Attracting Intra-marginal Traders across Multiple Markets

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Abstract

High-valued traders, or intra-marginal traders, can give a market higher transaction rates and generate more profit both for the traders and for the market maker. In previous CAT tournaments with competing market specialists, and in accordance with economic theory, markets using registration fee policies attract intra-marginal traders and drive out extra-marginal traders. We build a simple trader market selection game-theoretic model and simulation to determine how the Nash equilibrium (NE) changes across two markets when a registration fee is charged in one of them.

Introduction

Conventional economics often assumes a single market perspective, but multiple market systems occur frequently in the real world. However, research into the interactions between multiple markets is still in the early stages. For example, in a financially interconnected world, price volatility created in one country's stock market can quickly spread into other countries' stock markets.

We consider multiple market systems that trade the same good across multiple distinct markets. Each market can have different market policies including trading fees. Hasbrouck (1995) provides an example of such multiple markets as follows: "… since a share of IBM is the same whether purchased on the Midwest or Pacific Exchange, this is a particularly clear instance of multiple markets."

The annual CAT tournament, and the associated JCAT software platform (Niu et al, 2008), provide an impetus for exploring the properties of multiple market systems. In the CAT tournament, the software market specialists compete against each other to attract software agent traders. The winner is determined by the three-part scoring rule composed of (1) total profit earned from market fees, (2) market share, and (3) transaction success rate, or the ratio of successful trades in the market.

In the CAT tournament, traders have a two part decision process. They first make a *market selection* decision that determines which market to trade in, and then they determine their bidding price to trade in that market depending on their trading strategies. Market policies (including fees) are announced to all before traders make their market selection decision.

Niu et al. (2008) report that attracting intra-marginal traders is one crucial aspect of winning a CAT tournament. *Intra-marginal* traders are high-value traders whose private values are higher than the market equilibrium price for buyers and lower for sellers, while *extra-marginal* traders are low-value traders where buyer's private values are lower than the market equilibrium price and higher for sellers.

According to Niu et al., market specialists find it easier to match buyers and sellers when a market has many intra-marginal traders. While it is not surprising that buyers who are willing to buy high and sellers who are willing to sell low are more easily matched, intra-marginal traders can receive higher profits when they are matched to other intra-marginal traders. With higher trading profits, they are more likely to return to the market. Thus one key question for a market specialist is how to attract these high-valued traders. Using registration fees has been shown to be one simple, effective way to attract intra-marginal traders in the CAT tournament.

In this paper, we consider more formally why registration fees attract intra-marginal traders and drive extra-marginal traders away. We use a simple game-theoretic model of traders' market selection behavior. We hope to extend this simple framework to cover more interesting and complex trader behavior and market configurations in the future.

Our preliminary results show that extra-marginal traders are the first to leave the registration fee market when the market starts charging a positive registration fee. Intra-marginal traders with low profit margins are the next to leave the market as the registration fee increases, confirming observed CAT tournament trader behavior where intra-marginal traders stayed in the registration fee market while extra-marginals left.

Related Work

Price discrimination (Varian, 1989) occurs when the same good or service is sold by the same provider for different prices. Oi (1971) discusses an example of price discrimination using a two-part tariff that is often used by an amusement park. The owner of an amusement park faces an interesting pricing problem over whether to charge consumers a large lump-sum entrance fee and allow them to then ride all the rides for free or to charge them a zero entrance fee combined with pay-per-ride pricing.

Oi's model has similar aspects to a market considering charging a registration fee vs. charging a transaction fee. Both the registration fee and the entrance fee can be categorized as lump-sum fees, while the transaction fee on each transaction resembles the pay-per-ride fee. However, Oi clearly assumes monopolistic market power from the seller in his model. This might not be applicable to our problem because our markets face competitive landscape.

Hotelling's (1929) classical model of spatial competition provides another view of product differentiation based on location. Hotelling investigated how two ice cream sellers would find an optimal location on a fixed-length beach given that ice cream buyers always select the closest stand. Hotelling's law predicts that the two sellers will locate themselves in the middle of the beach, dividing the market share in half.

