A Hybrid FE/Spectral Analysis of Turbofan Aeroacoustics

Mike Giles

Mihai Duta

Alistair Laird

Oxford University Computing Laboratory

`http://www.comlab.ox.ac.uk`

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Introduction

The Trent 700 powers the Airbus A330 airliner.

Picture 99237708

Rolls-Royce

Motivation

- noise is becoming a critical issue for airlines, airports and governments
- tonal noise propagated forward of the engine is an important component
- ground noise level influenced by design of engine inlet
- computationally demanding analysis (high frequencies, large grids)
- rapid analysis tools (based on simplified modelling?) are of great value in design optimisation

Potential Flow Modelling

For turbofan inlets, potential flow approximations (inviscid, irrotational, uniform entropy) are accurate and give greatly reduced cost per grid point.

Potential flow methods split into two classes:

- frequency domain
  - original method developed for axisymmetric geometries
  - not used in 3D because of computational cost?
- time domain
  - more popular now, because of viability in 3D?

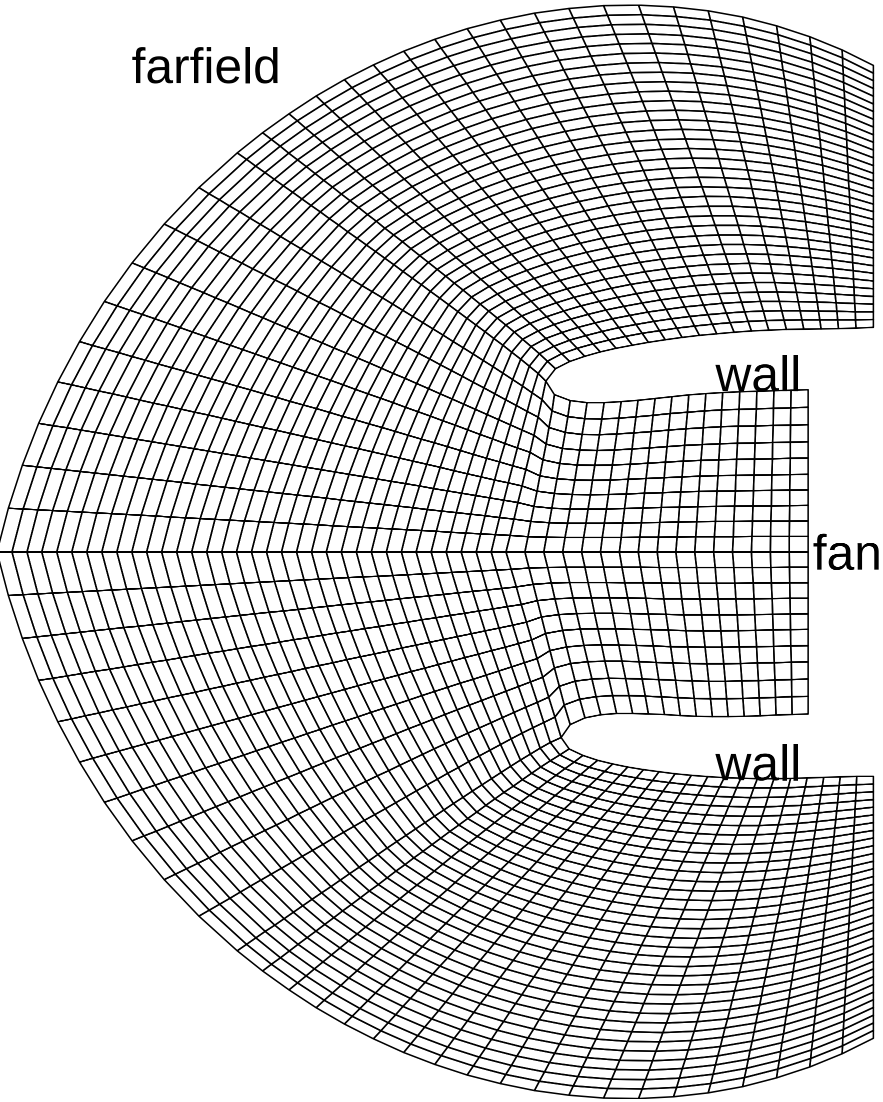
Potential Flow Modelling

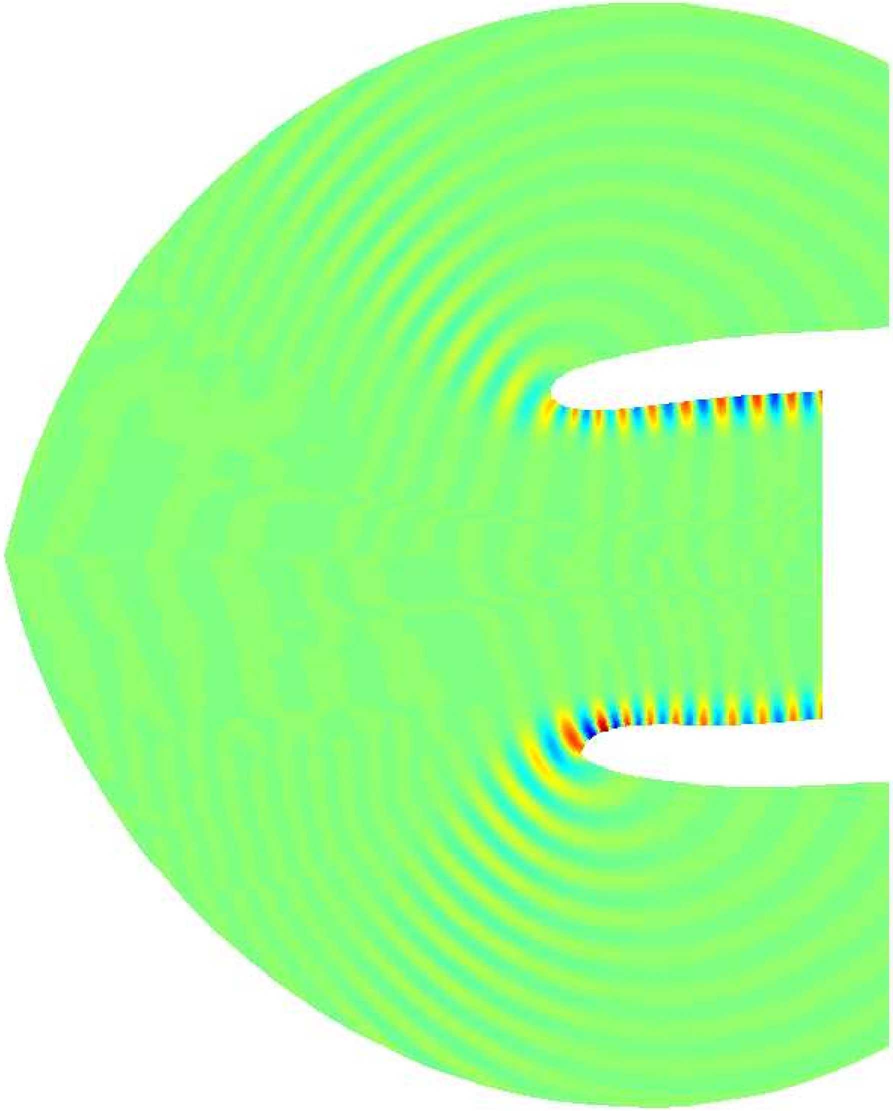
Our goal: extend frequency-domain finite element method to efficient computations for non-axisymmetric geometries

Two main elements:

- Fourier representation of circumferential variation in geometry and steady and unsteady flow fields
- iterative solution using a very effective preconditioner based on an axisymmetric geometry and steady flow field

Potential Flow Modelling





