On neutrino mass in left-right symmetric theories

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A master equation

Neutrino mass: oscillations, neutrinoless 2β decay, large scale structures, ...

Electroweak symmetry breaking scale: LHC physics

Neutrino Yukawa coupling: the way Dirac fermions get mass

$$m_\nu = v_L f - v^2 y (v_R f)^{-1} y$$

Sub-eV scale v_L versus Grand Unification scale v_R : seesaw mechanism

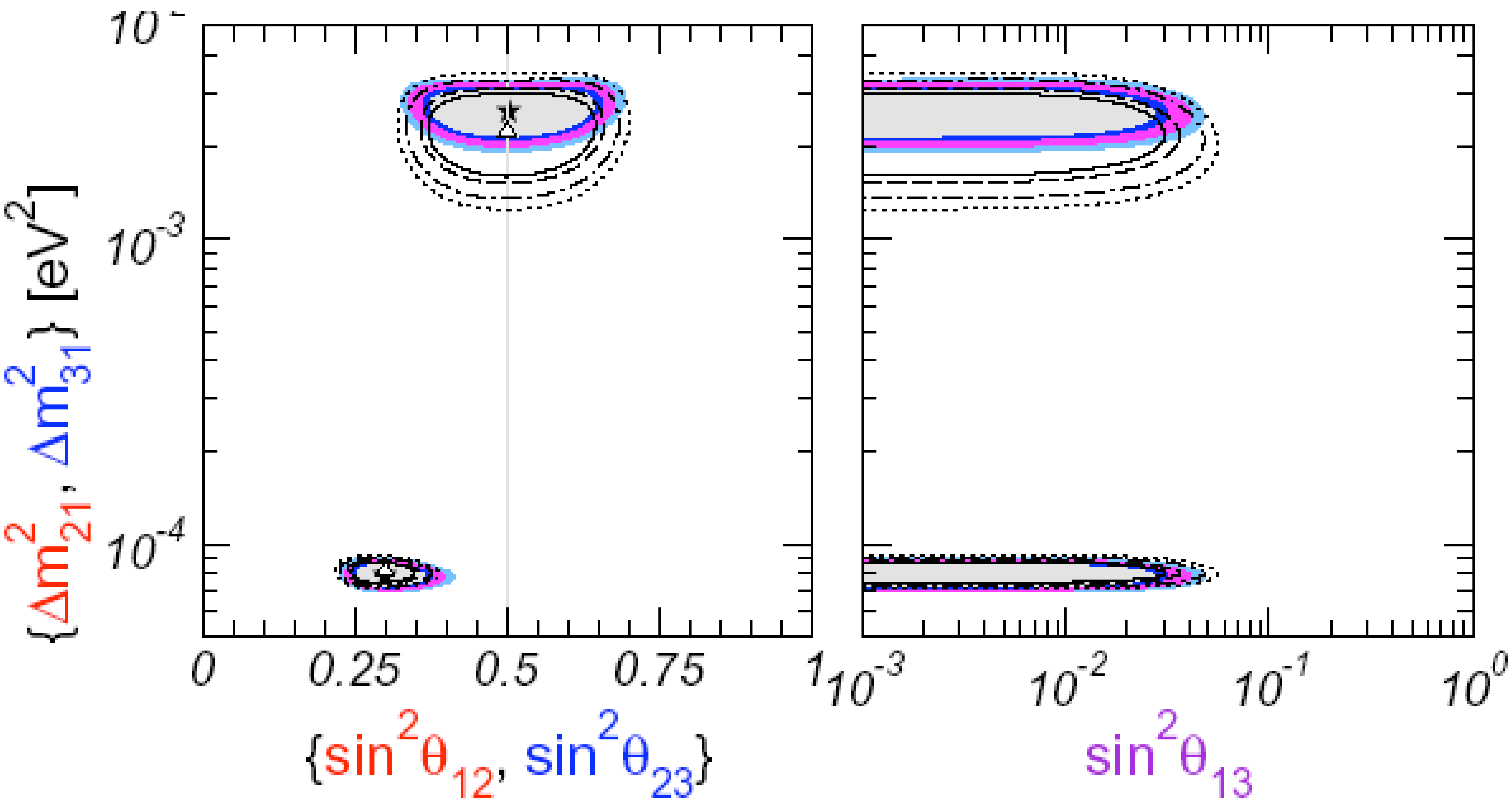
Majorana-type coupling: lepton number violation in a Left-Right symmetric way

Outline

- A theoretical perspective on present and future experimental results on the neutrino mass
- From tiny neutrino masses to energy scales beyond the Standard Model: the seesaw mechanism
- A non-minimal well-motivated framework: models with left-right gauge symmetry
- A bottom-up reconstruction of the super-heavy seesaw sector and its implications for
  - ✳ baryogenesis via leptogenesis
  - ✳ Grand Unification theories

Status of oscillations data

3 active light neutrinos (no sterile states): a global fit



Maltoni, Schwetz, Tortola, Valle, NJP 6 (2004) 122 [hep-ph/0405172 **v5**]

Mixing angle	Data	$\sin^2\theta_{exp}$ (at 2σ)	$\sin^2\theta_{Tri\text{-}Bi\text{-}Maximal}$
2-3	Atm - K2K - Minos	$0.50\left(^{+0.26}_{-0.24}\right)$	1/2
1-2	Solar - KamLAND	$0.30\left(^{+0.20}_{-0.13}\right)$	1/3
1-3	Chooz	0.000(+0.025)	0

Status of oscillations data

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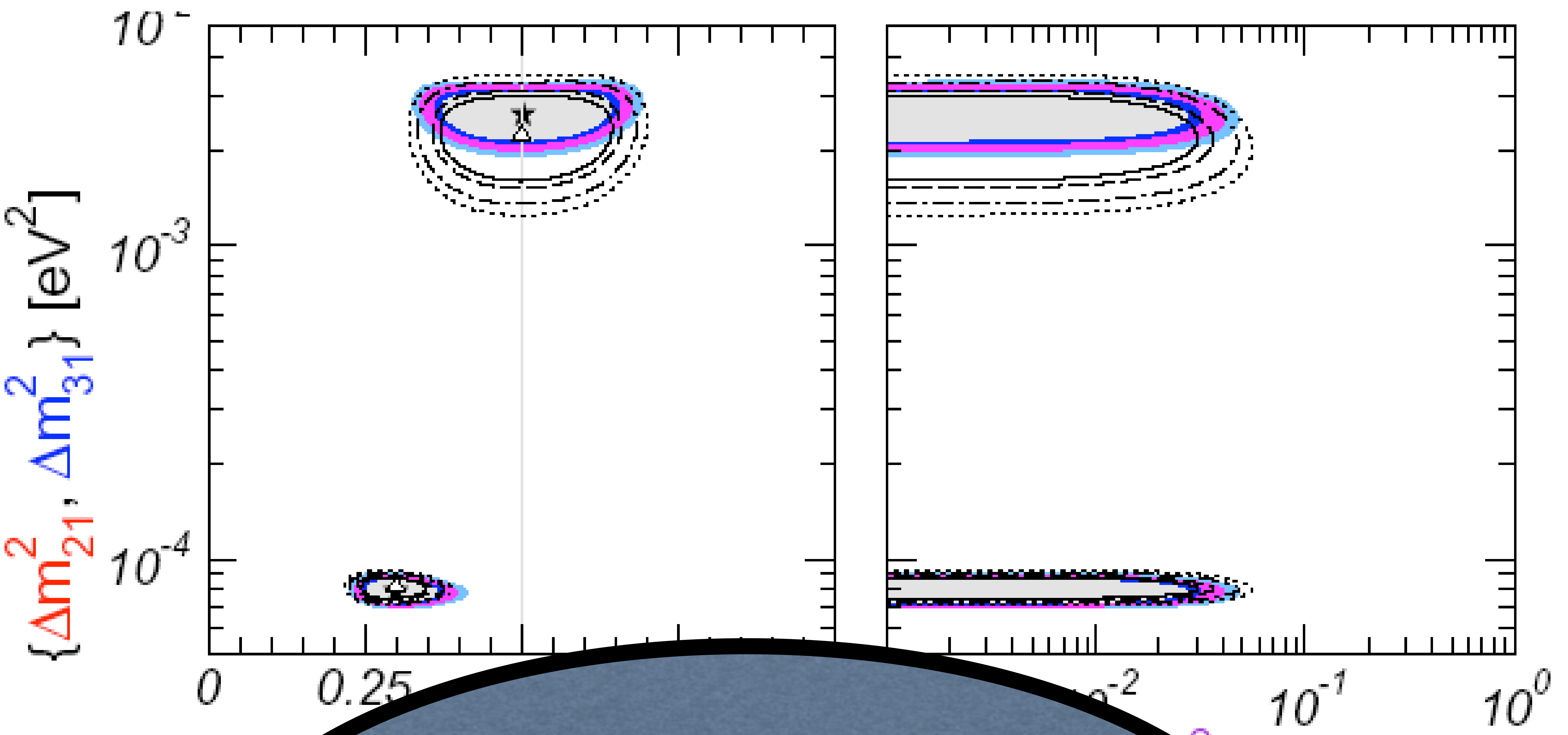


