1. Introduction

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Purpose of the Course

- To give you some insights about modelling and formal reasoning

- To show that programs can be correct by construction

- To show that modelling can be made practical

- To illustrate this approach with many examples

What you Will Learn

- By the end of the course you should be comfortable with:
    - Modelling (versus programming)
    - Abstraction and Refinement
    - Some mathematical techniques used for reasoning
    - The practice of proving as a means to construct programs
    - The usage of some proving tools

Preamble 1: The Vasa Disaster

The Story

- August 10, 1628: The Swedish warship Vasa sank.

- This was her maiden voyage.

- She sailed about 1,300 meters only in Stockholm harbor.

- 53 lives were lost in the disaster.

Problems with the Vasa Construction

1. Changing requirements (by King Gustav II Adolf).

2. Lack of specifications (by Ship Builder Henrik Hybertsson).

3. Lack of explicit design (by Subcontractor Johan Isbrandsson)

   (No scientific calculation of the ship stability)

4. Test outcome was not followed (by Admiral Fleming)

References

- Enter keywords "Vasa disaster" in Google

- The Vasa: A Disaster Story with Software Analogies.
  By Linda Rising.
  The Software Practitioner, January-February 2001.
  http://members.cox.net/risingl1/articles/Vasa.pdf

- Why the Vasa Sank: 10 Problems and Some Antidotes for Software Projects.
  By Richard E. Fairley and Mary Jane Willshire.
  IEEE Software, March-April 2003.
  http://www.cse.ogi.edu/ dfairley/The_vasa.pdf

Preamble 2: The Ariane 5 Disaster

- June 4, 1996: The launch vehicle Ariane 5 exploded.

- This was its maiden voyage.

- It flew for about 37 Sec only in Kourou’s sky.

- No injury in the disaster.

The Story

- Normal behavior of the launcher for 36 Sec after lift-off

- Failure of both Inertial Reference Systems almost simultaneously

- Strong pivoting of the nozzles of the boosters and Vulcain engine

- Self-destruction at an altitude of 4000 m (1000 m from the pad)

More Details

- Both inertial computers failed because of overflow on one variable

- This caused a software exception and stops these computers

- These computers sent post-mortem info through the bus

- Normally the main computer receives velocity info through the bus

- The main computer was confused and pivoted the nozzles

More Details (cont'd)

- The faulty program was working correctly on Ariane 4

- The faulty program was not tested for A5 (since it worked for A4)

- But the velocity of Ariane 5 is far greater than that of Ariane 4

- The faulty program happened to be useless for Ariane 5

- It was kept for commonality reasons

References

- Enter keywords "flight 501" in Google

- Ariane 5 flight 501 Inquiry Board Report:
  http://esamultimedia.esa.int/docs/esa-x-1819eng.pdf

- INRIA report challenging the Inquiry Board Report:
  ftp://ftp.inria.fr/INRIA/publication/publi-pdf/RR/RR-3079.pdf

Purpose of the First Introductory Lecture: Three Topics

1. About formal methods in general

2. About the requirement analysis (example)

3. About modelling

1. About formal methods in general

Formal Methods: a Great Confusion

- What are they used for?

- When are they to be used?

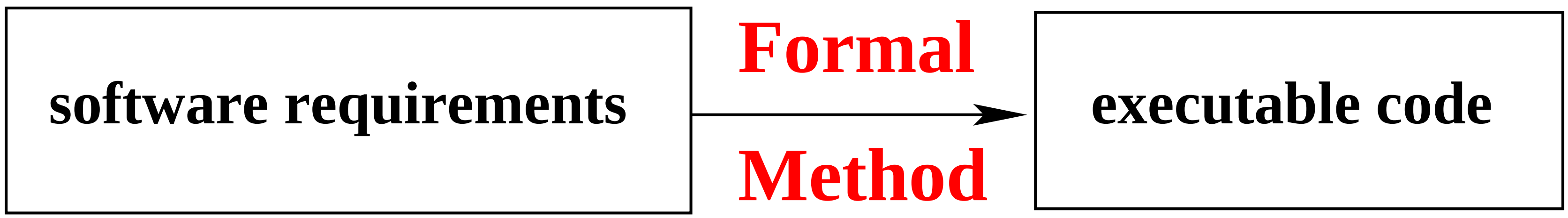
- Is UML a formal method?

- Are they needed when doing OO programming?

- What is their definition?

Purpose of Formal Methods

- Helping engineers in doing the following transformation:



- It does not seem to be different from ordinary programming

- A formal method is a systematic approach used to determine whether a program has certain wishful properties

- Different kinds of formal methods (according to this definition)
    - Type checking
    - Static analysis
    - Model checking
    - Theorem proving

Type Checking

- Controlling low level properties of variables in a program

- A type defines:
    - a set of values to be assigned to a variable
    - the operations that can be performed on a variable
    - the way a variable will be stored in the memory

- Type checking controls that:
    - value assignments to a variable is correct
    - the variable is used in authorized operations only

- It is done automatically within the compiler

- It is an automatic technique used for checking that a program will not have certain run-time errors

- Typical run-time errors detected:
    - Division by zero
    - Array bound overflow
    - Arithmetic overflow (floating point)

- The analysis is performed by abstracting the program variables and "executing" the resulting abstraction rather than the program itself

- The abstract interpretation may lead to false alarm

- The properties to be checked are not properties of programs

- They are properties of models of program

- Usually, these models denote finite state machines
  (state and transitions)

- The properties to be checked are often temporal properties
  (reachability)

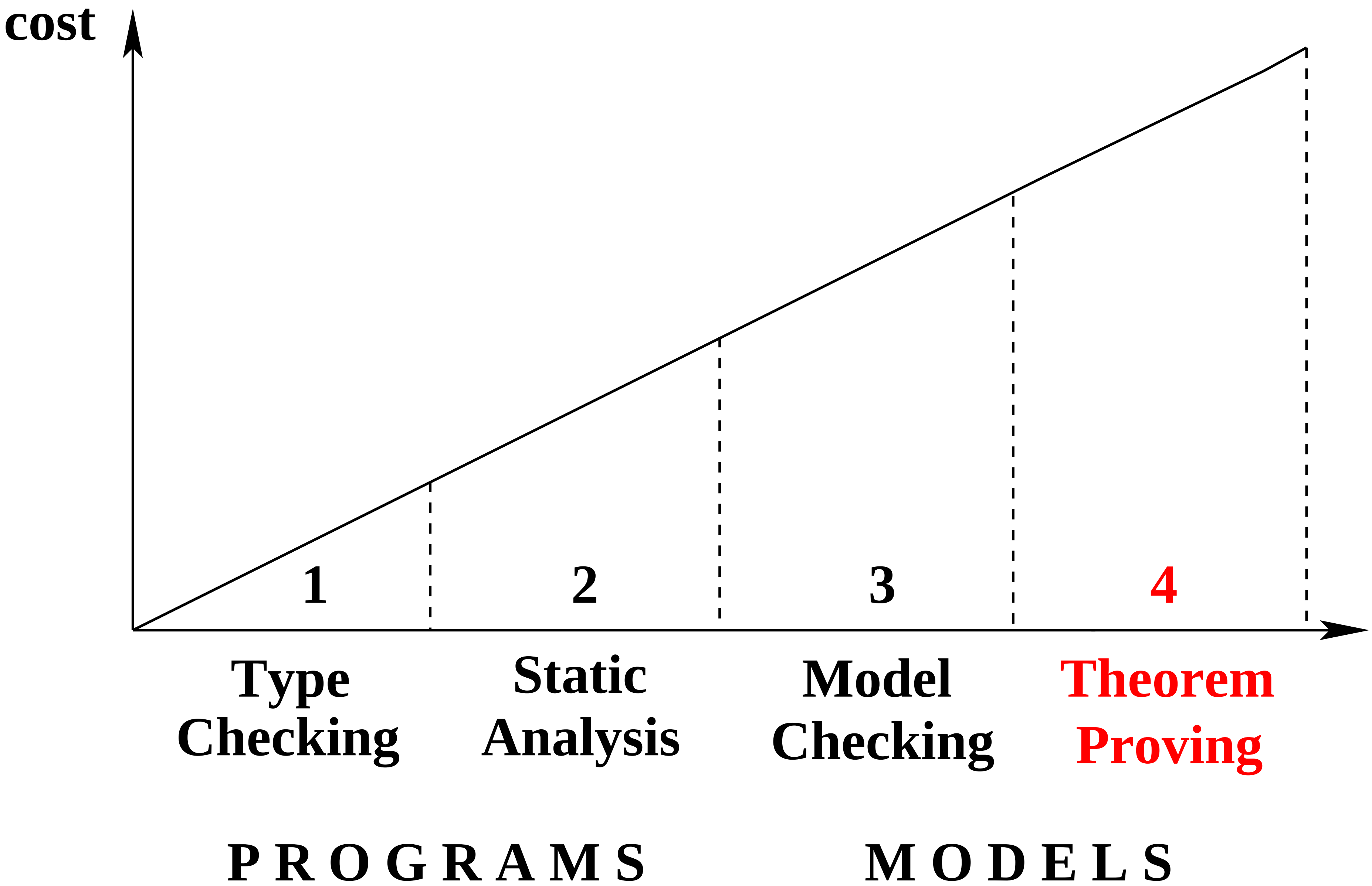
- Model checkers work automatically

- This is the approach I am going to develop in this course

- It concentrates on the construction of models by
  successive refinements

- The properties to be proved are parts of the models:
  invariants and refinement