Potential Flow Modelling

Given irrotational flow with uniform entropy, velocity is represented by the potential gradient $\nabla\phi$, while the density and speed of sound are given by

$$\left(\frac{\rho}{\rho_\infty}\right)^{\gamma-1} = \frac{c^2}{c_\infty^2} = 1 - (\gamma-1)\,\frac{q - q_\infty}{c_\infty^2},$$

where

$$q = \tfrac{1}{2}\left|\nabla\phi\right|^2 + \frac{\partial\phi}{\partial t}.$$

Steady Flow Equations

The mass conservation equation gives the p.d.e.

$$\nabla \cdot (\rho \, \nabla \phi) = 0,$$

in domain V, with Dirichlet b.c.'s on the far-field part of the boundary ∂V_0, and Neumann b.c.'s

$$\rho \, \frac{\partial \phi}{\partial n} = \beta,$$

on the walls ($\beta = 0$) and the fan boundary ($\beta \neq 0$).

Integration by parts then gives the weak form

$$\int_V \rho \, \nabla \phi \cdot \nabla w \, \mathrm{d}V - \int_{\partial V} \beta \; w \; \mathrm{d}S = 0, \;\; \forall w \in H_0^1(V).$$

Unsteady Flow Equations

Linearising the unsteady equations, and considering a single harmonic component $\widehat{\phi}\, e^{\mathrm{i}\omega t}$, the weak form of the unsteady equations is

$$\int_V \rho\, \nabla\widehat{\phi} \cdot \nabla w - \frac{\rho}{c^2}\left(\nabla\phi\cdot\nabla\widehat{\phi} + \mathrm{i}\,\omega\,\widehat{\phi}\right)\left(\nabla\phi\cdot\nabla w - \mathrm{i}\,\omega\, w\right)\,\mathrm{d}V$$

$$- \int_{\partial V} \widehat{\beta}\, w\,\mathrm{d}S = 0,$$

where

$$\widehat{\beta} = \rho\,\frac{\partial\widehat{\phi}}{\partial n} - \frac{\rho}{c^2}\left(\nabla\phi\cdot\nabla\widehat{\phi} + \mathrm{i}\,\omega\widehat{\phi}\right)\frac{\partial\phi}{\partial n}.$$

