Lepton g-2: Standard Model vs Measurements

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The present experimental values:

$$a_e = 1159652180.85\ (76) \times 10^{-12}$$

0.7 parts per billion !! Odom et al., PRL97 (2006) 030801

$$a_\mu = 116592080\ (63) \times 10^{-11}$$

0.5 parts per million !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018\ (17)$$

DELPHI at LEP2 - EPJC35 (2004) 159

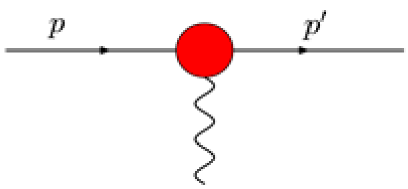
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

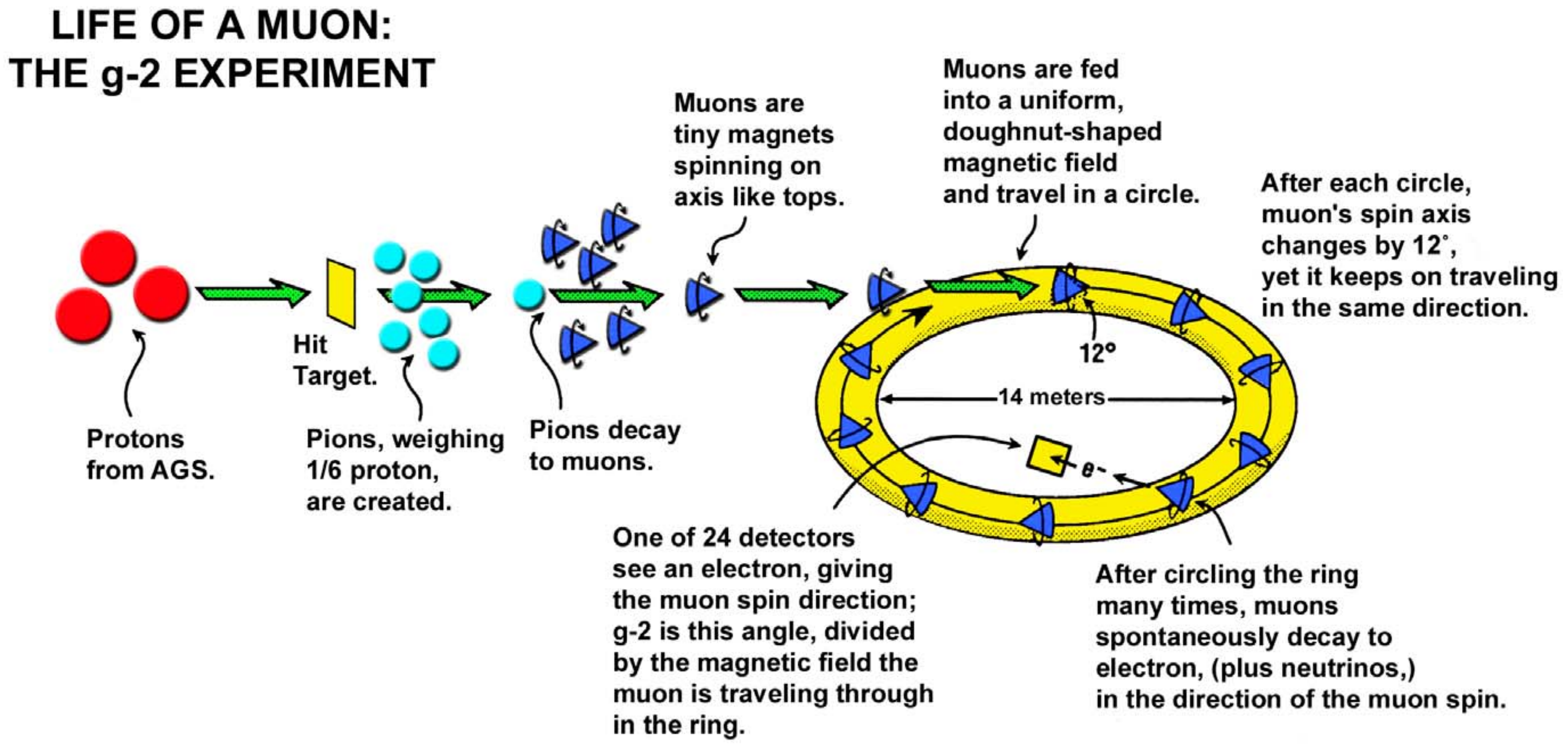
- Study the photon-lepton vertex:



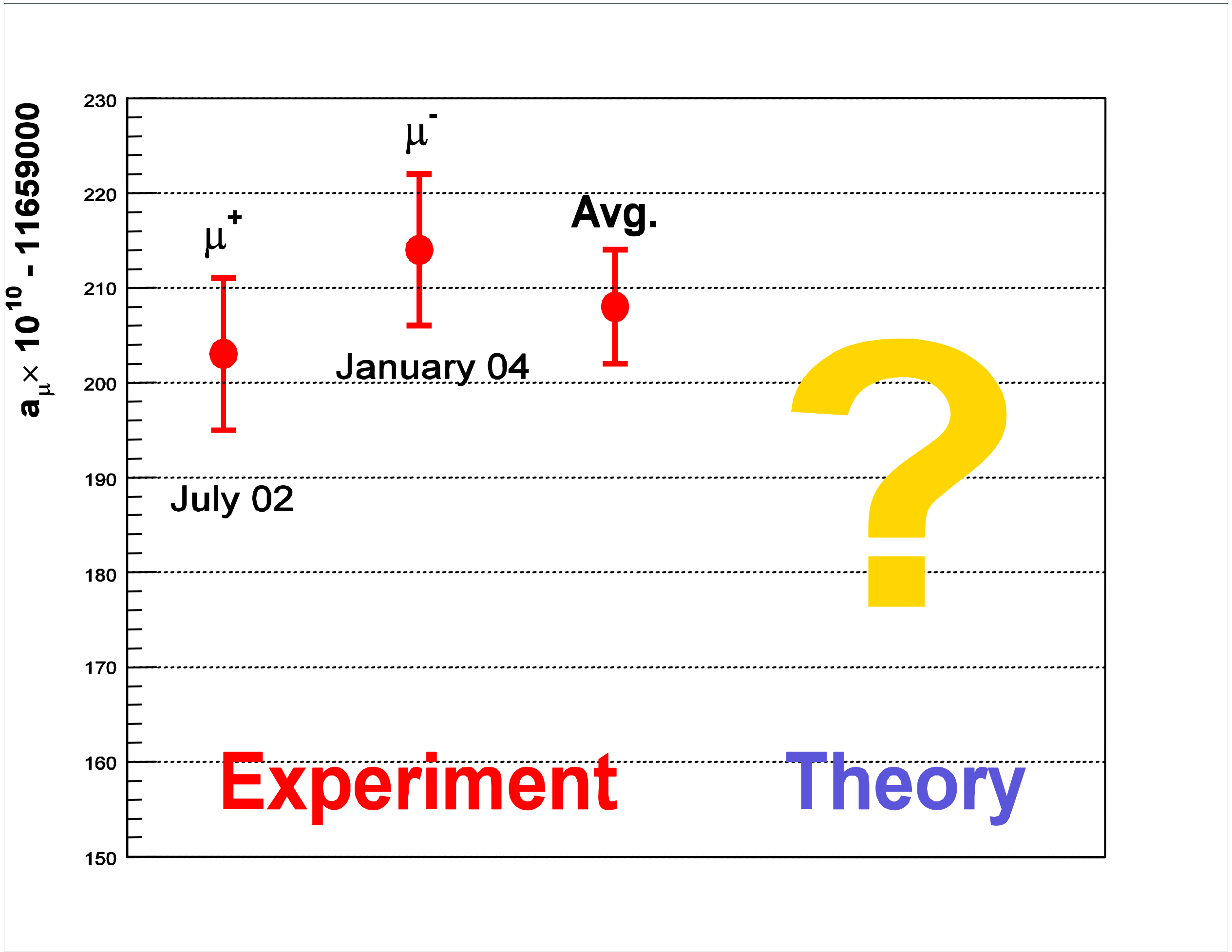
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \ldots\right] u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

