

Bounded Re-transmission Protocol

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Purpose of this Lecture

- The Bounded Re-transmission Protocol is a **file transfer protocol**
- This is a problem dealing with **fault tolerance**
- We suppose that the transfer channels are **unreliable**
- We present classical solutions to handle that problem: **timers**.
- We would like to see how we can **formalize such timers**



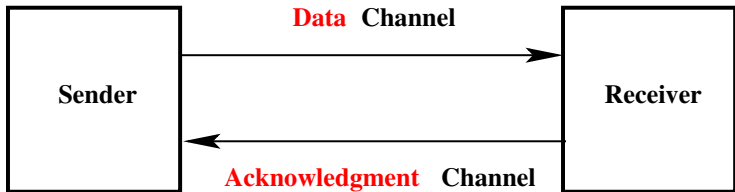
Outline

- 1 Requirements Document
- 2 Formal Development
 - Initial Model
 - First Refinement
 - Second Refinement
 - Third Refinement
- 3 What about Probability



The Bounded Retransmission Protocol

- A **sequential file** is transmitted from a **Sender** to a **Receiver**
- The file is transmitted **piece by piece** through a **Data Channel**
- After receiving some data, the Receiver sends an **acknowledgment**
- After receiving it, the Sender sends the **next piece of data**, etc.



- **Messages can be lost** in the Data or Acknowledgment channels



Requirements (1)

The goal of the BRP is to **totally** or **partially** transfer a certain non-empty original sequential file from one site to another.

FUN1

A **total transfer** means that the transmitted file is a **copy** of the original one.

FUN2

A **partial transfer** means that the transmitted file is a **genuine prefix** of the original one.

FUN3



Unreliability of the Communications (1)

- Messages can be lost in the Data or Acknowledgment channels
- The Sender starts a timer before sending a piece of data
- The timer wakes up the Sender after a delay d_l
- This occurs if the Sender has not received an acknowledgment in the meantime



Unreliability of the Communications (2)

- dI is guaranteed to be **greater than twice the transmission time**
- When waken up, the Sender is then **sure** that the **data** or the **acknowledgment** has been **lost**
- When waken up, the Sender **re-transmits the previous data**
- The Sender sends an **alternating bit** together with a **new data**
- This ensures that the Receiver **does not confuse** (?) a **new data** with a **retransmitted** one.



Abortion of Protocol at the Sender Site

- The Sender can re-transmit the same data **at most $MAX + 1$ times**
- After this, the Sender **decides to abort**
- **How does the Receiver know** that the Sender aborted?



Abortion of Protocol at the Receiver Site

- Each time the Receiver receives a **new** piece of data, it **starts a timer**
- The timer **wakes up** the Receiver after a delay $(MAX + 1) \times dl$
- This occurs if the Receiver **has not received a new data** in the meantime.
- After this delay, the Receiver is certain that **the Sender has aborted**
- Then the **Receiver aborts** too.



Final Situation of the Protocol

- At the end of the protocol, we might be in one of the **three situations**:

(1) The file **has been transmitted** entirely and the Sender **has received** the last acknowledgment

(2) The file **has been transmitted** entirely but the Sender **has not received** the last acknowledgment

(3) The file **has not been transmitted** entirely



Requirements (2)

Each site may end up in any of the **two situations**:

- either **it believes** that the protocol has **terminated successfully**,
- or **it believes** that the protocol has **aborted**

FUN4

When the **Sender** believes that the protocol has **terminated successfully** then the **Receiver** believes so too.

FUN5



Requirements (3)

However, it is possible for the **Sender** to believe that the protocol has **aborted** while the **Receiver** believes that it has **terminated successfully**.

FUN6

When the **Receiver** believes that the protocol has **terminated successfully**, this is because the original file has been **entirely copied** on the Receiver's site.

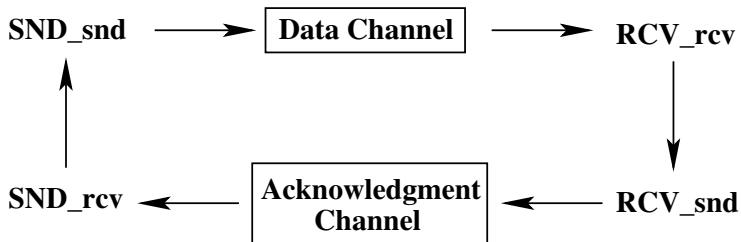
FUN7

When the **Receiver** believes that the protocol has **aborted**, this is because the original file has **not been copied entirely** on the Receiver's site.

FUN8



Pseudo-code for the Protocol



The Sender sends Data

SND_snd

when

SND_snd is waken up

then

Acquire data from Sender's file;
Store acquired data on Data Channel;
Store Sender's bit on Data Channel;
Start Sender's timer;
Activate Data Channel;

end



The Receiver Receives Data

```
RCV_rcv
  when
    Data Channel interrupt occurs
  then
    Acquire Sender's bit from Data Channel;
    if Sender's bit = Receiver's bit then
      Acquire Data from Data Channel;
      Store data on Receiver's file;
      Modify Receiver's bit;
      if data is not the last one then
        Start Receiver's timer;
      end
    end
  end
  Reset Data Channel Interrupt;
  Wake up RCV_snd;
end
```



The Receiver sends Acknowledgment

```
RCV_snd  
  when  
    RCV_snd is waken up  
  then  
    Activate Acknowledgment Channel;  
  end
```



The Sender Receives Acknowledgment

SND_rcv

when

Acknowledgment Channel interrupt occurs

then

Remove Data from Sender's file;

Reset retry counter;