These two ice cream sellers must attract consumers under a game-theoretic setting where one seller's location decision is dependent on the other's and vice versa. This setting can also be applied to our multiple market scenario where a registration fee market, for instance, must determine its registration fee while the other markets select and set their choice of fees.

However, Hotelling's model does not further develop a game-theoretic framework for consumer decision-making behavior. In Hotelling's model, transaction costs (in terms of distance travelled) are of primary interest for consumers and these costs can be evaluated by each consumer independently. But in our case, traders must consider the market selection behavior of the other traders in the multiple market system since traders can only make profits when transacting with other matched partners in a market.

Two-stage extensive game modeling, often employed by duopoly (or oligopoly) models such as the Stackelberg leadership model, better reflects the game-theoretic interactions between market specialists and traders. Such models can capture the CAT tournament activities where markets first publicly announce their fee policy and traders then make their market-selection decision. But Stackelberg's model does not cover how each trader's market-selection decision is dependent on all the other traders' market selections.

Problem Description

To maximize profits in a multimarket system such as the CAT tournament, traders must consider two key issues. First, a trader must be matched with another trader to transact. If a trader cannot be matched, its profits will be zero. Second, the trader must consider each market's market policies. For markets that provide the same expected trading opportunities, a trader gains more profit by selecting the market with a lower fee.

Our preliminary model uses truth-telling trading agents (i.e., agents that always bid their true private value) and a sealed-bid auction mechanism for each case described below. We introduce a simple normal-form, game-theoretic model to predict any interesting equilibrium. Starting from a very simplistic one-buyer-and-one-seller case, the model is extended into *n* cases in general.

The market-selection decision can be simplified into a two-stage extensive-form game where the market selects the fee policy in the first phase and a representative trader decides in the second phase whether to stay in the market or leave to the other market. However for our first pass, we further simplify this model to assume that the market "announces" the fee rather than determine its fee based on other market's expected decisions or expected trader payoff structures.

Based on the default CAT tournament settings, we assume the following:

- Trader private values (or willingness to pay) are drawn from linear demand and supply curve with the interval of [50, 150].
- Theoretical market equilibrium price is $p^* = 100$.
- Two free markets (M1 and M2) are assumed. Registration fee for M1 will be introduced later.
- Trader strategy is truth-telling.

In addition, the original market selection strategy for CAT tournament is based on N-armed bandit approach. Traders select the market with the highest profit history from trading for the probability of ε (exploitation) and randomly select a market for $1 - \varepsilon$ (exploration). The default value for ε in the CAT tournament is 0.9. In this study, traders simply select the market which gives the highest profit. We will consider fully incorporating the N-armed bandit market selection in future work since we focus on simplifying our first-pass game-theoretic modeling at this time.

Trader vs. Trader Perspective

Two Intra-marginal Traders Case For a simple example, consider an intra-marginal buyer with the private value of 125 and an intra-marginal seller with the private value of 75. If they are matched, the resulting transaction price is set to $p^* = 100$, giving the buyer a payoff of 125–100 and 100–75 to the seller. We assume here that both markets are free markets for simplicity. A normal-form game model can be constructed as in Table 1.

Table 1. Normal-form market selection model for one buyer and one seller with payoffs. Bold typeface denotes Nash equilibrium.

	Seller selects M1	Seller selects M2
Buyer selects M1	(**25, 25**)	(0, 0)
Buyer selects M1	(0, 0)	(**25, 25**)

It can be easily seen that this is a typical battle-of-the-sexes game. The resulting NE of the market selection strategy is (Buyer, Seller) = (M1, M1) or (Buyer, Seller) = (M2, M2). In other words, traders want to stay in the same market with the other trader.

Now the question is whether this battle-of-the-sexes framework can be extended into *n*-trader cases in general. We begin by considering four-trader cases with two buyers and two sellers, and work out the results by hand. We verify these results with a computer simulation that conducts a brute-force NE search allowing us to extend this game-theoretic framework into *n*-trader case in general.

