Precision phenomenology for the LHC

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Content:

- Motivation: LHC@NLO, why going to loops?
- One loop methods
- The GOLEM project
- First applications for LHC
- Evaluation of rational polynomials à la GOLEM
- Summary

The advent of the LHC era

LHC:

- Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, start 2007
- Long and Hard Calculations (for theorists)

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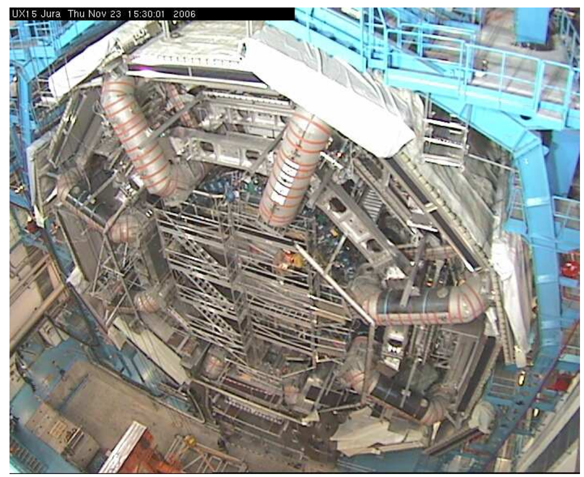

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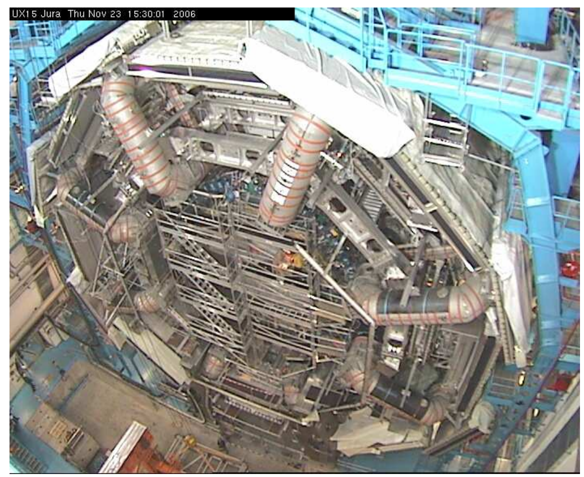

What will we see?

- nothing → extremely disturbing/interesting!
- Higgs boson + nothing → asks for high precision checks (ILC!)
- Higgs boson + something → investigate ”somthing” in SM background!

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Higgs sector and beyond

- LEP: Nonabelian structure and loops important $\Rightarrow$ bounds on M_{Top}, $\log(M_H)$
- Tevatron Run I & II: SM and nothing else!

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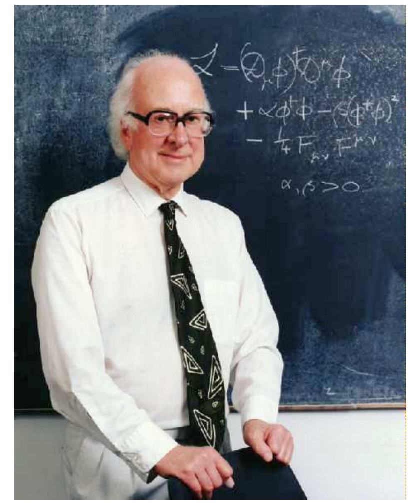

- Higgs sector not yet confirmed

$$V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3\ H^3 + \lambda_4\ H^4$$

SM: $\lambda_4 = \lambda_3 / v = 3\ M_H^2 / v^2$

- SM Higgs boson ⇒ 114.4 GeV $< m_H < 200$ GeV (!)

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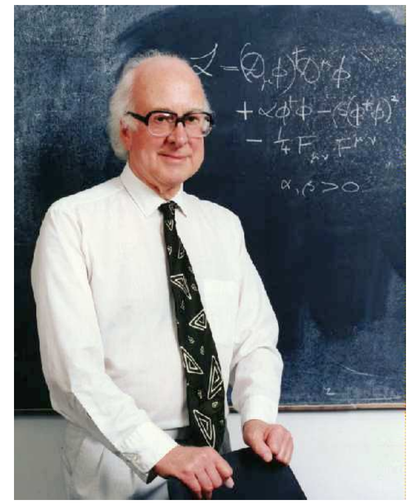

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- SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
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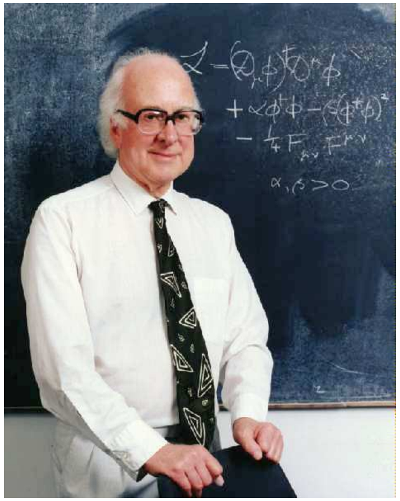

