Refinement

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May 22, 2010

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- The simplification should
  - focus on the intended purpose of the system
  - ignore details of how that purpose is achieved.
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- If the purpose is to provide some service, then
  - model what a system does from the perspective of the service users
  - 'users' might be computing agents as well as humans.
- If the purpose is to control, monitor or protect some phenomenon, then
  - the abstraction should focus on those phenomenon
  - in what way should they be controlled or protected?
  - why should they be monitored?

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- Abstraction and refinement together should allow us to manage system complexity in the design process

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   - Add variables and invariants
   - Extend existing events to act on additional variables
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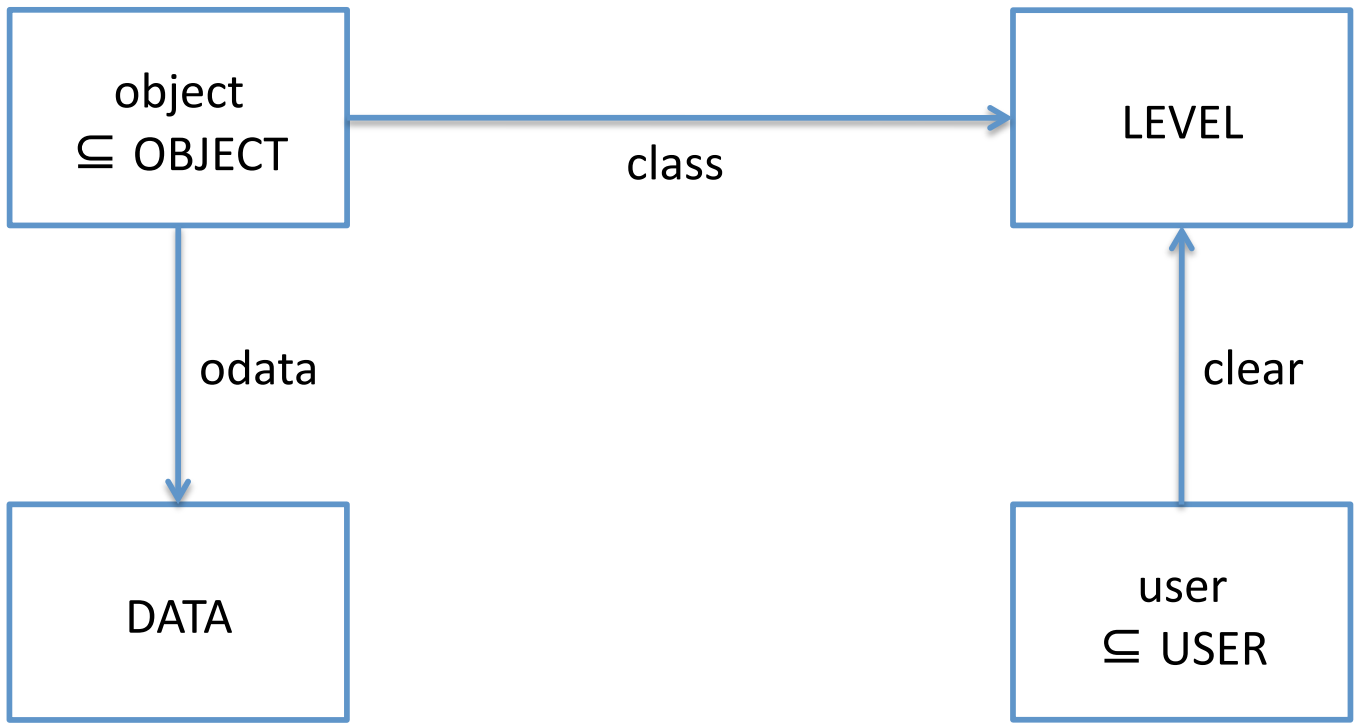
- Verification of 2, 3 and 4 requires gluing invariants that link abstract and concrete variables.

- Extension example: add ownership to secure database
- Extension with Guard Modification example: add tokens to access control system
- Variable replace example: replace disjoint sets by status function in building control system

Class diagram for secure database



Types and variables

sets $OBJECT \quad DATA \quad USER$
constants $LEVEL$
axioms $LEVEL = 1..10$

variables $object,\ user,\ odata,\ class,\ clear$
invariants

$$object \subseteq OBJECT$$
$$user \subseteq USER$$
$$odata \in object \rightarrow DATA$$
$$class \in object \rightarrow LEVEL$$
$$clear \in user \rightarrow LEVEL$$

initialisation

$object := \{\} \quad user := \{\} \quad odata := \{\} \quad class := \{\} \quad clear := \{\}$

Adding object ownership

Extend the database specification so that each object has an owner.