Maltoni, Schwetz, Tortola, Valle, NJP 6 (2004) 122 [hep-ph/0405172 **v5**]

Beams: OPERA, MINOS
Reactors: Double Chooz
SuperBeams: T2K, NOνA
...

Mixing		(at 2σ)	$\sin^2\theta_{\text{Tri-Bi-Maximal}}$
2-3		$(^{1+0.26}_{1-0.24})$	1/2
1-2		$0.30(^{1+0.20}_{1-0.13})$	1/3
1-3	Chooz	0.000(+0.025)	0

ν mass spectrum: open questions

$$\Delta m^2_{12} \equiv m^2_2 - m^2_1 = 7.9 \cdot 10^{-5}\text{eV}(1 \pm 0.09)$$

$$\Delta m^2_{23} \equiv |m^2_3 - m^2_2| = 2.4 \cdot 10^{-3}\text{eV}(1^{+0.21}_{-0.26})$$

- Future oscillation experiments may measure **sign(m_3^2 - m_2^2)**
- Absolute mass scale **m_i** unknown, but constrained by:
  - **tritium β decay**: m_i < 2.2 eV [Katrin 3 years: m_i < 0.2 eV]
  - **neutrinoless 2β decay**: m_{ee} < (0.3 ÷ 1.0) eV
    [Cuoricino & Nemo-3: m_{ee} < 0.1 eV]
  - **cosmological bounds**: $\Sigma_i m_i$ < (0.4 ÷ 0.7) eV
    [Planck CMB + lensing: $\sigma(\Sigma_i\, m_i) \approx 0.05$ eV ⇒ m_i determined!]
- If neutrinos are Majorana, two unknown CP violating phases **arg(m_1/m_2)** (enters m_{ee}) and **arg(m_3/m_2)** (not accessible)

Neutrino mass matrix

The most sound theoretical interpretation of all neutrino data: add to the Standard Model a **3x3 Majorana mass term**

$$m_\nu = U diag(m_1, m_2, m_3) U^T \ , \qquad U = U(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$$

- **Knowns**: θ_{12} , θ_{23} , $m_2^2 - m_1^2$ and $|m_3^2 - m_2|^2$; upper bounds on θ_{13} and $|m_i|$.
- **Unknowns**: θ_{13} , sign$(m_3^2 - m_2^2)$, $|m_i|$ and three CP phases, δ , arg(m_1/m_2) , arg(m_3/m_2)

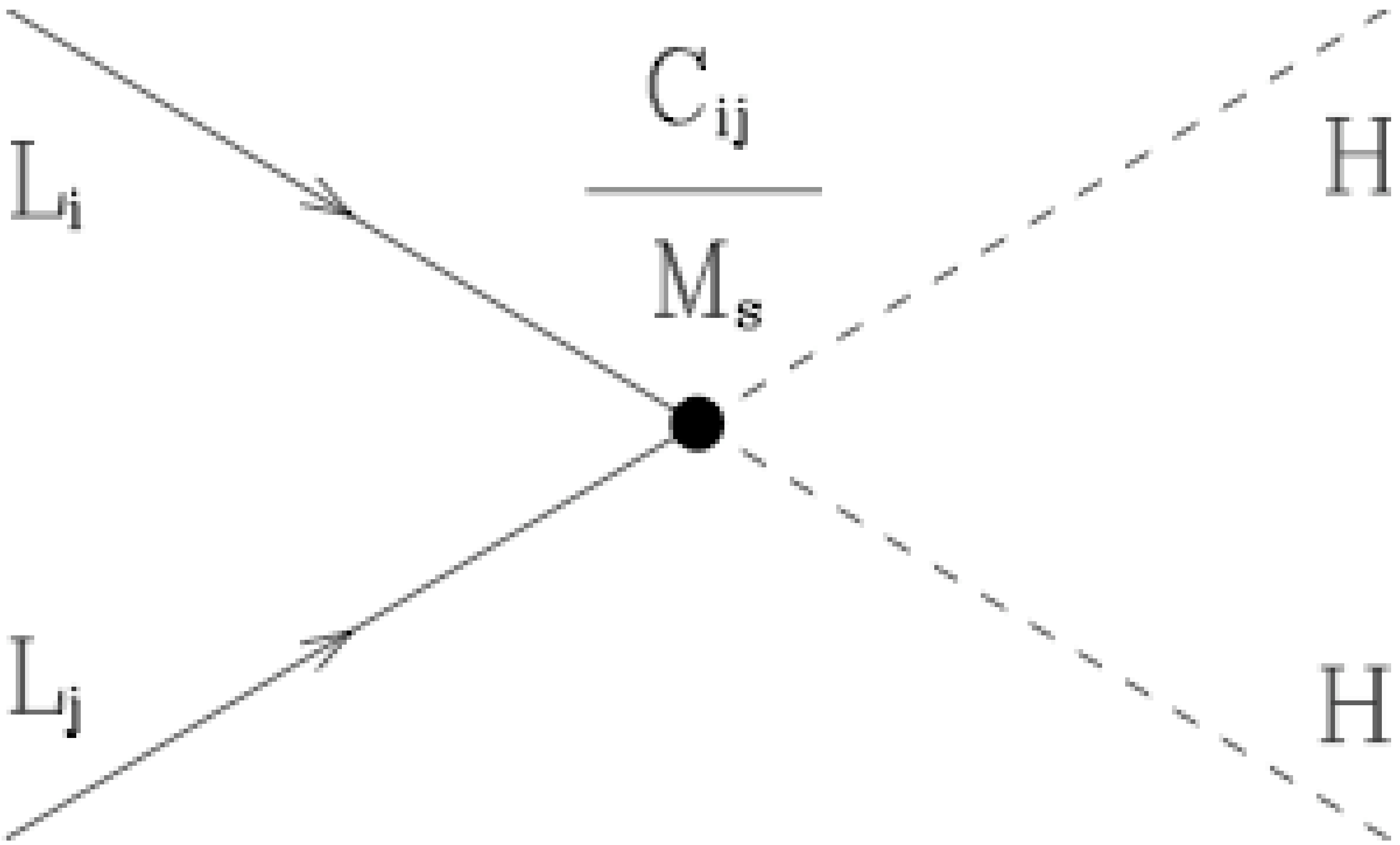
The **structure of m_ν** provides a crucial clue on the particle theory beyond the Standard Model

Theoretical priorities = fix the largest uncertainties in the structure of m_ν:

(I) mass spectrum
(II) Majorana CP phases
(III) mixing angles
(IV) Dirac CP phase

What ν physics beyond SM?