- At the end of the process, the most refined model is
  automatically translated into a program


cost
1
2
3
4
Type Checking
Static Analysis
Model Checking
Theorem Proving
P R O G R A M S
M O D E L S

Comparisons

- In 1 and 2, one works on programs

- In 3 and 4, one works on models

- In 1 and 4, you prove a property that is part of the object to analyze

- In 2 and 3, you prove a property that is proposed externally

Why Using Formal Methods with Proofs? (case 4)

- When there is nothing better available.

- When the risk is too high (e.g. in embedded systems).

- When people have already suffered enough.

- When people question their development process.

- Decision of using formal methods is always strategic.

Embedded Systems (1)

- It is to be opposed to a general purpose computer system like a PC Operating System

- The computer is encapsulated within the device it controls

- It is doing for ever a number of specific tasks

- Examples: Systems controlling
  - a portable telephone
  - an aircraft or a space ship
  - a driverless train
  - a nuclear reactor
  - ...

Embedded Systems (2)

- Such systems are working in close connection with an external often unpredictable environment (physical and human)

- Reliability is usually very important

- Error detection and recovery must be performed (degraded mode)

- Real-time constraints have to be taken into account

- Consequently, the software has to be developed with great care

Which Formal Method with Proofs?

- This is a difficult question.

- Today many formal methods vendors.

- "Formal method" has become a meaningless buzz word.

- “Formal” alone does not mean anything.

Questions to be Asked to Vendors

- Is there a theory behind your Formal Method with Proofs (FMP) ?

- What kind of language is your FMP using ?

- Does there exist any refinement mechanism in your FMP ?

- Have you got an efficient automatic prover ?

Claimed Difficulties in Using FMP

- You have to be a mathematician.

- Formalism is hard to master.

- Not visual enough (no boxes, arrows, etc.).

- People will not be able to do formal proofs.

Genuine Difficulties (my own view)

- You have to think a lot before final coding.

- Incorporation in development process.

- Model building is an elaborate activity.

- Reasoning by means of proof is necessary.

- Poor quality of requirement documents.

What About Other Engineering Disciplines

- Some mature engineering disciplines:
  - Avionics,
  - Civil engineering,
  - Mechanical engineering,
  - Train systems,
  - Ship building.

- Are there any equivalent approaches to Formal Methods with Proofs?

- Yes, BLUE PRINTS

What is a Blue Print?

- A certain representation of the system we want to build

- It is not a mock-up (although mock-ups can be very useful too)

- The basis is lacking (you cannot “drive” the blue print of a car)

- Allows to reason about the future system during its design

- Is it important? (according to professionals) YES

Reasoning about the Intended System?

- Defining and calculating its behavior (what it does)

- Incorporating constraints (what it must not do)

- Defining architecture

- Based on some underlying theories
    - strength of materials,
    - fluid mechanics,
    - gravitation,
    - etc.

Techniques of “Blue Printing”

- Using pre-defined conventions (often computerized these days)

- Conventions should help facilitate reasoning

- Adding details on more accurate versions

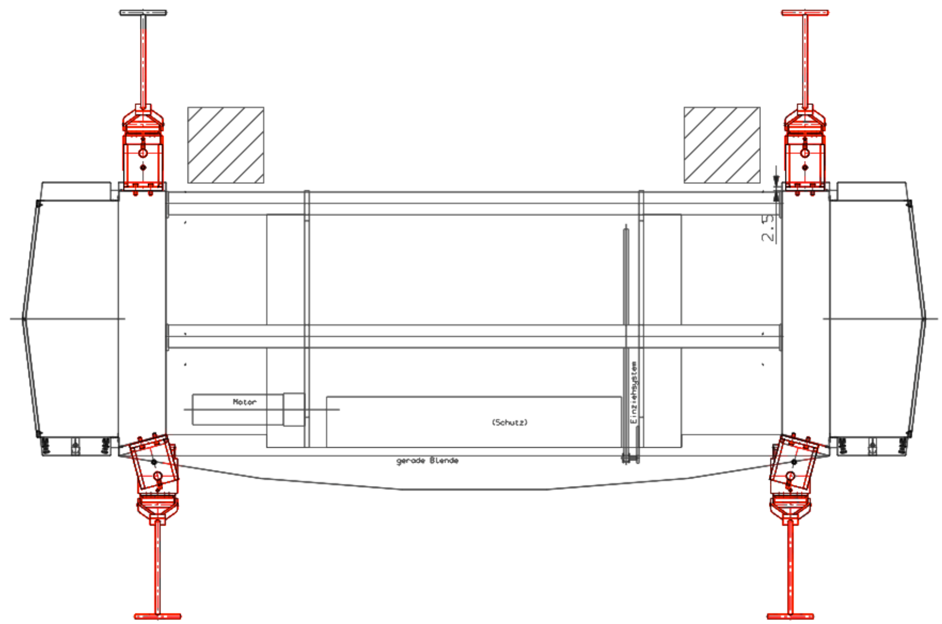
- Postponing choices by having some open options

- Decomposing one blue print into several

- Reusing “old” blue prints (with slight changes)

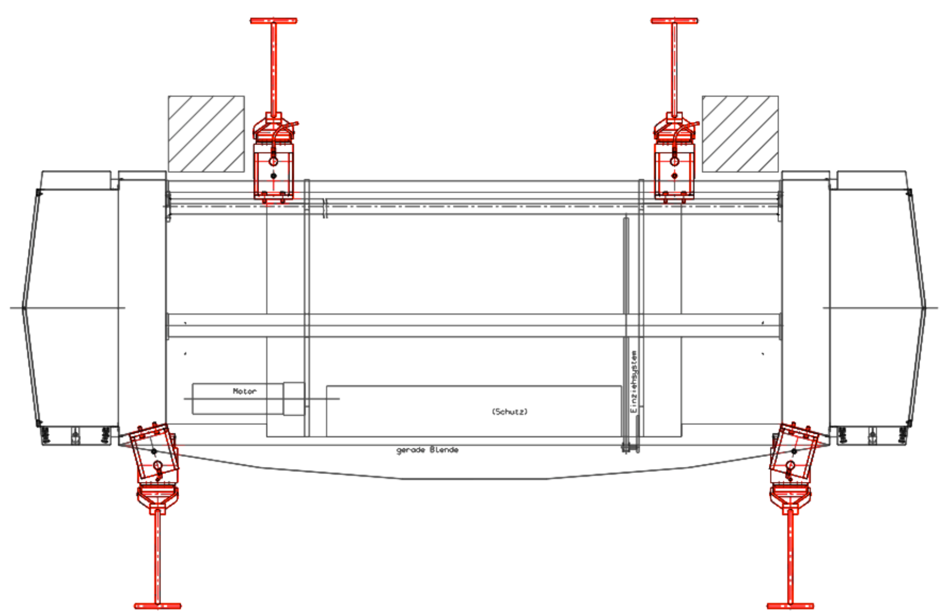
Befestigung bei guten Platzverhältnissen