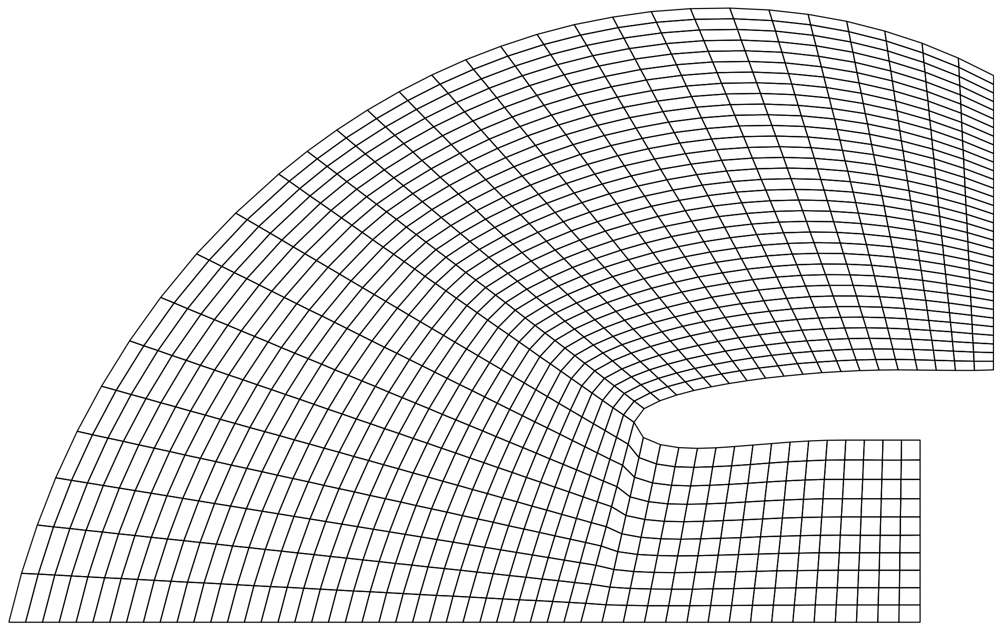
Unsteady Flow Equations

There are different models for $\widehat{\beta}$ on the different parts of the boundary:

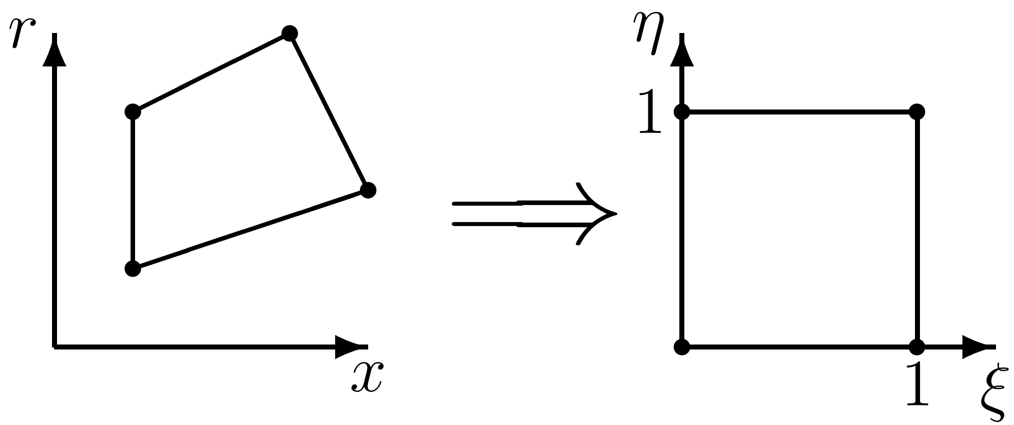
- fan boundary: unsteady flow is decomposed into a sum of incoming and outgoing eigenmodes;
- far-field boundary: high-frequency ray theory determines the angle at which the acoustic waves cross the boundary;
- walls: $\widehat{\beta} = 0$ on solid walls, but with acoustic liners (lots of small holes linked to a plenum) additional modelling gives a modified weak form boundary integral.

Axisymmetric Analysis

2D axisymmetric grid



with each cell mapped to a unit square



Axisymmetric Analysis

Standard finite element representation

$$
\begin{aligned}
x(\xi,\eta) &= \sum_n x_n \, N_n(\xi,\eta), \\
r(\xi,\eta) &= \sum_n r_n \, N_n(\xi,\eta), \\
\phi(\xi,\eta) &= \sum_n \phi_n \, N_n(\xi,\eta),
\end{aligned}
$$

with bi-quadratic elements and Gauss quadrature leads to a coupled system of nonlinear equations for the steady flow potential

$$R(\phi) = 0$$

which is solved by Newton iteration, with direct solution of the linear equations.