A non-zero Majorana neutrino mass may be introduced as the effect of the **unique dimension 5 effective operator**:



$$C_{ij}\frac{L_i L_j H H}{M_s} \Rightarrow (m_\nu)_{ij}\nu_i\nu_j$$

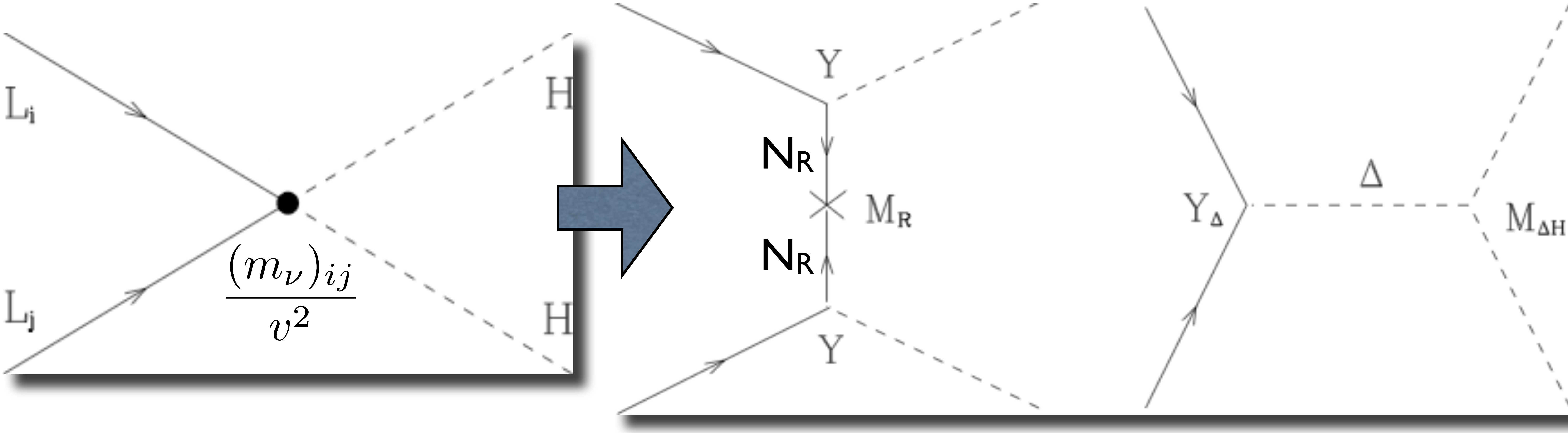
EWSB:
<H> = 174 GeV

A host of new physics candidates brings a contribution to neutrino mass through this operator:

$$m_\nu = \sum_i m_\nu^{(i)}$$

Seesaw mechanisms

Seesaw means to interpret the effective operator as the **exchange of a certain super-heavy particle**



Minkowski, Gell-Mann, Ramond, Slansky, Yanagida, Glashow, Mohapatra, Senjanovic, Magg, Wetterich, Lazarides, Shafi, Schecter, Valle, Foot, Lew, He, Joshi, Ma

Seesaw Mechanism in 3 possible versions:

[type I] SM singlet fermions N_R : $m_\nu \sim v^2 / M_R$

[type II] $SU(2)_L$ triplet scalars Δ : $m_\nu \sim v^2 / M_\Delta$

[type III] $SU(2)_L$ triplet fermions Σ : $m_\nu \sim v^2 / M_\Sigma$

From a mechanism to a theory

Seesaw explains (i) **smallness of ν mass**

(ii) **baryogenesis via leptogenesis**

However the heavy scale and the new particles are ad hoc...

minimal **Left-Right gauge symmetry**:

$\mathbf{SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \quad \rightarrow \quad SU(2)_L \times U(1)_Y$

extensions: SU_{422} , SO(10), …

Pati, Salam, Mohapatra, Senjanovic, Georgi, Fritzsch, Minkowski

(i) **right-handed neutrinos** are incorporated naturally

(ii) **maximal parity violation** can be understood

(iii) **Grand Unification** gives a rationale for the heavy scale

(iv) **supersymmetry** can be easily incorporated & **R-parity** is unbroken [if only (B-L)-even Higgs bosons acquire VEVs]

Left-Right symmetric ν mass

Fields:	L = (ν, e)	$L^c = (N^c, e^c)$	$\Phi = (H_u, H_d)$	Δ_L	Δ_R
$SU(2)_L$	2	1	2	3	1
$SU(2)_R$	1	2	2	1	3
$U(1)_{B-L}$	-1	1	0	2	-2

Lepton Yukawas: $\mathcal{L}_Y = yLL^c\Phi + \frac{f}{2}(LL\Delta_L + L^cL^c\Delta_R)$

(both y and f are 3x3 **symmetric matrices**)

VEVs: - $\mathbf{v_R = \langle\Delta_R^0\rangle}$ breaks SU_{221} into SU_{21}

- $\mathbf{v = \langle\Phi^0\rangle}$ breaks SU_{21} into $U(1)_{em}$

- $\mathbf{v_L = \langle\Delta_L^0\rangle} \sim v^2/M_\Delta$ is induced by EW breaking

Mass matrix in (ν, N) basis: $M_\nu = \begin{pmatrix} v_L f & vy \\ vy & v_R f \end{pmatrix}$

Left-Right symmetric seesaw

$$M_\nu = \begin{pmatrix} v_L f & vy \\ vy & v_R f \end{pmatrix}$$

Seesaw mechanisms:

$v \ll v_R$ (**Type I**)

$v_L \ll v$ (**Type II**)

Integrating out the super-heavy neutrinos N:

$$m_\nu = m_\nu^{II} + m_\nu^{I} = v_L f - v^2 y (v_R f)^{-1} y$$

Type I and II seesaw contributions to light neutrino masses are strictly **intertwined**

Several Left-Right models which are fully consistent up to GUT scale **do not contain other sources of ν mass**

LR seesaw: the parameter space

$$m_\nu = v_L f - v^2 y (v_R f)^{-1} y$$

- $v^2 = (174\ \text{GeV})^2$ (EWSB)
- $0 \leq v_L \leq$ GeV ($\Delta\rho \approx - 2\, v_L^2 / v^2$)
- TeV $\leq v_R \leq M_{Pl}$ (no RH weak currents)
- $0 \leq (m_\nu)_{ij} \leq$ eV : **partially known** from oscillations data
- $0 \leq y_{ij} \leq 1$: **in general unknown** Yukawa couplings, but
  - Minimal SUSY LR: $y = \tan\beta\, y_e$
  - Minimal SO(10): $y = y_u$
  - Seesaw + mSUGRA: $y_{ij} << 1$ to suppress, e.g., $\tau \to \mu\gamma$
- $0 \leq f_{ij} \leq 1$: **completely unknown** Yukawa couplings

Bottom-up approach: what is the structure of the matrix f ?
To what extent we can reconstruct $M_R = v_R f$?

Seesaw duality

$$m_\nu = v_L f - v^2 y (v_R f)^{-1} y$$

Consider a matrix **f** solution of the seesaw formula for a given set of all other parameters.

Define:

$$\hat{f} \equiv \frac{m_\nu}{v_L} - f$$

Then:

$$m_\nu = v_L \hat{f} - v^2 y (v_R \hat{f})^{-1} y$$

Duality: **f** solution if and only if $\hat{\mathbf{f}}$ is

Akhmedov & MF

Solutions of the seesaw equation come in pairs:

$$\mathbf{f} = \mathbf{f}_1, \hat{\mathbf{f}}_1, \mathbf{f}_2, \hat{\mathbf{f}}_2, \ldots$$

Multiple solutions

$$m_\nu = v_L f - v^2 y (v_R f)^{-1} y$$

- Seesaw formula **non-linear in f** : for 3 lepton generations, one finds **8 solutions for f** (4 dual pairs)

➡ The right-handed neutrino mass matrix has **8 possible structures, $M_R = v_R$ f**