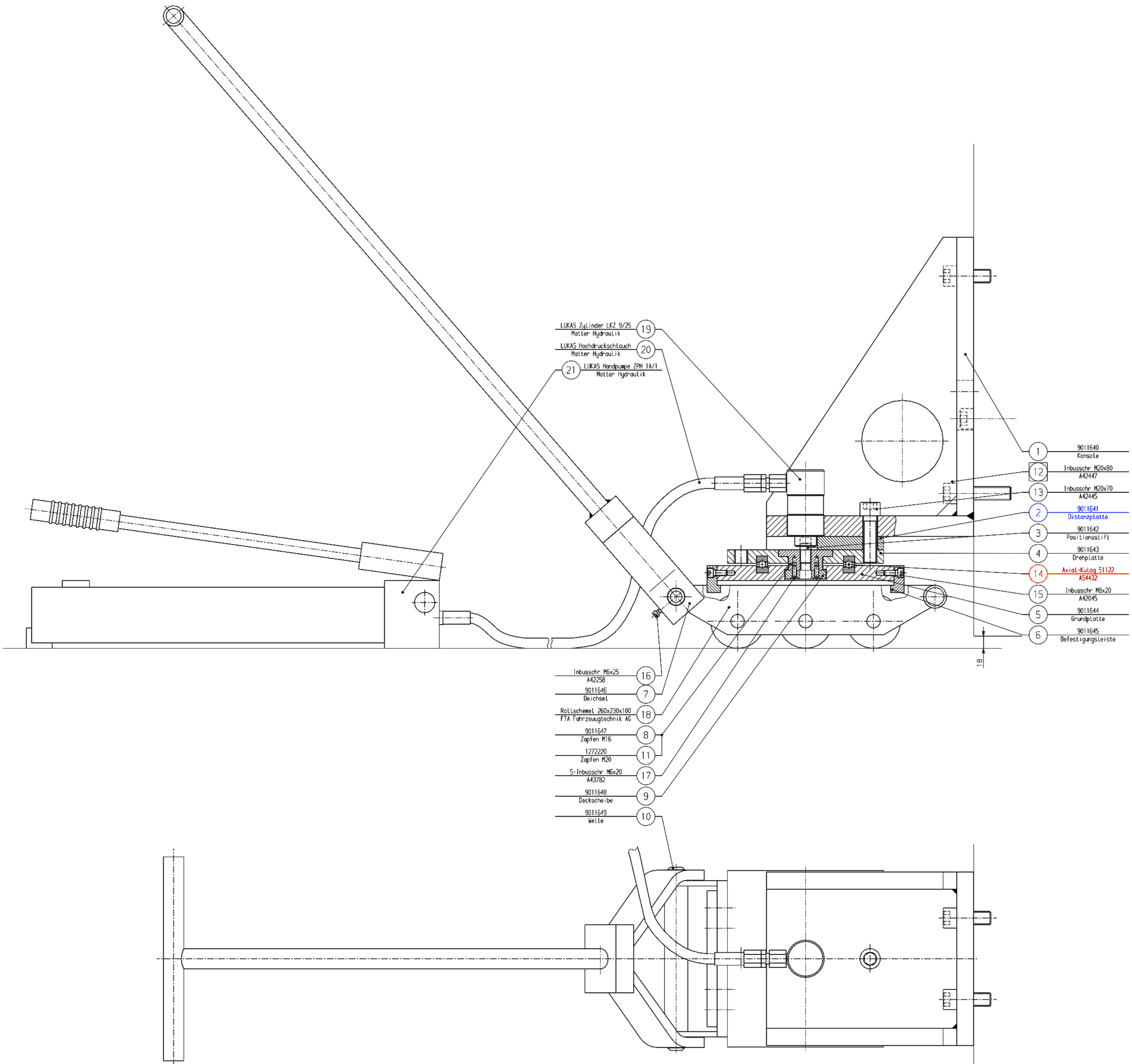
(1:20)



Befestigung bei schlechten Platzverhältnissen

(1:20)





19 LUKAS Zylinder LKZ 9/25
Matter Hydraulik
20 LUKAS Hochdruckschlauch
Matter Hydraulik
21 LUKAS Handpumpe ZPH 1A/1
Matter Hydraulik
1 9011640
Konsole
12 Inbusschr M20x80
A42447
13 Inbusschr M20x70
A42445
2 9011641
Distanzplatte
3 9011642
Positionsstift
4 9011643
Drehplatte
14 Axial-Kulag 51122
A54432
15 Inbusschr M8x20
A42045
5 9011644
Grundplatte
6 9011645
Befestigungsleiste
18
16 Inbusschr M6x25
A42258
7 9011646
Deichsel
18 Rollschemel 260x230x100
FTA Fahrzeuugtechnik AG
8 9011647
Zapfen M16
11 1272220
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17 S-Inbusschr M6x20
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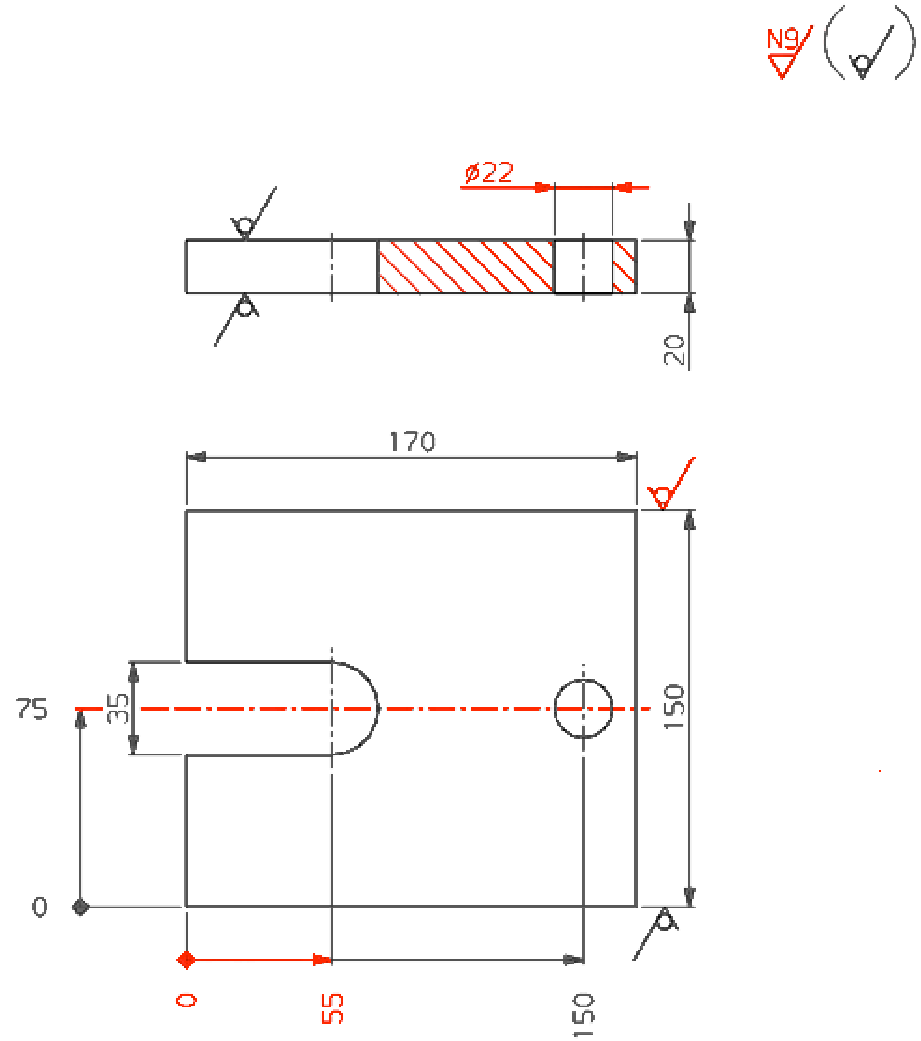
Deckscheibe
10 9011649
Welle



P–T

Zulässige Abweichungen ab Basisflächen
für Koordinatenmasse
ohne Toleranzangabe ± 0.15

Kanten ohne Massangaben 0,6 angeschrägt oder gerundet	Dat.-Aend.	Übrige Allg.-Tol.: DIN ISO 2768-mK EN ISO 13920-BF DIN 6935		CAD Zchg	Urspr.	Gez. . .	Gez. 09.04.2001 AO	Zchg.Art .
	.					Freit: .	Freit: BEM Freit2: JAN	

Allgemein-Toleranzen für Längenmasse	bis 6	über 6-30	über 30-120	über 120-400	über 400-1000	über 1000-2000
Spanende Bearbeitung	±0,3	±0,4	±0,6	±0,8	±1,2	±2,0
Sägen auf Länge	±0,5	±0,5	±0,6	±0,8	±1,2	±2,0
Blech- und Schweisskonstruktionen	±1,0	±1,0	±2,0	±2,0	±3,0	±4,0

Massstab 1:2

A 4

Bezugszchg. .

V/P-Zchg .

Rev .