Axisymmetric Analysis

The unsteady flow analysis for a prescribed circumferential mode number

$$\exp(\mathrm{i}\omega t + \mathrm{i}\kappa\theta)\ \widehat{\phi}(x, r)$$

with

$$\widehat{\phi}(\xi, \eta) = \sum_n \widehat{\phi}_n\, N_n(\xi, \eta),$$

leads to a linear system of equations

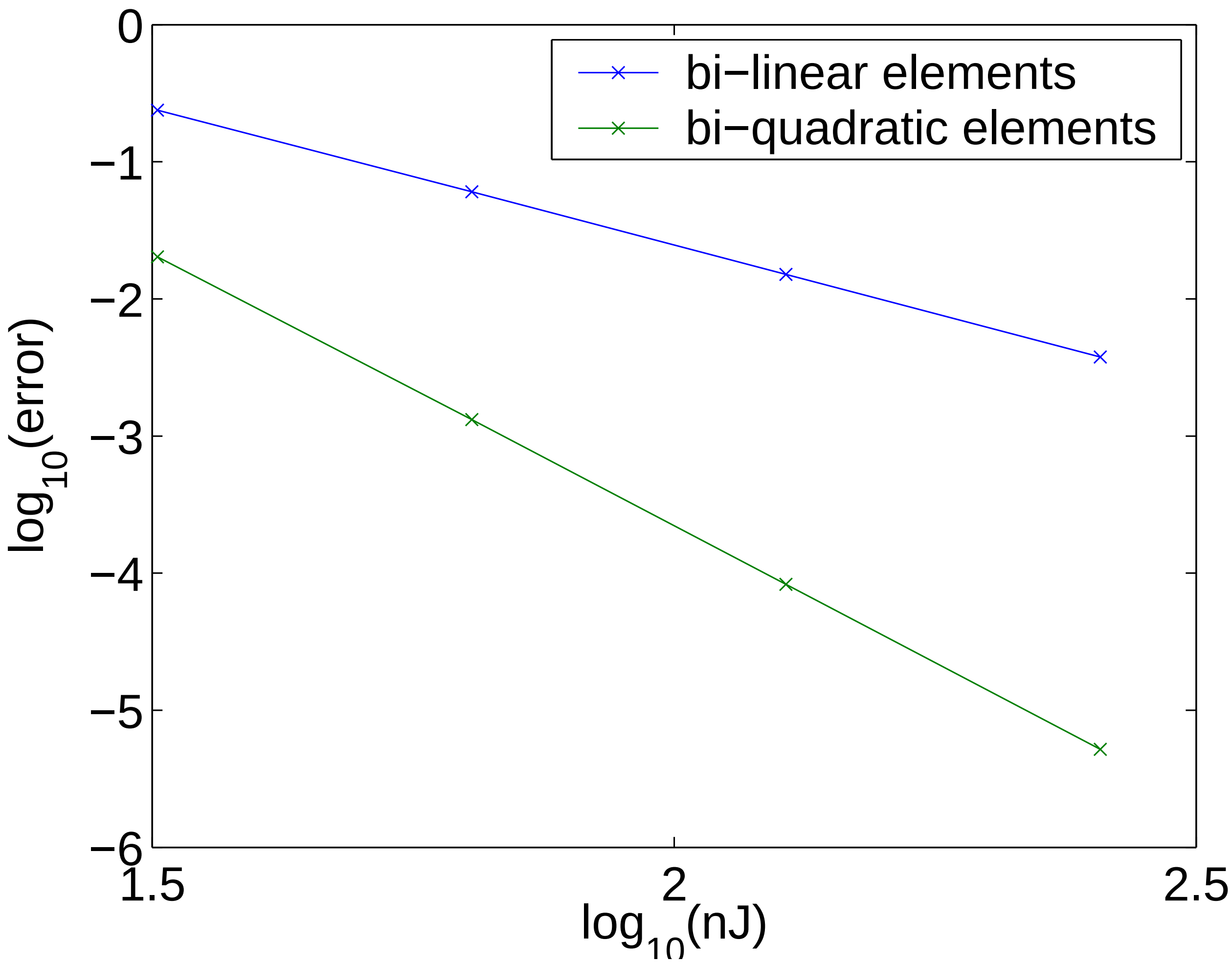
$$\widehat{L}\ \widehat{\phi} = \widehat{f}$$

with the forcing term $\widehat{f}$ coming from a prescribed acoustic excitation on the fan face.

Again, the equations are solved directly – efficient in 2D.

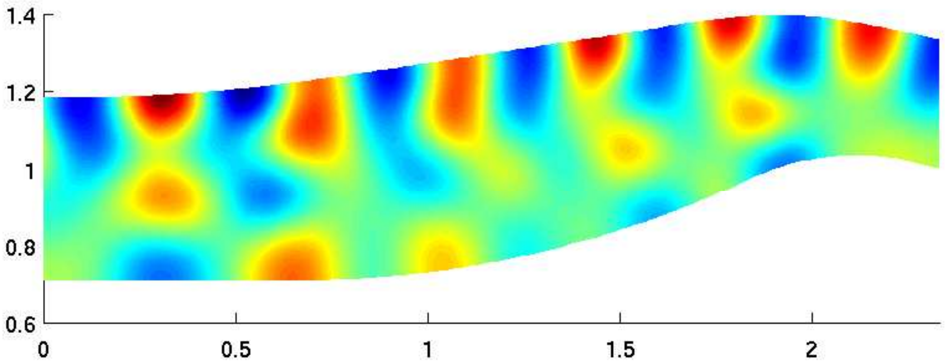
Axisymmetric Validation

Cylindrical duct grid convergence validation against analytic theory: length/radius = 2, $M = 0.4, \quad \omega = 6, \quad \kappa = 2$.

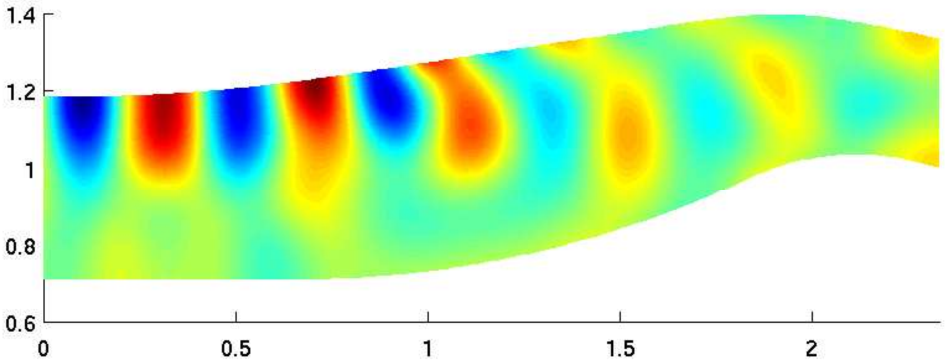


Axisymmetric Validation

Engine bypass duct validation against ACTRAN code, without and with acoustic liner



	ARCADIA	ACTRAN
mode 1	0.959	0.954
mode 2	0.134	0.132
mode 3	0.054	0.055



	ARCADIA	ACTRAN
mode 1	0.0455	0.0450
mode 2	0.1034	0.1029
mode 3	0.0093	0.0094

Non-axisymmetric Analysis

One way of handling non-axisymmetry is to use standard 3D finite elements:

$$\phi(\xi,\eta,\zeta) = \sum_n \phi_n \, N_n(\xi,\eta,\zeta)$$

The problem is how to solve the resulting linear system of equations.

