualities, while vague objects may consist of simple or compound qualities. There exist indeed some vague concepts which are termed multi-dimensional vague objects, such as the notion of vehicle, of machine, etc.. A final difference between the two categories of objects (iii) lies in the fact that some dual poles have an inherently precise nature. This is particularly the case of the individual/collective duality, which is susceptible to give rise to a very accurate definition.

2. The principle of dialectical indifference

From the notions of duality and of dual poles which have been just mentioned, we are in a position to define the notion
of a viewpoint related to a given duality or dual pole. Thus, we have first the notion of viewpoint corresponding to a
given A/Ā duality: it consists for example in the standpoint of the extension/restriction duality, or of the
qualitative/quantitative duality or of the diachronic/synchronic duality, etc.. It also follows the concept of point of view
related to a given pole of an A/Ā duality: we get then, for example (at the level of the extension/restriction duality) the
standpoint by extension, as well as the viewpoint by restriction. Similarly, the qualitative viewpoint or perspective
results from it, as well as the quantitative point of view, etc.. (at the level of the qualitative/quantitative duality). Thus,
when considering a given object o (either a concrete or an abstract object such as a proposition or a reasoning), we may
consider it in relation to various dualities, and at the level of the latter, relative to each of its two dual poles.

The underlying idea inherent to the viewpoints relative to a given duality, or to a given pole of a duality, is that each
of the two poles of the same duality, all things being equal, deserve an equal legitimacy. In this sense, if we consider an
object o in terms of a duality A/Ā, one should not favour one of the poles with respect to the other. To obtain an
objective point of view with respect to a given duality A/Ā, one should place oneself in turn from the perspective of the
pole A, and then from that of the pole Ā. For an approach that would only address the viewpoint of one of the two poles
would prove to be partial and truncated. The fact of considering in turn the perspective of the two poles, in the study of
an object o and of its associated reference class allows to avoid a subjective approach and to meet as much as possible
the needs of objectivity.

As we can see it, the idea underlying the concept of point of view can be formalized in a principle of dialectical
indifference, in the following way:

(PRINCIPLE OF DIALECTICAL INDIFFERENCE) When considering a given object o and the reference class E
associated with it, from the angle of duality A/Ā, all things being equal, it should be given equal weight to the
viewpoint of the A pole and the viewpoint of the Ā pole.

This principle is formulated in terms of a principle of indifference: if we consider an object o under the angle of an A/Ā
duality, there is no reason to favour the viewpoint from A with regard to the viewpoint from Ā, and unless otherwise
resulting from the context, we must weigh equally the viewpoints A and Ā. A direct consequence of this principle is
that if one considers the perspective of the A pole, one also needs to take into consideration the standpoint of the
opposite pole Ā (and vice versa). The need to consider both points of view, the one resulting from the A pole and the
other associated with the Ā pole, meets the need of analysing the object o and the reference class associated with it from
an objective point of view. This goal is achieved, as far as possible, by taking into account the complementary points of
view which are those of the poles A and Ā. Each of these viewpoints has indeed, with regard to a given duality A/Ā, an
equal relevance. Under such circumstances, when only the A pole or (exclusively) the pole Ā is considered, it consists
then of a one-sided perspective. Conversely, the viewpoint which results from the synthesis of the viewpoints
corresponding to both poles A and Ā is of a two-sided type. Basically, this approach proves to be dialectical in essence.
In effect, the step consisting of successively analysing the complementary views relative to a given reference class, is
intended to allow, in a subsequent step, a final synthesis, which results from the joint consideration of the viewpoints
corresponding to both poles A and Ā. In the present construction, the process of confronting the different perspectives
relevant to an A/Ā duality is intended to build cumulatively, a more objective and comprehensive standpoint than the
one, necessarily partial, resulting from taking into account those data that stem from only one of the two poles.

The definition of the dialectical principle of indifference proposed here refers to a reference class E, which is
associated with the object o. The reference class is constituted by a number of phenomena or objects. Several examples
can be given: the class of human beings who ever lived, the class of future events in the life of a person, the class of
body parts of a given person, the class of ravens, etc.. We shall consider in what follows, a number of examples.
Mention of such a reference class has its importance because its very definition is associated with the above-mentioned
duality A/Ā. In effect, the reference class can be defined either from the viewpoint of A or from the viewpoint of Ā.
Such a feature needs to be emphasized and will be useful in defining the bias which is associated with the very
definition of the principle of dialectical indifference: the one-sidedness bias.