Schutzvermerk DIN 34

Produkthierarchie .

DISTANZPLATTE

ZU 9011650

Werkstoff St.37K

Zeichnungsnummer 9011641

Rev –

Plot: AO, 30.08.2004, FR (Released)

2. About the requirement analysis (example)

What About BEFORE the Blue Print

- Define main objectives of future system

- $\boxed{\text{Define requirements}}$

- Study feasibility

- Place of requirement document
  - System life cycle
  - Difficulties and weak point
- Role of requirement document
  - Characterizing the requirement document
  - Some structuring rules

The Classical Development Cycle

1. Feasibility Study
2. Requirement Analysis
3. Technical Specification
4. Design

4. Coding
5. Test
6. Documentation
7. Maintenance

Difficulties

- Ensuring relative consistency between the phases

- Formal Methods could help (in the later phases)

- But still a problem in the earlier phases

- Weakest part: the requirement document

About the Requirement Document

- Importance of this document (due to its position in the life cycle)

- Obtaining a good requirement document is not easy:

    - missing points

    - too specific (over-specified)

- Requirement document are usually difficult to exploit

- There might exist some guidelines allowing us to better exploit it

About the Requirement Document (cont'd)

- Hence very often necessary to rewrite it

- It will cost a significant amount of time and money (but well spent)

- The famous specification change syndrome might disappear

Some Structuring Rules

- Two separate texts in the same document:
    - explanatory text: the why
    - reference text: the what

- Embedding the reference text within the explanation text

- The reference text eventually becomes the official document

- Must be signed by concerned parties

2.8 The Cantor-Bernstein Theorem.

If $a \preceq b$ *and* $b \preceq a$ *then* a *and* b *are equinumerous.*

This theorem was first conjectured by Cantor in 1895 and proved by Bernstein in 1898.

Proof: Since $b \preceq a$, then a has a subset c such that $b \approx c$.

. . .

$\square$

The Reference Text

- Contains the properties of the future system

- Made of short labeled “fragments” (traceability)

- Should be easy to read (different font) and easy to extract (boxed)

- About the abstraction levels (don’t care too much)

- The problem of over-specification (don’t care too much)

A Requirement Document (1)

- The system we are going to build is a piece of software connected to some equipment.

- There are two kinds of requirements:
  - those concerned with the equipment, labeled EQP,
  - those concerned with the function of the system, labeled FUN.

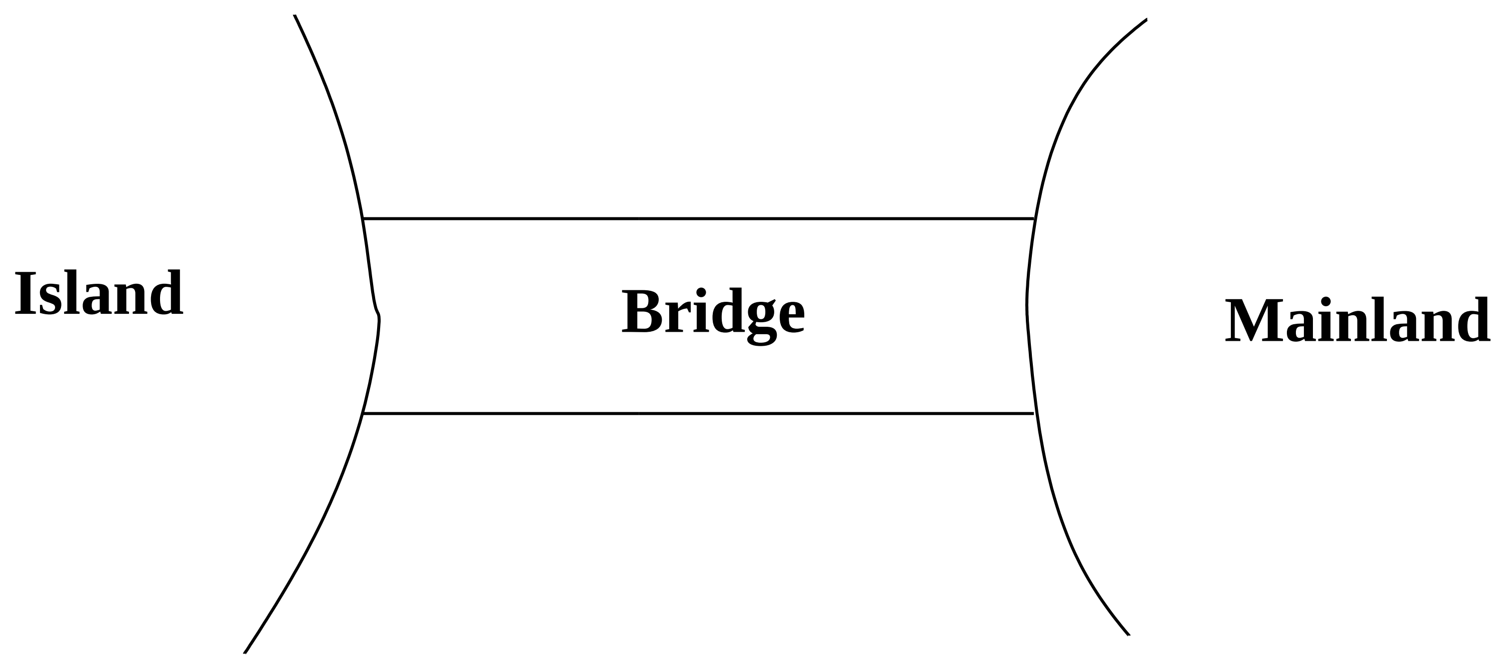
- The function of this system is to control cars on a narrow bridge.

- This bridge is supposed to link the mainland to a small island.

A Requirement Document (2)

The system is controlling cars on a bridge between the mainland and an island	FUN-1

- This can be illustrated as follows



A Requirements Document (3)

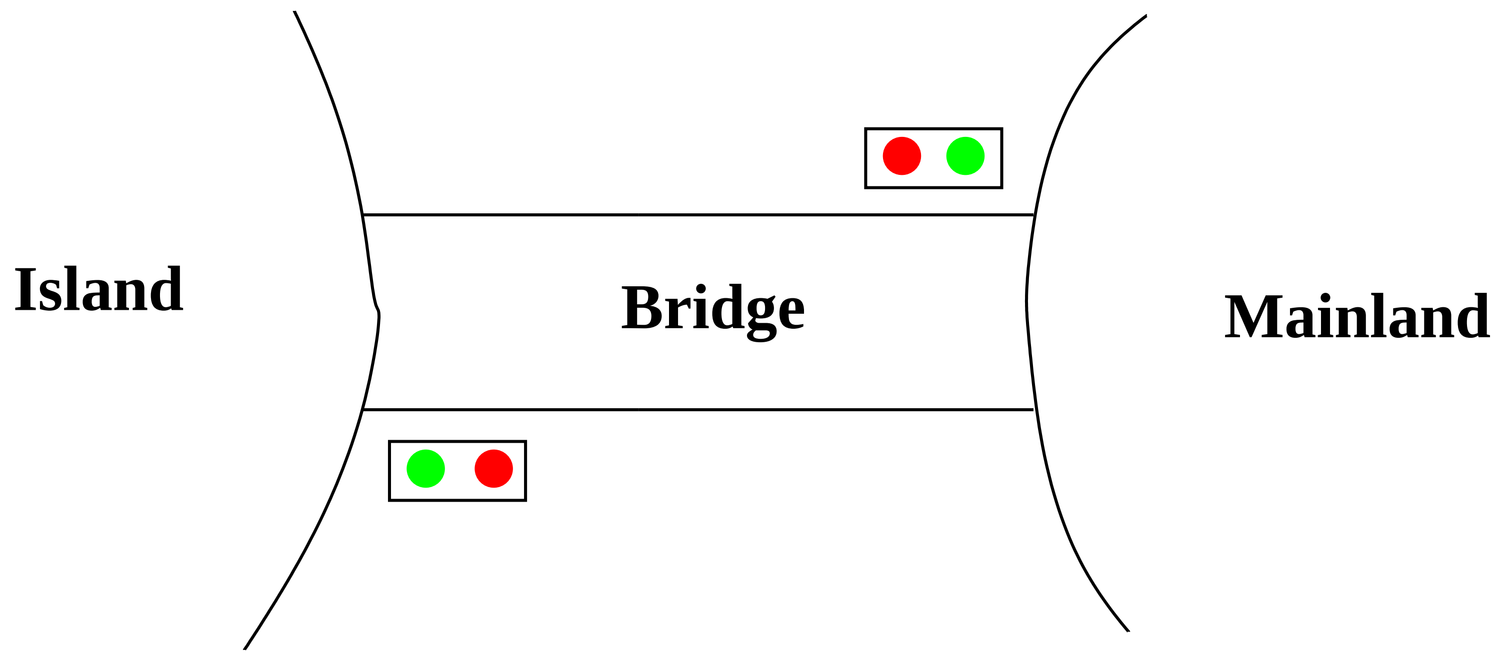
- The controller is equipped with two traffic lights with two colors.