Modify Sender's bit;

Wake up event SND_snd;

Reset Acknowledgment Channel interrupt;

if Sender's file is not empty **then**

Wake up event SND_snd

end

end



Timer Interrupt Occurs at Sender's Site

```
SND_timer
  when
    Sender's timer interrupt occurs
  then
    if retry counter is equal to MAX+1 then
      Abort protocol on Sender's site;
    else
      Increment retry counter;
      Wake up event SND_snd;
    end
  end
```



Timer Interrupt occurs at Receiver's Site

RCV_timer

when

Receiver's timer interrupt occurs

then

Abort protocol on Receiver's site

end



About the Pseudo-code

- Quite often, protocols are "specified" by such pseudo-codes
- In fact, such a pseudo-code raises a number of questions:
 - Are we sure that this description is correct?
 - Are we sure that this protocol terminates?
 - What kinds of properties should this protocol maintain?
- Hence the formal development which is presented now



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Refinement Strategy

- (1) FUN1, FUN2, FUN3: **partial transmission** of the file **in one shot**.
- (2) FUN4 to FUN8: each participant has **access to the other**
- (3) Introducing **unreliable channels** and **timers**.
- (4) **Optimize** protocol



Reminder (1)

The goal of the BRP is to **totally** or **partially** transfer a certain non-empty original sequential file from one site to another.

FUN1

A **total transfer** means that the transmitted file is a **copy** of the original one.

FUN2

A **partial transfer** means that the transmitted file is a **genuine prefix** of the original one.

FUN3



Reminder (2)

Each site may end up in any of the **two situations**:

- either **it believes** that the protocol has **terminated successfully**,
- or **it believes** that the protocol has **aborted**

FUN4

When the **Sender** believes that the protocol has **terminated successfully** then the **Receiver** believes so too.

FUN5



Reminder (3)

However, it is possible for the **Sender** to believe that the protocol has **aborted** while the **Receiver** believes that it has **terminated successfully**.

FUN6

When the **Receiver** believes that the protocol has **terminated successfully**, this is because the original file has been **entirely copied** on the Receiver's site.

FUN7

When the **Receiver** believes that the protocol has **aborted**, this is because the original file has **not been copied entirely** on the Receiver's site.

FUN8



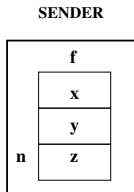
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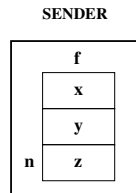


The Sender and the Receiver: a First View

INITIAL SITUATION



FINAL SITUATION



Initial Model: the Constants

- Set D denotes the objects in the files
- Constant n denotes the **size** of the **non-empty** file
- Constant f denotes the **original** file.

set: D

constants: n
 f

axm0_1: $0 < n$

axm0_2: $f \in 1 .. n \rightarrow D$



Initial Model: the Variables

- Variable i denotes the **size** of file g
- Variable g denotes the **transmitted file**.

variables: i
 g

inv0_1: $i \in 0..n$

inv0_2: $g \in 1..i \rightarrow D$



Reminder of Mathematical Conventions (1)

$x \in S$	set membership operator
\mathbb{N}	set of natural numbers: $\{0, 1, 2, 3, \dots\}$
$a .. b$	interval from a to b : $\{a, a + 1, \dots, b\}$ (empty when $b < a$)
$a \mapsto b$	pair constructing operator
$S \times T$	Cartesian product operator
$S \subseteq T$	set inclusion operator
$\mathbb{P}(S)$	power set operator

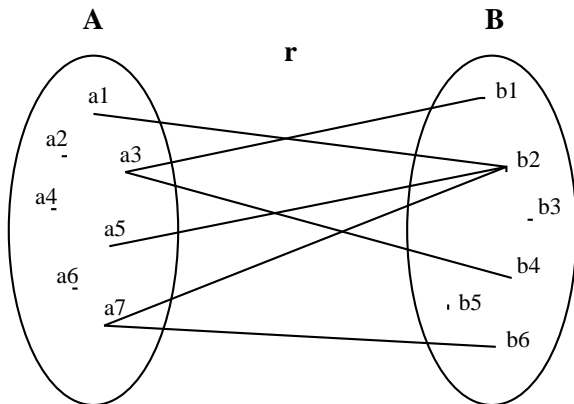


Reminder of Mathematical Conventions (2)

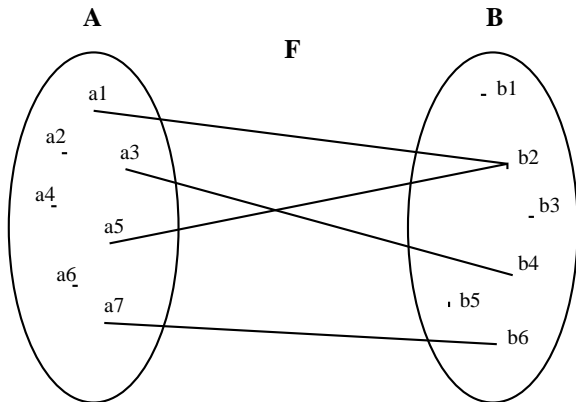
$S \leftrightarrow T$	set of binary relations from S to T
$S \rightarrow T$	set of total functions from S to T
$S \leftrightarrow\!\!\!\rightarrow T$	set of partial functions from S to T
$\text{dom}(r)$	domain of a relation r
$\text{ran}(r)$	range of a relation r