3 Characterisation of the one-sidedness bias

The previous formulation of the principle of dialectical indifference suggests straightforwardly an error of reasoning of
a certain type. Informally, such a fallacy consists in focusing on a given standpoint when considering a given object,
and of neglecting the opposite view. More formally, in the context described above, such a fallacy consists, when
considering an object o and the reference class associated with it, in taking into account the viewpoint of the A pole

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2 The present construction also applies to objects that are associated with several classes of reference. We shall limit ourselves
here, for the sake of simplicity, to one single reference class.
(respectively Ā), while completely ignoring the viewpoint corresponding to its dual pole Ā (respectively A) to define the reference class. We shall term one-sidedness bias such type of fallacy. The conditions of this type of bias, in violation of the principle of dialectical indifference, needs however to be clarified. Indeed, in this context, we can consider that there are some cases where the two-sidedness with respect to a given duality A/Ā is not required. Such is the case when the elements of the context do not presuppose conditions of objectivity and exhaustiveness of views. Thus, a lawyer who would only emphasise the evidence in defence of his/her client, while completely ignoring the evidence against him/her does not commit the above-mentioned type of error of reasoning. In such a circumstance, in fact, the lawyer would not commit a faulty one-sidedness bias, since it is his/her inherent role. The same would go in a trial for the prosecutor, who conversely, would only focus on the evidence against the same person, by completely ignoring the exculpatory elements. In such a situation also the resulting one-sidedness bias would not be inappropriate, because it follows well from the context that it consists well of the limited role assigned to the prosecutor. By contrast, a judge who would only take into account the evidence against the accused, or who would commit the opposite error, namely of only considering the exculpatory against the latter, would well commit an inappropriate one-sidedness bias because the mere role of the judge implies that he/she takes into account the two types of elements, and that his/her judgement is the result of the synthesis which is made.

In addition, as hinted at above, the mention of a reference class associated with the object o proves to be important. In effect, as we will have the opportunity to see it with the analysis of the following examples, the definition itself is associated with an A/Ā duality. And the reference class can be defined either from the viewpoint of A, or from the viewpoint of Ā. Such feature has the consequence that all objects are not likely to give rise to a one-sidedness bias. In particular, the objects that are not associated with a reference class that is itself likely to be envisaged in terms of an A/Ā duality, do not give rise to any such one-sidedness bias.

Before illustrating the present construction with the help of several practical examples, it is worth considering, at this stage, the one-sidedness bias which has been just defined, and which results from the very definition of the principle of dialectical indifference, in the light of several similar concepts. In a preliminary way, we can observe that a general description of this type of error of reasoning had already been made, in similar terms, by John Stuart Mill (On Liberty, II):

He who knows only his own side of the case, knows little of that. His reasons may be good, and no one may have been able to refute them. But if he is equally unable to refute the reasons on the opposite side; if he does not so much know what they are, he has no ground for preferring either opinion.

In the recent literature, some very similar concepts have also been described. It consists in particular of the dialectic bias notably described by Douglas Walton (1999). Walton (pp. 76-77) places then himself in the framework of the dialectical theory of bias, which opposes one-sided to two-sided arguments:

The dialectical theory of bias is based on the idea [...] that an argument has two sides. [...] A one-sided argument continually engages in pro-argumentation for the position supported and continually rejects the arguments of the opposed side in a dialogue. A two-sided (balanced) argument considers all arguments on both sides of a dialogue. A balanced argument weights each argument against the arguments that have been opposed to it.

Walton describes thus the dialectical bias as a one-sided perspective that occurs during the course of the argument. Walton emphasizes, though, that dialectic bias, which is universally common in human reasoning, does not necessarily constitute an error of reasoning. In line with the distinction between “good” and “bad” bias due to Antony Blair (1988), Walton considers that the dialectic bias is incorrect only under certain conditions, especially if it occurs in a context that is supposed to be balanced, that is to say where the two sides of the corresponding reasoning are supposed to be mentioned (p. 81):

Bad bias can be defined as “pure (one-sided) advocacy” in a situation where such unbalanced advocacy is normatively inappropriate in argumentation.

A very similar notion of one-sidedness bias is also described by Peter Suber (1998). Suber describes indeed a fallacy that he terms one-sidedness fallacy. He describes it as a fallacy which consists in presenting one aspect of the elements supporting a judgement or a viewpoint, by completely ignoring the other aspect of the relevant elements relating to the same judgement:

The fallacy consists in persuading readers, and perhaps ourselves, that we have said enough to tilt the scale of evidence and therefore enough to justify a judgment. If we have been one-sided, though, then we haven't yet said enough to justify a judgment. The arguments on the other side may be stronger than our own. We won't know until we examine them.

The error of reasoning consists then in taking only into account one viewpoint relating to the judgement in question, whereas the other viewpoint could as well prove to be decisive with regard to the conclusion to be drawn. Suber also
undertakes to provide a characterization of the one-sidedness fallacy and notes in particular that the fallacy of one-sidedness constitutes a valid argument. For its conclusion is true if its premises are true. Moreover, Suber notes, it appears that the argument is not only valid but sound. For when the premises are true, the conclusion of the argument can be validly inferred. However, as hinted at by Suber, the argument is defective due to the fact that a number of premises are lacking. This is essential because if the missing premises are restored within the argument, the resulting conclusion can be radically different.