$15K$ 2D nodes $\times$ 200 circumferentially $= 3M$ 3D nodes

- direct solution requires too much memory/CPU
- iterative solution is very slow due to indefiniteness of the matrix

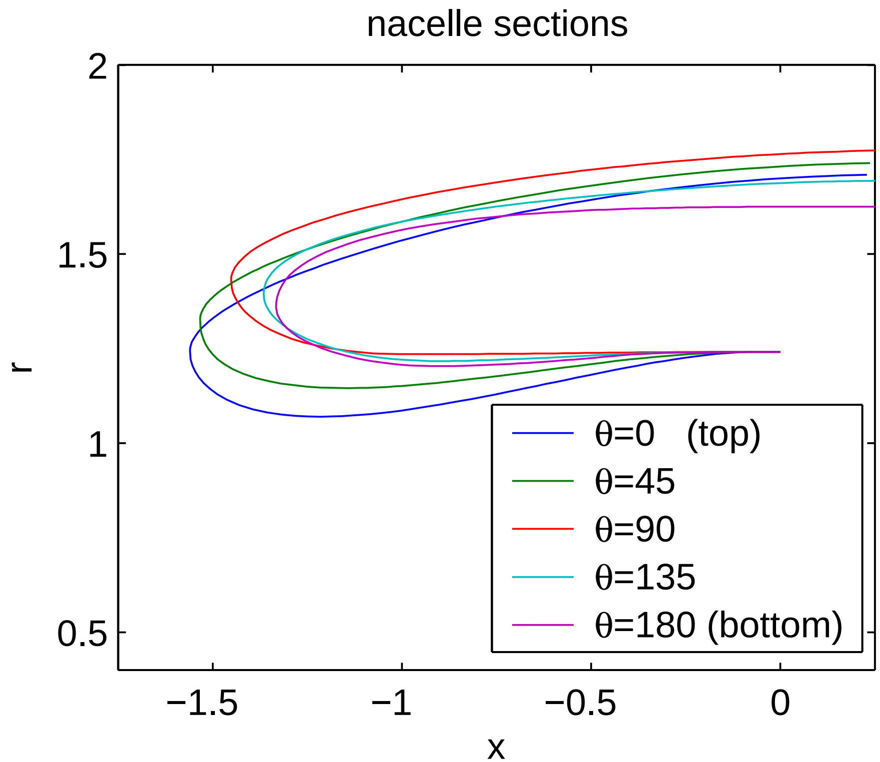
Non-axisymmetric Analysis

We have instead chosen to use spectral elements with $x, r, \phi, \widehat{\phi}$ expressed as Fourier series

$$\phi(\xi, \eta, \theta) = \sum_{m,n} \phi_{m,n} \, \exp(\mathrm{i}m\theta) \, N_n(\xi, \eta)$$

- double summation over 2D nodes n and circumferential modes m
- very few modes needed for mildly non-axisymmetric geometries, so the total number of DOFs is greatly reduced
- still significant issues about how to formulate spectral equations, and how to solve them.

Non-axisymmetric Geometry



- FFT of corresponding points on each section gives spectral geometric representation on nacelle
- this is extended to the mesh interior by interpolation

Non-Axisymmetric Analysis

- if geometry is axisymmetric, all of the circumferential modes are uncoupled — each requires a 2D calculation;
- if the geometry is not axisymmetric, all modes are coupled — for M modes, $O(M^3)$ direct solution cost, and $O(M^2)$ memory required;
- instead, solve iteratively (using QMR or CGNR) with a good preconditioner,

$$\left(\widehat{P}^{-1}\widehat{L}\right)\widehat{\phi} = \widehat{P}^{-1}\widehat{f},$$

with $\widehat{P} \approx \widehat{L}$ and easy to "invert".

Non-Axisymmetric Analysis

- novel idea is using $\widehat{P} = \widehat{L}_{\text{axi}}$ where $\widehat{L}_{\text{axi}}$ is the block-diagonal matrix corresponding to an axisymmetric geometry and steady flow.
- solving $\widehat{P}\,\widehat{v} = \widehat{b}$ requires a separate 2D calculation for each circumferential mode.
- memory requirement is minimised by using standard sparse matrix solution.
- CPU cost is minimised by performing LU factorisation once for each circumferential mode, then just back-solve at each iteration step.

Non-Axisymmetric Analysis

In more detail, one of the contributions to the matrix $\widehat{L}_{\text{axi}}$ from a single cell is

$$\int_0^1\int_0^1 \left\{\rho\,\nabla N_n\cdot\nabla N_{n'} - \frac{\rho}{c^2}\left(\nabla\phi\cdot\nabla N_n\right)\left(\nabla\phi\cdot\nabla N_{n'}\right)\right\}\, r\,|J|\,\mathrm{d}\xi\,\mathrm{d}\eta,$$

which is approximated through Gaussian quadrature as

$$\sum_i w_i\; f(\xi_i,\eta_i)$$

where $f(\xi_i,\eta_i)$ is the integrand evaluated at the Gauss points and w_i is the appropriate weight.