➡ For a given y, **8 structures of f induce the same m_ν**

- One may derive a complete analytic resolution of the **non-linear polynomial system of equations for f_{ij}**

method 1: Akhmedov & MF
method 2: Hosteins, Lavignac & Savoy, NPB 755 (2006) 137

Full analytic resolution

- Linearize by a rescaling parameter λ: **$f(m_{\alpha\beta}, y_{1,2,3}, v_L, v_R, \lambda)$**

$$f_{ij} = \frac{\lambda^2\left[(\lambda^2-Y^2)^2 - Y^2\lambda\det m + Y^4 S\right] m_{ij} + \lambda\left(\lambda^4 - Y^4\right) A_{ij} - Y^2\lambda^2(\lambda^2+Y^2)S_{ij}}{(\lambda^2-Y^2)^3 - Y^2\lambda^2(\lambda^2-Y^2)S - 2Y^2\lambda^3\det m}$$

$$Y^2 \equiv \frac{(y_1y_2y_3)^2}{x^3}\,, \quad S \equiv \sum_{k,l=1}^{3}\left(\frac{m_{kl}^2 x}{y_k y_l}\right)\,, \quad A_{ij} \equiv \frac{y_i y_j M_{ij}}{x}\,, \quad S_{ij} \equiv \sum_{k,l=1}^{3}\left(m_{ik}m_{jl}\frac{m_{kl}x}{y_k y_l}\right)\,,$$

$$x \equiv v_L v_R/v^2 \qquad m \equiv m_\nu/v_L \qquad M_{ij} \equiv \mathrm{minor}(m)_{ij}$$

- Non-linearity contained in a 8th order **equation for λ**

$$0 = \left[(\lambda^2-Y^2)^2 - Y^2\lambda^2 S\right]^2 - \lambda^2(\lambda^2+Y^2)^2 A$$
$$-Y^2\lambda^4(\det m)^2 - \lambda\left[\lambda^6 + Y^2\lambda^2(\lambda^2-Y^2)(5+S) - Y^6\right]\det m$$

- Seesaw duality ⇒ 4th order equation in **$z \equiv \lambda - Y^2/\lambda$**

$$0 = z^4 - \det m\, z^3 - (2Y^2S + A)z^2 - Y^2(8+S)\det m\, z + Y^2[Y^2S^2 - 4A - (\det m)^2]$$

A realistic numerical example

$$m \equiv \frac{m_\nu}{v_L} = \begin{pmatrix} 0 & 0.1 & -0.1 \\ \dots & 0.55 & 0.45 \\ \dots & \dots & 0.55 \end{pmatrix}$$

Tribimaximal mixing:
$\tan^2\theta_{23} = 1$
$\tan^2\theta_{12} = 0.5$
$\tan^2\theta_{13} = 0$
No CP violation

EX

Eigenvalues: -0.1, 0.2, 1
Normal hierarchy with $\Delta m^2_{sol} / \Delta m^2_{atm} = 0.031$

EX

TH

$v_L v_R = v^2$ (natural when scalar potential couplings are of order 1)
neglect CKM-like rotations (both charged lepton and neutrino Yukawa couplings diagonal in the same basis)
$y_1 = 10^{-2}$ $y_2 = 10^{-1}$ $y_3 = 1$ (inter-generation hierarchy analog to charged fermions)

$$m_\nu = v_L f - v^2 y (v_R f)^{-1} y$$

Given all these inputs, the 4 dual pairs of f structures are determined

The 8 reconstructed solutions

$$f_1 \approx \begin{pmatrix} 0 & 0.1 & 0 \\ \ldots & 0.5 & 0.4 \\ \ldots & \ldots & -0.9 \end{pmatrix} \qquad \hat{f}_1 \approx \begin{pmatrix} 0 & 0 & -0.1 \\ \ldots & 0 & 0 \\ \ldots & \ldots & 1.4 \end{pmatrix}$$

$$f_2 \approx \begin{pmatrix} 0 & 0.1 & 0 \\ \ldots & 0.6 & 0.3 \\ \ldots & \ldots & 1.5 \end{pmatrix} \qquad \hat{f}_2 \approx \begin{pmatrix} 0 & 0 & -0.1 \\ \ldots & -0.1 & 0.1 \\ \ldots & \ldots & -1.0 \end{pmatrix}$$

$$f_3 \approx \begin{pmatrix} 0 & 0.1 & 0.1 \\ \ldots & 0.6 & 0.2 \\ \ldots & \ldots & -0.3 \end{pmatrix} \qquad \hat{f}_3 \approx \begin{pmatrix} 0 & 0 & -0.2 \\ \ldots & -0.1 & 0.2 \\ \ldots & \ldots & 0.8 \end{pmatrix}$$

$$f_4 \approx \begin{pmatrix} 0 & 0.1 & -0.1 \\ \ldots & 0.5 & 0.4 \\ \ldots & \ldots & 0.9 \end{pmatrix} \qquad \hat{f}_4 \approx \begin{pmatrix} 0 & 0 & 0.04 \\ \ldots & 0 & -0.04 \\ \ldots & \ldots & -0.36 \end{pmatrix}$$

Features of the solutions

Consider a given pair of dual solutions:

$$f_4 \approx \begin{pmatrix} -0.001 & 0.105 & -0.14 \\ \dots & 0.56 & 0.49 \\ \dots & \dots & 0.88 \end{pmatrix} \qquad \hat{f}_4 \approx \begin{pmatrix} 0.001 & -0.005 & 0.04 \\ \dots & -0.01 & -0.04 \\ \dots & \dots & -0.33 \end{pmatrix}$$

Seesaw Duality: $f_4 + \hat{f}_4 = m = \begin{pmatrix} 0 & 0.1 & -0.1 \\ \dots & 0.55 & 0.45 \\ \dots & \dots & 0.55 \end{pmatrix}$

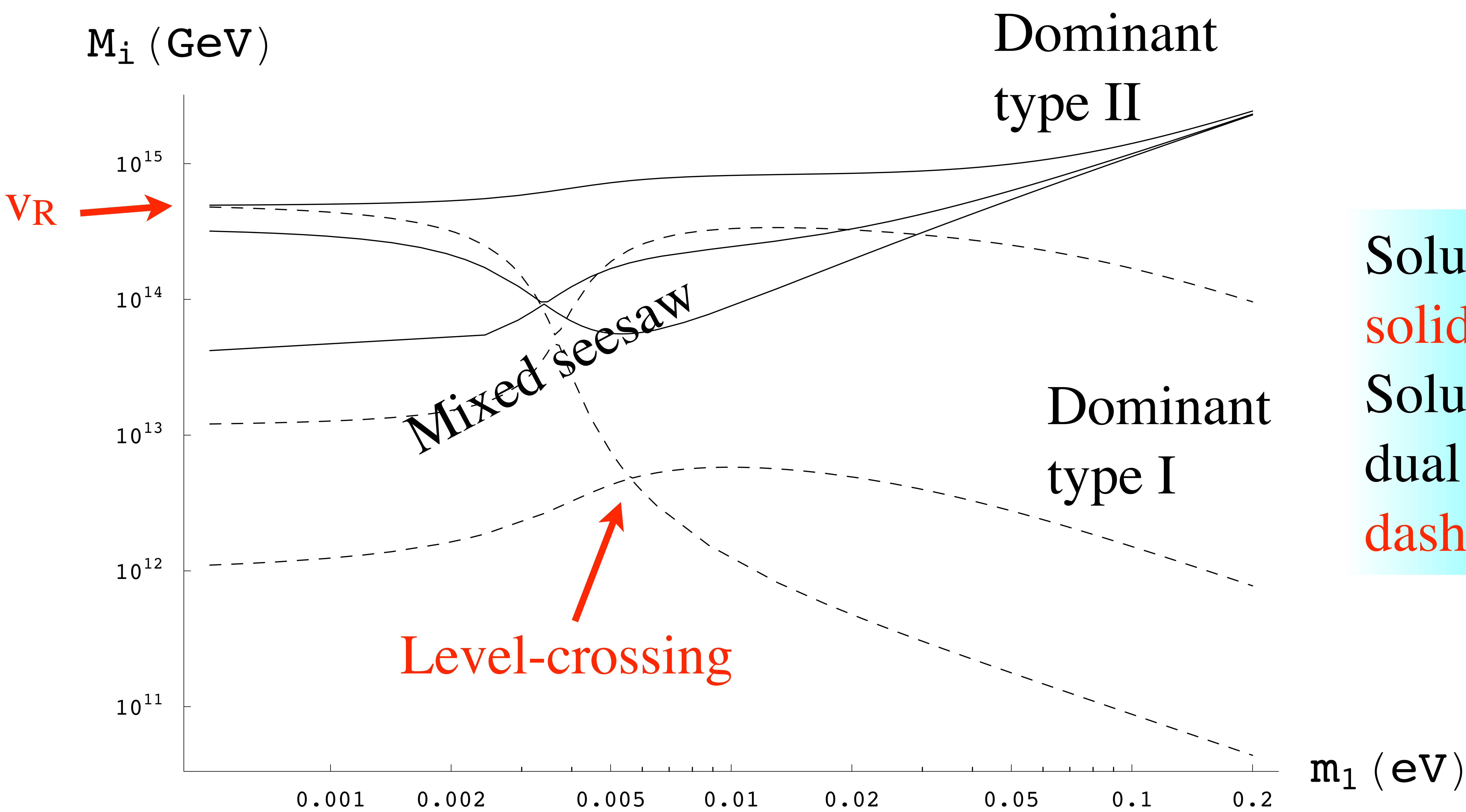
f_4 structure has dominant 23-block; large (but non-maximal) 2-3 mixing

Dual structure is hierarchical, with dominant 33-entry; small 2-3 mixing

One seesaw type dominance in m_{12}, m_{22}, m_{23} : type II in the case of f_4, type I in the dual case. **Mixed seesaw** in m_{11}, m_{13}, m_{33} .