4. An instance of the one-sidedness bias

To illustrate the above concepts, it is worth at this stage providing an example of the one-sidedness bias. To this end, consider the following instance, which is a form of reasoning, mentioned by Philippe Boulanger (2000, p. 3), who attributes it to the mathematician Stanislaw Ulam. The one-sidedness bias shows up in a deductive form. Ulam estimates that if a company were to achieve a level of workforce large enough, its performance would be paralysed by the many internal conflicts that would result. Ulam estimates that the number of conflicts between people would increase according to the square of the number of employees, while the impact on the work that would result would only grow as a function of the number of employees within a company becomes important. However, it turns out that Ulam's reasoning is fallacious, as Boulanger points out, for it focuses exclusively on the conflictual relations between employees. But the relationships among the company employees can well be confrontational, but may include as well collaborative relationships that are quite beneficial for the company. And so there is no reason to favour conflictual relationships with respect to collaborative ones. And when among $n^2$ relationships established between the company employees, some are genuine collaborative relationships, the effect is, instead, of improving business performance. Therefore, we can not legitimately conclude that it is not desirable that the workforce of a company reaches a large size.

For the sake of clarity, it is worth formalizing the above reasoning. It turns out thus that Ulam's reasoning can be described as follows:

\[(D1_\text{A}) \text{ if } \text{<a company has a large workforce>}
(D2_\text{A}) \text{ then } <n^2 \text{ conflictual relationships will result}>
(D3_\text{A}) \text{ then negative effects will result}
(D4_\text{A}) \text{ \therefore the fact that <a company has a large workforce> is bad}
\]

This type of reasoning has the structure of a one-sidedness bias, since it focuses only on conflicting relationships (the dissociation pole of the association/dissociation duality), by ignoring a parallel argument with the same structure that could legitimately be raised, focusing on collaborative relationships (the association pole), which is the other aspect relevant to this particular topic. This parallel argument goes as follows:

\[(D1_\text{A}) \text{ if } \text{<a company has a large workforce>}
(D2_\text{A}) \text{ then } <n^2 \text{ collaborative relationships will result}>
(D3_\text{A}) \text{ then positive effects will result}
(D4_\text{A}) \text{ \therefore the fact that <a company has a large workforce> is good}
\]

This finally casts light on how the two formulations of the argument lead to conflicting conclusions, i.e. $(D4_\text{A})$ and $(D4_\bar{\text{A}})$. At this point, it is worth noting the very structure of the conclusion of the above reasoning, which is as follows:

\[(D5_\bar{\text{A}}) \text{ the situation s is bad from the viewpoint of } \bar{\text{A}} (\text{dissociation})
\]

while the conclusion of the parallel reasoning is as follows:

\[(D5_\text{A}) \text{ the situation s is good from the viewpoint of } \text{A} (\text{association})
\]

But if the reasoning had been complete, by taking into account the two points of view, a different conclusion would have ensued:

\[(D5_\bar{\text{A}}) \text{ the situation s is bad from the viewpoint of } \bar{\text{A}} (\text{dissociation})
(D5_\text{A}) \text{ the situation s is good from the viewpoint of } \text{A} (\text{association})
\]

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Philippe Boulanger says (personal correspondence) that he heard Stanislaw Ulam develop this particular point in a conference at the University of Colorado.
(D6_{A,\tilde{A}})  the situation $s$ is bad from the viewpoint of $\tilde{A}$ (dissociation) and good from the viewpoint of $A$ (association)

(D7_{A,\tilde{A}})  the situation $s$ is neutral from the viewpoint of the duality $A/\tilde{A}$ (association/dissociation)

And such a conclusion turns out to be quite different from that resulting from (D5_{A}) and (D5_{\tilde{A}}).

Finally, we are in a position to replace the one-sidedness bias which has just been described in the context of the present model: the object $o$ is the above reasoning, the reference class is that of the relationships between the employees of a business, and the corresponding duality – allowing to define the reference class – is the dissociation/association duality.

5. Dichotomic analysis and meta-philosophy

The aforementioned principle of dialectic indifference and its corollary – one-sidedness bias – is likely to find applications in several domains\(^4\). We shall focus, in what follows, on its applications at a meta-philosophical level, through the analysis of several contemporary philosophical paradoxes. Meta-philosophy is that branch of philosophy whose scope is the study of the nature of philosophy, its purpose and its inherent methods. In this context, a specific area within meta-philosophy is the method to use to attach oneself to resolve, or make progress towards the resolution of philosophical paradoxes or problems. It is within this specific area that falls the present construction, in that it offers dichotomous analysis as a tool that may be useful to assist in the resolution of paradoxes or philosophical problems.