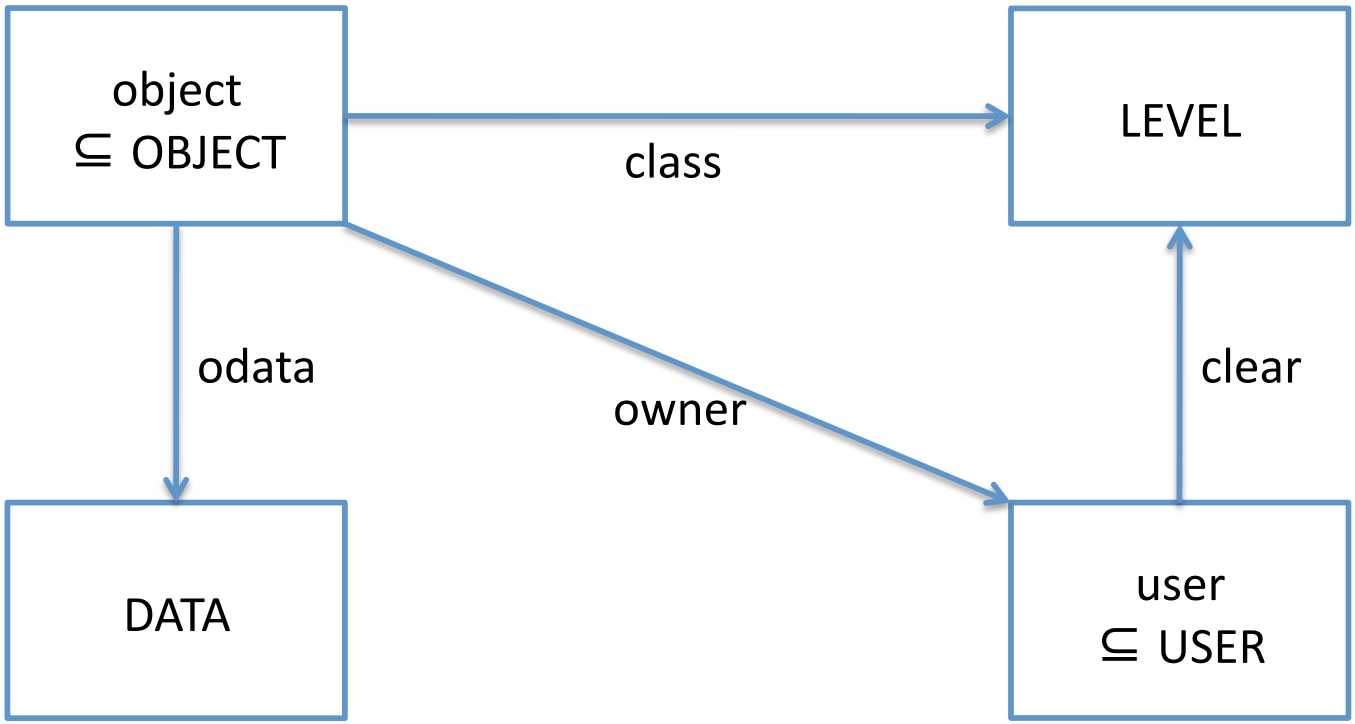
The clearance associated with that owner must be at least as high as the classification of the object.

Only the owner of an object is allowed to delete it.

What additional variables are required?

What events are affected?

Class diagram with ownership



Refinement

machine $m2$
refines $m1$
variables $object$, $user$, $odata$, $class$, $clear$, $owner$
invariants

$$owner \in object \rightarrow user$$

initialisation

$$\ldots \quad owner := \{\}$$

Adding users

$$
\begin{array}{l}
\mathit{AddUser} \;\hat{=}\; \\
\quad \textbf{any}\ u, c\ \textbf{where} \\
\qquad u \in \mathit{USER} \\
\qquad u \notin \mathit{user} \\
\qquad c \in \mathit{LEVEL} \\
\quad \textbf{then} \\
\qquad \mathit{user} := \mathit{user} \cup \{u\} \\
\qquad \mathit{clear}(u) := c \\
\quad \textbf{end}
\end{array}
$$

Do we need to modify this?

Adding objects

$$
\begin{array}{l}
\mathit{AddObject} \;\hat{=} \\
\quad \textbf{any } o, d, c \textbf{ where} \\
\qquad o \in \mathit{OBJECT} \\
\qquad o \notin \mathit{object} \\
\qquad d \in \mathit{DATA} \\
\qquad c \in \mathit{LEVEL} \\
\quad \textbf{then} \\
\qquad \mathit{object} := \mathit{object} \cup \{o\} \\
\qquad \mathit{odata}(o) := d \\
\qquad \mathit{class}(o) := c \\
\quad \textbf{end}
\end{array}
$$

Do we need to modify this?

Event Extension

$$
\begin{array}{l}
\mathit{AddObject}\ \ \textbf{extends}\ \ \mathit{AddObject}\ \ \hat{=} \\
\quad \textbf{any}\ u\ \textbf{where} \\
\quad\quad u \in \mathit{user} \\
\quad\quad \mathit{clear}(u) \geq \mathit{class}(o) \\
\quad \textbf{then} \\
\quad\quad \mathit{owner}(o) := u \\
\quad \textbf{end}
\end{array}
$$

Equivalent to

$$
\begin{array}{l}
\mathit{AddObject}\ \ \textbf{refines}\ \ \mathit{AddObject}\ \ \hat{=} \\
\quad \textbf{any}\ o, d, c, u\ \textbf{where} \\
\qquad o \in \mathit{OBJECT} \\
\qquad o \notin \mathit{object} \\
\qquad d \in \mathit{DATA} \\
\qquad c \in \mathit{LEVEL} \\
\qquad u \in \mathit{user} \\
\qquad \mathit{clear}(u) \geq \mathit{class}(o) \\
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\qquad \mathit{object} := \mathit{object} \cup \{o\} \\
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Other events to consider

- *Read*
- *Write*
- *ChangeClass*
- *ChangeClear*
- *RemoveUser*, *RemoveObject*

Do we need new events?