

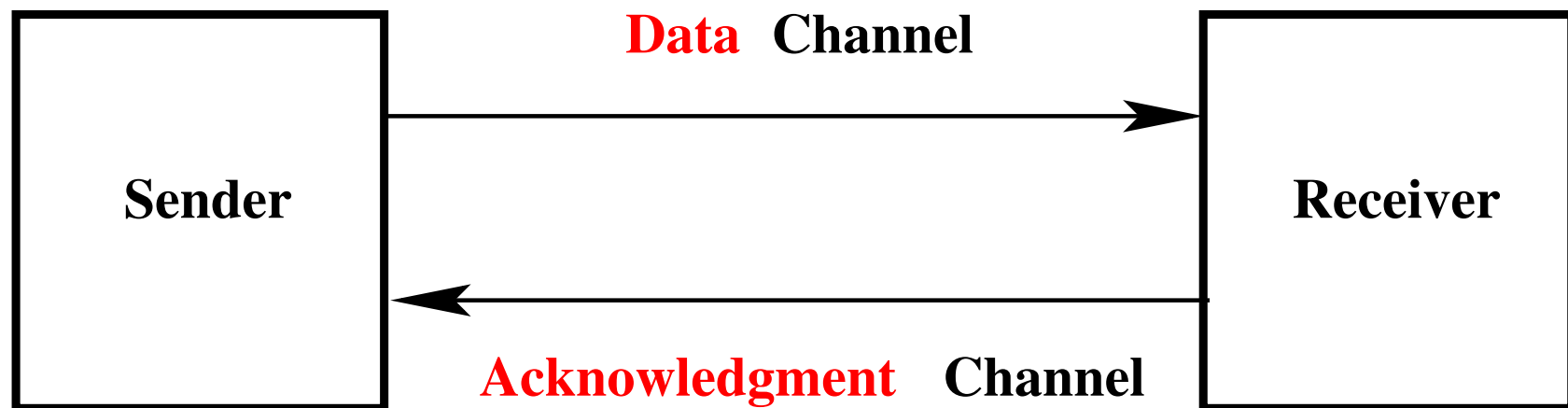
6. The Bounded Re-transmission Protocol

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2009

- The Bounded Re-transmission Protocol is a **file transfer protocol**
- This is a problem dealing with **fault tolerance**
- We suppose that the transfer channels are **unreliable**
- We present classical solutions to handle that problem: **timers**.
- We would like to see how we can **formalize such timers**

- A **sequential file** is transmitted from a **Sender** to a **Receiver**
- The file is transmitted **piece by piece** through a **Data Channel**
- After receiving some data, the Receiver sends an **acknowledgment**
- After receiving it, the Sender sends the **next piece of data**, etc.



- **Messages can be lost** in the Data or Acknowledgment channels

The goal of the BRP is to **totally** or **partially** transfer a certain non-empty original sequential file from one site to another.

FUN_1

A **total transfer** means that the transmitted file is a **copy** of the original one.

FUN_2

A **partial transfer** means that the transmitted file is a **genuine prefix** of the original one.

FUN_3

- Messages can be lost in the Data or Acknowledgment channels
- The Sender starts a timer before sending a piece of data
- The timer wakes up the Sender after a delay dl
- This occurs if the Sender has not received an acknowledgment in the meantime

- dl is guaranteed to be **greater than twice the transmission time**
- When waken up, the Sender is then **sure** that the **data** or the **acknowledgment** has been **lost**
- When waken up, the Sender **re-transmits the previous data**
- The Sender sends an **alternating bit** together with a **new data**
- This ensures that the Receiver **does not confuse** (?) a **new data** with a **retransmitted** one.

- The Sender can re-transmit the same data **at most $MAX + 1$ times**
- After this, the Sender **decides to abort**
- **How does the Receiver know** that the Sender aborted?

- Each time the Receiver receives a **new** piece of data, it **starts a timer**
- The timer **wakes up** the Receiver after a delay $(MAX + 1) \times dl$
- This occurs if the Sender **has not received a new data** in the meantime.
- After this delay, the Receiver is certain that **the Sender has aborted**
- Then the **Receiver aborts** too.

- At the end of the protocol, we might be in one of the **three situations**:
 - (1) The file **has been transmitted** entirely and the Sender **has received** the last acknowledgment
 - (2) The file **has been transmitted** entirely but the Sender **has not received** the last acknowledgment
 - (3) The file **has not been transmitted** entirely

Each site may end up in any of the **two situations**:

- either **it believes** that the protocol has **terminated successfully**,
- or **it believes** that the protocol has **aborted**

FUN_4

When the **Sender** believes that the protocol has **terminated successfully** then the **Receiver** believes so too.

FUN_5

However, it is possible for the **Sender** to believe that the protocol has **aborted** while the **Receiver** believes that it has **terminated successfully**.

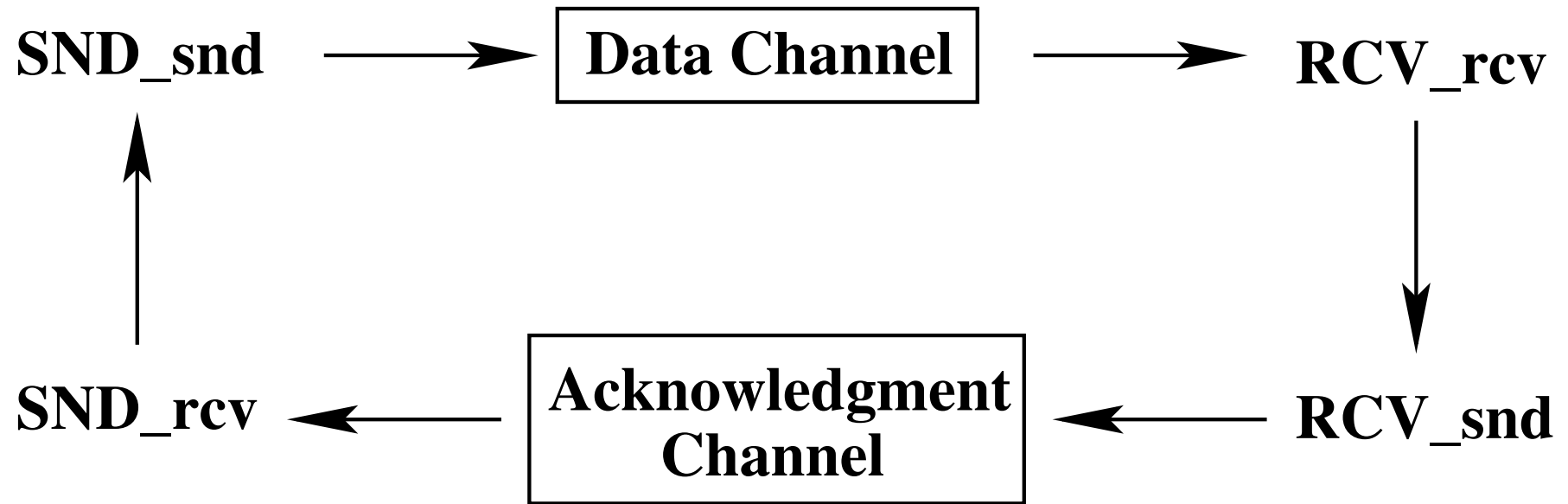
FUN_6

When the **Receiver** believes that the protocol has **terminated successfully**, this is because the original file has been **entirely copied** on the Receiver's site.

FUN_7

When the **Receiver** believes that the protocol has **aborted**, this is because the original file has **not been copied entirely** on the Receiver's site.