The dichotomous analysis as a methodology that can be used to search for solutions to some paradoxes and philosophical problems, results directly from the statement of the principle of dialectical indifference itself. The general idea underlying the dichotomous approach to paradox analysis is that two versions, corresponding to one and the other pole of a given duality, can be untangled within a philosophical paradox. The corresponding approach then is to find a reference class which is associated with the given paradox and the corresponding duality $A/\tilde{A}$, as well as the two resulting variations of the paradox that apply to each pole of this duality. Nevertheless, every duality is not suitable for this, as for many dualities, the corresponding version of the paradox remains unchanged, regardless of the pole that is being considered. In the dichotomous method, one focuses on finding a reference class and a relevant associated duality, such that the viewpoint of each of its poles actually lead to two structurally different versions of the paradox, or the disappearance of paradox from the point of view of one of the poles. Thus, when considering the paradox in terms of two poles $A$ and $\tilde{A}$, and if it has no effect on the paradox itself, the corresponding duality $A/\tilde{A}$ reveals itself therefore, from this point of view, irrelevant.

The dichotomous analysis is not by far a tool that claims to solve all philosophical problems, but only constitutes a methodology that is susceptible of shedding light on some of them. In what follows, we shall try to illustrate through several works of the author, how dichotomous analysis can be applied to progress towards the resolution of three contemporary philosophical paradoxes: Hempel's paradox, the surprise examination paradox and the Doomsday argument.

In a preliminary way, we can observe here that in the literature, there is also an example of dichotomous analysis of a paradox in David Chalmers (2002). Chalmers attempts then to show how the two-envelope paradox leads to two fundamentally distinct versions, one of which corresponds to a finite version of the paradox and the other to an infinite version. Such an analysis, although conceived of independently of the present construction can thus be characterized as a dichotomous analysis based on the finite/infinite duality.

![Finite Infinite](#)

The dual poles in David Chalmers’ analysis of the two-envelope paradox

6. Application to the analysis of the philosophical paradoxes

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At this point, it is worth applying the foregoing to the analysis of concrete problems. We shall illustrate this through the analysis of several contemporary philosophical paradoxes: Hempel's paradox, the surprise examination paradox and the Doomsday argument. We will endeavour to show how a problem of one-sidedness bias associated with a problem of definition of a reference class can be found in the analysis of the aforementioned philosophical paradoxes. In addition, we will show how the very definition of the reference class associated with each paradox is susceptible of being qualified with the help of the dual poles A and Ā of a given duality A/Ā as they have just been defined.

6.1. Application to the analysis of Hempel's paradox

Hempel's paradox is based on the fact that the two following assertions:

(H) All ravens are black
(H*) All non-black things are non-ravens

are logically equivalent. By its structure (H*) presents itself indeed as the contrapositive form of (H). It follows that the discovery of a black raven confirms (H) and also (H*), but also that the discovery of a non-black thing that is not a raven such as a red flame or even a grey umbrella, confirms (H*) and therefore (H). However, this latter conclusion appears paradoxical.

We shall endeavour now to detail the dichotomous analysis on which is based the solution proposed in Franceschetti (1999). The corresponding approach is based on finding a reference class associated with the statement of the paradox, which may be defined with the help of an A/Ā duality. If we scrutinise the concepts and categories that underlie propositions (H) and (H*), we first note that there are four categories: ravens, black objects, non-black objects and non-ravens. To begin with, a raven is precisely defined within the taxonomy in which it inserts itself. A category such as that of the ravens can be considered well-defined, since it is based on a precise set of criteria defining the species corvus corax and allowing the identification of its instances. Similarly, the class of black objects can be accurately described, from a taxonomy of colours determined with respect to the wave lengths of light. Finally, we can see that the class of non-black objects can also be a definition that does not suffer from ambiguity, in particular from the specific taxonomy of colours which has been just mentioned.

However, what about the class of non-ravens? What does constitute then an instance of a non-raven? Intuitively, a blue blackbird, a red flamingo, a grey umbrella and even a natural number, are non-ravens. But should we consider a reference class that goes up to include abstract objects? Should we thus consider a notion of non-raven that includes abstract entities such as integers and complex numbers? Or should we limit ourselves to a reference class that only embraces the animals? Or should we consider a reference class that encompasses all living beings, or even all concrete things, also including this time the artefacts? Finally, it follows that the initial proposition (H*) is susceptible of giving rise to several variations, which are the following:

(H1) All that is non-black among the corvids is a non-raven
(H2) All that is non-black among the birds is a non-raven
(H3) All that is non-black among the animals is a non-raven
(H4) All that is non-black among the living beings is a non-raven
(H5) All that is non-black among the concrete things is a non-raven
(H6) All that is non-black among the concrete and abstract objects is a non-raven