The muon g-2



E821 Homepage



$$a_\mu^{EXP} = (116592080 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$$

The QED contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ 0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ 24.05050964\ (43)\ (\alpha/\pi)^3$

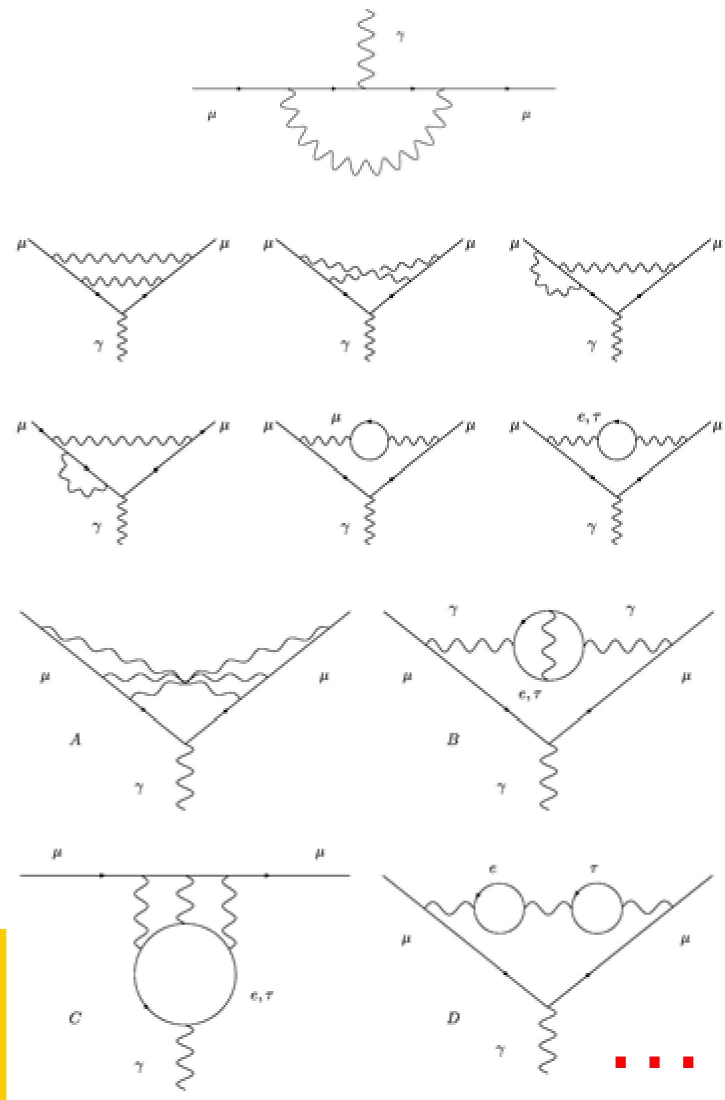
Barbieri, Laporta, Remiddi, … , Czarnecki, Skrzypek, MP '04

$+ 130.992\ (8)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04 & '05

$+ 663\ (20)\ (\alpha/\pi)^5$ In progress

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.



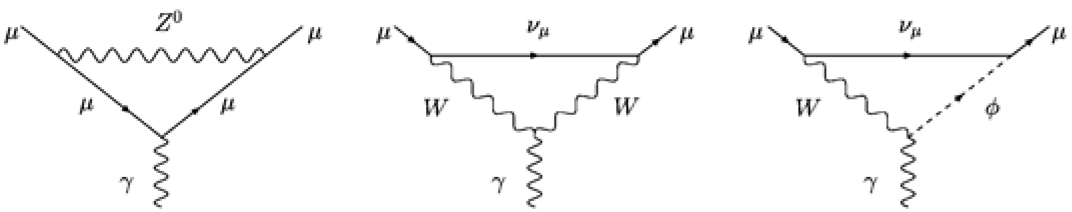
Adding up, I get:

$a_\mu^{QED} = 116584718.09\ (14)\ (08) \times 10^{-11}$

mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$

with $\alpha = 1/137.035999709\ (96)$ [0.7 ppb]

- **One-Loop Term:**



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

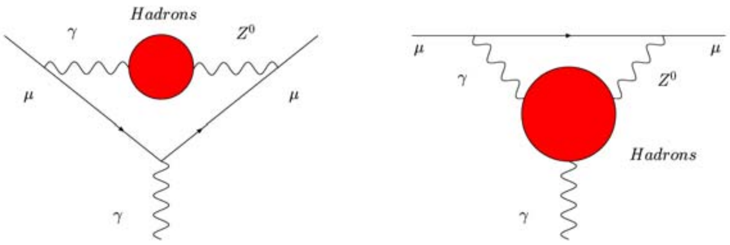
- **One-Loop plus Higher-Order Terms:**

a_μ^{EW} = 154 (2) (1) x 10^{-11}

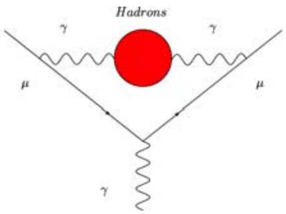
Kukhto et al. '92; Czarnecki, Krause & Marciano '95; Knecht, Peris, Perrottet & de Rafael '02; Czarnecki, Marciano & Vainshtein '02; Degrassi & Giudice '98; Heinemeyer, Stockinger & Weiglein '04; Gribouk & Czarnecki '05; Vainshtein '03.

