Summary of Event-B Modeling Notation

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Purpose of this Presentation

- Showing the structure of the Event-B modeling notation
- Machines, contexts, and events
- Presenting a small example

Model Developments with Event-B

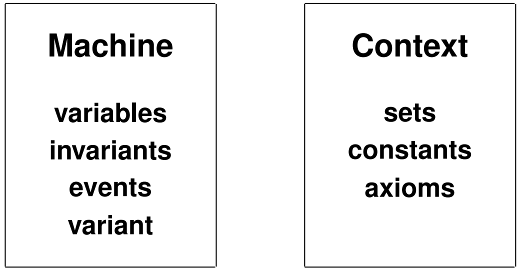
- Event-B is not a programming language (even very abstract)
- Event-B is a notation used for developing mathematical models of discrete transition systems
- Event-B is to be used together with the Rodin Platform

Model Developments with Event-B (cont'd)

- Such models, once finished, can be used to eventually construct:
  - sequential programs,
  - distributed programs,
  - concurrent programs,
  - electronic circuits,
  - <u>large systems involving a possibly fragile environment</u>,
  - etc.
- The <u>underlined statement</u> is an important case.
- In this presentation, we shall construct a small sequential program.

Machines and Contexts

- A model is made of several components
- A component is either a machine or a context:

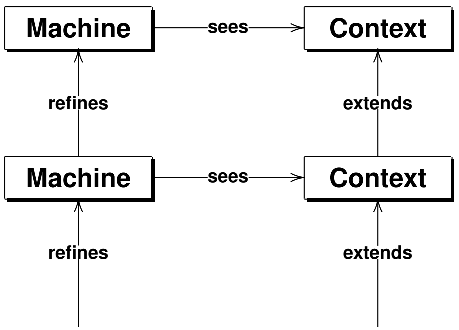


- Machines and contexts have names
- Such names must be distinct in a given model

Machines and Contexts (cont'd)

- Contexts contain the static structure of a discrete system (constants and axioms)
- Machines contain the dynamic structure of a discrete system (variables, invariants, and events)
- Machines see contexts
- Contexts can be extended
- Machines can be refined

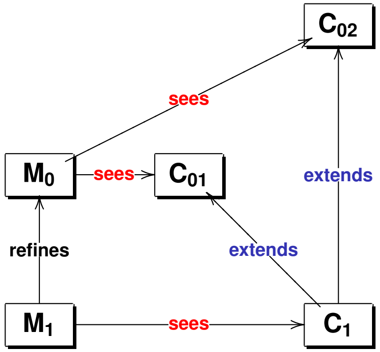
Relationship Between Machines and Contexts



Visibility Rules (can be Skipped at First Reading)

- A machine can see several contexts (or no context at all).
- A context may extend several contexts (or no context at all).
- A machine implicitly sees all contexts extended by a seen context.
- A machine only sees a context either explicitly or implicitly.
- A machine only refines at most one other machine.
- No cycle in the "refines" or "extends" relationships.

Example (can be Skipped at First Reading)



- $\mathbf{M_0}$ sees $\mathbf{C_{01}}$ and $\mathbf{C_{02}}$ explicitly.
- $\mathbf{M_1}$ sees $\mathbf{C_1}$ explicitly.
- $\mathbf{M_1}$ sees $\mathbf{C_{01}}$ and $\mathbf{C_{02}}$ implicitly.

Context Structure

```
context
  < context_identifier >
extends ⋆
  < context_identifier >
  ...
sets ⋆
  < set_identifier >
  ...
constants ⋆
  < constant_identifier >
  ...
axioms ⋆
  < label >: < predicate >
  ...
end
```

- Sections with "⋆" might be empty
- All keyword sections are predefined in the Rodin Platform
- All labels are generated automatically by the Rodin Platform (but can be modified)

Explaining Context Sections

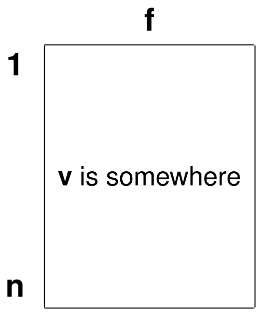
- "sets" lists various carrier sets, which define pairwise disjoint types
- The only property we can assume about a set is that it is not empty
- "constants" lists the different constants introduced in the context
- "axioms" defines the main properties of the constants
- axioms can be marked as "theorems" denotes derived properties (to be proved) from previously declared the axioms.