Right-handed neutrino masses

as a function of the **absolute scale of light neutrino masses**



Solution f_4: solid lines

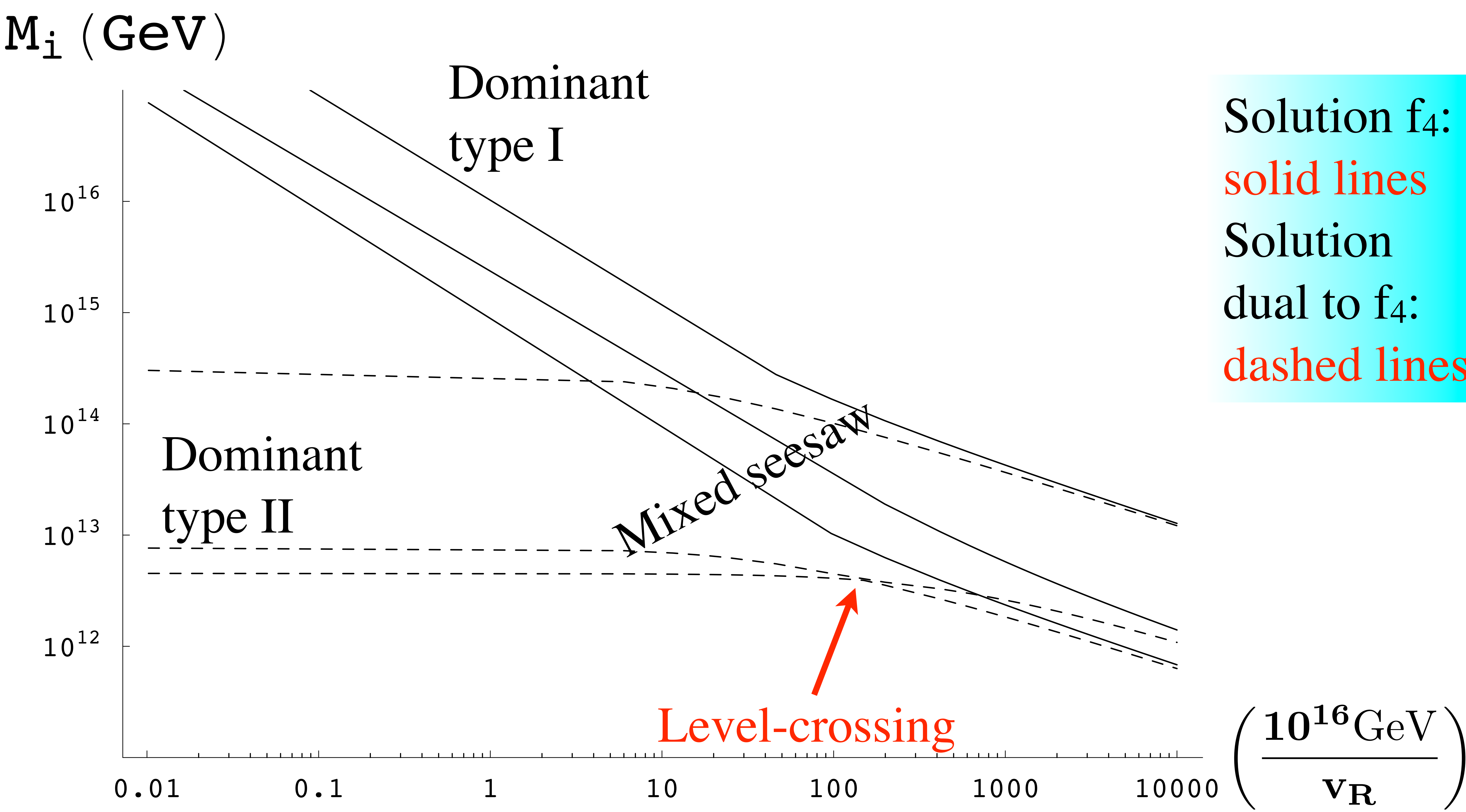
Solution dual to f_4: dashed lines

Normal hierarchy

Quasi-degeneracy

Right-handed neutrino masses

as a function of the **Left-Right symmetry breaking scale v_R**



Solution f_4: solid lines

Solution dual to f_4: dashed lines

Recipes for light RH neutrinos

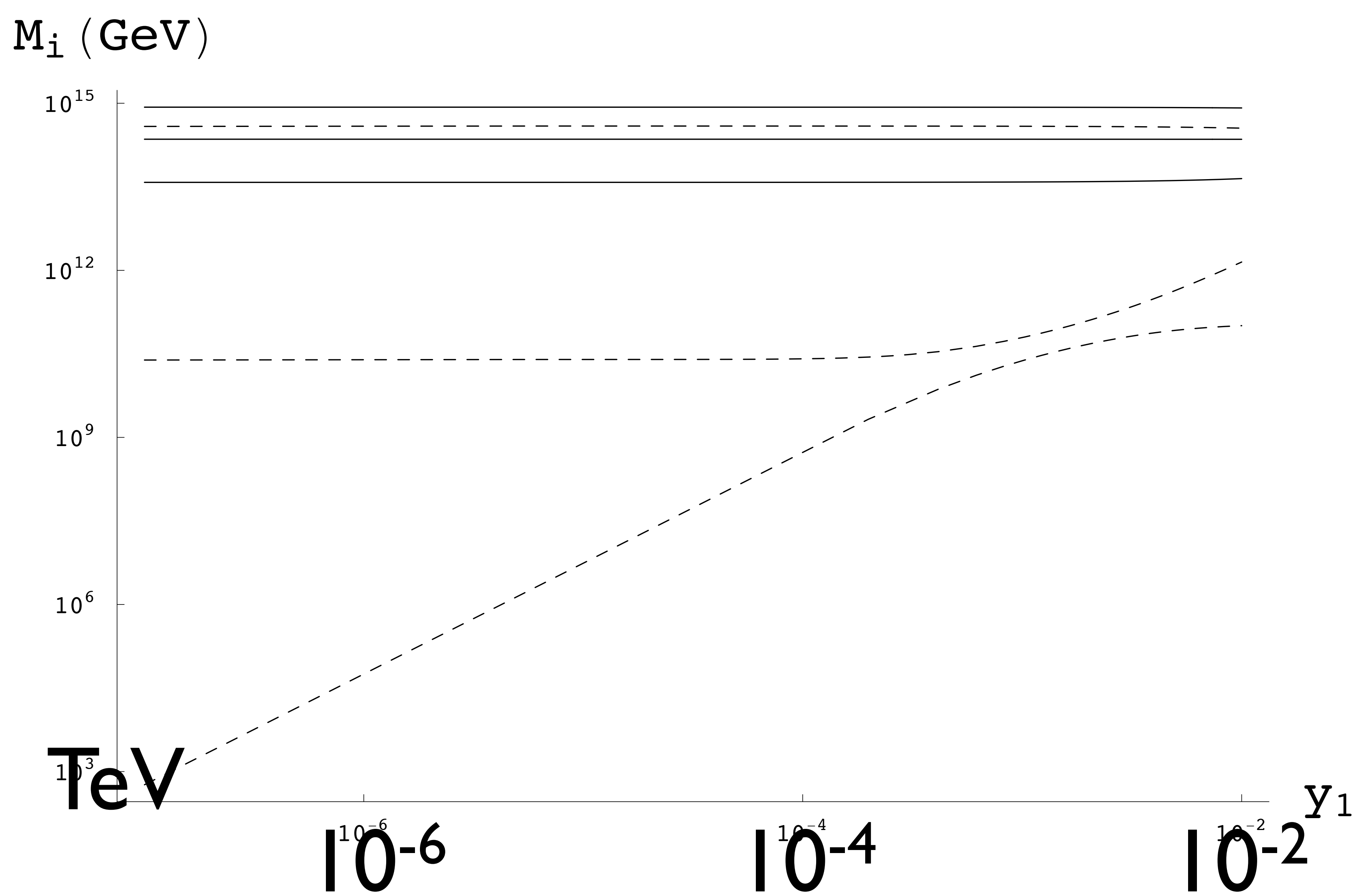
Interesting for (i) active-sterile oscillations
(ii) warm dark matter [Shaposhnikov, ...] (iii) low scale leptogenesis
(iv) direct detection at LHC [Del Aguila, ...]