FUN_8



SND_snd

when

SND_snd is waken up

then

Acquire data from Sender's file;

Store acquired data on Data Channel;

Store Sender's bit on Data Channel;

Start Sender's timer;

Activate Data Channel;

end

```
RCV_rcv
  when
    Data Channel interrupt occurs
  then
    Acquire Sender's bit from Data Channel;
    if Sender's bit = Receiver's bit then
      Acquire Data from Data Channel;
      Store data on Receiver's file;
      Modify Receiver's bit;
      if data is not the last one then
        Start Receiver's timer;
      end
    end
  end
  Reset Data Channel Interrupt;
  Wake up RCV_snd;
end
```

```
RCV_snd
```

```
  when
```

```
    RCV_snd is waken up
```

```
  then
```

```
    Activate Acknowledgment Channel;
```

```
  end
```

```
SND_rcv
  when
    Acknowledgment Channel interrupt occurs
  then
    Remove Data from Sender's file;
    Reset retry counter;
    Modify Sender's bit;
    Wake up event SND_snd;
    Reset Acknowledgment Channel interrupt;
    if Sender's file is not empty then
      Wake up event SND_snd
    end
  end
end
```



```
SND_timer
  when
    Sender's timer interrupt occurs
  then
    if retry counter is equal to MAX+1 then
      Abort protocol on Sender's site;
    else
      Increment retry counter;
      Wake up event SND_snd;
    end
  end
```

RCV_timer

when

Receiver's timer interrupt occurs

then

Abort protocol on Receiver's site

end

-
- Quite often, protocols are "specified" by such pseudo-codes
 - In fact, such a pseudo-code raises a number of questions:
 - Are we sure that this description is correct?
 - Are we sure that this protocol terminates?
 - What kinds of properties should this protocol maintain?
 - Hence the formal development which is presented now

(0) FUN_4: Defining the final "belief" situation

(1) and (2) FUN_5 and FUN_6: Connecting the "beliefs"

(3) FUN_1 to FUN_3, FUN_7 and FUN_8: Partial Transmission and final situation of the Receiver

(4) Introducing the Sender

(5) Introducing **unreliable channels** and **timers**.

The goal of the BRP is to **totally** or **partially** transfer a certain non-empty original sequential file from one site to another.

FUN_1

A **total transfer** means that the transmitted file is a **copy** of the original one.

FUN_2

A **partial transfer** means that the transmitted file is a **genuine prefix** of the original one.

FUN_3

Each site may end up in any of the **two situations**:

- either **it believes** that the protocol has **terminated successfully**,
- or **it believes** that the protocol has **aborted**

FUN_4

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FUN_6

When the **Receiver** believes that the protocol has **terminated successfully**, this is because the original file has been **entirely copied** on the Receiver's site.

FUN_7

When the **Receiver** believes that the protocol has **aborted**, this is because the original file has **not been copied entirely** on the Receiver's site.

FUN_8

Our initial model deals with requirements FUN-4:

Each site may end up in any of the **two situations**:

- either **it believes** that the protocol has **terminated successfully**,
- or **it believes** that the protocol has **aborted**

FUN_4

set: *STATUS*

constants: *working*
success
failure

axm0_1: *STATUS = {working, success, failure}*

axm0_2: *working ≠ success*

axm0_3: *working ≠ failure*

axm0_4: *success ≠ failure*

- Variables s_st and r_st denote the status of the participants (Sender and Receiver respectively).

variables: s_st
 r_st

inv0_1: $s_st \in STATUS$

inv0_2: $r_st \in STATUS$

- Initially, both participants are working
- Event "brp" is an "oserver" fired when both participants are not working

```
init
  s_st := working
  r_st := working
```

```
brp
  when
    s_st ≠ working
    r_st ≠ working
  then
    skip
  end
```

Next are two anticipated events:

```
SND_progress
  status
    anticipated
  when
    s_st = working
  then
    s_st ∈ {success, failure}
  end
```

```
RCV_progress
  status
    anticipated
  when
    r_st = working
  then
    r_st ∈ {success, failure}
  end
```

When the **Sender** believes that the protocol has **terminated successfully** then the **Receiver** believes so too.

FUN_5

However, it is possible for the **Sender** to believe that the protocol has **aborted** while the **Receiver** believes that it has **terminated successfully**.

FUN_6

inv1_1: $s_st = success \Rightarrow r_st = success$

```
SND_success
refines
  SND_progress
status
  convergent
when
  s_st = working
  r_st = success
then
  s_st := success
end
```

```
SND_failure
refines
  SND_progress
status
  convergent
when
  s_st = working
then
  s_st := failure
end
```

```
variant1: {success, failure} \ {s_st}
```

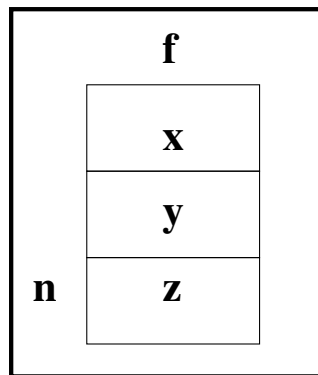
```
RCV_success
refines
  RCV_progress
status
  convergent
when
  r_st = working
then
  r_st := success
end
```

```
RCV_failure
refines
  RCV_progress
status
  convergent
when
  r_st = working
  s_st = failure
then
  r_st := failure
end
```

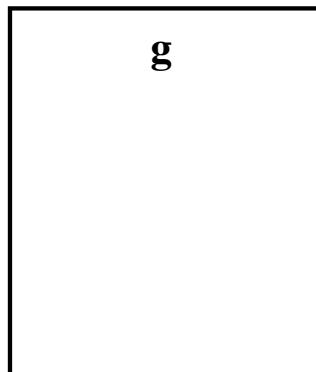
```
variant2: {success, failure} \ {r_st}
```

INITIAL SITUATION

SENDER

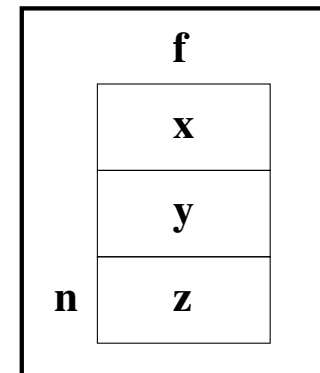


RECEIVER

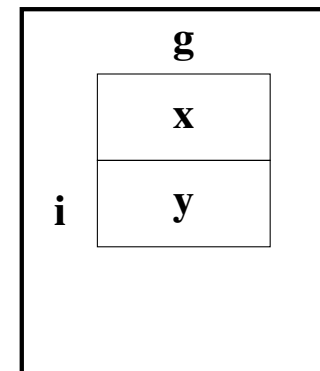


FINAL SITUATION

SENDER



RECEIVER



- Set D denotes the objects in the files
- Constant n denotes the size of the non-empty file
- Constant f denotes the original file.

set: D

constants: n
 f

axm1_1: $0 < n$

axm1_2: $f \in 1..n \rightarrow D$

- Variable r denotes the **size** of file g
- Variable g denotes the **transmitted file**.

variables: r
 g

inv3_1: $r \in 0 .. n$

inv3_2: $g = (1 .. r) \triangleleft f$

inv3_3: $r_{st} = success \Leftrightarrow r = n$

Both these events are **cheating**: they have access to $f(r + 1)$, $f(n)$, and n .