Non-Axisymmetric Analysis

Similarly, part of the contribution to the product $\widehat{L}\,\widehat{\phi}$ from a single cell is

$$\frac{1}{2\pi}\int_0^{2\pi}\int_0^1\int_0^1 \left\{\rho\,\nabla\widehat{\phi}\cdot\nabla\left(e^{-\mathrm{i}\,(\kappa+m)\theta}N_n\right) - \frac{\rho\,\omega^2}{c^2}\,\widehat{\phi}\,e^{-\mathrm{i}\,(\kappa+m)\theta}N_n\right\}\,|J|\,r\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\theta.$$

With Gaussian quadrature in ξ, η, this becomes

$$\frac{1}{2\pi}\int_0^{2\pi} e^{-\mathrm{i}\,m\theta}\sum_i w_i\left(f(\xi_i,\eta_i,\theta) - \mathrm{i}\,m\,g(\xi_i,\eta_i,\theta)\right)\,\mathrm{d}\theta,$$

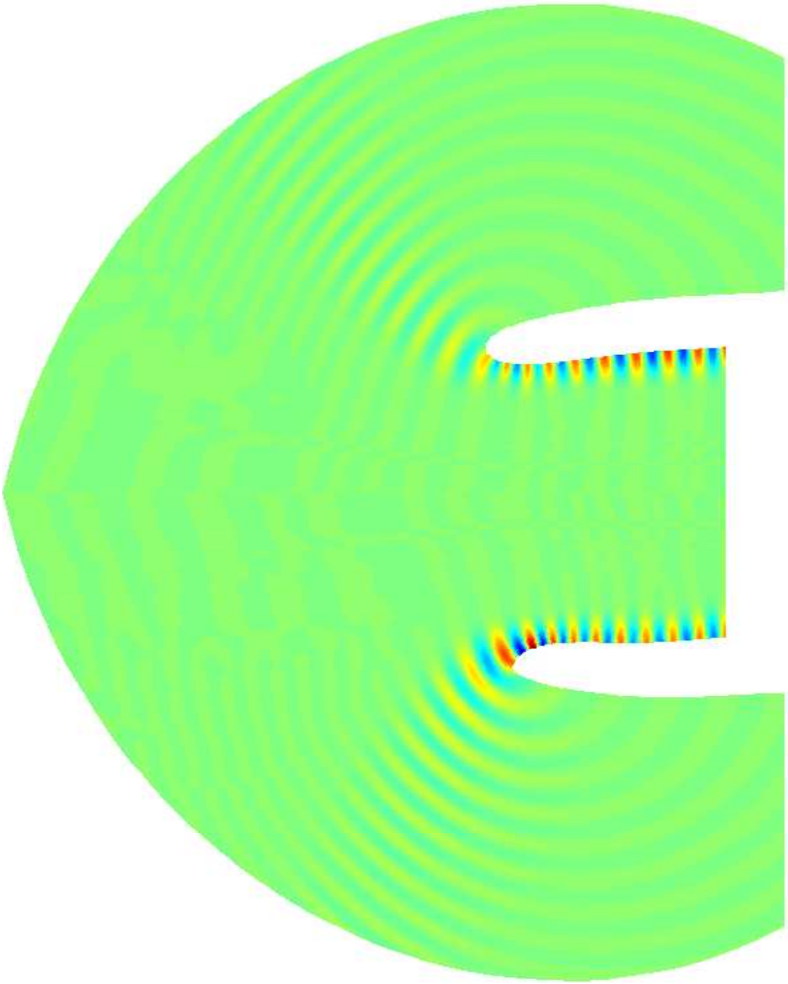
and the θ integration is approximated by an FFT.