Context Example

context
  ctx_0
sets
  D
constants
  n
  f
  v
axioms
  axm1 : $n \in \mathbb{N}$
  axm2 : $f \in 1..n \rightarrow D$
  axm3 : $v \in \mathrm{ran}(f)$
  thm1 : $n \in \mathbb{N}_1$
end

- A set D is defined in context **ctx_0**
- Moreover, three constants, n, f, and v, are defined in this context:
  - n is a natural number (**axm1**)
  - f is a total function from the interval $1 \,..\, n$ to the set D (**axm2**)
  - v is supposed to belong to the range of f (**axm3**)
- A theorem is proposed: n is a positive number (**thm1**)

Pictorial Representation of the Context



Machine Structure

```
machine
  < machine_identifier >
refines  *
  < machine_identifier >
sees  *
  < context_identifier >
  ...
variables
  < variable_identifier >
  ...
invariants
  < label >: < predicate >
  ...
events
  initialisation ...
  ...
variant  *
  < variant >
end
```

- Each machine has exactly one **initialisation** event
- All keyword sections are predefined in the Rodin Platform
- All labels are generated automatically by the Rodin Platform (but can be modified)

Explaining Machine Sections

- "**variables**" lists the state variables of the machine
- "**invariants**" states the properties of the variables
- Invariants are defined in terms the seen sets and constants
- invariants can be marked as "**theorems**" which are derivable from previously declared invariants and seen axioms
- "**events**" defines the dynamics of the transition system (slide 17)
- "**variant**" is explained later (slide 29)

Machine (and Context) Example

machine
 m_0a
sees
 ctx_0
variables
 i
invariants
 inv1 : $i \in 1\,..\,n$
events
 ...
end

context
 ctx_0
sets
 D
constants
 n
 f
 v
axioms
 axm1 : $n \in \mathbb{N}$
 axm2 : $f \in 1..n \rightarrow D$
 axm3 : $v \in \mathrm{ran}(f)$
 thm1 : $n \in \mathbb{N}_1$
end

- Machine **m_0a** sees the previously defined context **ctx_0**
- A variable i is defined
- i is a member of the interval $1\,..\,n$ (**inv1**)
- **events**: next slide

Event Structure

```
< event_identifier > ≙
  status
    {ordinary, convergent, anticipated}
  refines  *
    < event_identifier >
    ...
  any  *
    < parameter_identifier >
    ...
  where  *
    < label >: < predicate >
    ...
  with  *
    < label >: < witness >
    ...
  then  *
    < label >: < action >
    ...
  end
```

- Notice that keyword "**where**" becomes "**when**" in the Rodin Platform Pretty Print when there is no "**any**".
- Notice that keyword "**then**" becomes "**begin**" in the Rodin Platoform Pretty Print when there are no "**any**" and no "**where/when**".
- Again, all keyword sections are predefined in the Rodin Platform.
- All labels are generated automatically by the Rodin Platform (but can be modified)

Explaining Event Sections

- An event is a state transition in a discrete dynamic system.
- "**refines**" contains the name(s) of the refined event(s) (if any)
- Can be skipped at first reading:
  - Several refined events are possible in case of a merging refining event concentrating more than one refined event
  - Merged events must have the same actions

Explaining Event Sections (cont'd)

- "**status**" is either:
  - ordinary,
  - convergent: it has to decrease the variant (slide 29),
  - anticipated: to be convergent later in a refinement.
- "**any**" contains the parameters of the event (might be empty)
- "**where**" (or "**when**") contains the various guards of the event
- A guard is a necessary condition for an event to be enabled
- Guards can be marked as "theorems" which are derivable from invariants, seen axioms and previously declared guards.
- "**actions**" see next slide

Explaining Action Section

- An action describes the ways one or several state variables are modified by the occurrence of an event
- An action might be either deterministic or non-deterministic

Deterministic Action (Example)

- Here is the form of some deterministic actions on variables x, y and z:

$$\begin{aligned} x &:= x + y \\ y &:= y - x - z \end{aligned}$$

- Notice that x and y should be distinct.
- Actions are supposed to be "performed" in parallel
- Variables x and y are assigned to $x + y$ and $y - x - z$ respectively
- Variable z is used but not modified by these actions

First Form of Non-deterministic Action (Example)

$$x, y :\mid \; x' > x \;\; \wedge \;\; y' < x'$$

- On the LHS of operator $:\mid$, we have two distinct variables
- On the RHS, we have a, so-called, before-after predicate
- The RHS contains occurrences of x and y (before values) and primed occurrences x' and y' (after values)
- As a result (in this example):
  - x is assigned a value greater than its previous value
  - y is assigned a value smaller than that, x', assigned to x

Second Form of Non-deterministic Action (Example)

$$x :\in \; \{x + 1, y - 2, z + 3\}$$

- Here x is assigned any value from the set $\{x + 1, y - 2, z + 3\}$

The Most General Form of an Action

- The second form of non-deterministic action is equivalent to the following first form:

$$x :\mid \; x' \in \{x + 1, y - 2, z + 3\}$$

- Likewise, a deterministic action has an equivalent non-deterministic form:

$$x, y :\mid \; x' = x + y \;\wedge\; y' = y - x - z$$

- The non-det. first form can thus always be assumed (by the tools)

Event Examples of Machine **m_0a**

- This machine is the model specification of a searching program

```
machine
   m_0a
sees
   ctx_0
variables
   i
invariants
   inv1 :  i ∈ 1 .. n
events
   . . .
end
```

```
initialisation  ≙
   status
      ordinary
   begin
      act1 :   i := 1
   end
```