A Binary Relation r from a Set A to a Set B



A Partial Function F from a Set A to a Set B



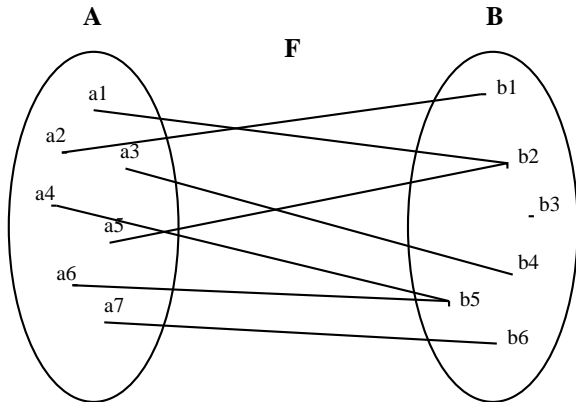
$$F = \{a1 \mapsto b2, a3 \mapsto b4, a5 \mapsto b2, a7 \mapsto b6\}$$

$$\text{dom}(F) = \{a1, a3, a5, a7\}$$

$$\text{ran}(F) = \{b2, b4, b6\}$$



A Total Function F from a Set A to a Set B



$$\text{dom}(F) = A$$



Initial model: a Single Event (no Protocol)

- Event **brp** describes the situation at the **end of the protocol**
- It only says that the file might be **partially transmitted**
- It is made of a **non-deterministic assignment**

```
init
  i := 0
  g := ∅
```

```
brp
  i, g :| i' ∈ 0 .. n ∧
         g' = (1 .. i') ◁ f
```

- Operator **:|** is to be read: "**become such that ...**"



Informal Meaning

brp

$$i, g :| \left(\begin{array}{l} i' \in 0..n \\ g' = (1..i') \triangleleft f \end{array} \right)$$

i and g are assigned **any values i' and g'** such that the following holds:

$$i' \in 0..n \wedge g' = (1..i') \triangleleft f$$

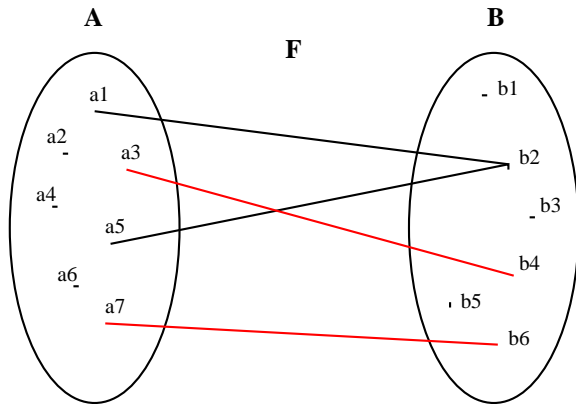


More Mathematical Conventions: Restrictions

$s \triangleleft r$	domain restriction operator
$s \triangleleft r$	domain subtraction operator
$r \triangleright t$	range restriction operator
$r \triangleright t$	range subtraction operator