The system has two traffic lights with two colors: green and red	EQP-1

A Requirements Document (4)

- One of the traffic lights is situated on the mainland and the other one on the island. Both are close to the bridge.

- This can be illustrated as follows:



A Requirements Document (5)

The traffic lights control the entrance to the bridge at both ends of it	EQP-2

- Drivers are supposed to obey the traffic light by not passing when a traffic light is red.

Cars are not supposed to pass on a red traffic light, only on a green one	EQP-3

- There are also some car sensors situated at both ends of the bridge.

- These sensors are supposed to detect the presence of cars intending to enter or leave the bridge.

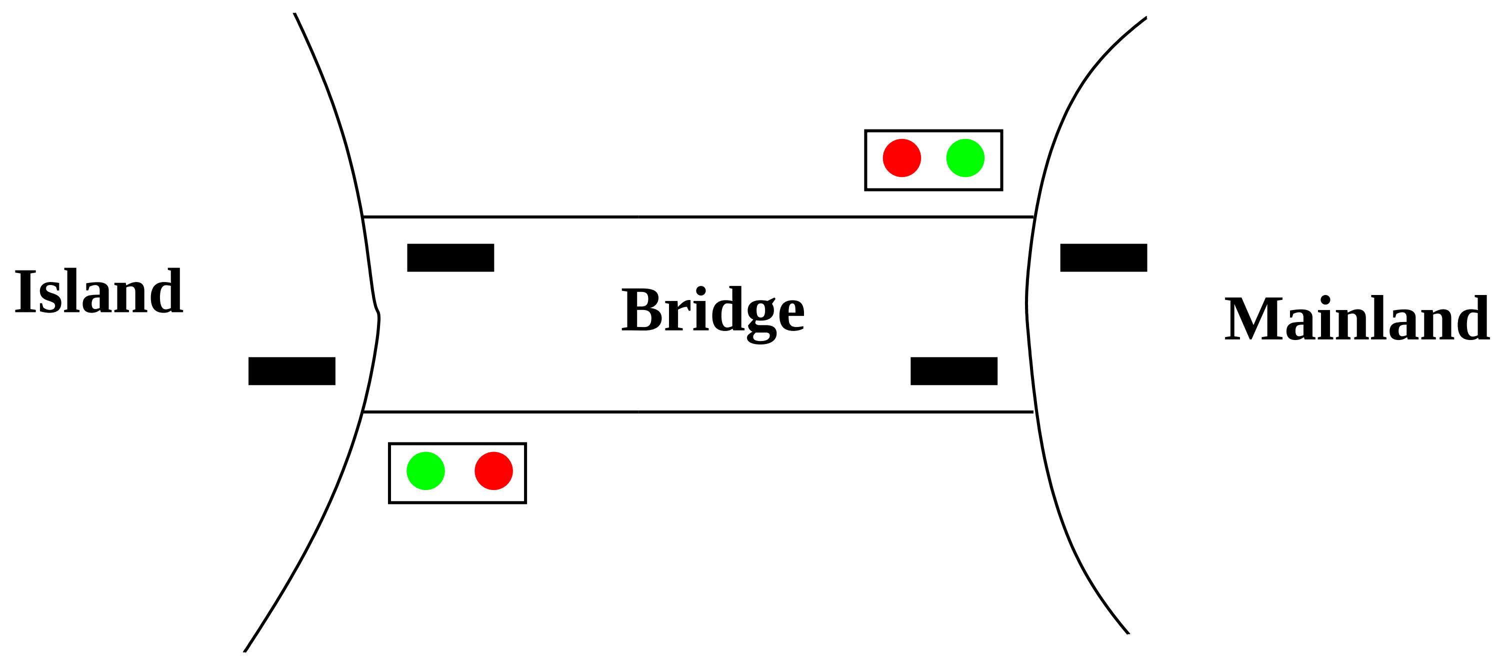
- There are four such sensors. Two of them are situated on the bridge and the other two are situated on the mainland and on the island.

The system is equipped with four car sensors each with two states: on or off	EQP-4

A Requirements Document (7)

The sensors are used to detect the presence of cars entering or leaving the bridge	EQP-5

- The pieces of equipment can be illustrated as follows:



A Requirements Document (8)

- This system has two main constraints:
  - the number of cars on the bridge and the island is limited
  - the bridge is one way.

The number of cars on the bridge and the island is limited	FUN-2

The bridge is one way or the other, not both at the same time	FUN-3

The Reference Document (1)

The system is controlling cars on a bridge between the mainland and an island	FUN-1
The number of cars on the bridge and the island is limited	FUN-2
The bridge is one way or the other, not both at the same time	FUN-3

The system has two traffic lights with two colors: green and red	EQP-1

The traffic lights control the entrance to the bridge at both ends of it	EQP-2

Cars are not supposed to pass on a red traffic light, only on a green one	EQP-3

The Reference Document (3)

The system is equipped with four car sensors each with two states: on or off	EQP-4

The sensors are used to detect the presence of cars entering or leaving the bridge	EQP-5

Taxonomy of Properties (incomplete)

- Functional

- Safety

- Equipment

- Degraded modes

- Availability

- Delays

Style of Reference Text (incomplete)

- Short natural language statements

- Tables (“data” description)

- Transition diagrams

- Mathematical formulae

- Physical units table

- . . .

3. About modelling

Back to the Questions about Formal Methods

- What are they used for?

- When are they to be used?

- Is UML a formal method?

- Are they needed when doing OO programming?

- What is their definition?

Definitions of Formal Methods Used in this Course

- Formal methods are techniques for building and studying blue prints ADAPTED TO OUR DISCIPLINE

Our discipline is: design of hardware and software SYSTEMS

- Such blue prints are now called models

- Reminder:
  - Models allow to reason about a FUTURE system
  - The basis is lacking (hence you cannot “execute” a model)

- Reminder (cont'd):
  - Using pre-defined conventions
  - Conventions should help facilitate reasoning (more to come)

- Consequence: Using ordinary discrete mathematical conventions:
  - Classical Logic (Predicate Calculus)
  - Basic Set Theory (sets, relations and functions)

- Such conventions will be reviewed in subsequent lectures

Systems we are Interested to Develop

- a “classical” piece of software
- an electronic circuit
- a file transfer protocol
- an airline booking system
- a PC operating system
- a nuclear plant controller
- a SmartCard electronic purse
- a launch vehicle flight controller
- a driverless train controller
- a mechanical press controller
- etc.

Characterizing such Systems (general)

- They are made of many parts

- They interact with a possibly hostile environment

- They involve several executing agents

- They require a high degree of correctness

- There construction spreads over several years

- Their specifications are subjected to many changes

Two Real Examples

- Fully automatic train systems

- Paris metro line 14 (October 1998)

- Roissy Airport shuttle (March 2007)

- In each case, the safety critical part only is done with this approach of Formal Method with Proofs (B)

Line length	8.5 km
Number of Stops	8
Time interval between two trains	115 s
Speed	40 km/h
Number of trains	17
Passengers per day	$350,000$

Line length	3.3 km
Number of Stops	5
Time interval between two trains	105 s
Speed	26 km/h
Number of trains	14
Passengers per hour	$2,000$

Comparing the Examples (1)

	Paris	Roissy
Number of final ADA lines (from B)	$86,000$	$158,000$
Number of proofs	$27,800$	$43,610$
Percentage of interactive proofs	8.1	3.3
Interactive proofs in Man.Month	7.1	4.6

Comparing the Examples (2)

- Man.month calculated with:
    - 15 interactive proofs per man.day
    - 21 days in a month

- In both cases, no unit tests and no integration tests

- Reinforcing global tests (catastrophic scenarios)

-Important differences in the software requirements:
    - Paris: specially done for the project
    - Roissy: adaptation from O’Hare Airport (problems)

Characterizing such Systems (more technical)

- These systems operate in a discrete fashion

- Their dynamical behavior can be abstracted by:
    - A succession of steady states
    - Intermixed with sudden jumps