Thus, it turns out that the statement of Hempel's paradox and in particular of proposition (H*) is associated with a reference class, which allow to define the non-ravens. Such a reference class can be assimilated to corvids, birds, animals, living beings, concrete things, or to concrete and abstract things, etc.. However, in the statement of Hempel's paradox, there is no objective criterion for making such a choice. At this point, it turns out that one can choose such a reference class restrictively, by assimilating it for example to corvids. But in an equally legitimate manner, we can choose a reference class more extensively, by identifying it for example to the set of concrete things, thus notably including umbrellas. Why then choose such or such reference class defined in a restrictive way rather than another one extensively defined? Indeed, we are lacking a criterion allowing to justify the choice of the reference class, whether we proceed by restriction or by extension. Therefore, it turns out that the latter can only be defined arbitrarily. But the choice of such a reference class proves crucial because depending on whether you choose such or such class reference, a given object such as a grey umbrella will confirm or not (H*) and therefore (H). Hence, if we choose the reference class by extension, thus including all concrete objects, a grey umbrella will confirm (H). On the other hand, if we choose such a reference class by restriction, by assimilating it only to corvids, a grey umbrella will not confirm (H). Such a difference proves to be essential. In effect, if we choose a definition by extension of the reference class, the paradoxical effect inherent to Hempel's paradox ensues. By contrast, if we choose a reference class restrictively defined, the paradoxical effect disappears.
The dual poles in the reference class of the non-ravens within Hempel's paradox

The foregoing permits to describe accurately the elements of the preceding analysis of Hempel's paradox in terms of one-sidedness bias such as it has been defined above: to the paradox and in particular to proposition (H*) are associated the reference class of non-ravens, which itself is susceptible of being defined with regard to the extension/restriction duality. However, for a given object such as a grey umbrella, the definition of the reference class by extension leads to a paradoxical effect, whereas the choice of the latter by restriction does not lead to such an effect.

6.2. Application to the analysis of the surprise examination paradox

The classical version of the surprise examination paradox (Quine 1953, Soresen 1988) goes as follows: a teacher tells his students that an examination will take place on the next week, but they will not know in advance the precise date on which the examination will occur. The examination will thus occur surprisingly. The students reason then as follows. The examination cannot take place on Saturday, they think, otherwise they would know in advance that the examination would take place on Saturday and therefore it could not occur surprisingly. Thus, Saturday is eliminated. In addition, the examination can not take place on Friday, otherwise the students would know in advance that the examination would take place on Friday and so it could not occur surprisingly. Thus, Friday is also ruled out. By a similar reasoning, the students eliminate successively Thursday, Wednesday, Tuesday and Monday. Finally, every day of the week is eliminated. However, this does not preclude the examination of finally occurring by surprise, say on Wednesday. Thus, the reasoning of the students proved to be fallacious. However, such reasoning seems intuitively valid. The paradox lies here in the fact the students' reasoning is apparently valid, whereas it finally proves inconsistent with the facts, i.e. that the examination can truly occur by surprise, as initially announced by the professor.

In order to introduce the dichotomous analysis (Franceschi 2005) that can be applied to the surprise examination paradox, it is worth considering first two variations of the paradox that turn out to be structurally different. The first variation is associated with the solution to the paradox proposed by Quine (1953). Quine considers then the student's final conclusion that the examination can not take place surprisingly on any day of the week. According to Quine, the student's error lies in the fact of not having envisaged from the beginning that the examination could take place on the last day. Because the fact of considering precisely that the examination will not take place on the last day finally allows the examination to occur by surprise on the last day. If the student had also considered this possibility from the beginning, he would not have been committed to the false conclusion that the examination can not occur surprisingly.

The second variation of the paradox that proves interesting in this context is the one associated with the remark made by several authors (Hall 1999, Williamson 2000), according to which the paradox emerges clearly when the number n of units is large. Such a number is usually associated with a number n of days, but we may as well use hours, minutes, seconds, etc. An interesting feature of the paradox is indeed that it emerges intuitively more significantly when large values of n are involved. A striking illustration of this phenomenon is thus provided by the variation of the paradox that corresponds to the following situation, described by Timothy Williamson (2000, p 139).

Advance knowledge that there will be a test, fire drill, or the like of which one will not know the time in advance is an everyday fact of social life, but one denied by a surprising proportion of early work on the Surprise Examination. Who has not waited for the telephone to ring, knowing that it will do so within a week and that one will not know a second before it rings that it will ring a second later?

The variation described by Williamson corresponds to the announcement made to someone that he/she will receive a phone call during the week, but without being able to determine in advance at what exact second the latter event will occur. This variation highlights how surprise may occur, in a quite plausible way, when the value of n is high. The unit of time considered here by Williamson is the second, in relation with a time duration that corresponds to one week. The corresponding value of n here is very high and equal to 604800 (60 x 60 x 24 x 7) seconds. However, it is not necessary to take into account a value as large of n, and a value of n equal to 365, for example, should also be well-suited.