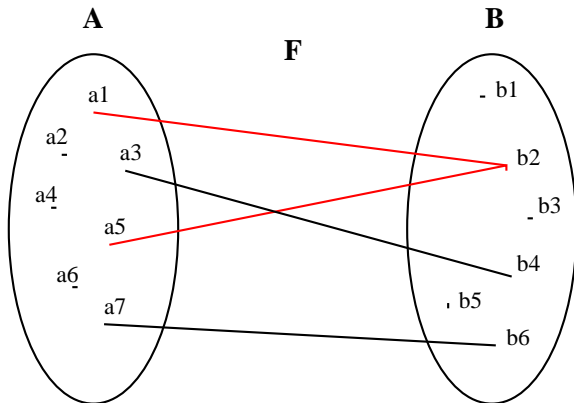
The Domain Restriction Operator



$$\{a_3, a_7\} \triangleleft F$$



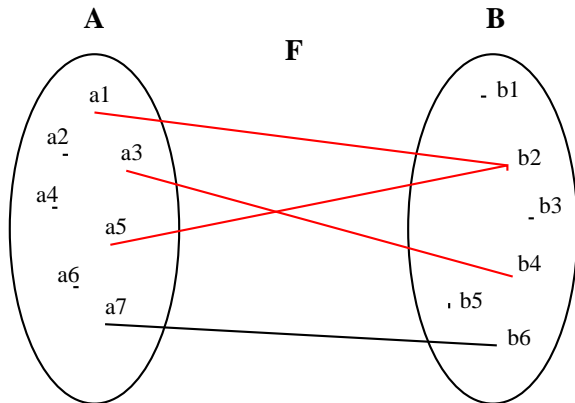
The Domain Subtraction Operator



$$\{a_3, a_7\} \triangleleft F$$



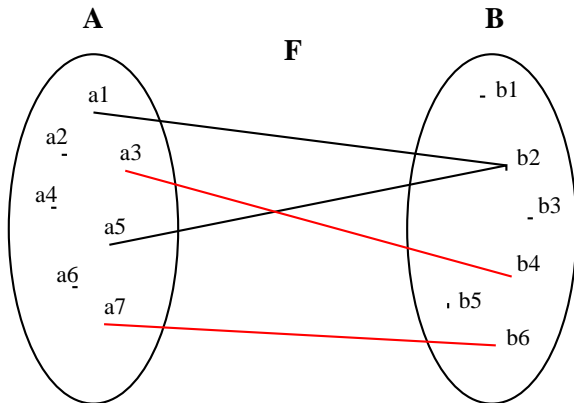
The Range Restriction Operator



$$F \triangleright \{b2, b4\}$$



The Range Subtraction Operator

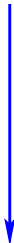


$$F \triangleright \{b2\}$$



The Abstract Situation

init



brp

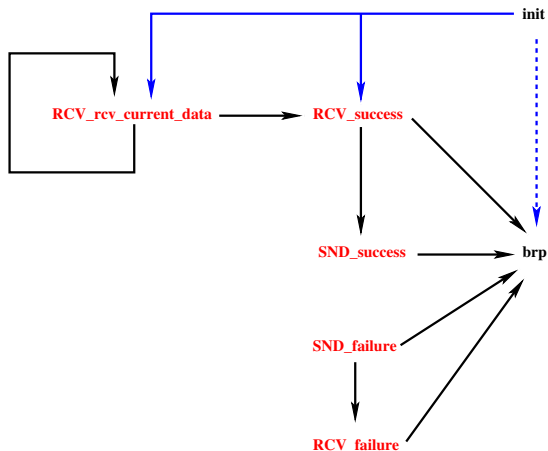


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First Refinement: Introducing New Events



About new Events in a Refinement

- They allow to **observe** the (future) system with a **finer time grain**
- Analogies with a **microscope** or a **parachute**
- They **refine** the (implicit) event **doing nothing** (skip)
- They must **not take control for ever** (exhibiting a variant)



First Refinement: Defining more Constants

set: *STATUS*

constants: ...
working
success
failure

axm1_1: *STATUS* = {*working*, *success*, *failure*}

axm1_2: *working* \neq *success*

axm1_3: *working* \neq *failure*

axm1_4: *success* \neq *failure*



First Refinement: Variables

- Variables i and g are replaced by **variables r and h**
- Variable r denotes the **size** of the transmitted file
- Variable h denotes the **transmitted file**
- Variables s_st and r_st denote the status of the participants (Sender and Receiver respectively).

variables: r
 h
 s_st
 r_st



First Refinement: Invariants (1)

- Variables h is a prefix of constant f (invariant **inv1_1** and **inv1_2**)

$$\mathbf{inv1_1: } r \in 0 .. n$$

$$\mathbf{inv1_2: } h = (1 .. r) \triangleleft f$$



First Refinement: Invariants (2)

- The **typing** of variables s_st and r_st is **implicit** (FUN4)
- Requirements FUN7 and FUN8 (Receiver's belief is true) is taken care invariant **inv1_3**
- Requirements FUN5 and FUN6 (Sender's status) are taken care by invariant **inv_4**

$$\mathbf{inv1_3:} \quad r_st = success \Leftrightarrow r = n$$

$$\mathbf{inv1_4:} \quad s_st = success \Rightarrow r_st = success$$



First Refinement: the Events (1)

- Initialisation

```
init
   $r := 0$ 
   $h := \emptyset$ 
   $r\_st := \textit{working}$ 
   $s\_st := \textit{working}$ 
```



First Refinement: the Events (2)

- Event **(concrete_)brp** now **does nothing**
- We give **witnesses** for the abstract after values i' and g'

$$\text{(abstract_)}\text{brp}$$
$$i, g : | \left(\begin{array}{l} i' \in 0 .. n \\ g' = (1 .. i') \triangleleft a \end{array} \right)$$

```
(concrete_)brp
  when
    r_st ≠ working
    s_st ≠ working
  with
    i' : i' = r
    g' : g' = h
  then
    skip
  end
```



First Refinement: the Events (3)

```
RCV_rcv_current_data
  when
    r_st = working
    r + 1 < n
  then
    r := r + 1
    h := h ∪ {r + 1 ↦ f(r + 1)}
  end
```

- This event is "cheating" (accessing constant f)
- This new event maintains invariant **inv1_3** and it refines **skip**

inv1_3: $r_st = success \Leftrightarrow r = n$



First Refinement: the Events (4)

```
RCV_success
  when
     $r\_st = working$ 
     $r + 1 = n$ 
  then
     $r\_st := success$ 
     $r := r + 1$ 
     $h := h \cup \{n \mapsto f(n)\}$ 
  end
```

```
RCV_failure
  when
     $r\_st = working$ 
     $s\_st = failure$ 
  then
     $r\_st := failure$ 
  end
```

- These new events are **cheating** (accessing f and s_st)
- These new events **maintain inv1_3 and inv1_4** and they refine **skip**

```
inv1_3:  $r\_st = success \Leftrightarrow r = n$   
inv1_4:  $s\_st = success \Rightarrow r\_st = success$ 
```



First Refinement: the Events (5)

```
SND_success
when
  s_st = working
  r_st = success
then
  s_st := success
end
```

```
SND_failure
when
  s_st = working
then
  s_st := failure
end
```

- Event SND_success is **cheating** (accessing *r_st*)
- Event SND_success **maintains invariant inv1_4**

```
inv1_4: s_st = success  $\Rightarrow$  r_st = success
```