Higgs mass, M_top error, three-loop nonleading logs

Hadronic loop uncertainties:



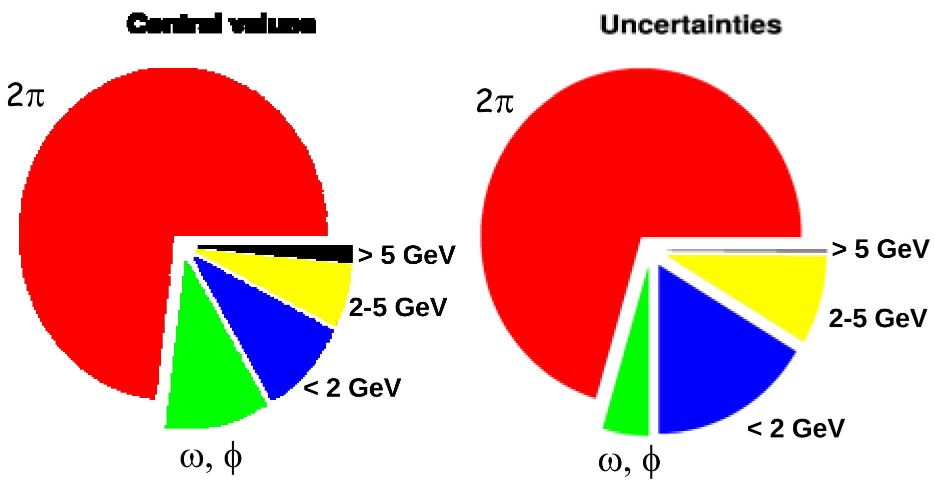
a_μ: leading hadronic contribution



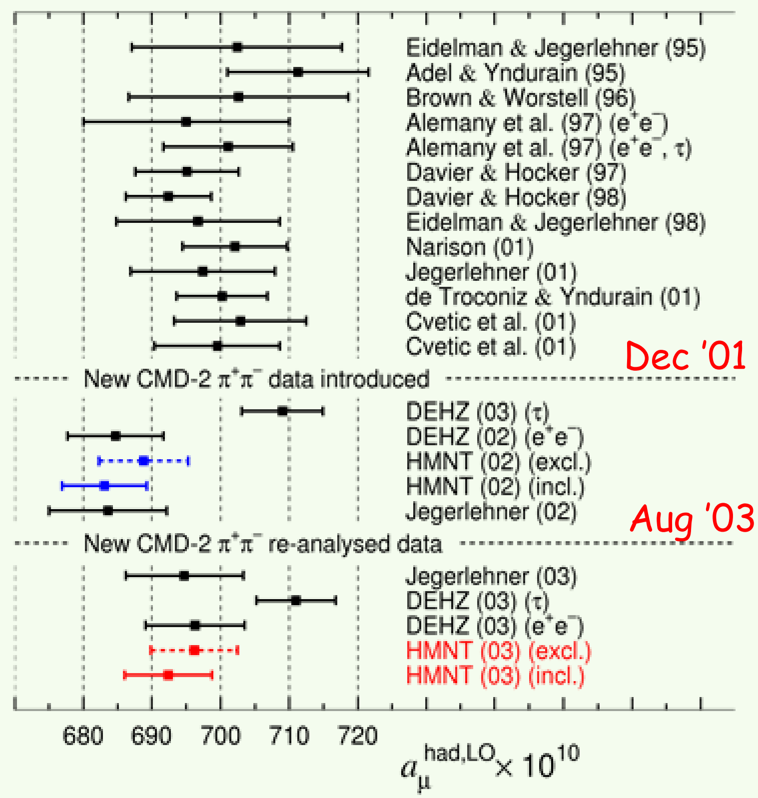
$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\,\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)\,R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961; Gourdin & de Rafael 1969



S. Eidelman, ICHEP 06, Moscow, July-August 2006



Hagiwara et al., PRD 69 (2004) 093003

Data from $e^+ e^-$ (Energy scan and ISR)

$a_\mu^{HLO} = 6909\,(39)_{exp}\,(19)_{rad}\,(7)_{qcd} \times 10^{-11}$ S. Eidelman, ICHEP 06, July 06 (DEHZ 06)

$= 6934\,(53)_{exp}\,(35)_{rad} \times 10^{-11}$ A. Hoecker, ICHEP 04, hep-ph/0410081 (DEHZ 04)

$= 6921\,(56) \times 10^{-11}$ F. Jegerlehner, hep-ph/0608329

$= 6944\,(48)_{exp}\,(10)_{rad} \times 10^{-11}$ de Troconiz, Yndurain, PRD71 (2005) 73008

$= 6894\,(42)_{exp}\,(18)_{rad} \times 10^{-11}$ Hagiwara, Martin, Nomura, Teubner, hep-ph/0611102

- Radiative Corrections (Luminosity, ISR, Vacuum Polarization, **FSR**) are a very delicate issue! All under control?
- CMD2's new (1998) $\pi^+\pi^-$ data presented at EPS 2005 and at Novosibirsk '06 agree well with their earlier (1995) ones.
- SND's $\pi^+\pi^-$ data released in 2005 have been reanalyzed (RC fixed, σ decreased – see hep-ex/0605013). There is now good agreement with the $\pi^+\pi^-$ data of CMD2.

a_μ: leading hadronic contribution - iii

- RADIATIVE RETURN: KLOE & BABAR. The collider operates at fixed energy but s_π can vary continuously. It is an important independent method!
- Some discrepancies between KLOE's and CMD2's results, even if their contributions to a_μ^{HLO} are similar (see table).
- SND's JETP101 (2005) 1053 data were significantly higher than KLOE's ones above the ρ peak, but they then decreased.
- Comparison in the range $s_\pi \in [0.37, 0.93]$ GeV^2:

$a_\mu^{\pi\pi} = (3786 \pm 27_{stat} \pm 23_{sys+th}) \times 10^{-11}$	CMD2 (95)	PLB578 (2004) 285
$a_\mu^{\pi\pi} = (3771 \pm 19_{stat} \pm 27_{sys+th}) \times 10^{-11}$	CMD2 (95+98)	S.Eidelman, ICHEP '06
$a_\mu^{\pi\pi} = (3756 \pm 8_{stat} \pm 48_{sys+th}) \times 10^{-11}$	KLOE	G.Venanzoni, ICHEP '04
$a_\mu^{\pi\pi} = (3768 \pm 13_{stat} \pm 47_{sys+th}) \times 10^{-11}$	SND (revised)	S.Eidelman, ICHEP '06