- The possible number of state changes are enormous

- Usually such systems never halt

- They are called discrete transition systems

- Test reasoning (a vast majority): VERIFICATION

- Blue Print reasoning (a very few): CORRECT CONSTRUCTION

Test Reasoning: VERIFICATION

- Based on laboratory execution
- Obvious incompleteness
- The oracle is usually missing
- Properties to be checked are chosen a posteriori
- Re-adapting and re-shaping after testing
- Reveals an immature technology

Blue Print Reasoning: CORRECT CONSTRUCTION

- Based on a formal model: the “blue print”

- Gradually describing the system with the needed precision

- Relevant Properties are chosen a priori

- Serious thinking made on the model, not on the final system

- Reasoning is validated by proofs

- Reveals a mature technology

Blue Print Reasoning: Outcome of Proving

- The proof succeeds

- The proof fails but refutes the statement to prove
    - the model is erroneous: it has to be modified

- The proof fails but is probably provable
    - the model is badly structured: it has to be reorganized

- The proof fails and is probably not provable nor refutable
    - the model is too poor: it has to be enriched

- Rules of Thumb:

n lines of final code implies $n/3$ proofs

95% of proofs discharged automatically

5% of proofs discharged interactively

350 interactive proofs per man-month

- 60,000 lines of final code $\leadsto$ 20,000 proofs $\leadsto$ 1,000 int. proofs
- 1,000 interactive proofs $\leadsto$ 1000/350 $\simeq$ 3 man-months
- Far less expensive than heavy testing

The State of a Model

- A discrete model is first made of a state

- The state is represented by some constants and variables

- Constants are linked by some properties

- Variables are linked by some invariants

- Properties and invariants are written using set-theoretic expressions

The Events of a Model (Transitions)

- A discrete model is also made of a number of events

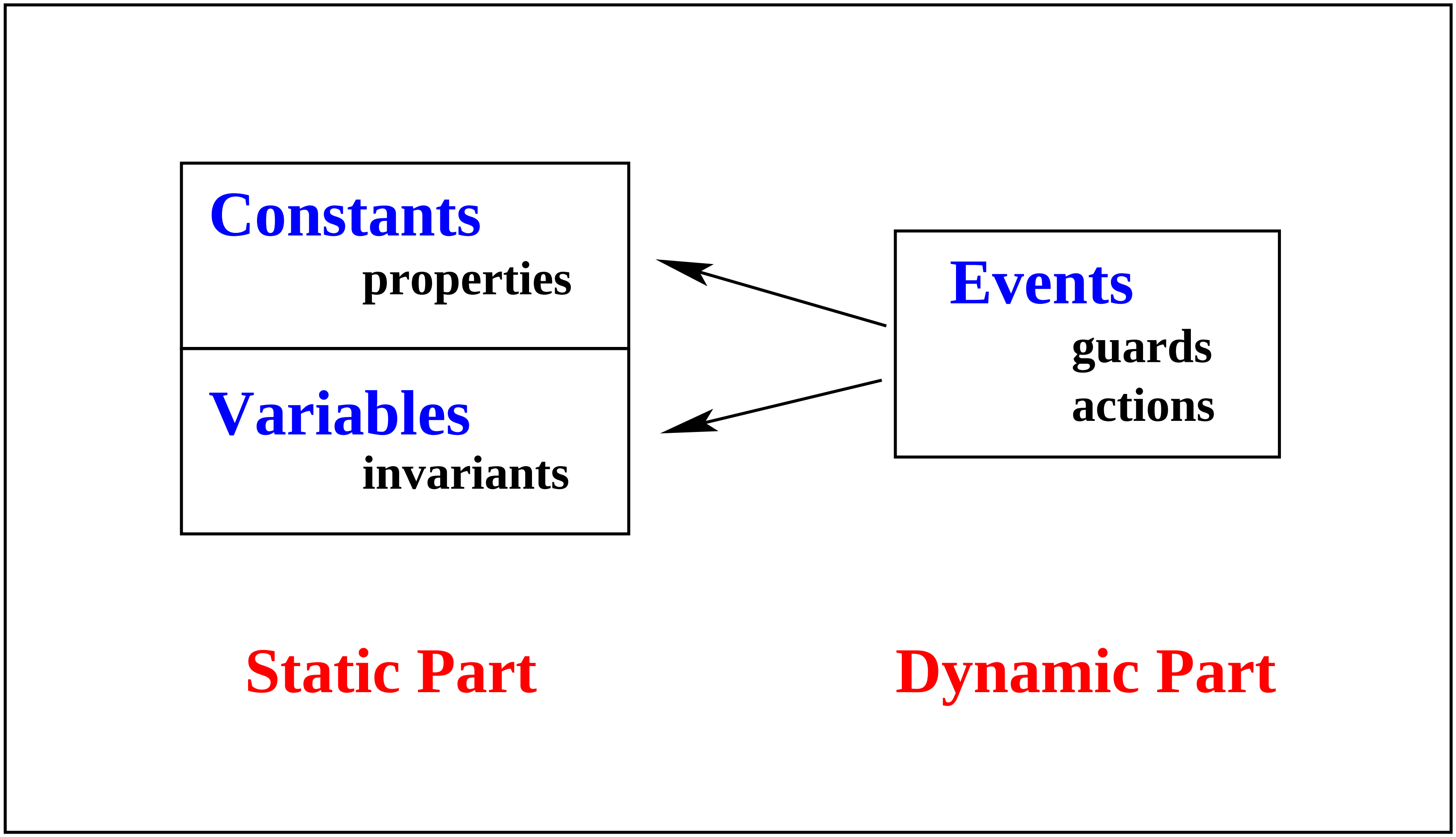
- An event is made of a guard and an action

- The guard denotes the enabling condition of the event

- The action denotes the way the state is modified by the event

- Guards and actions are written using set-theoretic expressions

A Model Schematic View



Operational Interpretation

- An event execution is supposed to take no time

- Thus, no two events can occur simultaneously

- When all events have false guards, the discrete system stops

- When some events have true guards, one of them is chosen non-deterministically and its action modifies the state

- The previous phase is repeated (if possible)

```
Initialize;
while (some events have true guards) {
   Choose one such event;
   Modify the state accordingly;
}
```

Comments on the Operational Interpretation

- Stopping is not necessary: a discrete system may run for ever

- This interpretation is just given here for informal understanding

- The meaning of such a discrete system will be given by the proofs which can be performed on it (next lectures)

- Formalization contains models of:
  - the future software components
  - the future equipments surrounding these components

- The overall model construction can be very complex

- Three techniques can be used to master this complexity
  - refinement
  - decomposition
  - generic instantiation