The fact that two versions of the paradox that seem a priori quite different coexist suggests that two structurally different versions of the paradox could be inextricably intertwined within the surprise examination paradox. In fact, if we analyse the version of the paradox that leads to Quine's solution, we find that it has a peculiarity: it is likely to occur for a value of n equal to 1. The corresponding version of the professor's announcement is then as follows: “An examination will take place tomorrow, but you will not know in advance that this will happen and therefore it will occur surprisingly.” Quine's analysis applies directly to this version of the paradox for which n = 1. In this case, the student's error resides, according to Quine, in the fact of having only considered the hypothesis: (i) “the examination will take place tomorrow and I predict that it will take place.” In fact, the student should also have considered three cases: (ii) “the examination will not take place tomorrow, and I predict that it will take place” (iii) “the examination will not take
place tomorrow and I do not predict that it will take place” (iv) “the examination will take place tomorrow and I do not predict that it will take place.” And the fact of having envisaged hypothesis (i), but also hypothesis (iv) which is compatible with the professor’s announcement would have prevented the student to conclude that the examination would not finally take place. Therefore, as Quine stresses, it is the fact of having only taken into account the hypothesis (i) that can be identified as the cause of the fallacious reasoning.

As we can see it, the very structure of the version of the paradox on which Quine’s solution is based has the following features: first, the non-surprise may actually occur on the last day, and second, the examination may also occur surprisingly on the last day. The same goes for the version of the paradox where \( n = 1 \): the non-surprise and the surprise may occur on day \( n \). This allows to represent such structure of the paradox with the following matrix \( S[k, s] \) (where \( k \) denotes the day on which the examination takes place and \( S[k, s] \) denotes whether the corresponding case of non-surprise \( (s = 0) \) or surprise \( (s = 1) \) is possible (in this case, \( S[k, i] = 1 \) ) or not (in this case, \( S[k, i] = 0 \) )):

<table>
<thead>
<tr>
<th>day</th>
<th>non-surprise</th>
<th>surprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Matrix structure of the version of the paradox corresponding to Quine’s solution for \( n = 7 \) (one week)

<table>
<thead>
<tr>
<th>day</th>
<th>non-surprise</th>
<th>surprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Matrix structure of the version of the paradox corresponding to Quine’s solution for \( n = 1 \) (one day)

Given the structure of the corresponding matrix which includes values that are equal to 1 in both cases of non-surprise and of surprise, for a given day, we shall term joint such a matrix structure.

If we examine the above-mentioned variation of the paradox set by Williamson, it presents the particularity, in contrast to the previous variation, of emerging neatly when \( n \) is large. In this context, the professor’s announcement corresponding for example to a value of \( n \) equal to 365, is the following: “An examination will take place in the coming year but the date of the examination will be a surprise.” If such a variation is analysed in terms of the matrix of non-surprise and of surprise, it turns out that this version of the paradox has the following properties: the non-surprise cannot occur on the first day while the surprise is possible on this very first day; however, on the last day, the non-surprise is possible whereas the surprise is not possible.

<table>
<thead>
<tr>
<th>day</th>
<th>non-surprise</th>
<th>surprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Matrix structure of the version of the paradox corresponding to Williamson’s variation for \( n = 365 \) (one year)

The foregoing allows now to identify precisely what is at fault in the student’s reasoning, when applied to this particular version of the paradox. Under these circumstances, the student would then have reasoned as follows. The surprise cannot occur on the last day but it can occur on day 1, and the non-surprise can occur on the last day, but cannot occur on the first day. These are proper instances of non-surprise and of surprise, which prove to be disjoint. However, the notion of surprise is not captured exhaustively by the extension and the anti-extension of the surprise. But such a definition is consistent with the definition of a vague predicate, which is characterized by an extension and an anti-extension which are mutually exclusive and non-exhaustive. Thus, the notion of surprise associated with a disjoint structure is that of a vague notion. Thus, the student’s error of reasoning at the origin of the fallacy lies in not having taken into account the fact that the surprise is in the case of a disjoint structure, a vague concept and includes therefore
the presence of a penumbra corresponding to borderline cases between non-surprise and surprise. Hence, the mere consideration of the fact that the surprise notion is here a vague notion would have prohibited the student to conclude that $S[k, 1] = 0$, for all values of $k$, that is to say that the examination can not occur surprisingly on any day of the period.

Finally, it turns out that the analysis leads to distinguish between two independent variations with regard to the surprise examination paradox. The matrix definition of the cases of non-surprise and of surprise leads to two variations of the paradox, according to the joint/disjoint duality. In the first case, the paradox is based on a joint definition of the cases of non-surprise and of surprise. In the second case, the paradox is grounded on a disjoint definition. Both of these variations lead to a structurally different variation of the paradox and to an independent solution. When the variation of the paradox is based on a joint definition, the solution put forth by Quine applies. However, when the variation of the paradox is based on a disjoint definition, the solution is based on the prior recognition of the vague nature of the concept of surprise associated with this variation of the paradox.

The dual poles in the class of the matrices associated with the surprise examination paradox

As we finally see it, the dichotomous analysis of the surprise examination paradox leads to consider the class of the matrices associated with the very definition of the paradox and to distinguish whether their structure is joint or disjoint. Therefore, it follows an independent solution for each of the resulting two structurally different versions of the paradox.