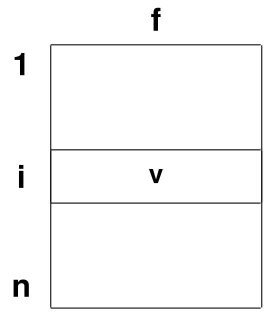
```
search  ≙
   status
      ordinary
   any
      k
   where
      grd1 : k ∈ 1 .. n
      grd2 : f(k) = v
   then
      act1 :   i := k
   end
```

- Event **search** assigns to i

- any value k such that $f(k) = v$,

- provided k is in interval $1 \,..\, n$

Pictorial Representation of the State after "search"



Another Machine m_0b

```
machine
   m_0b
sees
   ctx_0
variables
   i
invariants
   inv1 :  i ∈ 1 .. n
events
   . . .
end
```

```
initialisation  ≙
   status
      ordinary
   begin
      act1 :   i := 1
   end
```

```
search  ≙
   status
      ordinary
   begin
      act1 :   i :| i' ∈ 1 .. n ∧ f(i') = v
   end
```

- The only difference between **m_0a** and **m_0b** is in event **search**
- i is assigned non-deterministically a values i' such that $i' \in 1 \,..\, n$ and $f(i') = v$
- Notice that event **search** has no guard

Explaining Event Sections (cont'd)

- "**with**" contains the witnesses of a refining event.
- A witness has to be provided in a refining event
  - for each disappearing parameter of the refined event (see m_1a)
  - after value of each disappearing variable.
- The witness for parameter a is defined as follows $a : P(a)$ where $P(a)$ is a predicate involving a
- The witness for after value of variable b is defined as follows $b' : P(b')$ where $P(b')$ is a predicate involving b'
- For a deterministic witness $P(x)$ is $x = E$ (with E free of x)

Variant

- The variant of a machine is either a natural number expression or a finite set expression
- It has to be present in any machine with convergent events
- A numeric variant must be decreased by all convergent events
- A set variant must be made strictly included in its previous value by all convergent events

Refinement Machine m_1a Refining Machine m_0a

```
machine
   m_1a
refines
   m_0a
sees
   ctx_0
variables
   i
   j
invariants
   inv1 :  j ∈ 0 .. n − 1
   inv2 :  v ∉ f[1 .. j]
   thm1 :  v ∈ f[j + 1 .. n]
variant
   n − j
events
   . . .
end
```

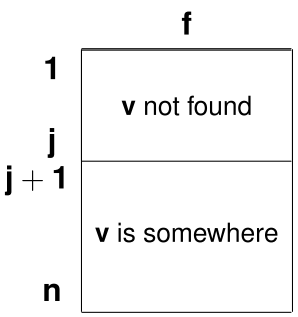
```
initialisation ≙
  status  ordinary
  begin
    act1 :  i := 1
    act2 :  j := 0
  end
```

```
search ≙
  status  ordinary
  refines
    search
  when
    grd1 : f(j + 1) = v
  with
    k :  j + 1 = k
  then
    act1 :  i := j + 1
  end
```

```
progress ≙
  status  convergent
  when
    grd1 : f(j + 1) ≠ v
  then
    act1 :  j := j + 1
  end
```

- A new variable j is introduced
- Notice invariant **inv2** and theorem **thm1**
- Notice the **with** section in event **search**
- A new **convergent** event **progress** is introduced
- Notice the numeric **variant** $n - j$

Pictorial Representation of the State



Refinement Machine m_1b Refining Machine m_0b

```
machine
   m_1b
refines
   m_0b
sees
   ctx_0
variables
   i
   j
invariants
   inv1 :  j ∈ 0 .. n − 1
   inv2 :  v ∉ f[i .. j]
   thm1 :  v ∈ f[j + 1 .. n]
variant
   j .. n
events
   . . .
end
```

```
initialisation ≙
  status  ordinary
  begin
    act1 :  i := 1
    act2 :  j := 0
  end
```

```
search ≙
  status  ordinary
  refines
    search
  when
    grd1 : f(j + 1) = v
  then
    act1 :  i := j + 1
  end
```

```
progress ≙
  status  convergent
  when
    grd1 : f(j + 1) ≠ v
  then
    act1 :  j := j + 1
  end
```

- The **with** section in event **search** is not needed
- Notice the finite set **variant** $j .. n$
- These are the only differences with refining machine **m_1a**

Constructing the Final Program

- A sequential program can be constructed from **m_1a** (or **m_1b**)
- This is done by applying a number of event merging rules (NOT DEFINED HERE)
- The application of these rules yields the following program:

$i, j := 1, 0$;	**initialisation**
while $f(j+1) \neq v$ **do**	
$\quad j := j + 1$	**progress**
end ;	
$i := j + 1$	**search**

Exercise

- Modify refinement **m_1a** (or **m_1b**) in order to obtain the following final program from the same specification **m_0a** (or **m_0b**):

$i, j := 1, n + 1$;	**initialisation**
while $f(j-1) \neq v$ **do**	
$\quad j := j - 1$	**progress**
end ;	
$i := j - 1$	**search**