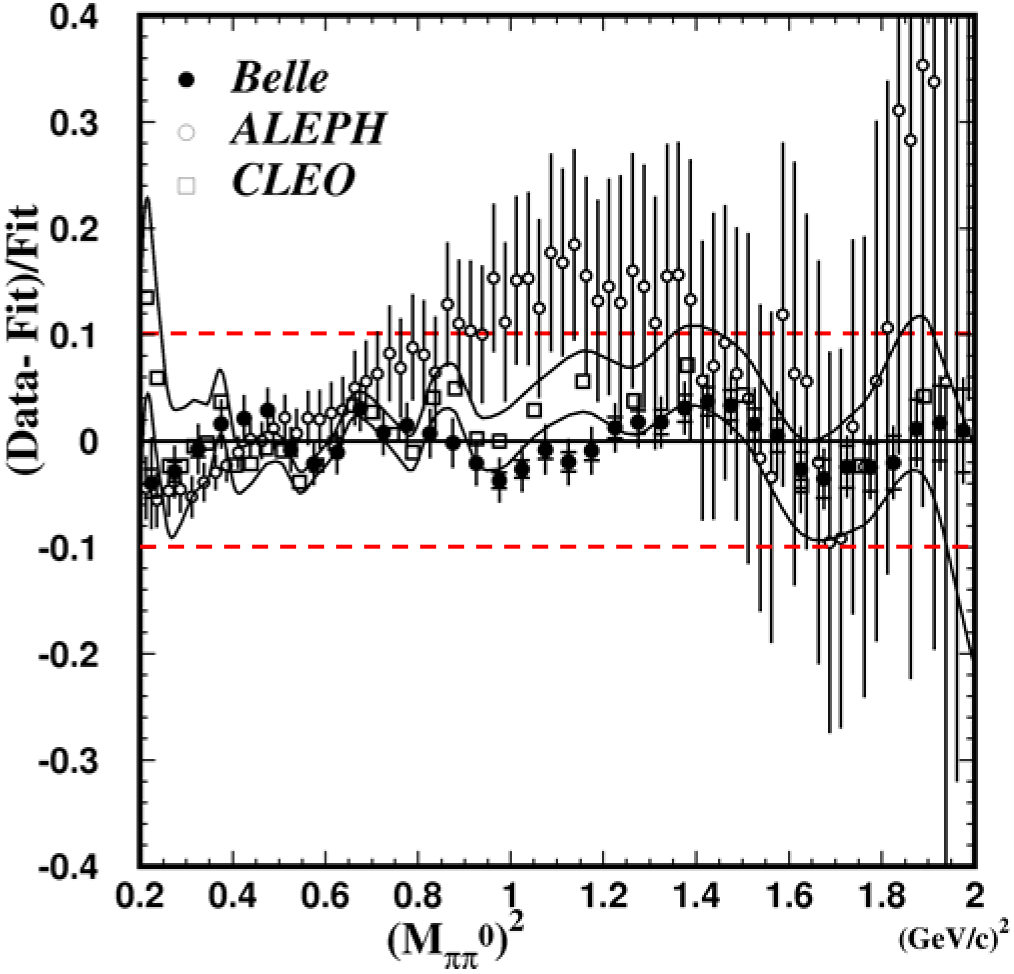
Tau Data (ALEPH, CLEO, OPAL and BELLE)

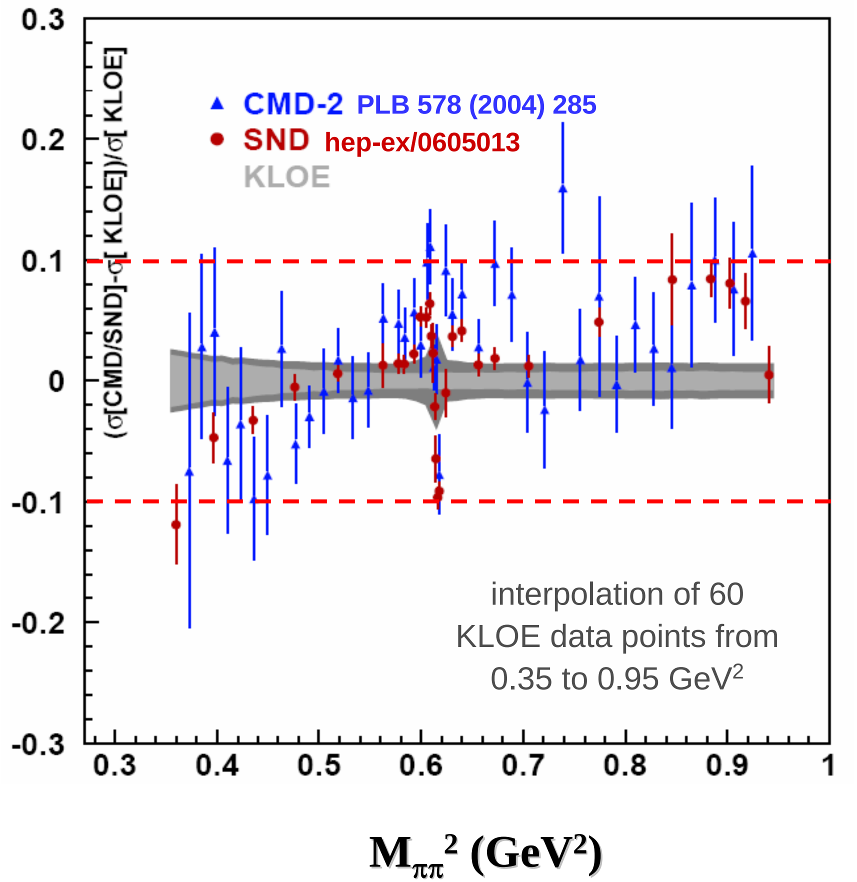
- The tau data of ALEPH and CLEO are significantly higher than CMD2 e+e- ones above ~ 0.85 GeV. KLOE confirms this discrepancy with the tau data.
- In the same region, SND no longer agrees with ALEPH.
- The preliminary tau results of BELLE seem to be in better agreement with e+e- data.
- Latest value (Davier, Eidelman, Hoecker & Zhang, EPJC31 (2003) 503):

$$a_\mu^{HLO} = 7110\ (58) \times 10^{-11}$$

- Inconsistencies in the e^+e^- or tau data? Are all possible isospin-breaking effects properly taken into account?? (Marciano & Sirlin '88; Cirigliano, Ecker & Neufeld '01-02, Flores-Baez et al. '06).
- Help from Lattice calculations??