6.3. Application to the analysis of the Doomsday Argument

The Doomsday argument, attributed to Brandon Carter, was described by John Leslie (1993, 1996). It is worth recalling preliminarily its statement. Consider then proposition (A):

(A) The human species will disappear before the end of the XXIst century

We can estimate, to fix ideas, to 1 on 100 the probability that this extinction will occur: $P(A) = 0.01$. Let us consider also the following proposition:

(Ā) The human species will not disappear at the end of the XXIst century

Let also $E$ be the event: I live during the 2010s. We can also estimate today to 60 billion the number of humans that ever have existed since the birth of humanity. Similarly, the current population can be estimated at 6 billion. One calculates then that one human out of ten, if event $A$ occurs, will have known of the 2010s. We can then estimate accordingly the probability that humanity will be extinct before the end of the twenty-first century, if I have known of the 2010s: $P(E, A) = 6 \times 10^7 / 6 \times 10^9 = 0.1$. By contrast, if humanity passes the course of the twenty-first century, it is likely that it will be subject to a much greater expansion, and that the number of human will be able to amount, for example to $6 \times 10^{12}$. In this case, the probability that humanity will not be not extinct at the end of the twenty-first century, if I have known of the 2010s, can be evaluated as follows: $P(E, \overline{A}) = 6 \times 10^7 / 6 \times 10^{12} = 0.001$. At this point, we can assimilate to two distinct urns – one containing 60 billion balls and the other containing 6,000,000,000,000 – the total human populations that will result. This leads to calculate the posterior probability of the human species' extinction before the end of the XXIst century, with the help of Bayes' formula:

$$P'(A) = \frac{P(A) \times P(E, A)}{P(A) \times P(E, A) + P(\overline{A}) \times P(E, \overline{A})} = \frac{(0.01 \times 0.1)}{(0.01 \times 0.1 + 0.99 \times 0.001)} = 0.5025.$$  
Thus, taking into account the fact that I am currently living makes pass the probability of the human species' extinction before 2150 from 1% to 50.25%. Such a conclusion appears counter-intuitive and is in this sense, paradoxical.

It is worth now describing how a dichotomous analysis (Franceschi, 1999, 2009) can be applied to the Doomsday Argument. We will endeavour, first, to point out how the Doomsday Argument has an inherent reference class problem definition linked to a duality $A/\overline{A}$. Consider then the following statement:

(A) The human race will disappear before the end of the XXIst century

Such a proposition presents a dramatic, apocalyptic and tragic connotation, linked to the imminent extinction of the

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5 The analysis of the Doomsday Argument from the perspective of the reference class problem is performed in detail by Leslie (1996). But Leslie's analysis aims at showing that the choice of the reference class, by extension or restriction does not affect the conclusion of the argument itself.
human species. It consists here of a prediction the nature of which is catastrophic and quite alarming. However, if we scrutinise such a proposition, we are led to notice that it conceals an inaccuracy. If the time reference itself – the end of the twenty-first century – proves to be quite accurate, the term “human species” itself appears to be ambiguous. Indeed, it turns out that there are several ways to define it. The most accurate notion in order to define the “human race” is our present scientific taxonomy, based on the concepts of genus, species, subspecies, etc.. Adapting the latter taxonomy to the assertion (A), it follows that the ambiguous concept of “human species” is likely to be defined in relation to the genus, the species, the subspecies, etc.. and in particular with regard to the homo genus, the homo sapiens species, the homo sapiens sapiens subspecies, etc.. Finally, it follows that assertion (A) is likely to take the following forms:

\[(A_h) \quad \text{The Homo genus will disappear before the end of the XXIst century} \]
\[(A_{hs}) \quad \text{The Homo sapiens species will disappear before the end of the XXIst century} \]
\[(A_{hss}) \quad \text{The Homo sapiens sapiens subspecies will disappear before the end of the XXIst century} \]

At this stage, reading these different propositions leads to a different impact, given the original proposition (A). For if (A_h) presents well in the same way as (A) a quite dramatic and tragic connotation, it is not the case for (A_{hss}). Indeed, such a proposition that predicts the extinction of our current subspecies homo sapiens sapiens before the end of the twenty-first century, could be accompanied by the replacement of our present human race with a new and more advanced subspecies than we could call homo sapiens supersapiens. In this case, the proposition (A_{hss}) would not contain any tragic connotation, but would be associated with a positive connotation, since the replacement of an ancient race with a more evolved species results from the natural process of evolution. Furthermore, by choosing a reference class even more limited as that of the humans having not known of the computer (homo sapiens sapiens antecomputeris), we get the following proposition:

\[(A_{hss}) \quad \text{The infra-subspecies Homo sapiens sapiens antecomputeris will disappear before the end of the XXIst century} \]

which is no longer associated at all with the dramatic connotation inherent to (A) and proves even quite normal and reassuring, being devoid of any paradoxical or counterintuitive nature. In this case, in effect, the disappearance of the infra-subspecies homo sapiens sapiens antecomputeris is associated with the survival of the much-evolved infra-subspecies homo sapiens sapiens postcomputeris. It turns out then that a restricted class of reference coinciding with an infra-subspecies goes extinct, but a larger class corresponding to a subspecies (homo sapiens sapiens) survives. In this case, we observe well the Bayesian shift described by Leslie, but the effect of this shift proves this time to be quite innocuous.