- Assume tiny first generation Yukawa coupling: $M_1 \sim y_1^2$

- Lower left-right symmetry breaking scale v_R at few TeVs with $v_L v_R << v^2$; it follows $y_i < 10^{-5}$ and $M_i \sim v_R$

- Lower v_R keeping $v_L v_R \approx v^2$; both y and f couplings need to be very small: $M_i << v_R$

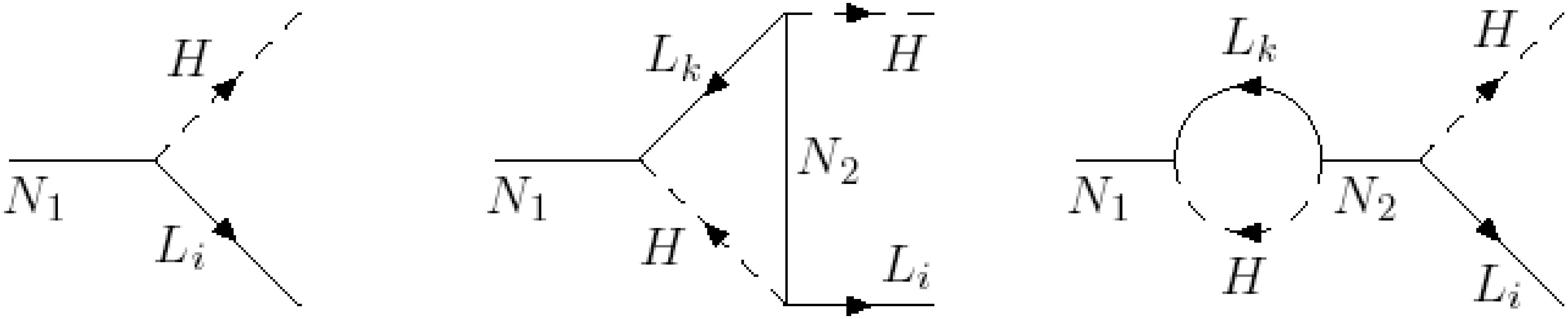
- Take v_R at few TeVs with couplings y of order one arranged in such a way to cancel in type I seesaw



$$y f^{-1} y =$$

$$\begin{pmatrix} r^2 & ir \\ ir & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \begin{pmatrix} r^2 & ir \\ ir & -1 \end{pmatrix} = 0$$

Baryogenesis via Leptogenesis



- Majorana mass term M_R N N for super-heavy neutrinos N_i violates Lepton Number

- N_1 decays at T ≈ M_1 out-of-equilibrium generating a lepton asymmetry by the interference between decay amplitudes at tree level and 1-loop: $\epsilon_L \sim [\, \Gamma(N_1 \to LH) - \Gamma(N_1 \to L^* H^*) \,]$

- Standard Model B+L violating effects at T > v convert lepton into baryon asymmetry. Since $[n_B/s]_{exp} \approx 10^{-10} \leq 10^{-3}\, \epsilon_L$, one needs $\epsilon_L \geq 10^{-7} \div 10^{-6}$

Leptogenesis in Left-Right models

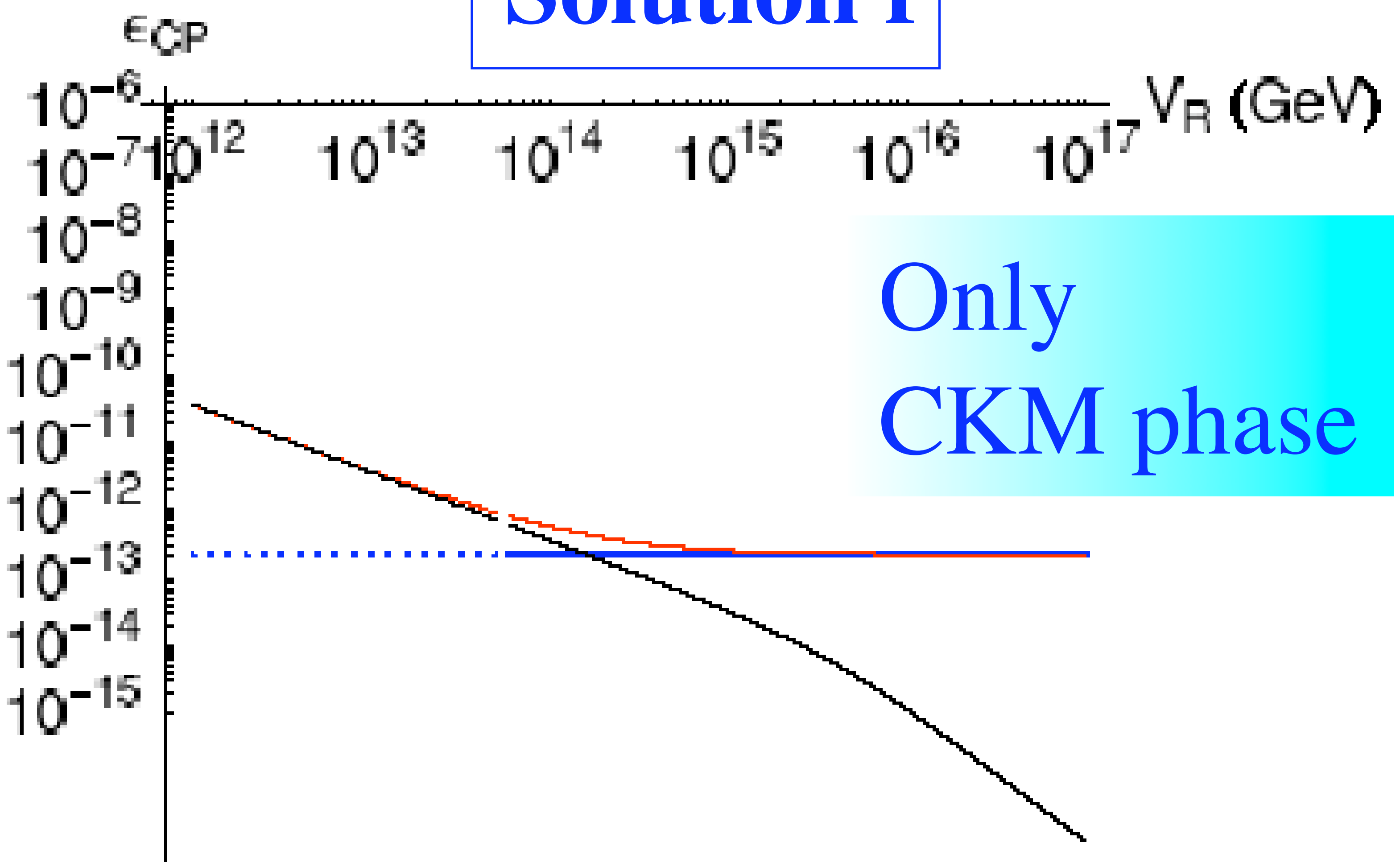
- The needed lepton asymmetry may be produced either by the type I seesaw sector (N_i decays), by the type II seesaw sector (Δ_L decays), or by their interplay.
- LR symmetry implies that the same matrix f determines both N_i masses and Δ_L coupling to leptons: more predictivity
- The 8 possible structures of f can be discriminated by their ability to achieve Baryogenesis via Leptogenesis
- Seesaw duality provides new options to enhance the asymmetry: (i) solutions where M_R is not hierarchical, (ii) solutions with quasi-degenerate masses, (iii) extra sources of asymmetry in the LR breaking sector, ...