a_μ: leading hadronic contribution - v

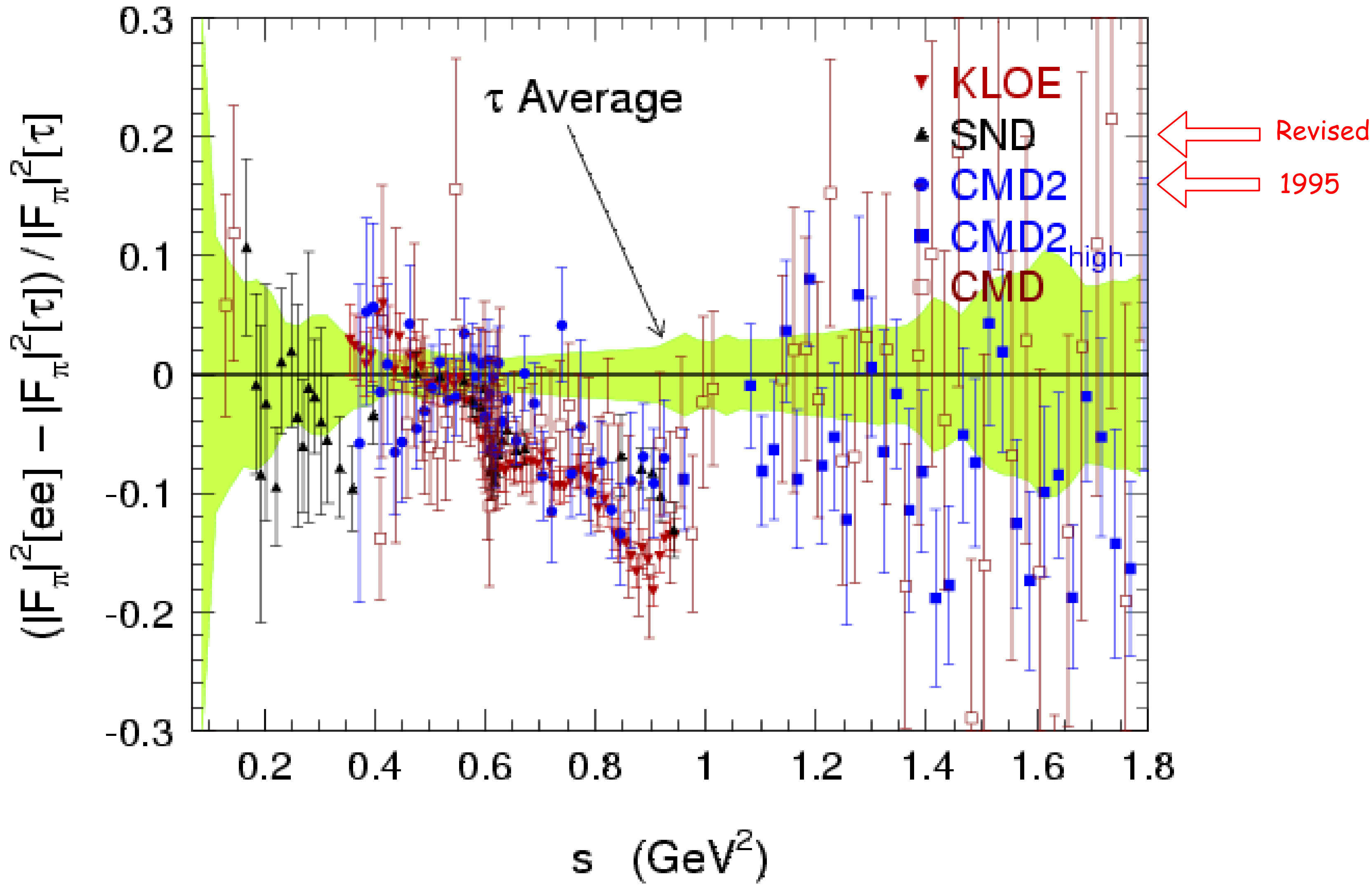




M. Fujikawa, TAU 06, September '06

A. Denig, Lepton Moments 06, June '06

a_μ: leading hadronic contribution - vi



M. Davier at TAU06, Pisa, September '06

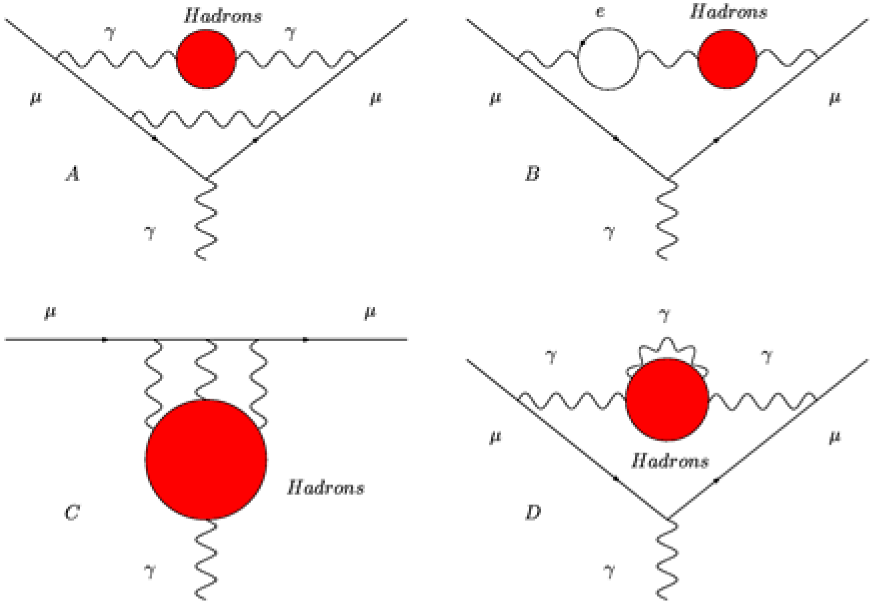
Vacuum Polarization

$O(\alpha^3)$ contribution of diagrams containing hadronic vacuum polarization insertions:

$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$

Krause '96, Alemany et al. '98, Hagiwara et al. '03,'06

Shifts by ~ -3×10^{-11} if tau data are used instead of the e^+e^- ones Davier & Marciano '04



Light-by-Light

The contribution of the hadronic l-b-l diagrams had a troubled life. The latest values vary between:

$a_\mu^{HHO}(lbl) = +\ 80\ (40) \times 10^{-11}$ Knecht & Nyffeler '02

$a_\mu^{HHO}(lbl) = +136\ (25) \times 10^{-11}$ Melnikov & Vainshtein '03

based on Hayakawa & Kinoshita '98 & '02; Bijnens, Pallante and Prades '96 & '02; Knecht, Nyffeler, Perrottet & de Rafael '02.