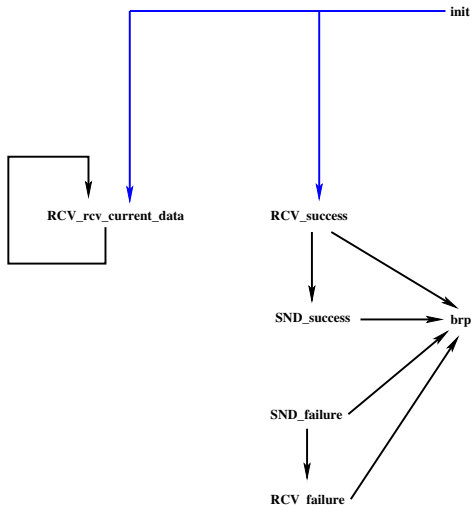


Outline

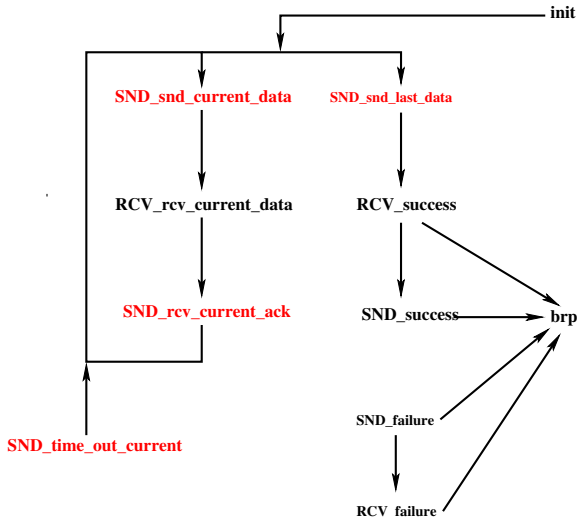
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Second Refinement: Introducing More Events



Second Refinement: Introducing More Events



Second Refinement: More Variables

- Variable s is the Sender **pointer** sent to the Receiver
- Variable d is the **data sent** to the Receiver
- Variable w is the Sender **activation bit**
- When w is **TRUE** it means the Sender has **just received the acknowledgement**
- When w is **FALSE** it means the Sender has **sent the information to the Receiver**

variables: ...
 w
 s
 d

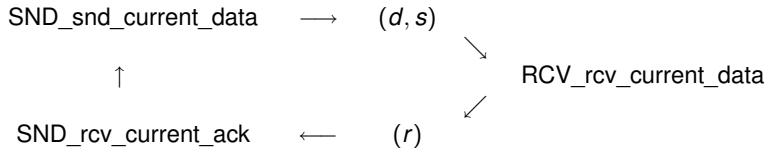
inv2_1: $s \in 0..n-1$

inv2_2: $r \in s..s+1$

inv2_3: $w = \text{FALSE} \Rightarrow d = f(s+1)$



The Main Communication



Second Refinement: the Events (1)

init

$r := 0$

$h := \emptyset$

$r_st := \text{working}$

$s_st := \text{working}$

$s := 0$

$d \in D$

$w := \text{TRUE}$

brp

when

$r_st \neq \text{working}$

$s_st \neq \text{working}$

then

skip

end



Second Refinement: the Events (2)

- **New Events**: the Sender prepares **data d** and **pointer s** to be sent

SND_snd_current_data

when

$s_st = \textit{working}$

$w = \text{TRUE}$

$s + 1 < n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

end

SND_snd_last_data

when

$s_st = \textit{working}$

$w = \text{TRUE}$

$s + 1 = n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

end

- These events clearly **refine skip** and maintain invariant **inv2_3**

inv2_3: $w = \text{FALSE} \Rightarrow d = f(s + 1)$



Second Refinement: the Events (3)

- The Receiver receives data d and pointer s . It sends pointer r .

```
RCV_rcv_current_data
  when
     $r\_st = working$ 
     $w = FALSE$ 
     $r = s$ 
     $r + 1 < n$ 
  then
     $r := r + 1$ 
     $h := h \cup \{r + 1 \mapsto d\}$ 
  end
```

```
RCV_success
  when
     $r\_st = working$ 
     $w = FALSE$ 
     $r = s$ 
     $r + 1 = n$ 
  then
     $r\_st := success$ 
     $r := r + 1$ 
     $h := h \cup \{r + 1 \mapsto d\}$ 
  end
```

- The Receiver **still cheats**: it accesses constant n



Refinement of RCV_rcv_current_data

```
(abstract-)RCV_rcv_current_data
when
  r_st = working
  r + 1 < n
then
  r := r + 1
  h := h ∪ {r + 1 ↦ f(r + 1)}
end
```

```
(concrete-)RCV_rcv_current_data
when
  r_st = working
  w = FALSE
  r = s
  r + 1 < n
then
  r := r + 1
  h := h ∪ {r + 1 ↦ d}
end
```

- Observe **guard strengthening**
- This invariant helps proving **event refinement**

inv2_3: $w = \text{FALSE} \Rightarrow d = f(s + 1)$



Refinement of RCV_success

(abstract-)RCV_success

when

$r_st = working$

$r + 1 = n$

then

$r_st := success$

$r := r + 1$

$h := h \cup \{n \mapsto f(n)\}$

end

(concrete-)RCV_success

when

$r_st = working$

$w = FALSE$

$r = s$

$r + 1 = n$

then

$r_st := success$

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

end

- Observe **guard strengthening**
- This invariant helps proving **event refinement**

inv2_3: $w = FALSE \Rightarrow d = f(s + 1)$



Second Refinement: the Events (4)

- The first event is **new**. It clearly **refines skip**
- The activation bit is set to TRUE (activating SND_snd_current_data)
- The Sender receives acknowledgment (pointer r)

```
SND_rcv_current_ack
  when
     $s\_st = working$ 
     $w = FALSE$ 
     $s + 1 < n$ 
     $r = s + 1$ 
  then
     $w := TRUE$ 
     $s := s + 1$ 
  end
```

```
SND_success
  when
     $s\_st = working$ 
     $w = FALSE$ 
     $s + 1 = n$ 
     $r = s + 1$ 
  then
     $s\_st := success$ 
  end
```



Refinement of **SND_success**

(abstract-)SND_success

when

$s_st = working$

$r_st = success$

then

$s_st := success$

end

(concrete-)SND_success

when

$s_st = working$

$w = FALSE$

$s + 1 = n$

$r = s + 1$

then

$s_st := success$

end

- The presence of **inv1_3** ensures that the **guard is strengthened**

inv1_3: $r_st = success \Leftrightarrow r = n$



Second Refinement: the Events (5)