- Refinement allows us to build model gradually

- We shall build an ordered sequence of more precise models

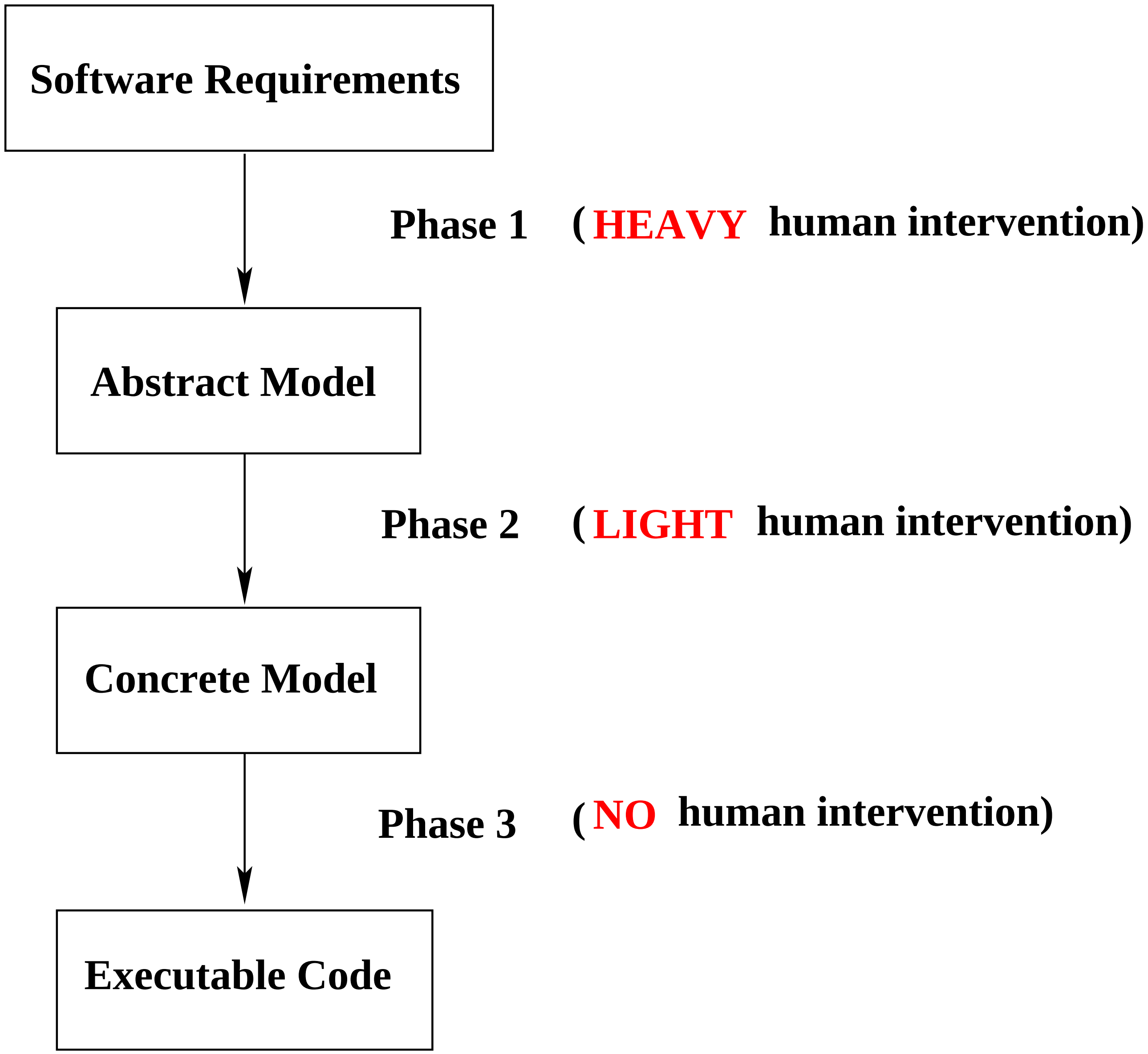
- Each model is a refinement of the one preceding it

- A useful analogy: looking through a microscope

- Spatial as well as temporal extensions

- Data refinement

Examples of Phases in Formal Development



Phase 1: Constructing the Abstract Model (1)

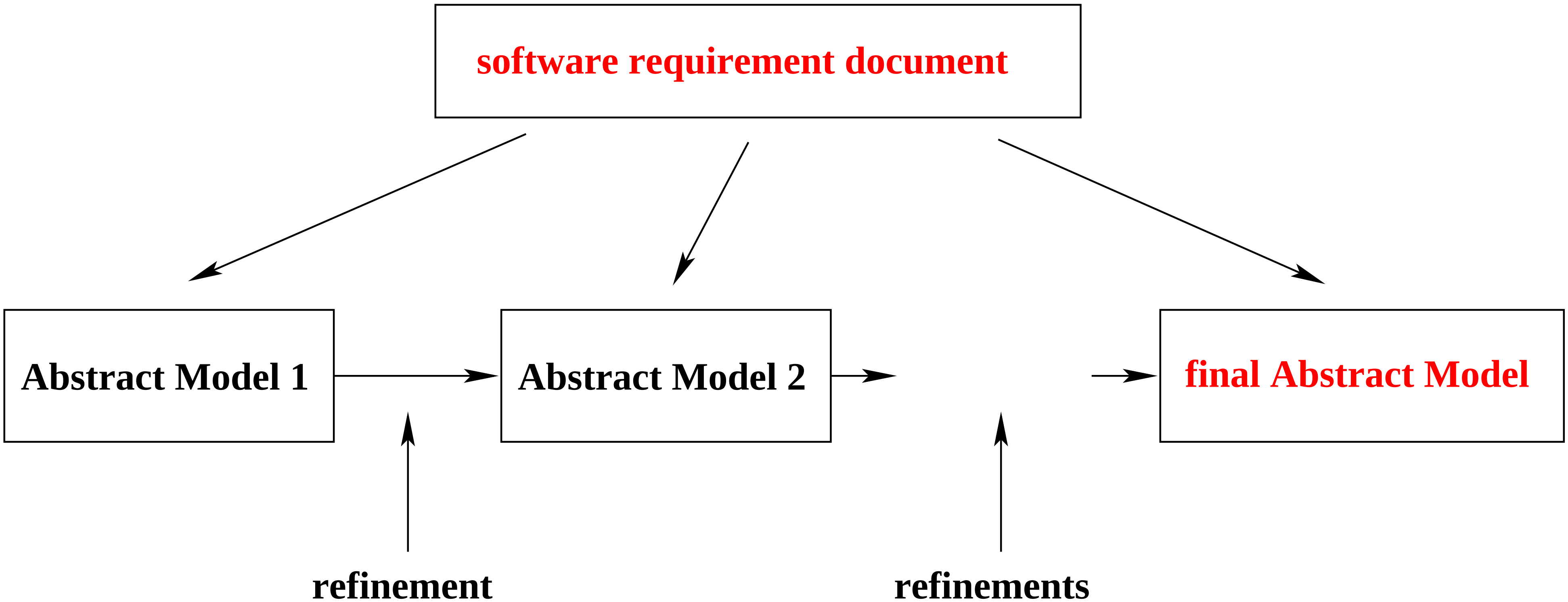
- The software requirement document is given

- Details of this document are gradually extracted

- The Abstract Model is thus constructed by successive refinements

- Each refinement is an independent model

Phase 1: Constructing the Abstract Model (2)



Phase 2: Constructing the Concrete Model (1)

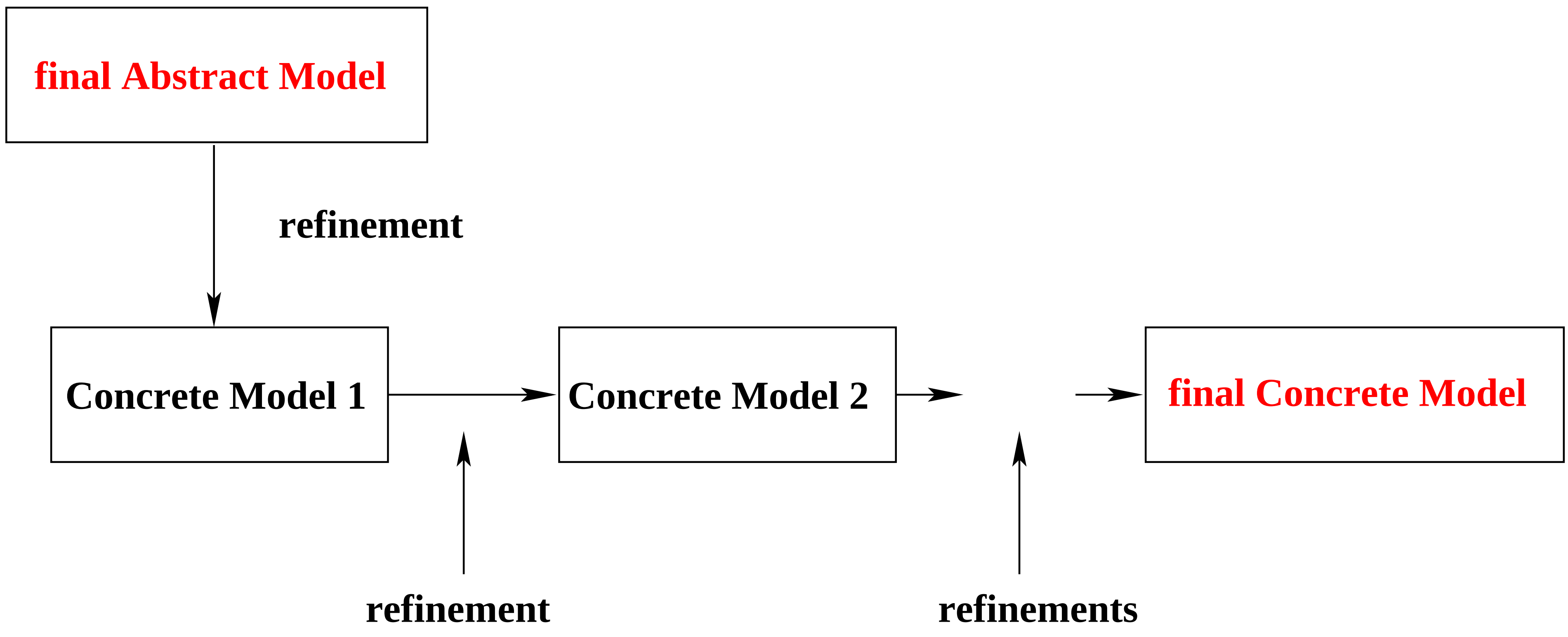
- The point of departure is now the final abstract model

- The concrete model construction is done in a similar manner:
  by successive refinements

- the set-theoretic constructs are gradually transformed into
  computerizable objects: finite scalars, arrays, pointers, etc.