RCV_rcv_current_data

status

convergent

when

$r_st = working$

$r + 1 < n$

then

$r := r + 1$

$g := g \cup \{r + 1 \mapsto f(r + 1)\}$

end

RCV_success

when

$r_st = working$

$r + 1 = n$

then

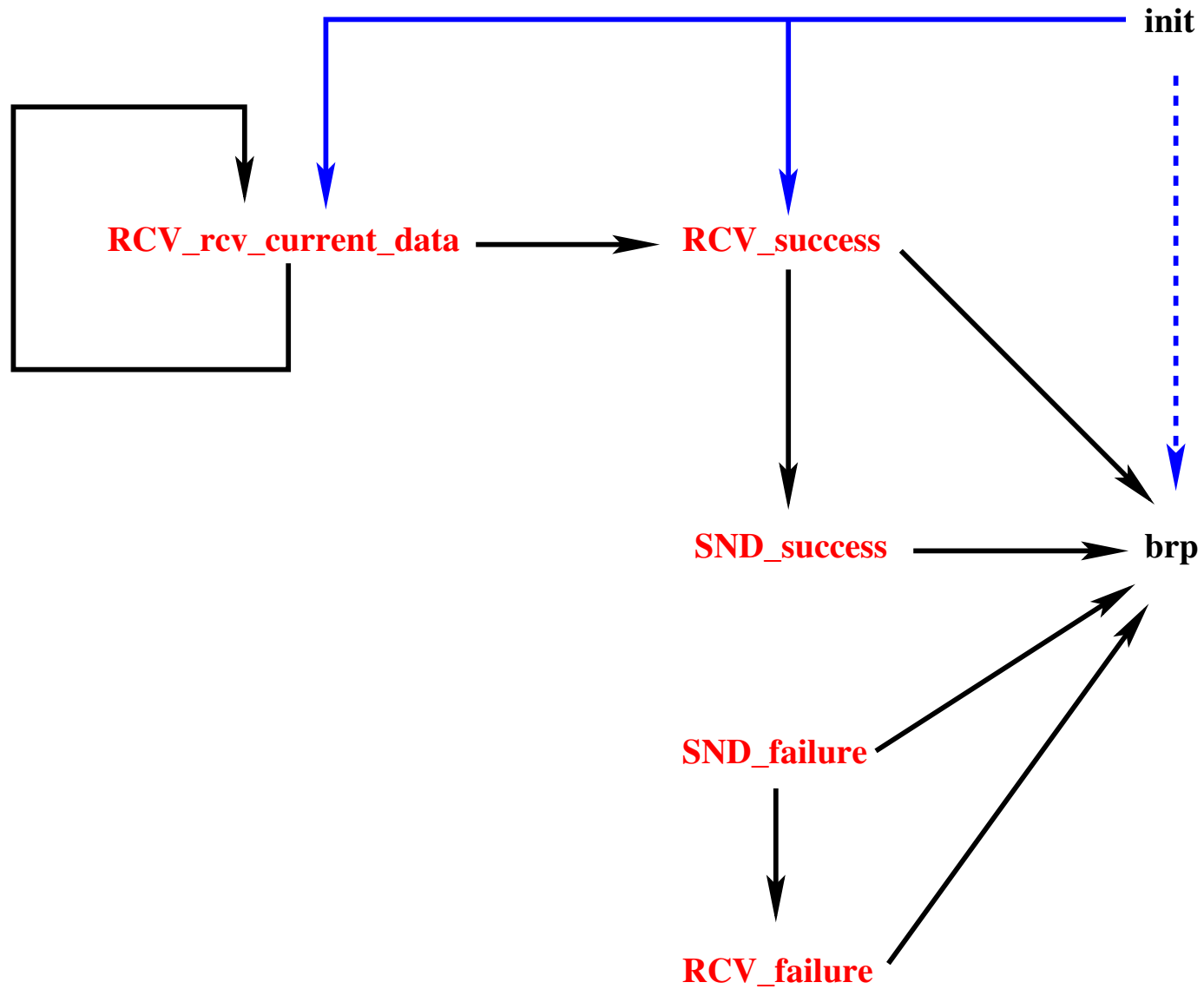
$r_st := success$

$r := r + 1$

$g := g \cup \{r + 1 \mapsto f(n)\}$

end

variant3: $n - r$



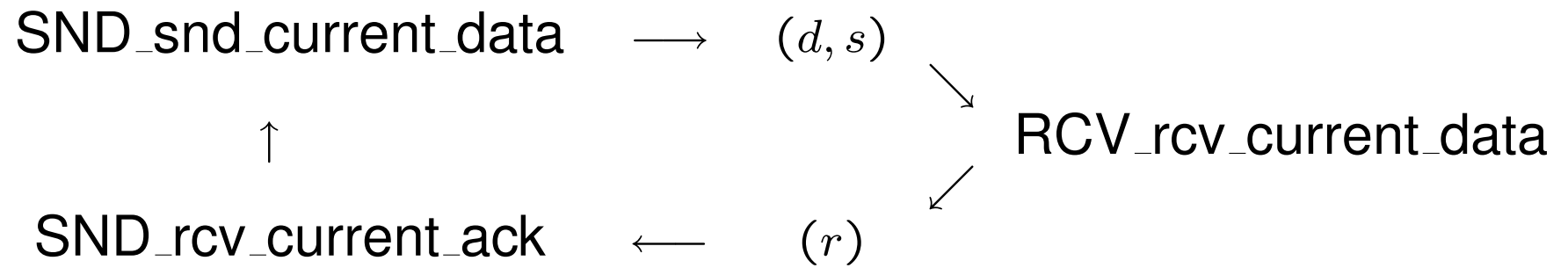
- Variable s is the Sender **pointer** sent to the Receiver
- Variable d is the **data sent** to the Receiver
- Variable w is the Sender **activation bit**
- When w is **TRUE** it means the Sender has **just received the acknowledgement**
- When w is **FALSE** it means the Sender has **sent the information to the Receiver**

variables: \dots
 w
 s
 d

inv4_1: $s \in 0 .. n - 1$

inv4_2: $r \in s .. s + 1$

inv4_3: $w = \text{FALSE} \Rightarrow d = f(s + 1)$



init

$r := 0$

$g := \emptyset$

$r_st := working$

$s_st := working$

$s := 0$

$d \in D$

$w := \text{TRUE}$

brp

when

$r_st \neq working$

$s_st \neq working$

then

skip

end

- **New Events**: the Sender prepares **data d** to be sent

```
SND_snd_data
  when
    s_st = working
    w = TRUE
  then
    d := f(s + 1)
    w := FALSE
  end
```

- These events clearly **refine skip** and maintain invariant **inv4_3**

inv4_3: $w = \text{FALSE} \Rightarrow d = f(s + 1)$

- The Receiver receives data d and pointer s . It sends pointer r .

```
RCV_rcv_current_data
  when
     $r\_st = working$ 
     $w = FALSE$ 
     $r = s$ 
     $r + 1 < n$ 
  then
     $r := r + 1$ 
     $g := g \cup \{r + 1 \mapsto d\}$ 
  end
```

```
RCV_success
  when
     $r\_st = working$ 
     $w = FALSE$ 
     $r = s$ 
     $r + 1 = n$ 
  then
     $r\_st := success$ 
     $r := r + 1$ 
     $g := g \cup \{r + 1 \mapsto d\}$ 
  end
```

- The Receiver **still cheats**: it accesses constant n and boolean w

(abstract-)RCV_rcv_current_data

when

$r_st = working$

$r + 1 < n$

then

$r := r + 1$

$g := g \cup \{r + 1 \mapsto f(r + 1)\}$

end

(concrete-)RCV_rcv_current_data

when

$r_st = working$

$w = \mathbf{FALSE}$

$r = s$

$r + 1 < n$

then

$r := r + 1$

$g := g \cup \{r + 1 \mapsto d\}$

end

- Observe **guard strengthening**
- This invariant helps proving **event refinement**

inv4_3: $w = \mathbf{FALSE} \Rightarrow d = f(s + 1)$

```

(abstract-)RCV_success
when
   $r\_st = working$ 
   $r + 1 = n$ 
then
   $r\_st := success$ 
   $r := r + 1$ 
   $h := h \cup \{n \mapsto f(n)\}$ 
end

```

```

(concrete-)RCV_success
when
   $r\_st = working$ 
   $w = \mathbf{FALSE}$ 
   $r = s$ 
   $r + 1 = n$ 
then
   $r\_st := success$ 
   $r := r + 1$ 
   $h := h \cup \{r + 1 \mapsto d\}$ 
end

```

- Observe **guard strengthening**
- This invariant helps proving **event refinement**

inv4_3: $w = \mathbf{FALSE} \Rightarrow d = f(s + 1)$

- The first event is **new**. It clearly **refines skip**
- The activation bit is set to **TRUE** (activating the event "SND_snd_data")
- The Sender receives acknowledgment (pointer r)

```
SND_rcv_current_ack
when
   $s\_st = working$ 
   $w = FALSE$ 
   $s + 1 < n$ 
   $r = s + 1$ 
then
   $w := TRUE$ 
   $s := s + 1$ 
end
```

```
SND_success
when
   $s\_st = working$ 
   $w = FALSE$ 
   $s + 1 = n$ 
   $r = s + 1$ 
then
   $s\_st := success$ 
end
```

```
(abstract-)SND_success
when
  s_st = working
  r_st = success
then
  s_st := success
end
```