- This new events will receive a full explanation in the next refinement

```
SND_time_out_current
  when
    s_st = working
    w = FALSE
  then
    w := TRUE
  end
```



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Third Refinement: Introducing more Activation Bits

- At most one activation bit is **TRUE** at a time

variables: ...
db
ab
v

inv3_1: $w = \text{TRUE} \Rightarrow db = \text{FALSE}$

inv3_2: $w = \text{TRUE} \Rightarrow ab = \text{FALSE}$

inv3_3: $w = \text{TRUE} \Rightarrow v = \text{FALSE}$

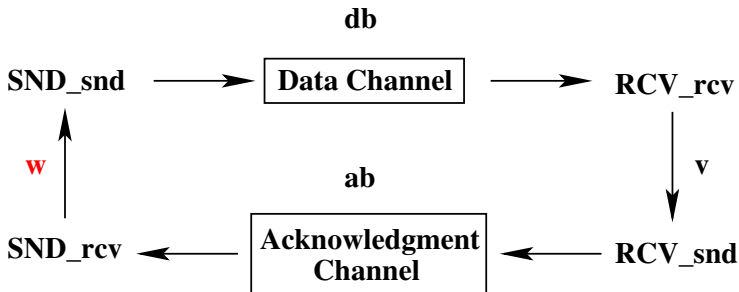
inv3_4: $db = \text{TRUE} \Rightarrow ab = \text{FALSE}$

inv3_5: $db = \text{TRUE} \Rightarrow v = \text{FALSE}$

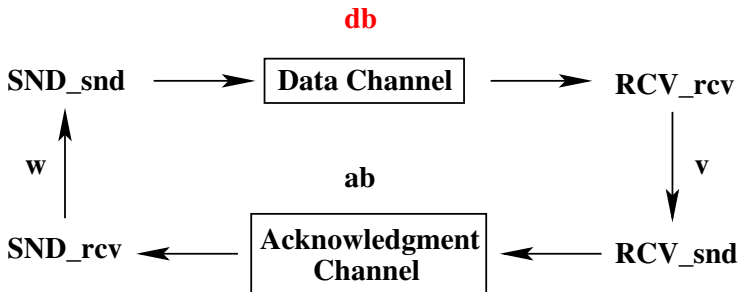
inv3_6: $ab = \text{TRUE} \Rightarrow v = \text{FALSE}$