Four Intra-marginal Traders Case Assume there are two intra-marginal buyers (B1, B2) with the private values of 140 and 120, and another two intra-marginal sellers (S1, S2) with the private values of 60 and 80 respectively. Table 2 shows the payoff matrix of the normal game form. Unlike the two-player case in Table 1, each market can now have different equilibibrium prices depending on the types of traders visiting the market and how they are matched by the market. Also note that traders were matched under sealed-bid auction rule.

Table 2 Market selection model for four intra-marginal traders. Bold typeface denotes Nash equilibrium.

	S1 selects M1	S2 selects M2
B1 selects M1	**(40, 40, 20, 20)** $p_1^* = 100, p_2^* = NA$	(30, 0, 0, 30) $p_1^* = 110, p_2^* = NA$
B1 selects M1	(0, 30, 30, 0) $p_1^* = 90, p_2^* = NA$	**(40, 40, 20, 20)** $p_1^* = 100, p_2^* = 100$
B2 selects M1, S2 selects M1		

	S1 selects M1	S1 selects M2
B1 selects M1	(40, 40, 0, 0) $p_1^* = 100, p_2^* = NA$	(0, 0, 0, 0) $p_1^* = NA, p_2^* = NA$
B1 selects M2	(30, 30, 30, 30) $p_1^* = 90, p_2^* = 110$	(40, 40, 0 ,0) $p_1^* = NA, p_2^* = 100$
B2 selects M1, S2 selects M2		

	S1 selects M1	S2 selects M2
B1 selects M1	(40, 40, 0, 0) $p_1^* = 100, p_2^* = NA$	(30, 30, 30, 30) $p_1^* = 110, p_2^* = 90$
B1 selects M2	(0, 0, 0, 0) $p_1^* = NA, p_2^* = NA$	(40, 40, 0, 0) $p_1^* = NA, p_2^* = 100$
B2 selects M2, S2 selects M1		

	S1 selects M1	S2 selects M2
B1 selects M1	**(40, 40, 20, 20)** $p_1^* = 100, p_2^* = 100$	(0, 30, 30, 0) $p_1^* = NA, p_2^* = 90$
B1 selects M2	(30, 0, 0, 30) $p_1^* = NA, p_2^* = 110$	**(40, 40, 20, 20)** $p_1^* = 100, p_2^* = 100$
B2 selects M2, S2 selects M2		

The resulting Nash equilibria for (B1, S1, B2, S2) are (M1, M1, M1, M1), (M1, M1, M2, M2), (M2, M2, M1, M1), and (M2, M2, M2, M2). In these Nash equilibria, the intra-marginal traders tend to pair together so that the higher-valued B1 and S1 move together, and similarly for the lower-valued B2 and S2.

Two Intra-marginal and Two Extra-marginal Traders Case

Now consider the case of two intra-marginal traders and two extra-marginal traders. The buyers (B1, B2) have the private values of 140 and 90 respectively and the sellers (S1, S2) have the private values of 60 and 110.

Table 3 Market selection model for two intra-marginal traders and two extra-marginal traders. Bold typeface denotes Nash equilibrium.

	S1 selects M1	S2 selects M2
B1 selects M1	**(40, 40, 0, 0)** $p_1^* = 100, p_2^* = NA$	(15, 0, 0, 15) $p_1^* = 125, p_2^* = NA$
B1 selects M1	(0, 15, 15, 0) $p_1^* = 75, p_2^* = NA$	**(40, 40, 0, 0)** $p_1^* = NA, p_2^* = 100$
B2 selects M1, S2 selects M1		

	S1 selects M1	S1 selects M2
B1 selects M1	**(40, 40, 0, 0)** $p_1^* = 100, p_2^* = NA$	(0, 0, 0, 0) $p_1^* = NA, p_2^* = NA$
B1 selects M2	(15, 15, 15, 15) $p_1^* = 75, p_2^* = 125$	**(40, 40, 0 ,0)** $p_1^* = NA, p_2^* = 100$
B2 selects M1, S2 selects M2		