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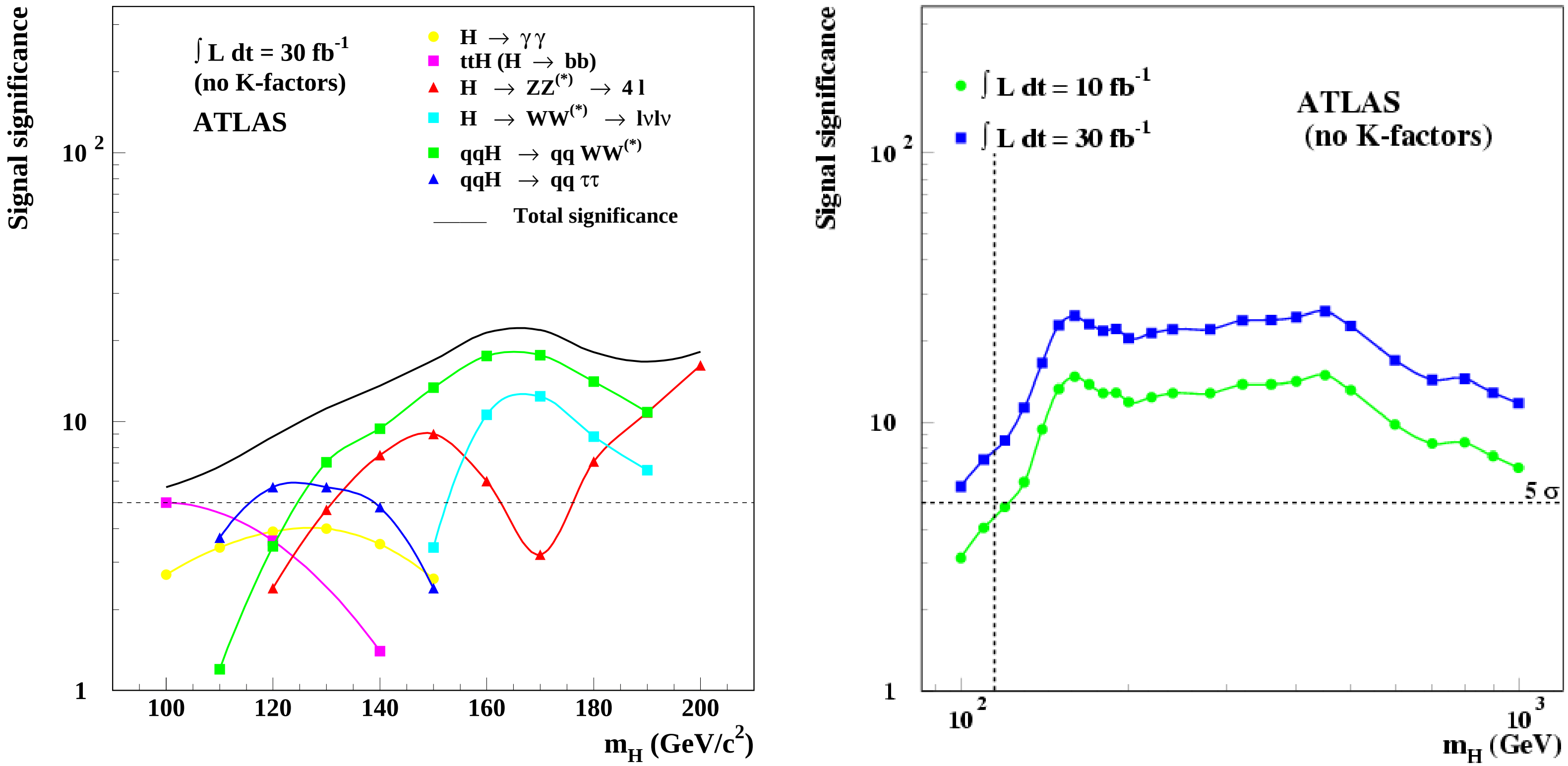
- BSM $\Rightarrow$ something around 1 TeV (?)

Hierarchy/finetuning problem not a convincing argument!
$m_H^2|_{\mathrm{phys}} = m_H^2|_{\mathrm{bare}} +$ loop effects $= m_H^2|_{\mathrm{bare}} + 1/\epsilon + \kappa\, M_{\mathrm{GUT}}^2 \log(M_{\mathrm{GUT}}^2)$

Discovery potential of the Higgs boson at the LHC



- LHC designed to find the Higgs boson up to $m_H \sim 1$ TeV
- $m_H < 2\,m_Z$ most difficult
- $2\,m_Z < m_H < 1$ TeV "gold plated mode" $H \to ZZ \to \mu\mu\mu\mu$
- $m_H \sim 1$ TeV perturbative approach ceases to be valid

Nothing seen at LHC...