Non-Axisymmetric Analysis

Mode   Circumferential node

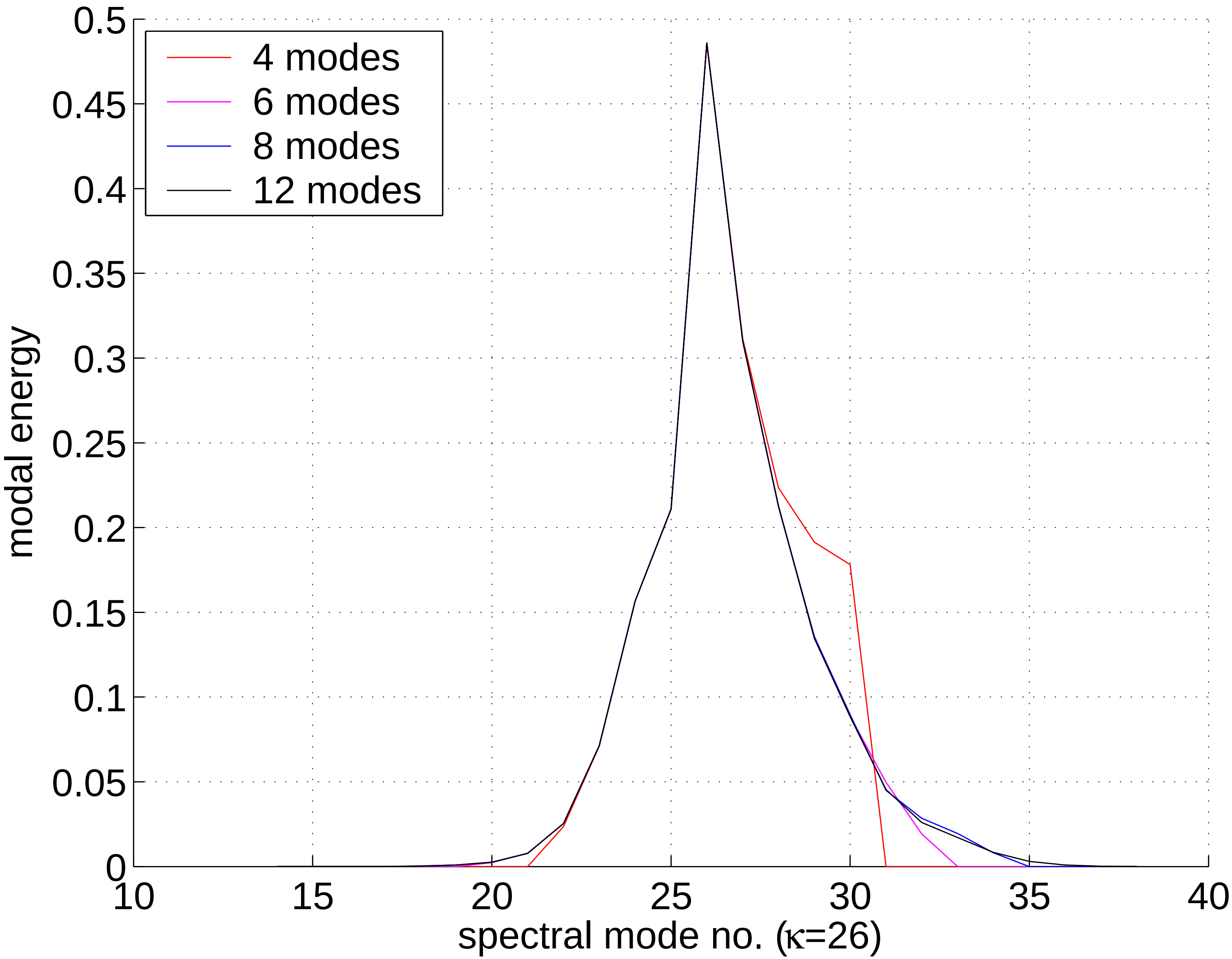
$$
\begin{tikzcd}
\boxed{\widehat{\phi}_m} \arrow[r] & \boxed{\widehat{\phi}(\theta),\ \dfrac{\partial\widehat{\phi}}{\partial\theta}} \arrow[d] \\
\boxed{f_m - \mathrm{i}\, m\, g_m} & \boxed{f(\theta),\ g(\theta)} \arrow[l]
\end{tikzcd}
$$