This may become the ultimate limitation of the SM prediction.

a_μ: Standard Model vs. Experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

$a_\mu^{SM} \times 10^{11}$	$(a_\mu^{EXP} - a_\mu^{SM}) \times 10^{11}$	σ		HLO Reference
116591763 (60)	317 (87)	3.7	$\langle 3.2 \rangle$	[1] (e^+e^-)
116591775 (69)	305 (93)	3.3	$\langle 2.8 \rangle$	[2] (e^+e^-)
116591798 (63)	282 (89)	3.2	$\langle 2.7 \rangle$	[3] (e^+e^-)
116591748 (61)	332 (88)	3.8	$\langle 3.4 \rangle$	[4] (e^+e^-)
116591961 (70)	119 (95)	1.3	$\langle 0.7 \rangle$	[5] (τ)

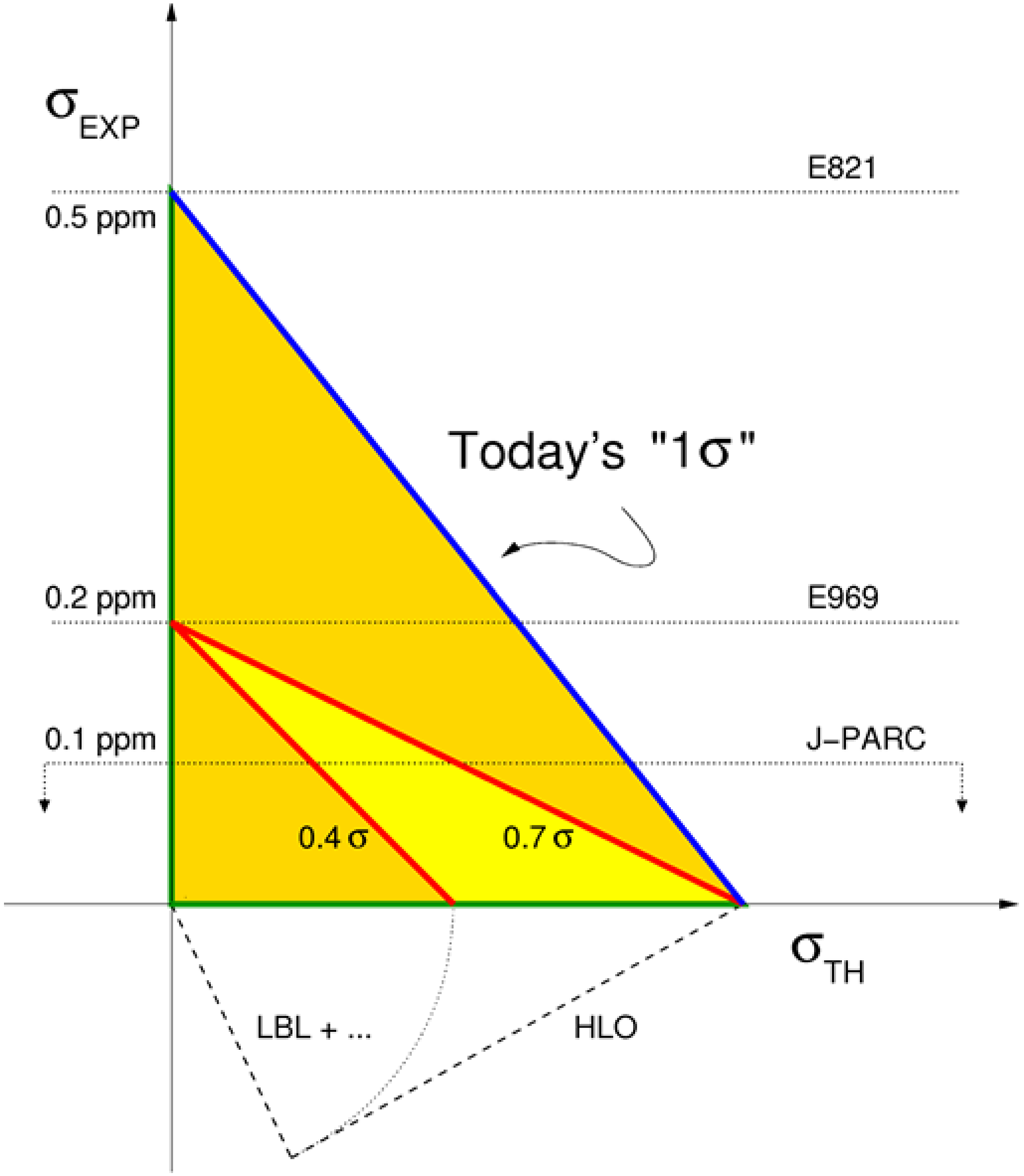
$a_\mu^{HHO}(lbl) = 80\ (40) \times 10^{-11}$ except angle brackets.

$a_\mu^{HHO}(lbl) = 136\ (25) \times 10^{-11}$

[1] S. Eidelman @ICHEP06 July 2006 (DEHZ06, update of DEHZ03, ref. [5]).
[2] F. Jegerlehner, hep-ph/0608329, August 2006.
[3] J.F. de Troconiz and F.J. Yndurain, PRD71 (2005) 073008.
[4] Hagiwara, Martin, Nomura & Teubner, hep-ph/0611102, November 2006.
[5] Davier, Eidelman, Hoecker and Zhang, EPJC31 (2003) 503.

The th. error is now the same (or even smaller) as the exp. one!

The future of a_μ?



The electron g-2

The electron g-2…

$a_e^{SM} = (1/2)(\alpha/\pi) - 0.328\ 478\ 444\ 002\ 90(60)\ (\alpha/\pi)^2$

Schwinger 1948 — Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$A_2^{(4)}(m_e/m_\mu) = 5.197\ 386\ 70\ (28) \times 10^{-7}$

$A_2^{(4)}(m_e/m_\tau) = 1.837\ 62\ (60) \times 10^{-9}$

$+\ 1.181\ 234\ 016\ 827\ (19)\ (\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel; Mohr & Taylor '05; MP '06

$A_2^{(6)}(m_e/m_\mu) = -7.373\ 941\ 64\ (29) \times 10^{-6}$

$A_2^{(6)}(m_e/m_\tau) = -6.5819\ (19) \times 10^{-8}$

$A_3^{(6)}(m_e/m_\mu,\ m_e/m_\tau) = 1.909\ 45\ (62) \times 10^{-13}$

$-\ 1.7283\ (35)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio July '05.

$+\ 0.0\ (3.8)\ (\alpha/\pi)^5$ In progress (12672 diagrams!)

Mohr & Taylor '05; Kinoshita & Nio, in progress.