	S1 selects M1	S2 selects M2
B1 selects M1	**(40, 40, 0, 0)** $p_1^* = 100, p_2^* = NA$	(15, 15, 15, 15) $p_1^* = 110, p_2^* = 90$
B1 selects M2	(0, 0, 0, 0) $p_1^* = NA, p_2^* = NA$	**(40, 40, 0, 0)** $p_1^* = NA, p_2^* = 100$
B2 selects M2, S2 selects M1		

	S1 selects M1	S2 selects M2
B1 selects M1	**(40, 40, 0, 0)** $p_1^* = 100, p_2^* = NA$	(0, 15, 15, 0) $p_1^* = NA, p_2^* = 75$
B1 selects M2	(15, 0, 0, 15) $p_1^* = NA, p_2^* = 125$	**(40, 40, 0, 0)** $p_1^* = NA, p_2^* = 100$
B2 selects M2, S2 selects M2		

Table 3 shows eight Nash equilibria of :

- (M1, M1, M1, M1), (M1, M1, M1, M2), (M1, M1, M2, M1), (M1, M1, M2, M2)
- (M2, M2, M1, M1), (M2, M2, M1, M2), (M2, M2, M2, M1), (M2, M2, M2, M2)

This result can be summarized as intra-marginal trader pair (B1, S1) either selects (M1, M1) or (M2, M2) and extra marginal traders B2 and S2 do not actually care which market they are in. As can be seen in Table 2, the intra-marginal trading pair of (B1, S1) wants to stay in the same market. Thus, an interim conclusion from NE can be drawn that intra-marginal traders stay in the same market to maximize their transaction profit.

Experiment

Extending the Model into *n*-Trader Case

Because of the high degree of complexity of analyzing NE by hand, we use a brute-force NE search program. The pseudocode for calculating the trader payoffs and searching Nash equilibrium is shown in Figure 1. We first confirmed that our simulation generated the same NE results presented in Table 2 and 3.

```
Generate traders with given private value setup;
Setup market policy for all two markets;
// such as registration fee, sealed-bid auction

/* build payoff tables */
For all possible market selection permutation:
  Traders are allocated into markets;
  Find p* for each market;
    // sealed-bid auction applied
    // Note that p1* and p2* can be different

  For each trader_i in all traders:
    Calculate payoffs for trader_i;
  End for
End for

/* check and print out Nash equilibrium */
For all possible market selection permutation:
  checkNE = true;
  For each trader_i in all traders:
    If trader_i_payoff(current_mkt_selection)
      < trader_i_payoff(the_other_mkt_selection):
    Then
      checkNE = false;
      Break off from the inner for loop;
    End if
  End for

  If checkNE is still true:
  Then
    Record the current mkt selection as NE;
    Print the mkt selection;
  End if
End for
```

Figure 1 Pseudocode for searching Nash equilibria for multiple number of traders.

Incorporation of Registration Fee

Using the simulation, we now consider several different registration fee values and their effect on the resultant NE. Recall that the registration-fee markets tend to attract intra-marginal traders, depending on the fee amount and which other markets are present.

Table 4 shows the private values of the traders in the experimental setup. Our traders consist of four intra-marginals and four extra-marginals. The theoretical market equilibrium price is $p^* = 100$.