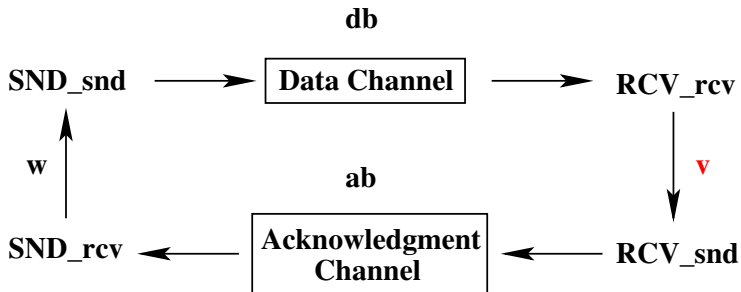
Activation bits at work



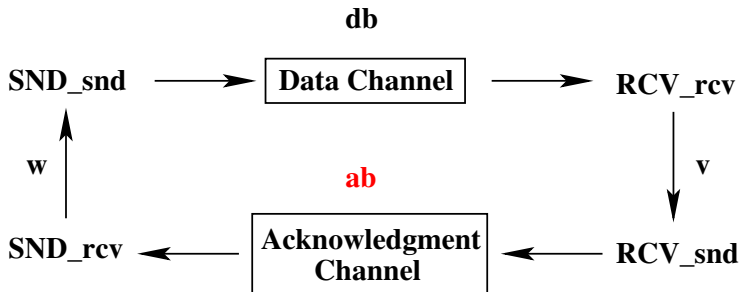
Activation bits at work



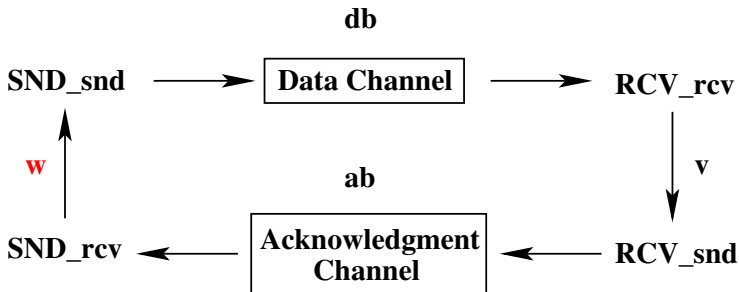
Activation bits at work



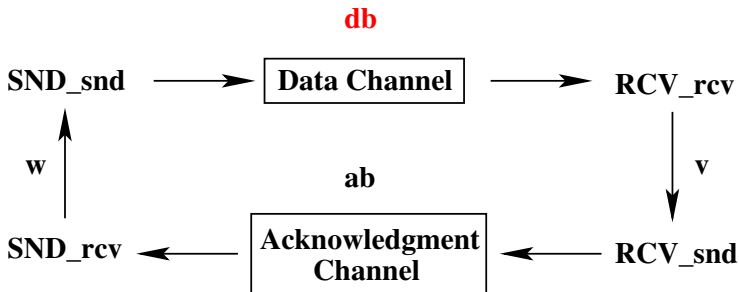
Activation bits at work



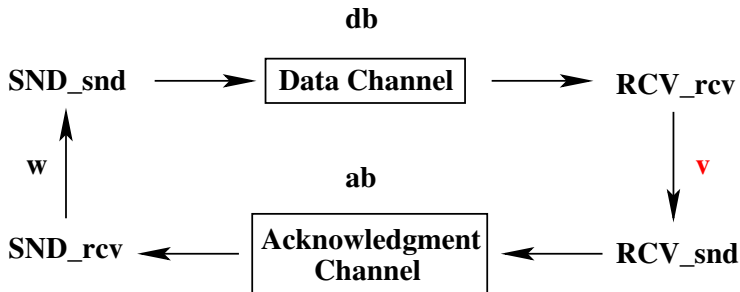
Activation bits at work



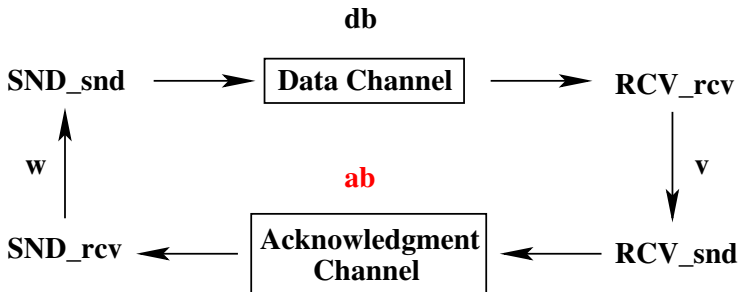
Activation bits at work



Activation bits at work



Activation bits at work



Third Refinement: Introducing the Last Item Indicator

- These invariants define the **last data indicator**

variables: ...
l

inv3_7: $db = \text{TRUE} \wedge r = s \wedge l = \text{FALSE} \Rightarrow r + 1 < n$

inv3_8: $db = \text{TRUE} \wedge r = s \wedge l = \text{TRUE} \Rightarrow r + 1 = n$

- This bit is sent by the Sender to the Receiver
- When equal to TRUE, this bit indicates that the sent item is the last one



Third Refinement: Introducing the Retry Counter c

- Constant MAX denotes the maximum number of retries
- The sender fails iff the retry counter c exceeds MAX (**inv3_10**)

constants: ...
 MAX

axm3_1: $MAX \in \mathbb{N}$

variables: ...
 c

inv3_9: $c \in 0 .. MAX + 1$

inv3_10: $c = MAX + 1 \Leftrightarrow s_st = failure$



Third Refinement: the Events (1)

```
init
  r := 0
  h :=  $\emptyset$ 
  r_st := working
  s_st := working
  s := 0
  d  $\in$  D
  w := TRUE
  db := FALSE
  ab := FALSE
  v := FALSE
  l := FALSE
  c := 0
```

```
brp
  when
    r  $\neq$  working
    s  $\neq$  working
  then
    skip
  end
```



Third Refinement: the Events (2)

SND_snd_current_data

when

$s_st = \text{working}$

$w = \text{TRUE}$

$s + 1 < n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

$db := \text{TRUE}$

$l := \text{FALSE}$

end

SND_snd_last_data

when

$s_st = \text{working}$

$w = \text{TRUE}$

$s + 1 = n$

then

$d := f(s + 1)$

$w := \text{FALSE}$

$db := \text{TRUE}$

$l := \text{TRUE}$

end



Third Refinement: New Events

- Daemons are **breaking the channels**

```
DMN_data_channel
  when
    db = TRUE
  then
    db = FALSE
  end
```

```
DMN_ack_channel
  when
    ab = TRUE
  then
    ab = FALSE
  end
```

- A **failure** is characterized by **all activation bits being FALSE**



Third Refinement: the Events (3)

SND_time_out_current

when

s_st = working

w = FALSE

ab = FALSE

db = FALSE

v = FALSE

c < MAX

then

w := TRUE

c := c + 1

end

SND_failure

when

s_st = working

w = FALSE

ab = FALSE

db = FALSE

v = FALSE

c = MAX

then

s_st := failure

c := c + 1

end



- Sender aborts after $MAX + 1$ tries

Third Refinement: the Events (4)

RCV_rcv_current_data

when

$r_st = working$

$db = TRUE$

$r = s$

$! = FALSE$

then

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

$db := FALSE$

$v := TRUE$

end

RCV_success

when

$r_st = working$

$db = TRUE$

$r = s$

$! = TRUE$

then

$r_st := success$

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

$db := FALSE$

$v := TRUE$

end

Reminder: $!$ is the **last data indicator**



Third Refinement: Guard Strengthening (1)

(abstract-)RCV_rcv_current_data

when

$r_st = \text{working}$

$w = \text{FALSE}$

$r = s$

$r + 1 < n$

then

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

end

(concrete-)RCV_rcv_current_data

when

$r_st = \text{working}$

$db = \text{TRUE}$

$r = s$

$l = \text{FALSE}$

then

$r := r + 1$

$h := h \cup \{r + 1 \mapsto d\}$

$db := \text{FALSE}$

$v := \text{TRUE}$

end

inv3_1': $db = \text{TRUE} \Rightarrow w = \text{FALSE}$

inv3_7: $db = \text{TRUE} \wedge r = s \wedge l = \text{FALSE} \Rightarrow r + 1 < n$