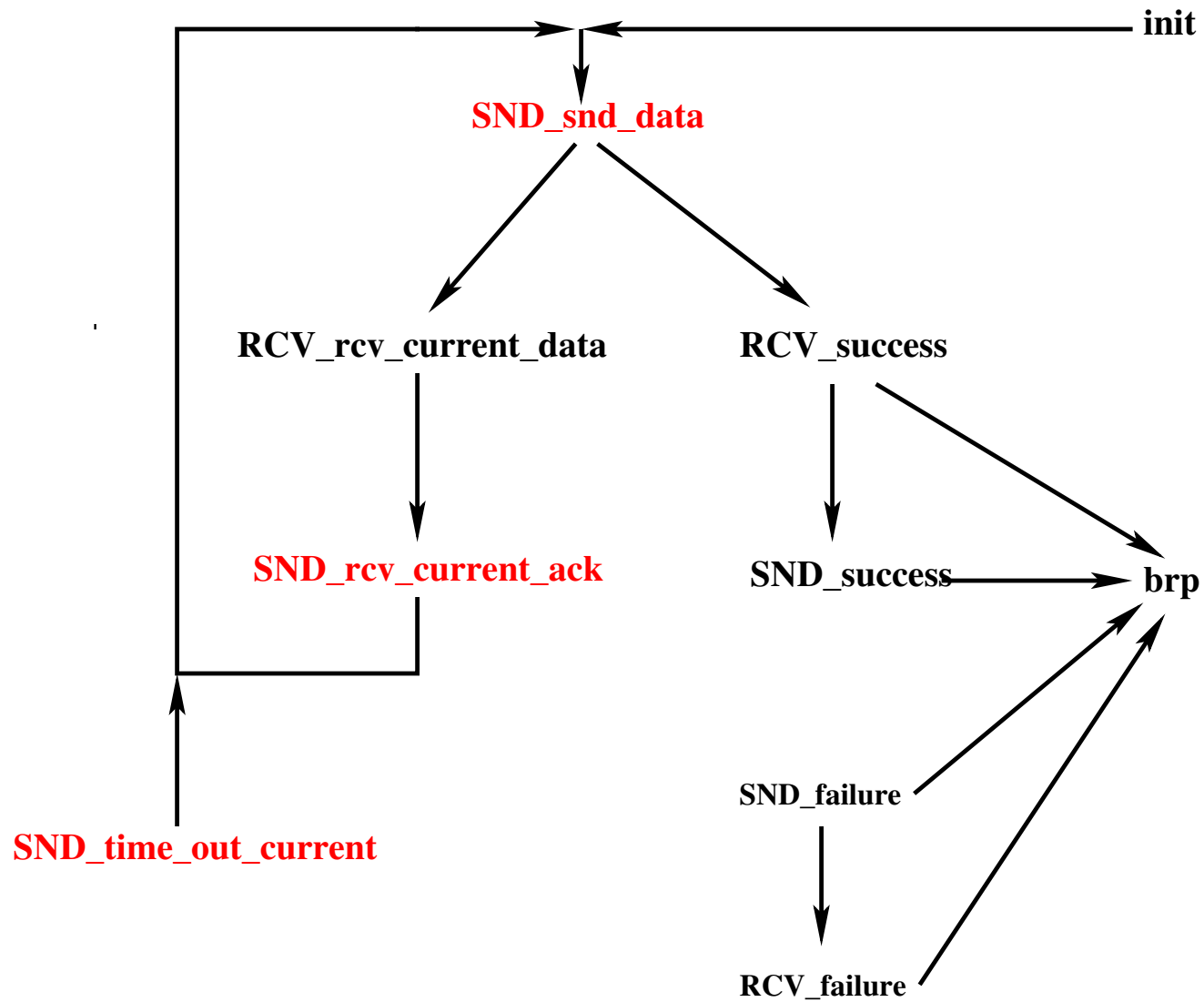
```
(concrete-)SND_success
when
  s_st = working
  w = FALSE
  s + 1 = n
  r = s + 1
then
  s_st := success
end
```

- The presence of **inv1_3** ensures that the **guard is strengthened**

```
inv3_3: r_st = success  $\Leftrightarrow$  r = n
```

- This new events will receive a full explanation in the next refinement

```
SND_time_out_current
  when
    s_st = working
    w = FALSE
  then
    w := TRUE
  end
```



- **At most** one activation bit is **TRUE at a time**

variables: ...
 db
 ab
 v

inv5_1: $w = \text{TRUE} \Rightarrow db = \text{FALSE}$

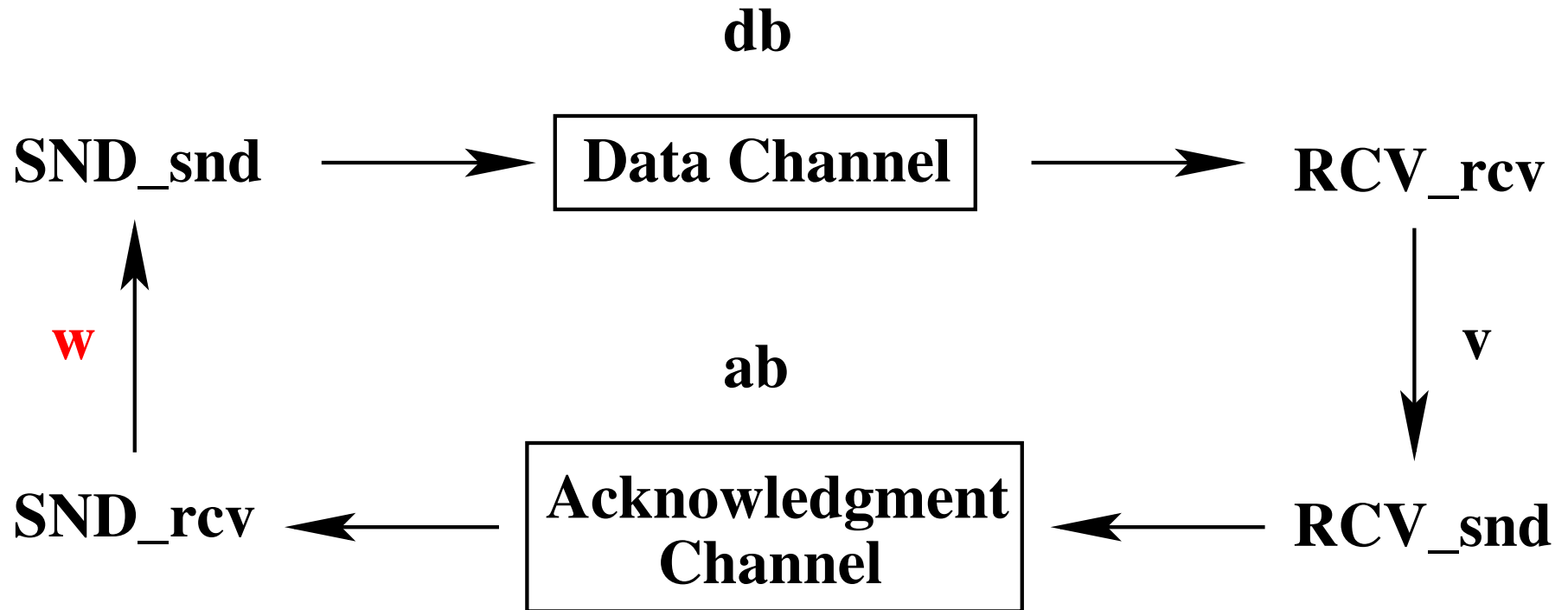
inv5_2: $w = \text{TRUE} \Rightarrow ab = \text{FALSE}$

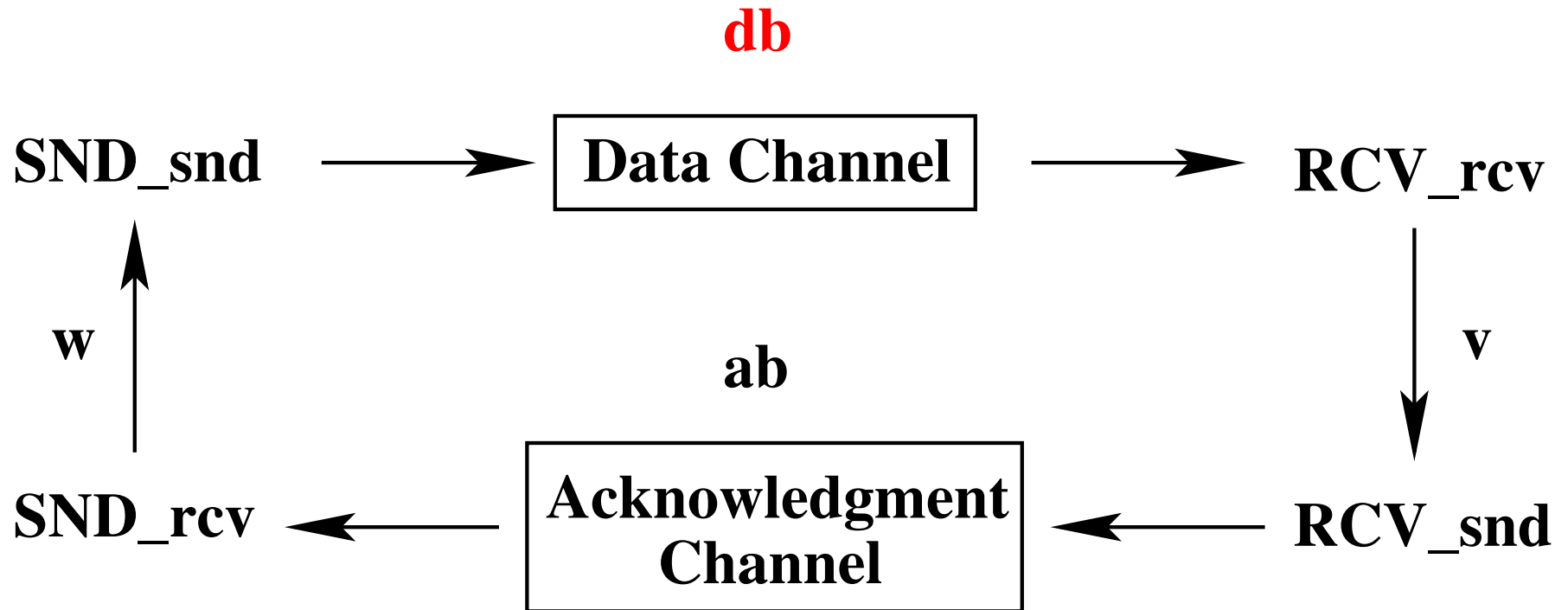
inv5_3: $w = \text{TRUE} \Rightarrow v = \text{FALSE}$