$+\ 1.671\ (19) \times 10^{-12}$ Hadronic

Mohr & Taylor '05; Davier & Hoecker '98, Krause '97, Knecht '03

$+\ 0.0297\ (5) \times 10^{-12}$ Electroweak

Mohr & Taylor '05; Czarnecki, Krause, Marciano '96

... and the best determination of alpha

- The new measurement of the electron g-2 is:

  $a_e^{exp} = 1159652180.85\ (76) \times 10^{-12}$ Odom et al, PRL97 (2006) 030801

  vs. old (factor of 6 improvement, 1.7σ difference):

  $a_e^{exp} = 1159652188.3\ (4.2) \times 10^{-12}$ Van Dyck et al, PRL59 (1987) 26

  Equating $a_e^{SM}(\alpha) = a_e^{exp} \rightarrow$ best determination of alpha to date:

α^{-1} = 137.035 999 709 (12)(30)(2)(90) [0.7ppb] Gabrielse et al, '06; MP '06

(12) → δC_4^{qed}, (30) → δC_5^{qed}, (2) → δa_e^{had}, (90) → δa_e^{exp}

- Compare it with other determinations:

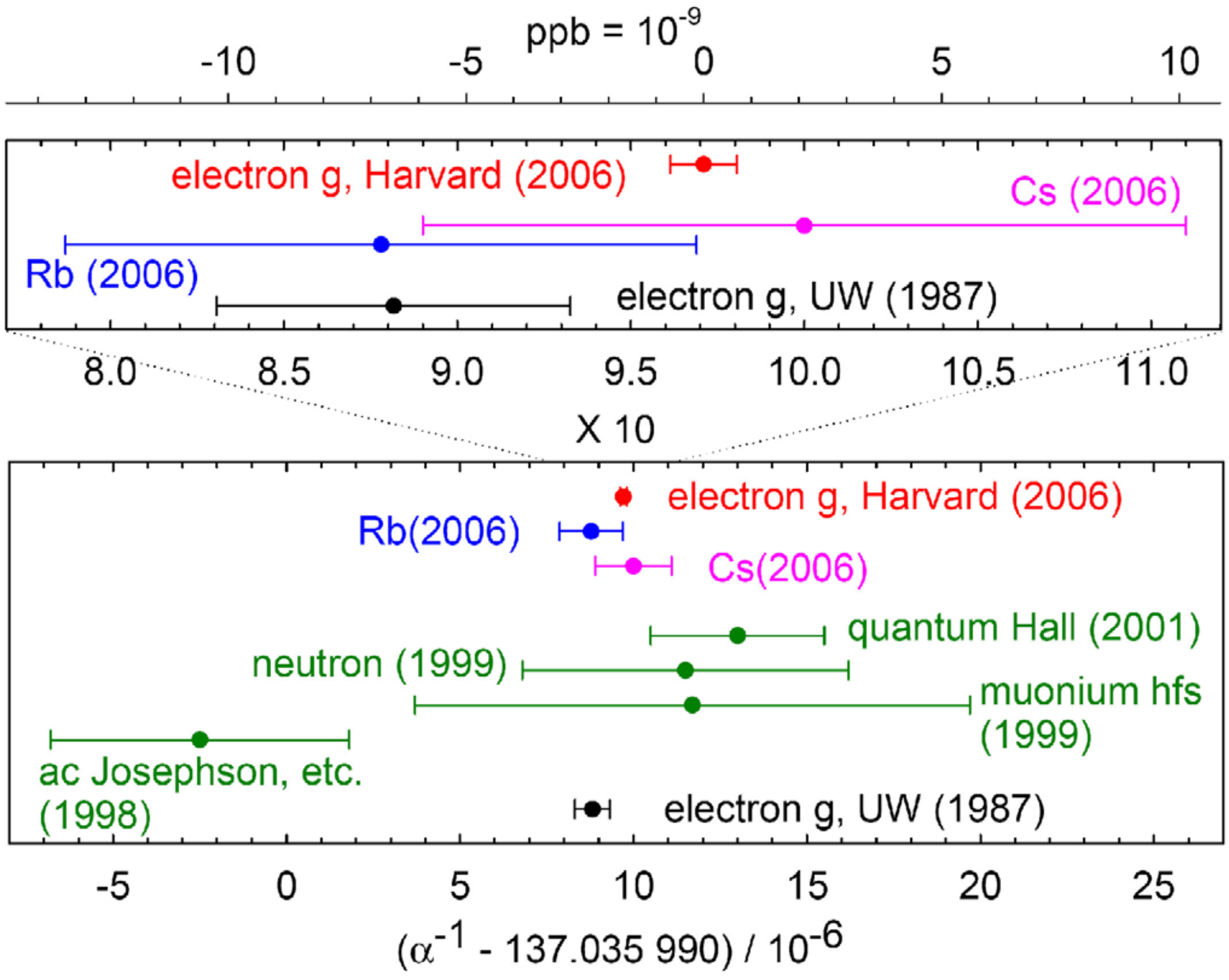
α^{-1} = 137.036 000 00	(110)	[8.0 ppb]	PRA73 (2006) 032504 (Cs)
α^{-1} = 137.035 998 78	(91)	[6.7 ppb]	PRL96 (2006) 033001 (Rb)

between 0.3 and 1 σ $\rightarrow$ beautiful test of QED at 4-loop level!

Old values were:

α^{-1} = 137.035 998 83	(50)	[3.6 ppb]	CODATA '98 based on UW '87
α^{-1} = 137.035 999 11	(46)	[3.3 ppb]	CODATA '02 = PDG'04 = PDG '06

Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL97 (2006) 030802

The g-2 of the tau

The QED contribution to a_τ

- $a_\tau^{QED} = (1/2)(\alpha/\pi) + 2.057\,457\,(93)\,(\alpha/\pi)^2$

  Schwinger 1948 — Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; Samuel, Li & Mendel '91; Narison '01; MP '06

  $$A_1^{(4)} = -0.328\,478\,965\,579\ldots$$
  $$A_2^{(4)}(m_\tau/m_e) = 2.024\,284\,(55)$$
  $$A_2^{(4)}(m_\tau/m_\mu) = 0.361\,652\,(38)$$

  $+\ 57.9315\,(27)\,(\alpha/\pi)^3$

  Kinoshita, Barbieri, Laporta, Remiddi, … ; Samuel, Li & Mendel '91; Narison '01; MP '06

  $$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$
  $$A_2^{(6)}(m_\tau/m_e) = 46.3921\,(15)$$
  $$A_2^{(6)}(m_\tau/m_\mu) = 7.01021\,(76)$$
  $$A_3^{(6)}(m_\tau/m_e, m_\tau/m_\mu) = 3.347\,97\,(41)$$

  New (hep-ph/0606174)

  $+\ ?\,(??)\,(\alpha/\pi)^4$

  Who? When??