Engine Nacelle

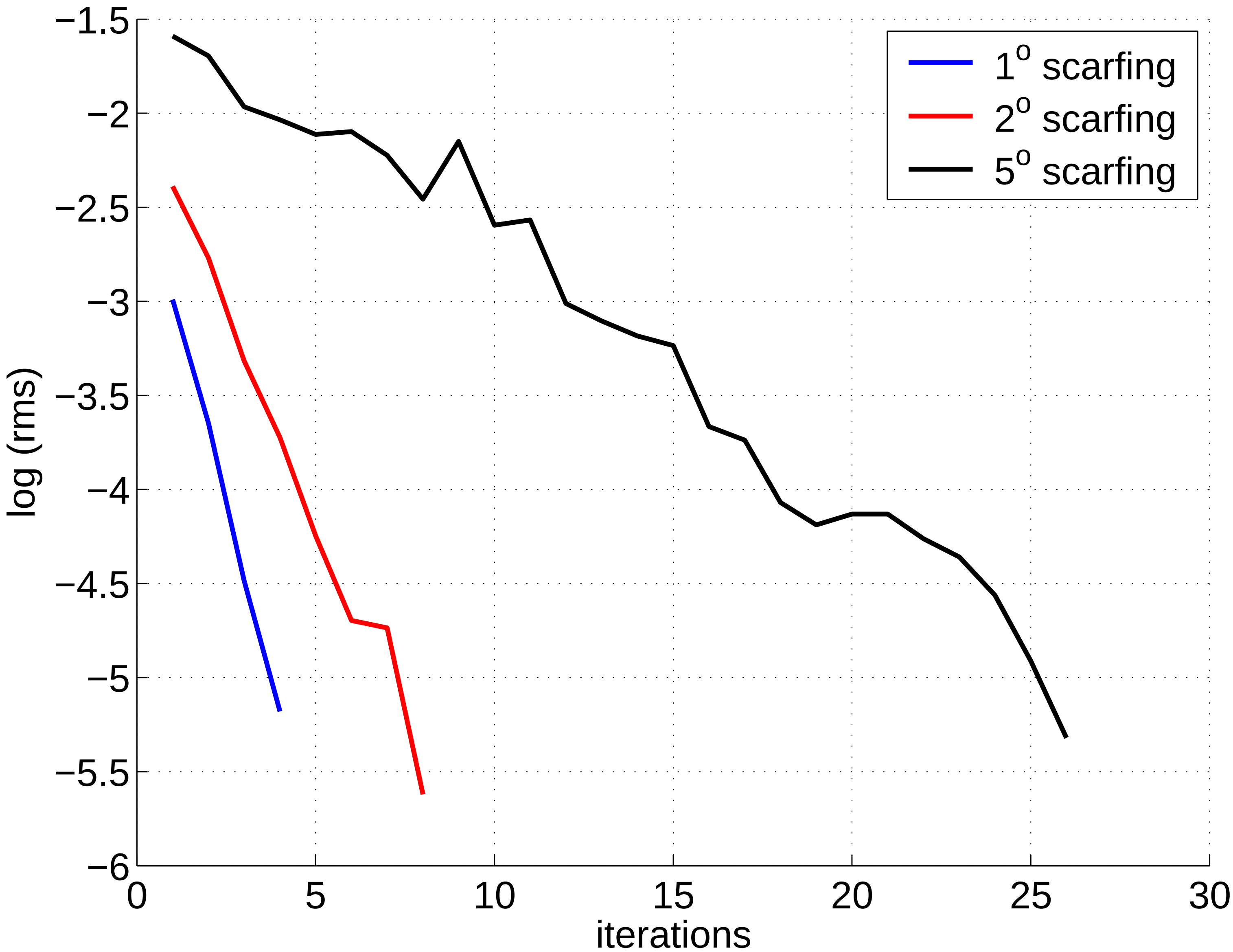


- $M_\infty = 0.3$
- $M_{\text{fan}} = 0.4$
- $\kappa = 26$
- $\dfrac{\omega R}{c_\infty} = 30$
- 15,000 grid nodes (equivalent to 3M in 3D)

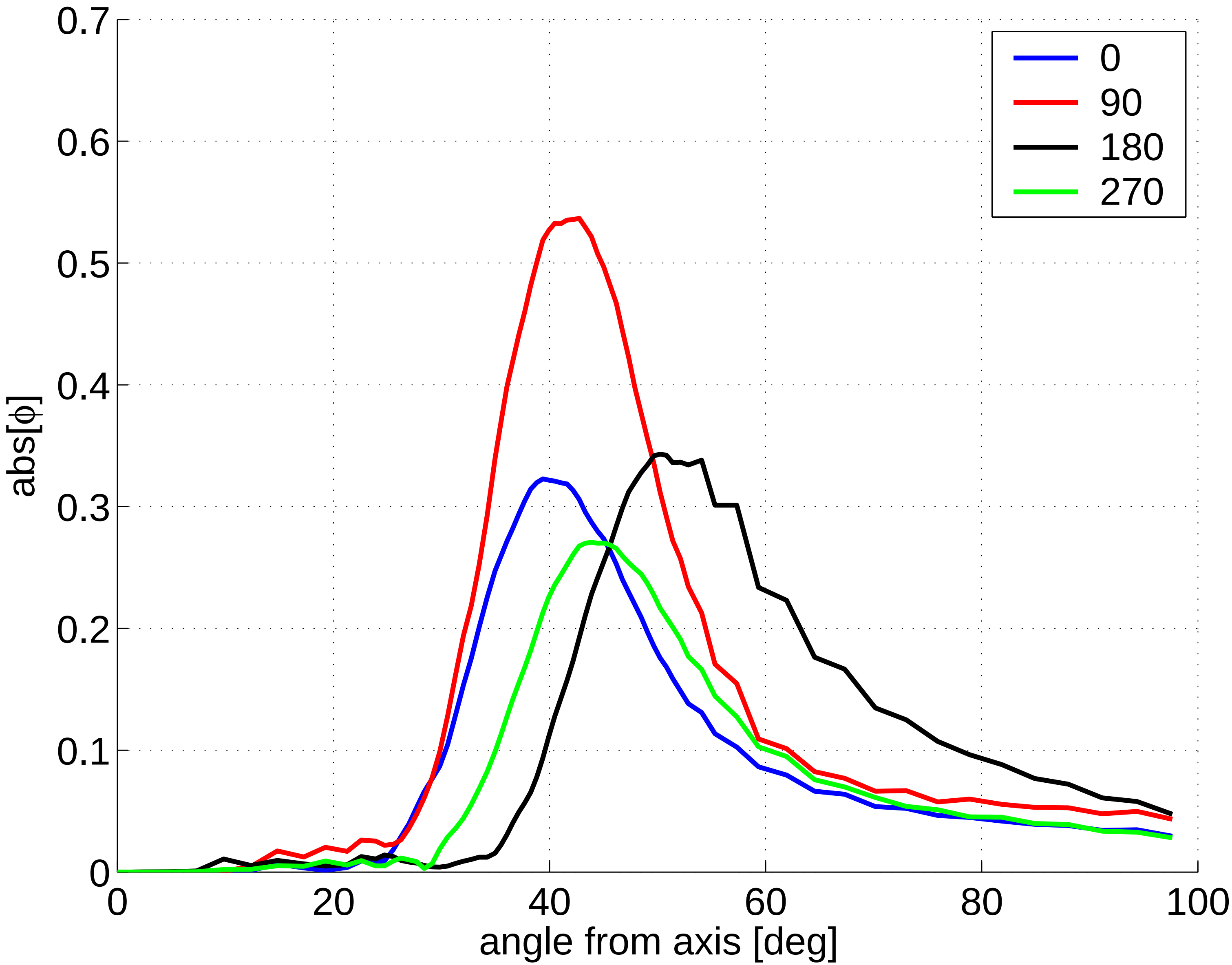
Spectral convergence



Iterative convergence



Far-field radiation pattern: 5^o



Conclusions

- hybrid FE/spectral representation very suitable for this niche application
- weak non-axisymmetry exploited by using axisymmetric preconditioning for iterative solution
- have also developed an adjoint capability for design optimisation (e.g. optimisation of shape of nacelle, or configuration of acoustic liner)

- general lesson? — if a difficult problem is closely related to a simple problem, the simple problem can be used as a very effective preconditioner