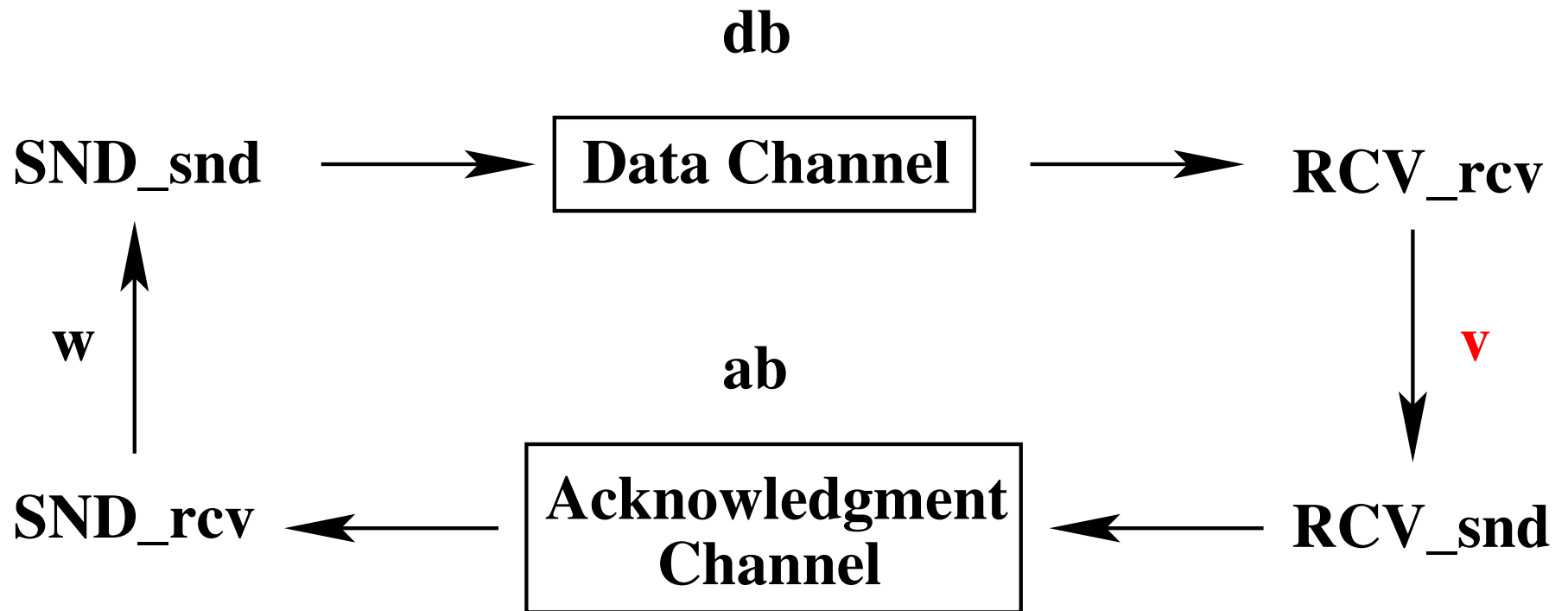
inv5_4: $db = \text{TRUE} \Rightarrow ab = \text{FALSE}$

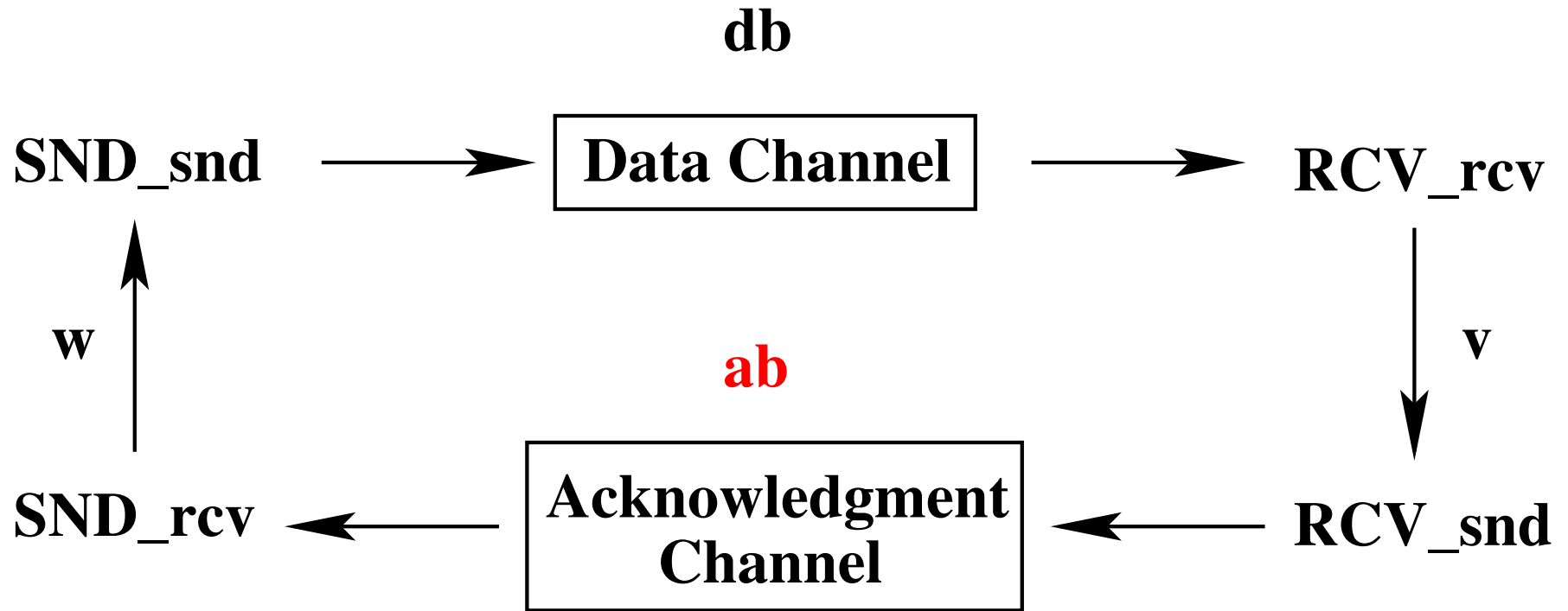
inv5_5: $db = \text{TRUE} \Rightarrow v = \text{FALSE}$