Thus, the choice of the reference class for proposition (A) proves to be essential for the paradoxical nature of the conclusion associated with the Doomsday Argument. If one chooses then an extended reference class for the very definition of humans, associated with e.g. the Homo genus, one gets the dramatic and disturbing nature associated with proposition (A). By contrast, if one chooses such a reference class restrictively, by associating it for example with the infra-subspecies homo sapiens sapiens antecomputeris, a reassuring and normal nature is now associated with the proposition (A) underlying the Doomsday Argument.

Finally, we are in a position to replace the foregoing analysis in the present context. The very definition of the reference class of the “humans” associated with the proposition (A) inherent to the Doomsday Argument is susceptible of being made according to the two poles of the extension/restriction duality. An analysis based on a two-sided perspective leads to the conclusion that the choice by extension leads to a paradoxical effect, whereas the choice by restriction of the reference class makes this paradoxical effect disappear.

The dual poles within the reference class of “humans” in the Doomsday Argument

The dichotomous analysis, however, as regards the Doomsday argument, is not limited to this. Indeed, if one examines the argument carefully, it turns out that it contains another reference class which is associated with another duality. This can be demonstrated by analysing the argument raised by William Eckhardt (1993, 1997) against the Doomsday argument. According to Eckhardt, the human situation corresponding to DA is not analogous to the two-urn case described by Leslie, but rather to an alternative model, which can be termed the consecutive token dispenser. The consecutive token dispenser is a device that ejects consecutively numbered balls at regular intervals: “(...) suppose on each trial the consecutive token dispenser expels either 50 (early doom) or 100 (late doom) consecutively numbered
tokens at the rate of one per minute.” Based on this model, Eckhardt (1997, p. 256) emphasizes that it is impossible to make a random selection, where there are many individuals who are not yet born within the corresponding reference class: “How is it possible in the selection of a random rank to give the appropriate weight to unborn members of the population?” The strong idea of Eckhardt underlying this diachronic objection is that it is impossible to make a random selection when there are many members in the reference class who are not yet born. In such a situation, it would be quite wrong to conclude that a Bayesian shift in favour of the hypothesis (A) ensues. However, what can be inferred rationally in such a case is that the initial probability remains unchanged.

At this point, it turns out that two alternative models for modelling the analogy with the human situation corresponding to the Doomsday argument are competing: first, the synchronic model (where all the balls are present in the urn when the draw takes place) recommended by Leslie and second, Eckhardt's diachronic model, where the balls can be added in the urn after the draw. The question that arises is the following: is the human situation corresponding to the Doomsday argument in analogy with (i) the synchronic urn model, or with (ii) the diachronic urn model? In order to answer, the following question arises: does there exist an objective criterion for choosing, preferably, between the two competing models? It appears not. Neither Leslie nor Eckhardt has an objective motivation allowing to justify the choice of their own favourite model, and to reject the alternative model. Under these circumstances, the choice of one or the other of the two models – whether synchronic or diachronic – proves to be arbitrary. Therefore, it turns out that the choice within the class of the models associated with the Doomsday argument is susceptible of being made according to the two poles of the synchronic/diachronic duality. Hence, an analysis based on a two-sided viewpoint leads to the conclusion that the choice of the synchronic model leads to a paradoxical effect, whereas the choice of the diachronic model makes this latter paradoxical effect disappear.

The dual poles within the models' class of the Doomsday Argument

Finally, given the fact that the above problem related to the reference class of the humans and its associated choice with in the extension/restriction duality only concerns the synchronic model, the structure of the dichotomous analysis at two levels concerning the Doomsday Argument can be represented as follows:

Structure of embedded dual poles Diachronic/Synchronic and Extension/Restriction for the Doomsday Argument

As we can see it, the foregoing developments implement the form of dialectical contextualism that has been described above by applying it to the analysis of three contemporary philosophical paradoxes. In Hempel's paradox, the reference class of the non-ravens is associated with proposition (H*), which itself is susceptible of being defined with regard to the extension/restriction duality. However, for a given object x such as a grey umbrella, the definition of the reference class by extension leads to a paradoxical effect, whereas the choice of the latter reference class by restriction eliminates this specific effect. Secondly, the matrix structures associated with the surprise examination paradox are analysed from the angle of the joint/disjoint duality, thus highlighting two structurally distinct versions of the paradox, which themselves admit of two independent resolutions. Finally, at the level of the Doomsday argument, a double dichotomistic analysis shows that the class of humans is related to the extension/restriction duality, and that the paradoxical effect that is evident when the reference class is defined by extension, dissolves when the latter is defined by restriction. It turns out, second, that the class of models can be defined according to the synchronic/diachronic duality; a paradoxical effect is associated with the synchronic view, whereas the same effect disappears if we place
ourselves from the diachronic perspective.

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References