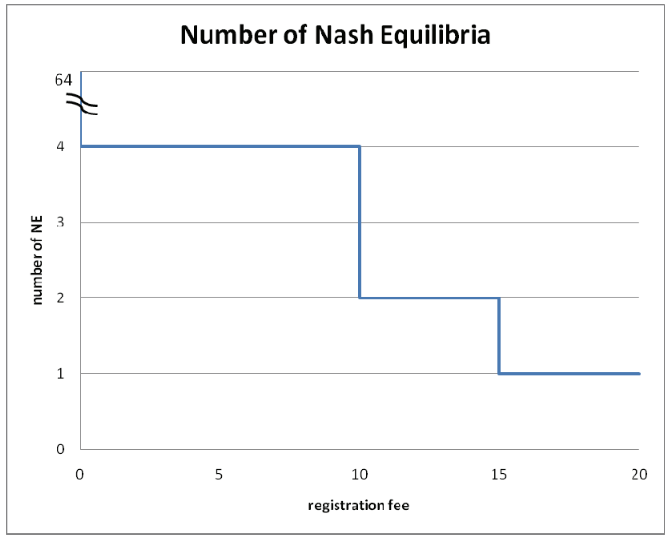

Table 4 Trader private value setup for eight-trader experiment

Index	1	2	3	4
Buyer	140	110	80	50
Seller	60	90	120	150

Figure 2 shows the changes in the number of Nash equilibria for different registration fees. When the registration fee is zero, the model produces 64 Nash equilibria where the intra-marginal trader pairs (Bn, Sn) select the same market and the extra-marginal traders do not care which market they are in.

Separation between Intra-marginal Traders and Extra-marginals into Different Markets However, market selection behaviors of extra-marginal traders change as the market begins to charge a registration fee. For the registration fee r with $0 < r \leq 10$, it was observed that extra-marginal traders go only to the free market while intra-marginals keep the pairing behavior in either market. The resulting four Nash equilibria are shown in Table 5.

When the registration-fee market cannot match extra-marginal traders, these extra-marginal traders incur negative profits (i.e., they must pay the registration fee even though they do not make a trade). On the other hand, even if not matched, these extra-marginal traders end up earning zero profit and thus staying in the free market.

Within this registration fee range, the equilibrium market selection behaviors of the intra-marginal traders are not affected. In pairs, they end up staying either in the registration fee market or in the free market. However, this trend is changed when the registration fee is increased over 10.

Table 5 Nash equilibria for trader market selection between registration fee market (M1) and free market (M2). Registration *fee r is set to 0 < r ≤ 10. Second rows for each Nash equilibrium denote the trader payoff.*

Trader	B1	S1	B2	S2	B3	S3	B4	S4
NE 1	M1 40-*r*	M1 40-*r*	M1 10-*r*	M1 10-*r*	M2 0	M2 0	M2 0	M2 0
NE 2	M1 40-*r*	M1 40-*r*	M2 10	M2 10	M2 0	M2 0	M2 0	M2 0
NE 3	M2 40	M2 40	M1 10-*r*	M1 10-*r*	M2 0	M2 0	M2 0	M2 0
NE 4	M2 40	M2 40	M2 10	M2 10	M2 0	M2 0	M2 0	M2 0

Low-value Intra-marginal Traders Leaving the Registration Fee Market Now when the registration fee r is increased into the interval of $10 < r \leq 15$, the number of Nash equilibria are reduced to two as shown in Table 6.

Table 6 Nash equilibria for trader market selection between registration fee market (M1) and free market (M2). Registration fee r is set to 10 < r ≤ 15. Second rows for each Nash equilibrium denote the trader payoff.

Trader	B1	S1	B2	S2	B3	S3	B4	S4
NE 1	M1 40-*r*	M1 40-*r*	M2 10	M2 10	M2 0	M2 0	M2 0	M2 0
NE 2	M2 40	M2 40	M2 10	M2 10	M2 0	M2 0	M2 0	M2 0

The reduction originates from the intra-marginal trader pair of (B2, S2) having lower private values, namely (110, 90), than the trading pair (B1, S1) with (140, 60). At the market equilibrium price of $p^* = 100$, the (B2, S2) pair have the lower profit margin of 10 (140 – 100 for B1, 100 – 90 for B2). When the registration fee over 10 is charged, the (B2, S2) traders will incur negative profit in the registration fee market and thus will want to leave the registration fee market.