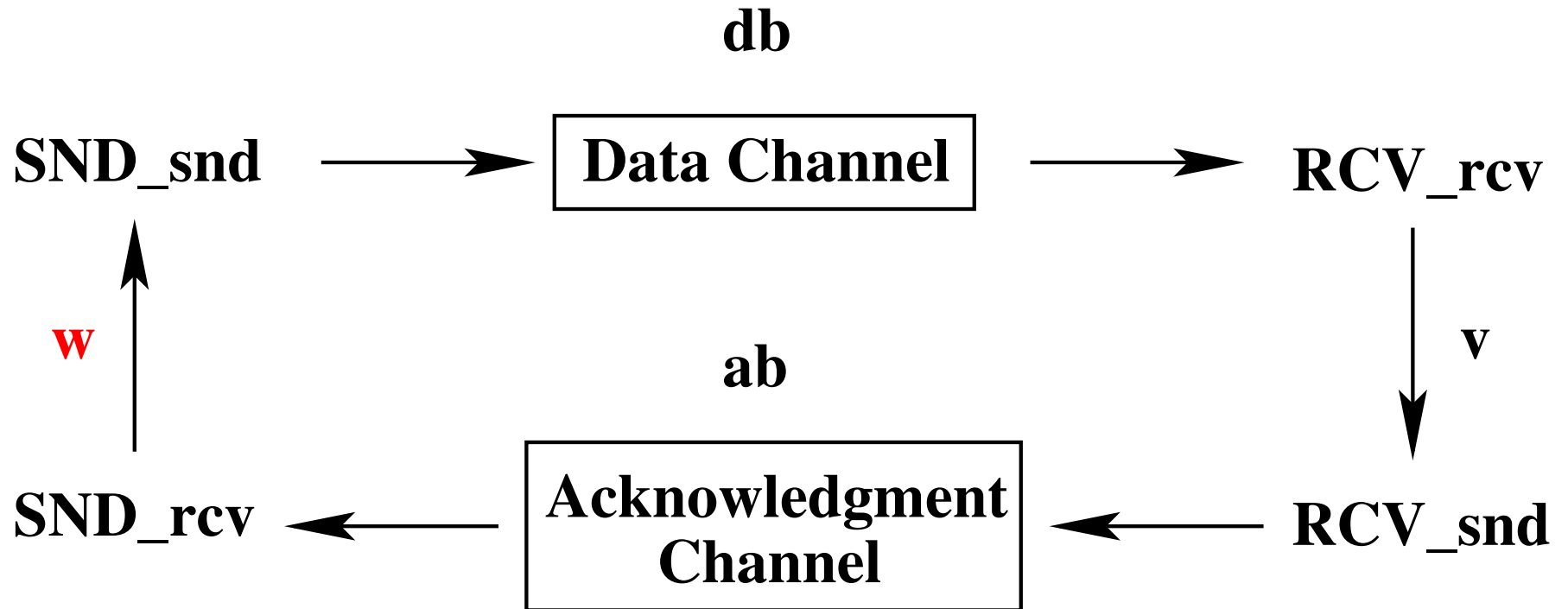
inv5_6: $ab = \text{TRUE} \Rightarrow v = \text{FALSE}$