Third Refinement: Guard Strengthening (2)

```
(abstract-)RCV_success
when
  r_st = working
  w = FALSE
  r = s
  r + 1 = n
then
  r := r + 1
  h := h ∪ {r + 1 ↦ d}
end
```

```
(concrete-)RCV_success
when
  r_st = working
  db = TRUE
  r = s
  l = TRUE
then
  r_st := success
  r := r + 1
  h := h ∪ {r + 1 ↦ d}
  db := FALSE
  v := TRUE
end
```

inv3_1': $db = \text{TRUE} \Rightarrow w = \text{FALSE}$

inv3_8: $db = \text{TRUE} \wedge r = s \wedge l = \text{TRUE} \Rightarrow r + 1 = n$



Third Refinement: the Events (5)

```
RCV_rcv_retry
when
   $db = \text{TRUE}$ 
   $r \neq s$ 
then
   $db := \text{FALSE}$ 
   $v := \text{TRUE}$ 
end
```

```
RCV_snd_ack
when
   $v = \text{TRUE}$ 
then
   $v := \text{FALSE}$ 
   $ab := \text{TRUE}$ 
end
```

```
RCV_failure
when
   $r\_st = \text{working}$ 
   $c = \text{MAX} + 1$ 
then
   $r\_st := \text{failure}$ 
end
```



Third Refinement: the Events (6)

```
SND_rcv_current_ack
  when
    s_st = working
    ab = TRUE
     $s + 1 < n$ 
  then
    w := TRUE
    s := s + 1
    c := 0
    ab := FALSE
  end
```

```
SND_success
  when
    s_st = working
    ab = TRUE
     $s + 1 = n$ 
  then
    s_st := success
    c := 0
    ab := FALSE
  end
```



Third Refinement: Guard Strengthening (1)

```
(abstract-)SND_rcv_current_ack
when
   $s\_st = working$ 
   $w = FALSE$ 
   $s + 1 < n$ 
   $r = s + 1$ 
then
   $w := TRUE$ 
   $s := s + 1$ 
end
```

```
(concrete-)SND_rcv_current_ack
when
   $s\_st = working$ 
   $ab = TRUE$ 
   $s + 1 < n$ 
then
   $w := TRUE$ 
   $s := s + 1$ 
   $c := 0$ 
   $ab := FALSE$ 
end
```

inv3_2': $ab = TRUE \Rightarrow w = FALSE$

- In order to prove guard strengthening we need invariant **inv3_11**, and invariant **inv3_12** is needed to prove **inv3_11**

inv3_11: $ab = TRUE \Rightarrow r = s + 1$

inv3_12: $v = TRUE \Rightarrow r = s + 1$



Third Refinement: Guard Strengthening (2)

```
(abstract-)SND_success
when
  s_st = working
  w = FALSE
  s + 1 = n
  r = s + 1
then
  s_st := success
end
```

```
(concrete-)SND_success
when
  s_st = working
  ab = TRUE
  s + 1 = n
then
  s_st := success
  c := 0
  ab := FALSE
end
```

inv3_2': $ab = \text{TRUE} \Rightarrow w = \text{FALSE}$

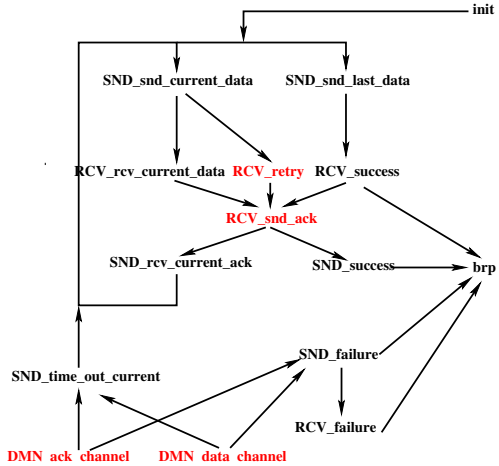
- In order to prove guard strengthening we need invariant **inv3_11**, and invariant **inv3_12** is needed to prove **inv3_11**

inv3_11: $ab = \text{TRUE} \Rightarrow r = s + 1$

inv3_12: $v = \text{TRUE} \Rightarrow r = s + 1$



Final Synchronization of the Events

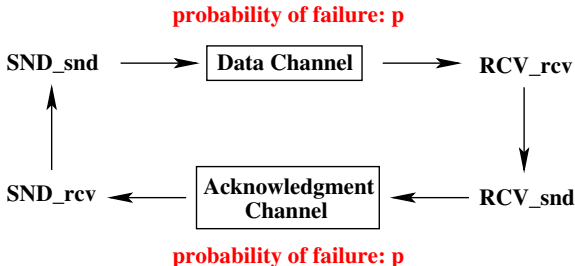


Outline

- 1 Requirements Document
- 2 Formal Development
 - Initial Model
 - First Refinement
 - Second Refinement
 - Third Refinement
- 3 What about Probability



Computing Probabilities



- We would like to compute the probability of success
- It is a function of:
 - p : probability of failure for one channel
 - n : size of the file
 - $MAX + 1$: number of re-tries



Computing Probabilities

Failure on one channel	p
Success on one channel	$1 - p$
Success on both channels	$(1 - p)^2$
Fails on one try	$1 - (1 - p)^2$
Fails on $MAX + 1$ tries	$(1 - (1 - p)^2)^{MAX+1}$
Succ. on $MAX + 1$ tries	$1 - (1 - (1 - p)^2)^{MAX+1}$
Success for n data	$(1 - (1 - (1 - p)^2)^{MAX+1})^n$

$$p = .1$$
$$MAX = 5$$
$$n = 100$$

$$.995$$



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$$\begin{aligned} p &= .1 \\ MAX &= 5 \\ n &= 100 \end{aligned} \quad .995$$



Computing Probabilities

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Computing Probabilities

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