Detailed studies by: Hosteins, Lavignac & Savoy, NPB 755 (2006) 137; Akhmedov, Blennow, Hallgren, Konstandin, Ohlsson, hep-ph/0612194.

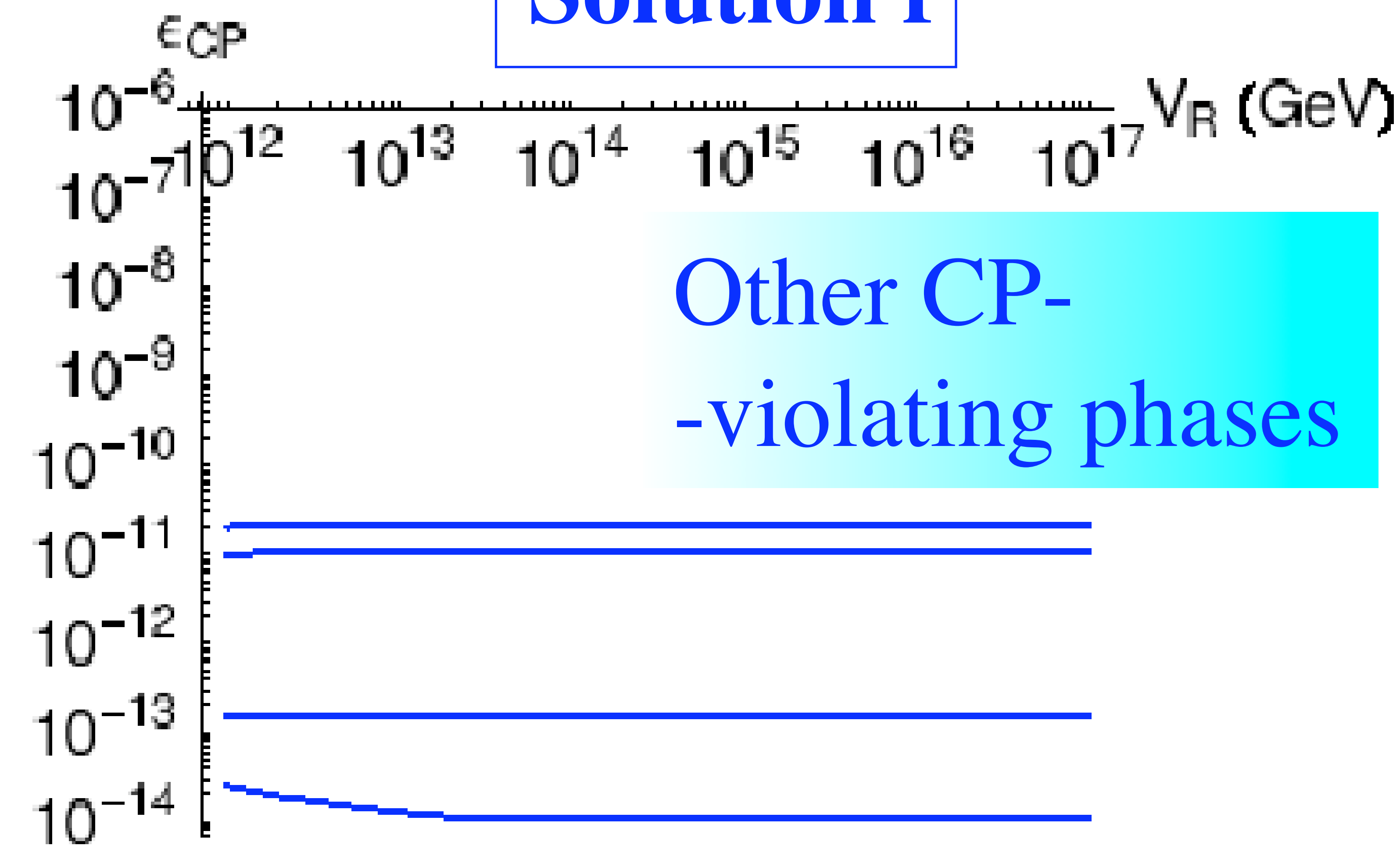
Multiple options for leptogenesis

Hosteins, Lavignac & Savoy, NPB 755 (2006) 137

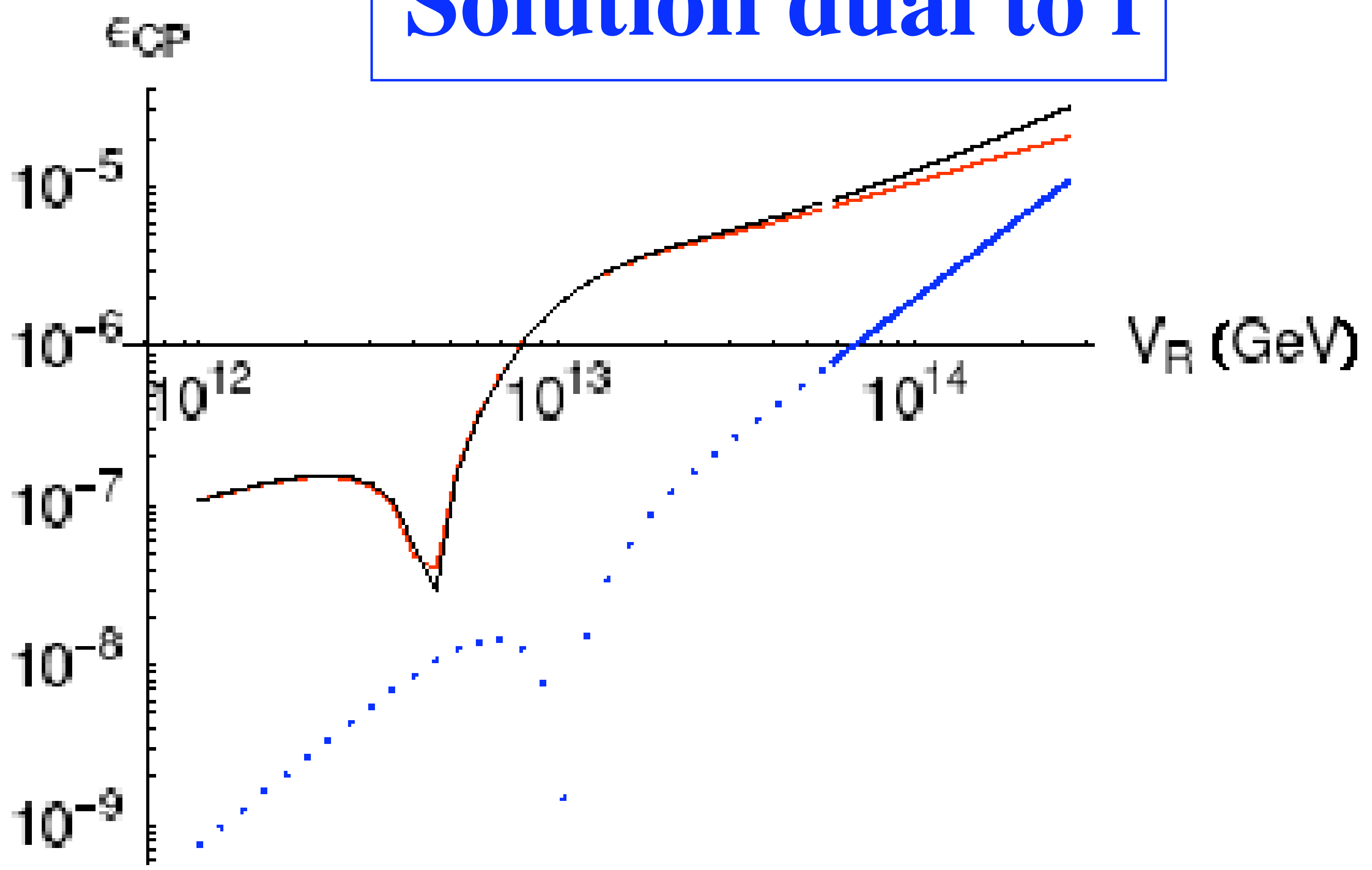
Solution f



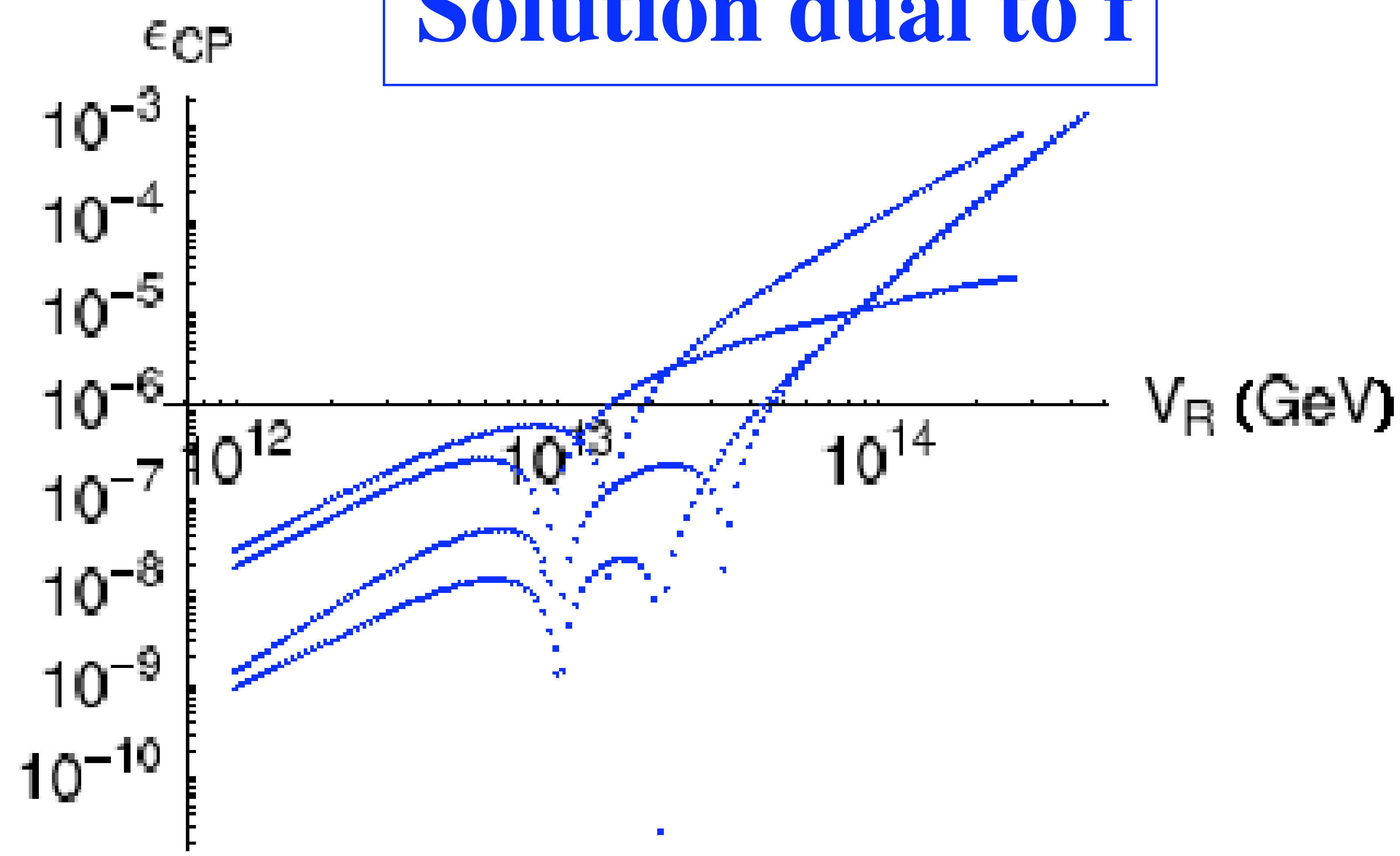
Solution f



Solution dual to f



Solution dual to f



Grand Unification à la SO(10)

$$SU(3)_c \times [SU(2)_L \times SU(2)_R \times U(1)_{B-L}] \subset SO(10)$$

All SM fermions + N's sit in the same multiplet **16$_F$**

Neutrino Majorana masses from a unique coupling:

$$f\ \mathbf{16}_F \mathbf{16}_F \overline{\mathbf{126}}_H \ \ni\ f(LL\Delta_L + L^c L^c \Delta_R)$$

Neutrino Dirac masses can receive several contributions:

$$vy = \langle \mathbf{10}_H \rangle y_{10} + \langle \mathbf{120}_H \rangle y_{120} + \langle \overline{\mathbf{126}}_H \rangle f$$

Even if f contributes to y the seesaw can be written as:

$$m'_\nu = v_L f - v^2 y' (v_R f)^{-1} y'$$

Therefore there are always multiple solutions for f.
Duality holds if only 10s and 126s (only 120s) contribute to y.

Minimal Supersymmetric SO(10)

Renormalizable Yukawas from one 10_H and one 126_H only

Babu, Mohapatra, Clark, Kuo, Nakagawa, Bajc, Senjanovic, Vissani, Melfo, Aulakh, Girdhar, Macesanu, Goh, Ng, Dutta, Mimura, Bertolini, Frigerio, Malinsky, ...

$$\mathcal{L}_Y = \mathbf{16}_F \left(y\, \mathbf{10}_H + f\, \overline{\mathbf{126}}_H \right) \mathbf{16}_F$$

Neutrino sector: $y = y_u$
$\Rightarrow$ 8 solutions for f

However, y and f strongly constrained by charged fermion masses and CKM mixing angles

$$M_u = y \langle 10_H \rangle^u + f \langle \overline{126}_H \rangle^u$$
$$M_d = y \langle 10_H \rangle^d + f \langle \overline{126}_H \rangle^d$$