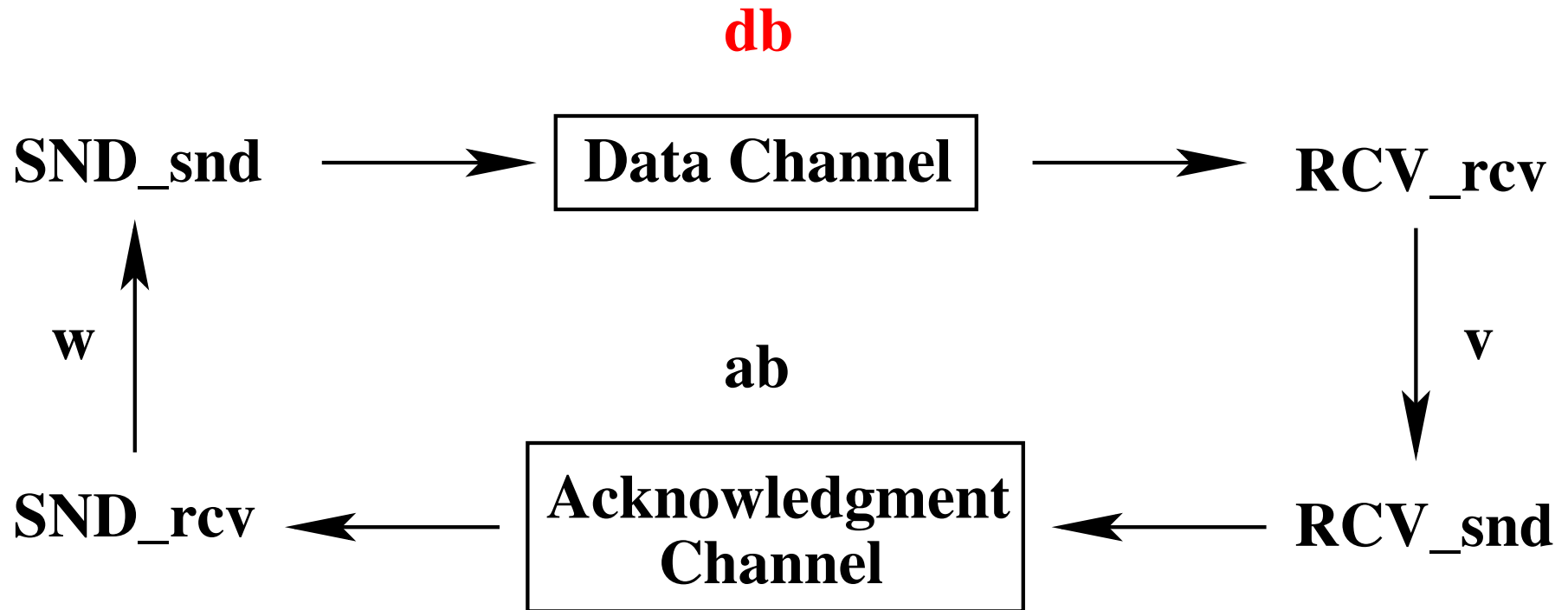


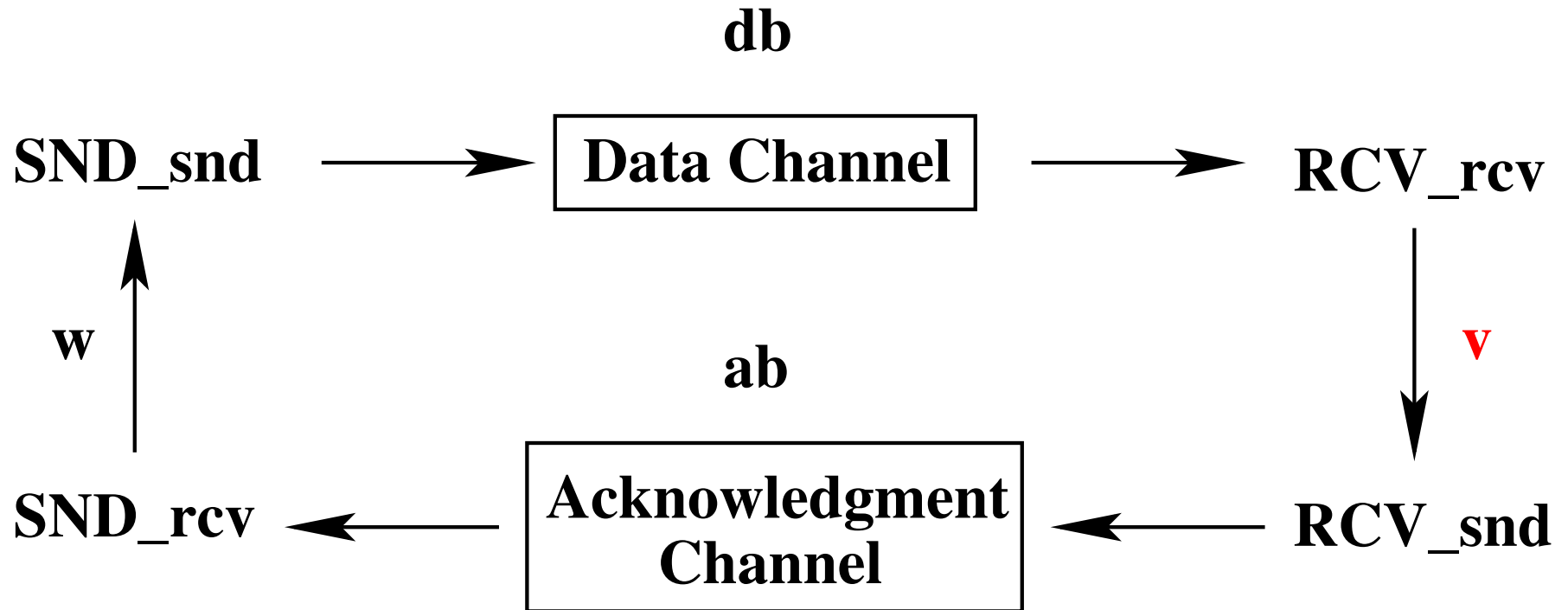


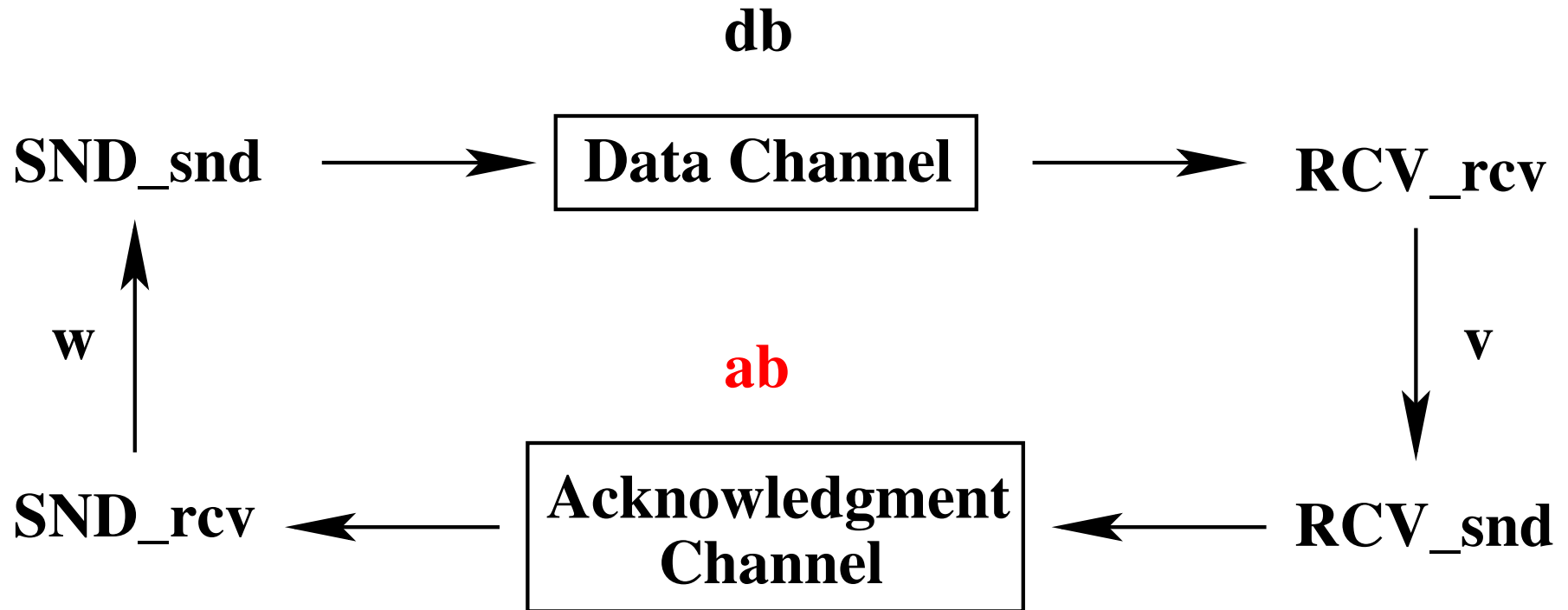












- These invariants define the **last data indicator**

variables: \dots
 l

inv5_7: $db = \text{TRUE} \wedge r = s \wedge l = \text{FALSE} \Rightarrow r + 1 < n$

inv5_8: $db = \text{TRUE} \wedge r = s \wedge l = \text{TRUE} \Rightarrow r + 1 = n$

- This bit is sent by the Sender to the Receiver
- When equal to **TRUE**, this bit indicates that the sent item is the last one

- Constant MAX denotes the maximum number of retries
- The sender fails iff the retry counter c exceeds MAX (**inv5_10**)

constants: ...
 MAX

axm5_1: $MAX \in \mathbb{N}$

variables: ...
 c

inv5_9: $c \in 0 .. MAX + 1$

inv5_10: $c = MAX + 1 \Leftrightarrow s_st = failure$

init

$r := 0$

$g := \emptyset$

$r_st := working$

$s_st := working$

$s := 0$

$d \in D$

$w := \text{TRUE}$

$db := \text{FALSE}$

$ab := \text{FALSE}$

$v := \text{FALSE}$

$l := \text{FALSE}$

$c := 0$

brp

when

$r \neq working$

$s \neq working$

then

skip

end

SND_snd_current_data

refines

SND_snd_data

when

$s_st = working$

$w = \text{TRUE}$

$s + 1 < n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

$db := \text{TRUE}$

$l := \text{FALSE}$

end

SND_snd_last_data

refines

SND_snd_data

when

$s_st = working$

$w = \text{TRUE}$

$s + 1 = n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

$db := \text{TRUE}$

$l := \text{TRUE}$

end

- Daemons are **breaking the channels**

```
DMN_data_channel
  when
    db = TRUE
  then
    db = FALSE
  end
```

```
DMN_ack_channel
  when
    ab = TRUE
  then
    ab = FALSE
  end
```

- A **failure** is characterized by **all activation bits being FALSE**

SND_time_out_current

when

s_st = working

w = FALSE

ab = FALSE

db = FALSE

v = FALSE

c < MAX

then

w := TRUE

c := c + 1

end

SND_failure

when

s_st = working

w = FALSE

ab = FALSE

db = FALSE

v = FALSE

c = MAX

then

s_st := failure

c := c + 1

end

- Sender aborts after $MAX + 1$ tries

Rcv_rcv_current_data

when

r_st = working

db = TRUE

r = s

l = FALSE

then

r := r + 1

h := h ∪ {r + 1 ↦ d}

db := FALSE

v := TRUE

end

Rcv_success

when

r_st = working

db = TRUE

r = s

l = TRUE

then

r_st := success

r := r + 1

h := h ∪ {r + 1 ↦ d}

db := FALSE

v := TRUE

end

Reminder: *l* is the last data indicator

(abstract-)RCV_rcv_current_data

when

$r_st = working$

$w = FALSE$

$r = s$

$r + 1 < n$

then

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

end

(concrete-)RCV_rcv_current_data

when

$r_st = working$

$db = TRUE$

$r = s$

$l = FALSE$

then

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

$db := FALSE$

$v := TRUE$

end

inv5_1': $db = TRUE \Rightarrow w = FALSE$

inv5_7: $db = TRUE \wedge r = s \wedge l = FALSE \Rightarrow r + 1 < n$

```

(abstract-)RCV_success
when
   $r\_st = working$ 
   $w = \text{FALSE}$ 
   $r = s$ 
   $r + 1 = n$ 
then
   $r := r + 1$ 
   $h := h \cup \{r + 1 \mapsto d\}$ 
end

```

```

(concrete-)RCV_success
when
   $r\_st = working$ 
   $db = \text{TRUE}$ 
   $r = s$ 
   $l = \text{TRUE}$ 
then
   $r\_st := success$ 
   $r := r + 1$ 
   $h := h \cup \{r + 1 \mapsto d\}$ 
   $db := \text{FALSE}$ 
   $v := \text{TRUE}$ 
end

```

inv5_1': $db = \text{TRUE} \Rightarrow w = \text{FALSE}$

inv5_8: $db = \text{TRUE} \wedge r = s \wedge l = \text{TRUE} \Rightarrow r + 1 = n$


```
RCV_rcv_retry
when
   $db = \text{TRUE}$ 
   $r \neq s$ 
then
   $db := \text{FALSE}$ 
   $v := \text{TRUE}$ 
end
```

```
RCV_snd_ack
when
   $v = \text{TRUE}$ 
then
   $v := \text{FALSE}$ 
   $ab := \text{TRUE}$ 
end
```

```
RCV_failure
when
   $r\_st = \textit{working}$ 
   $c = \textit{MAX} + 1$ 
then
   $r\_st := \textit{failure}$ 
end
```

```
SND_rcv_current_ack
when
  s_st = working
  ab = TRUE
   $s + 1 < n$ 
then
  w := TRUE
  s := s + 1
  c := 0
  ab := FALSE
end
```

```
SND_success
when
  s_st = working
  ab = TRUE
   $s + 1 = n$ 
then
  s_st := success
  c := 0
  ab := FALSE
end
```

```
(abstract-)SND_rcv_current_ack
when
  s_st = working
  w = FALSE
  s + 1 < n
  r = s + 1
then
  w := TRUE
  s := s + 1
end
```

```
(concrete-)SND_rcv_current_ack
when
  s_st = working
  ab = TRUE
  s + 1 < n
then
  w := TRUE
  s := s + 1
  c := 0
  ab := FALSE
end
```

inv5_2': $ab = \text{TRUE} \Rightarrow w = \text{FALSE}$

- In order to prove guard strengthening we need invariant **inv5_11**

inv5_11: $ab = \text{TRUE} \Rightarrow r = s + 1$

inv5_12: $v = \text{TRUE} \Rightarrow r = s + 1$

- Invariant **inv5_12** is needed to prove **inv5_11**

```
(abstract-)SND_success
when
  s_st = working
  w = FALSE
  s + 1 = n
  r = s + 1
then
  s_st := success
end
```

```
(concrete-)SND_success
when
  s_st = working
  ab = TRUE
  s + 1 = n
then
  s_st := success
  c := 0
  ab := FALSE
end
```