- The non-deterministic operations are gradually transformed into
  programming constructs: conditional, loops, procedure calls, etc.

Phase 2: Constructing the Concrete Model (2)



Phase 3: Getting the Executable Code (1)

- The Point of Departure is now the final concrete model

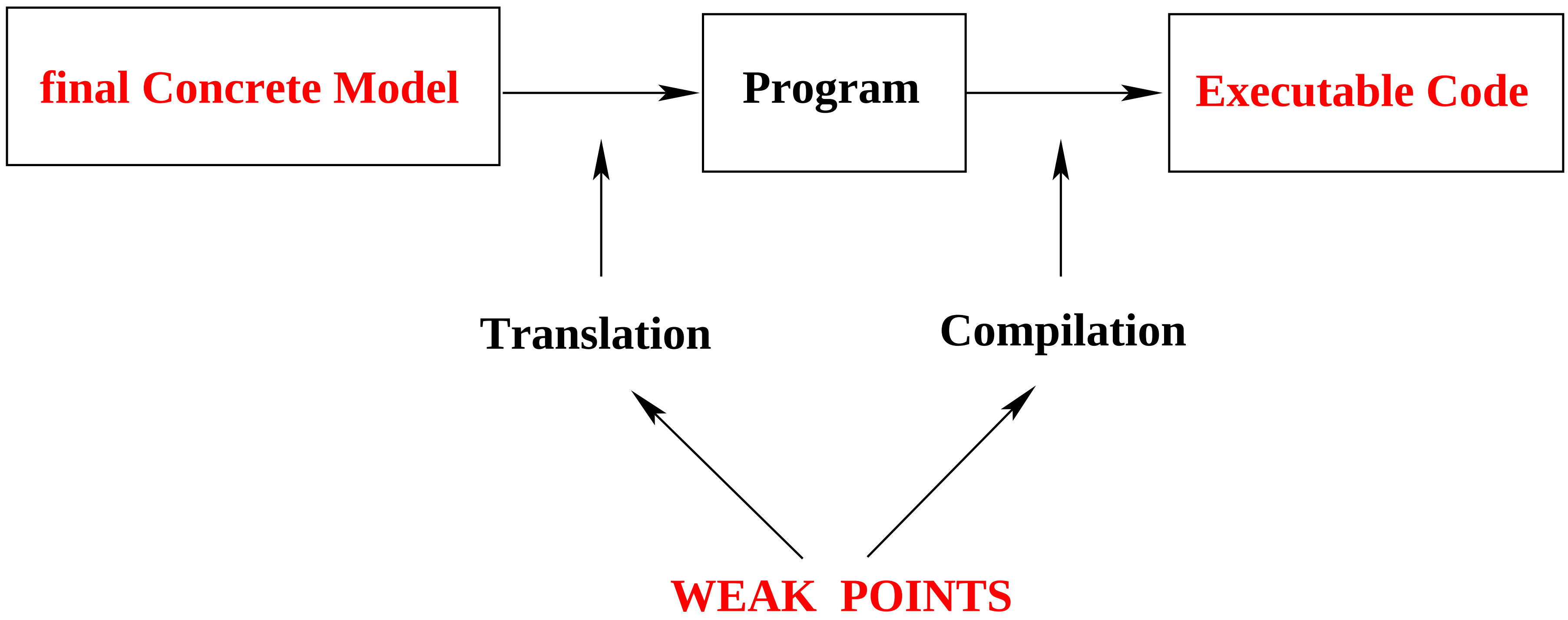
- The concrete model is checked by a translatability tool

- The concrete model is translated into a classical program

- The classical program is compiled into executable code

- Translation and compilation are weak points (more later)

Phase 3: Getting the Executable Code (2)



Summary of the Main Principles

- Three phases:
  - Constructing the abstract model
  - Constructing the concrete model
  - Constructing the executable code

- Two main concepts:
  - Refinement
  - Proof

Summary of the Tools

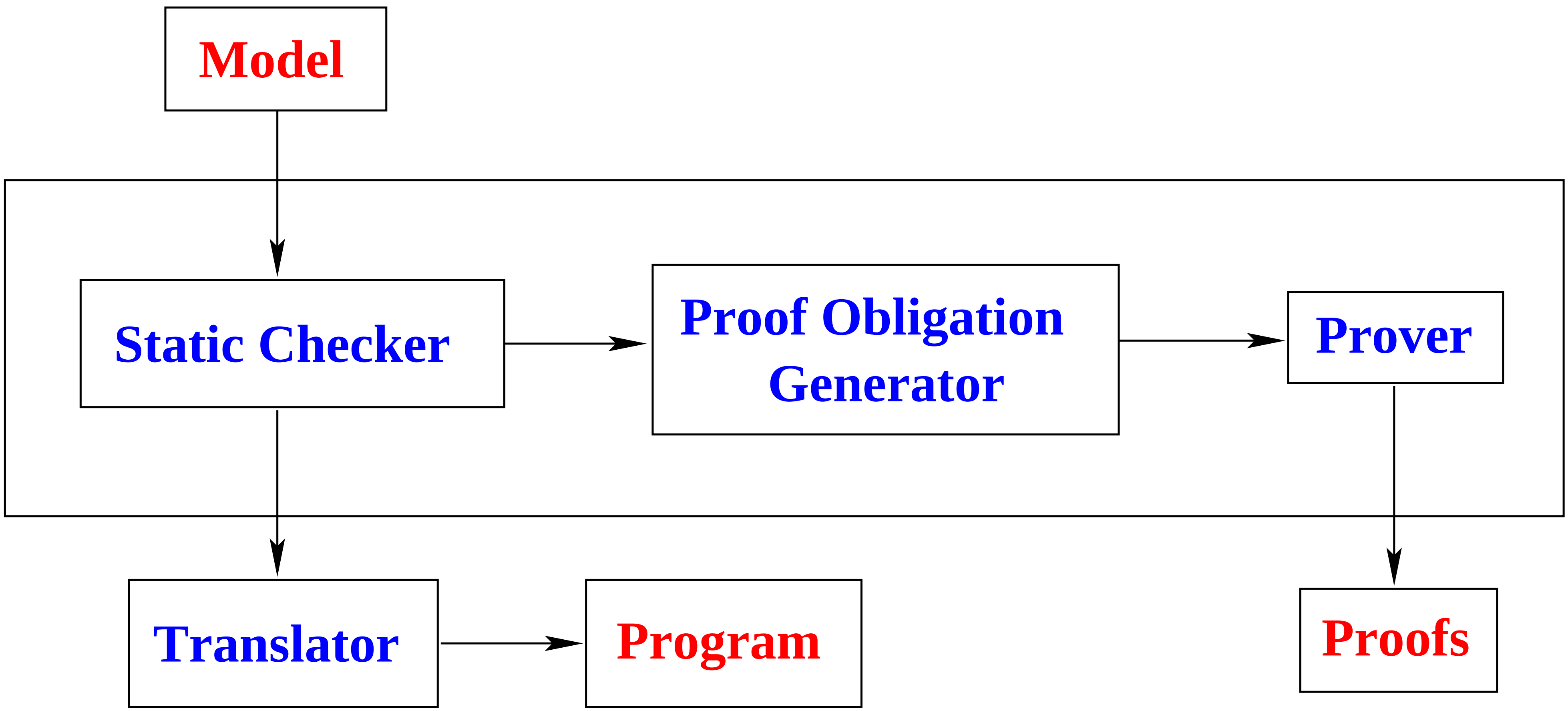
- The Static Checker

- The Proof Obligation Generator

- The Prover

- The Translator from B to a programming language

Summary of the Tools



Decomposition

- Refinement does not solve alone the problem of complexity

- Models can become very large: large state and many events

- A large model can be decomposed into smaller ones

- Decomposed models communicates

- Decomposed models can then be refined further and so on

Generic Instantiation

- Models can be parameterized with carrier sets and constants

- Constants are specified by means of properties

- Analogy with algebraic theories (i.e. group theory)

- Such models are said to be generic

- They can be instantiated