To verify this, consider a case where (B2, S2) pair stay in M1 while (B1, S1) pair stay either in M1 or M2 and other extra-marginal traders stick to M2. Let B2's payoff be π and the registration fee r be *10.1*. When B2 stays with S2 in M1, B2's payoff is $\pi(M1) = -0.1 = (110 - 100) - 10.1$ since the transaction price becomes $p^* = 100$ when B2 is matched with S2 in M1. Now when B2 switches from M1 to M2, B2 cannot be matched into any transaction in M2 and obtains zero profit of $\pi(M2) = 0$. Thus it follows that $\pi(M1) < \pi(M2)$ for B2 and B2 staying in M1 cannot be a Nash equilibrium.

In other words, a registration fee higher than 10 drove payoffs for (B2, S2) to be negative. Although (B2, S2) could not have any other trading possibilities in the free market, (B2, S2) had to switch to the free market to avoid negative net profit.

All Intra-marginals and Extra-marginal Traders Willing to Leave the Registration Fee Market When registration fee increases to $r > 15$, the high-value intra-marginal trader pair (B1, S1) now leaves the registration fee market. The resulting market selection equilibrium is shown in Table 7.

Table 7 Nash equilibria for trader market selection between registration fee market (M1) and free market (M2). Registration fee r is set to r > 15. Second row denotes the trader payoff.

Trader	B1	S1	B2	S2	B3	S3	B4	S4
NE 1	M2 40	M2 40	M2 10	M2 10	M2 0	M2 0	M2 0	M2 0

Presumably the high level of registration fee finally started to harm the trading profit for the (B1, S1) pair. However, unlike in the previous case with Table 6, the registration fee of 15 is not greater than the payoff of 30 for B1 or S1, assuming the market equilibrium price of $p^* = 100$.

At this moment we consider why (B1, S1) switches over to M2 while the trader seems to make a net positive profit with the payoff of 30 (assuming $p^* = 100$) and the registration fee of 15. Let B1's payoff be π and the registration fee r be 15.1 for convenience. Assume that (B1, S1) are in M1 and other traders are in M2. When B1 stays in the registration fee market, $\pi(M1) = 24.9 = (140 - 100) - 15.1$ since B1 and S1 match into a transaction with $p^* = 100$. When B1 switches to M2, B1 can now have a trading match with a seller S2 unlike B2 in $10 < r \leq 15$ case. (Note that S2's ask is 90) The resulting transaction price between B1 and S2 becomes $p^* = 115 = (140 + 90) / 2$ and the profit for B1 is now $\pi(M2) = 25 = 140 - 115$. Therefore, the market selection of B1 staying in M1 cannot be a Nash equilibrium since $\pi(M1) < \pi(M2)$ for B1 and the early market switch comes from across pair matching between B1 and S2.

Conclusion

To study the effects of a registration fee in the multiple market system framework, we develop a simple game-theoretic model for trader market-selection behavior. Starting with two free markets, our simulation shows that as the market registration fee increases, it has the effect of driving out extra-marginal traders first. As the registration fee further increases, intra-marginal traders started to be affected and switched to the other free market. The resulting number of Nash equilibria drop as the registration fee increases.

Clearly this is very preliminary work in the direction of more formally modeling the effects of market policy on trader market selection behavior. However, in the future we plan to explore the intertwined effects of different market policies and trader populations on Nash equilibrium controlling more of the attributes and considering more complex market configurations.

One primary implication about trader behavior from our initial findings is that intra-marginal traders seem to "pair-up" and end up in the same market (under NE). Within a given level of registration fees (up to 10), intra-marginals were still willing to stay in the registration fee market, while extra-marginals immediately switch to the other free market. Of course, this pairing behavior was promoted by our selection of a sealed-bid auction clearing policy rather than a continuous double auction. We intend to investigate further how much impact this choice of clearing policy has on the traders' pairing behavior.

In addition, Nash equilibrium is not necessarily the best construct for fully modeling the dynamics of trader's market-selection decisions. In Nash equilibrium, individual players do not want to deviate from equilibrium since they leave their trading partner behind, generally resulting in sub-optimal payoffs. Coalition-proof Nash equilibrium concepts appears to be a more natural model for market-selection decisions since single traders might be willing to deviate if their trading party also deviates.

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