$$M_e = y \langle 10_H \rangle^d - 3 f \langle \overline{126}_H \rangle^d$$

The **global fit of fermion masses and mixing** (including neutrinos) is intricate and very constrained.

Most recent analysis: **a perfect fit is possible, but the required heavy mass spectrum is incompatible with gauge coupling unification.**

Bertolini, Malinsky & Schwetz, PRD 73 (2006) 115012
(see also Aulakh & Garg, NPB 757 (2006) 47)

Non-minimal SUSY SO(10)

126_H plus two 10_H multiplets

Hosteins, Lavignac & Savoy, hep-ph/0606078

1) **Up versus down**: two 10_H distinguish vy = M_u from $M_d = M_e$

2) The 8 dual solutions for f may be derived **from neutrino sector**

3) For each viable f, one may compute

(i) lepton asymmetry (ii) lepton flavor violation bounds

(iii) **correction to $M_d \neq M_e$**, which remains difficult

126_H plus one 10_H and one 120_H

Aulakh, hep-ph/0602132, 0607252
Grimus,Kuhbock,Lavoura, hep-ph/0603259, 0607197

1) Fit with 10_H and 120_H alone **M_u , M_d and M_e**
(there is some small tension for first generation masses).

2) Derive the 8 dual solutions for f **from neutrino sector**

3) Select the (possibly) unique structure for f which achieves a good **fit of m_e, m_u, m_d**.

CONCLUSIONS

- The understanding of neutrino mass relies on the identification of its dominant source.

- If the new physics if Left-Right symmetric, type I+II seesaw stands up firmly as the unique candidate.

- Bottom-up reconstruction of the superheavy seesaw sector: duality among 8 different structures.

- Numerical & analytic reconstruction of the 8 structures allows to investigate different options for:

- Baryogenesis via Leptogenesis

- Grand Unified Theories

- flavor symmetries, lepton flavor violation, etc...