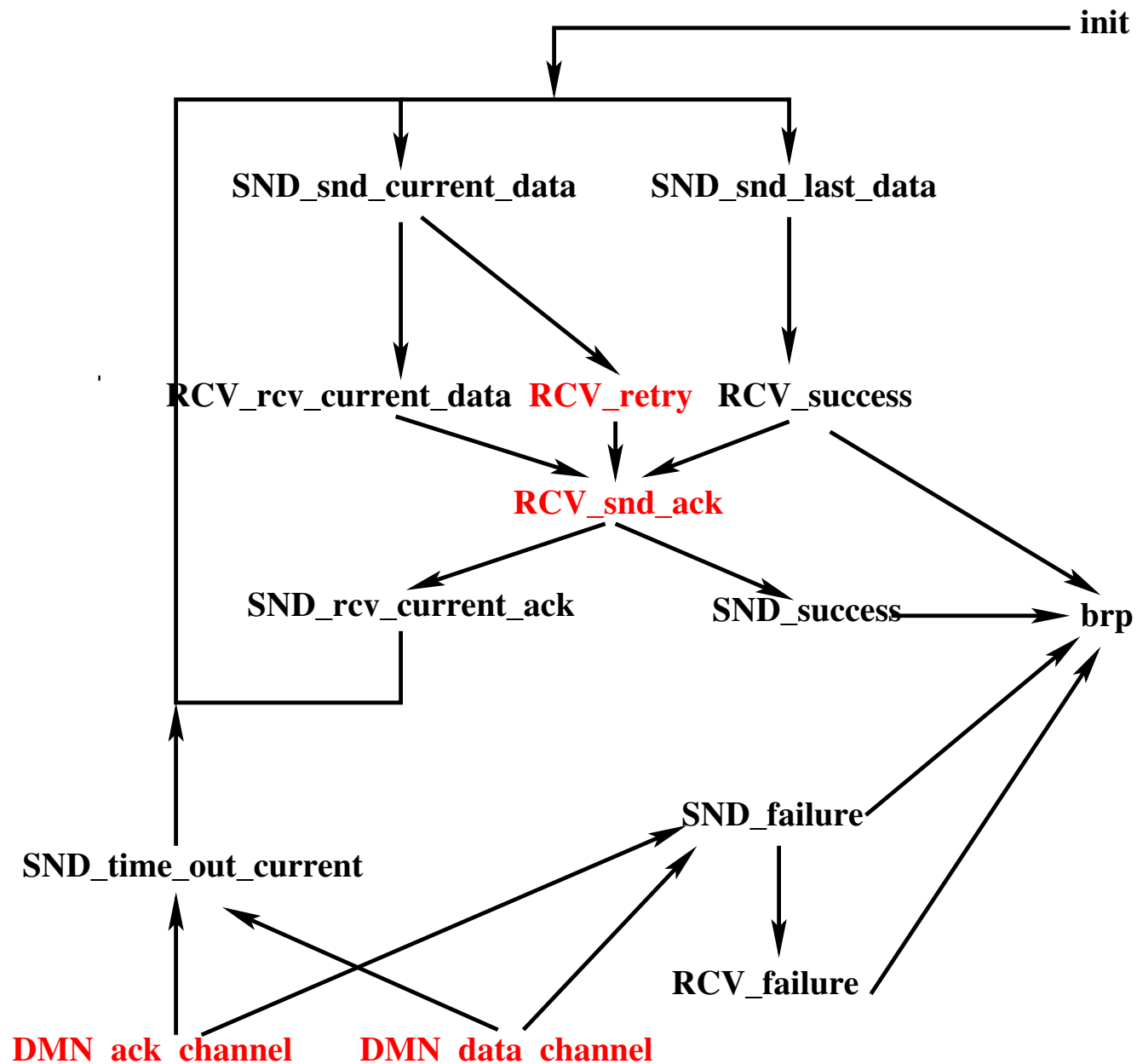
inv5_2': $ab = \text{TRUE} \Rightarrow w = \text{FALSE}$

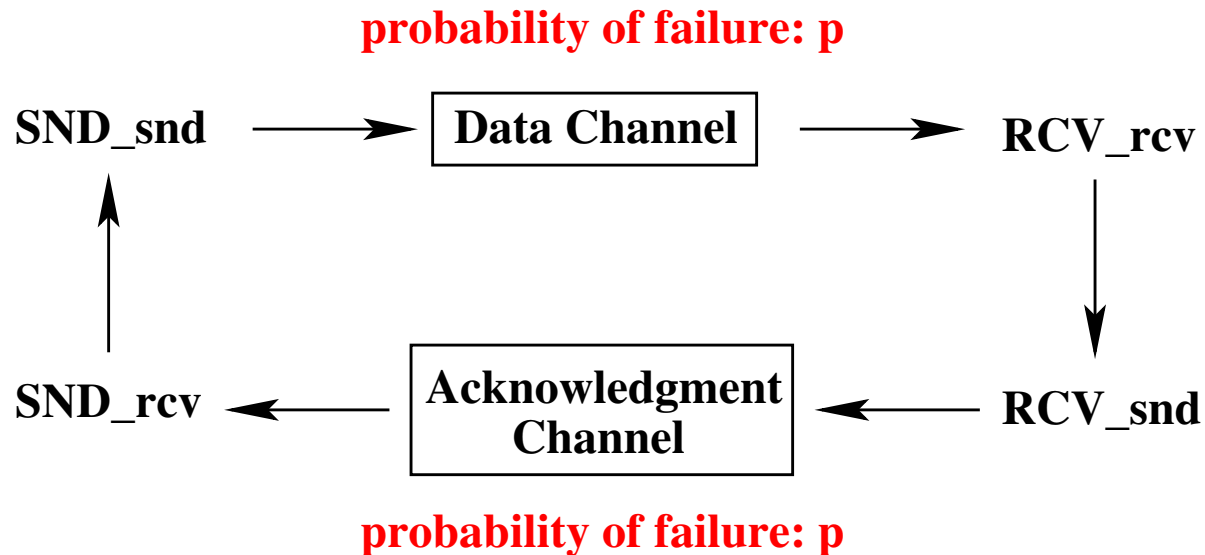
- In order to prove guard strengthening we need invariant **inv5_11**

inv5_11: $ab = \text{TRUE} \Rightarrow r = s + 1$

inv5_12: $v = \text{TRUE} \Rightarrow r = s + 1$

- Invariant **inv5_12** is needed to prove **inv5_11**





- We would like to compute the probability of success
- It is a function of:
 - p : probability of failure for one channel
 - n : size of the file
 - $MAX + 1$: number of re-tries

Failure on one channel

p

Failure on one channel p

Success on one channel $1 - p$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$
Fails on $MAX + 1$ tries	$(1 - (1 - p)^2)^{MAX+1}$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$
Fails on $MAX + 1$ tries	$(1 - (1 - p)^2)^{MAX+1}$
Succ. on $MAX + 1$ tries	$1 - (1 - (1 - p)^2)^{MAX+1}$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$
Fails on $MAX + 1$ tries	$(1 - (1 - p)^2)^{MAX+1}$
Succ. on $MAX + 1$ tries	$1 - (1 - (1 - p)^2)^{MAX+1}$
Success for n data	$(1 - (1 - (1 - p)^2)^{MAX+1})^n$

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$
Fails on $MAX + 1$ tries	$(1 - (1 - p)^2)^{MAX+1}$
Succ. on $MAX + 1$ tries	$1 - (1 - (1 - p)^2)^{MAX+1}$
Success for n data	$(1 - (1 - (1 - p)^2)^{MAX+1})^n$

$$p = .1$$

$$MAX = 5$$

$$n = 100$$

$$.995$$