- Adding up →

  $a_\tau^{QED} = 117324\,(2) \times 10^{-8}$

  2×10^{-8}: estimate of missing 4-loop

  $[\alpha\ 1/137.035999709\,(96)]$

The EW and Hadronic corrections to a_τ (are large)

EW (1- and 2-loop) corrections

$$a_\tau^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\tau^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\tau^2}{M_{Z,W,H}^2}\right)\right] = 55.1\,(1) \times 10^{-8}$$

Studenikin '90, included

Higgs mass (and τ mass error)

a_τ^{EW} (1-loop) = 55.09×10^{-8}
a_τ^{EW} (2-loop frm) = -4.68×10^{-8}
a_τ^{EW} (2-loop bos) = -3.06×10^{-8}

$a_\tau^{EW} = 47.4\,(5)(2) \times 10^{-8}$ New

Higgs mass, M_top error

Hadronic loop uncertainties, missing 3-loop contributions

Samuel, Li & Mendel '91: $55.60(2)\times10^{-8}$; Czarnecki, Krause & Marciano '95; Czarnecki & Krause '97; Narison '01; Eidelman, Giacomini, Ignatov & MP '06

Hadronic corrections:

$a_\tau^{HLO} = 360\,(30)(10) \times 10^{-8}$	Samuel, Li & Mendel '91
$a_\tau^{HLO} = 343.3\,(9.1) \times 10^{-8}$	Eidelman & Jegerlehner '95
$a_\tau^{HLO} = 353.6\,(4.0) \times 10^{-8}$	Narison 2001
$a_\tau^{HLO} = 351.7\,(3.9) \times 10^{-8}$	Eidelman, Giacomini, Ignatov & MP '06, preliminary
$a_\tau^{HHO}(\text{vac}) = 7.6(2) \times 10^{-8}$	Krause '96
$a_\tau^{HHO}(\text{lbl}) = 38\,(7) \times 10^{-8}$	Rescaling of a_μ^{HHO}(lbl) of Melnikov&Vainshtein

The SM prediction of the tau (g-2)

- Adding up, we get the complete Standard Model prediction:

a_τ^{SM}	=	117324	(2)	$\times\ 10^{-8}$	QED
	+	47.4	(0.5)	$\times\ 10^{-8}$	EW
	+	351.7	(3.9)	$\times\ 10^{-8}$	HLO
	+	46	(7)	$\times\ 10^{-8}$	HHO

$$a_\tau^{SM} = 117769\ (8) \times 10^{-8}$$

	Muon	Tau
a^{EW}/a^{HAD}	1/45	1/8
$a^{EW}/\delta a^{HAD}$	3	6

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics… …if only we could measure it!

Conclusions

- Beautiful examples of interplay between theory and experiment:
  g_e probed at <ppt! → α and extraordinary test of QED's validity;
  g_μ probed at <ppb → test of the full SM and "New Physics";
  g_τ well... theory is ahead of experiment! Great NP sensitivity.

- a_μ: The discrepancy Δ(Exp-SM) is more than 3 σ, if e^+e^- data are used (recent CMD2 and SND results are already included!).

- a_μ: With tau data, Δ(Exp-SM) ~ 1 σ only! The e^+e^- vs tau puzzle is still unsolved. Unaccounted isospin viol. corrections? Problems in the e^+e^- or τ data? Revised SND no longer agrees with Aleph; Preliminary Belle's τ data seem to be in better agreement with e^+e^-. More work and data needed from KLOE, Babar, Belle...

- Future: Further improvements in a_e^{EXP}? Possibilities for a_τ^{EXP}??
  a_μ: QED and EW sectors ready for E969 challenge! Hadronic sector needs more work and future exp. results: VEPP-2000 (DAFNE-2?). An improvement by a factor of 2 is challenging but possible! The effort is certainly worth the opportunity a_μ is providing us to unveil (or just constrain) "New Physics" effects!

The End

The effective fine-structure constant at the scale s is given by:

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha} \quad \text{with} \quad \Delta\alpha = \Delta\alpha_{lep} + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

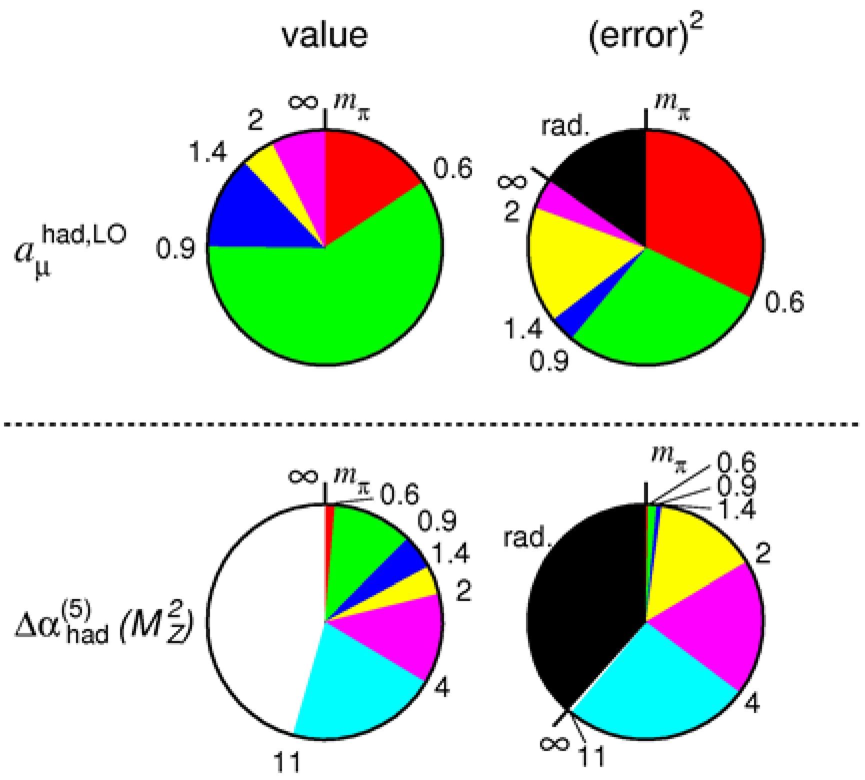
The light quarks part is determined by:

$$\Delta\alpha^{(5)}_{had}(M_z^2) = -\frac{\alpha M_z^2}{3\pi}\int_{s_{thr}}^{\infty} ds \frac{R(s)}{s(s-M_z^2-i\epsilon)}.$$

Progress due to significant improvement of the data (mostly CMD-2 and BES):

$\Delta\alpha_{had}^{(5)}(M_z^2)$ =

0.02800 (70)	Eidelman, Jegerlehner'95
0.02761 (36)	Burkhardt, Pietrzyk 2001
0.02755 (23)	Hagivara et al., 2004
0.02758 (35)	Burkhardt, Pietrzyk 6-05



Hagivara et al., PRD69 (2004) 093003