...due to undetectable invisible matter?

- $H \longrightarrow$ singlet matter and missing energy signal completely washed out
- Scalar singlets generic objects, Cold Dark Matter candidate
- Look for excess from $PP \rightarrow H + 2\text{jets} \rightarrow \not{E} + 2\text{jets}$
- $\not{E}$ Background control crucial

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- Higgs physics non-decoupling!
- Tree-level unitarity, Lattice studies, $1/N$ expansion $\Rightarrow m_H <\sim 1$ TeV
- $m_H \to \infty$: Look for excess in $W_L W_L \to W_L W_L$ scattering
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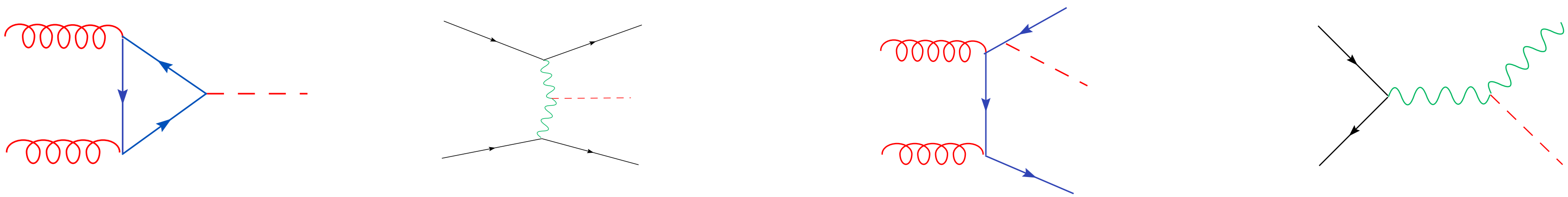
Immediate questions:

- What are the invisible decay channels?
- What fakes a light Higgs boson in the precision observables?

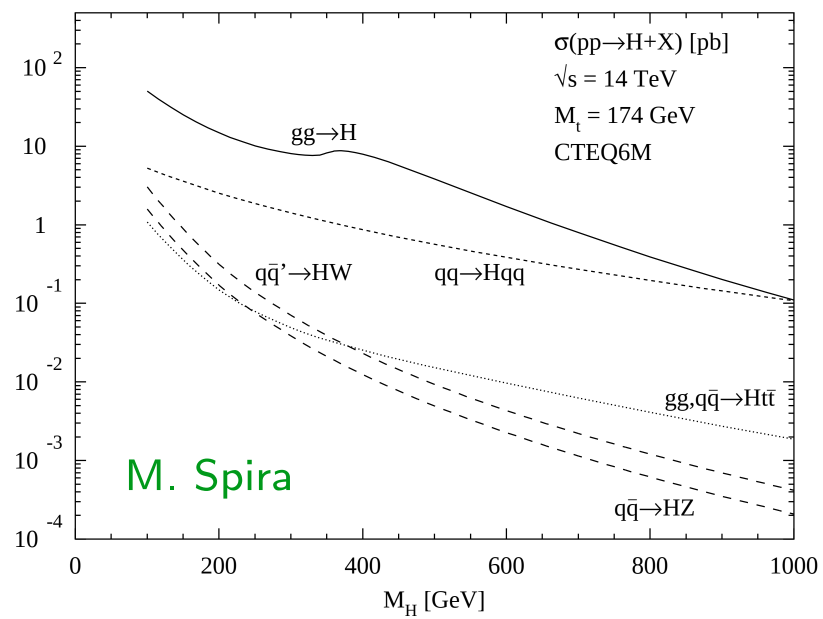
Conclusion: Nothing@LHC $\Rightarrow$ Manifestation of New physics!

Higgs boson + nothing...



Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$



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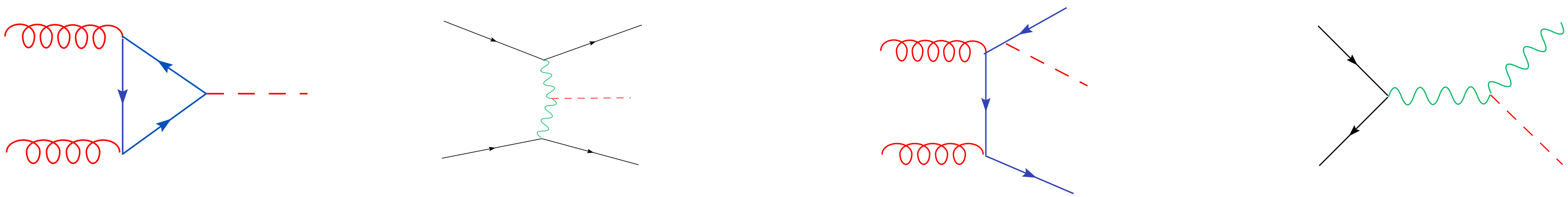

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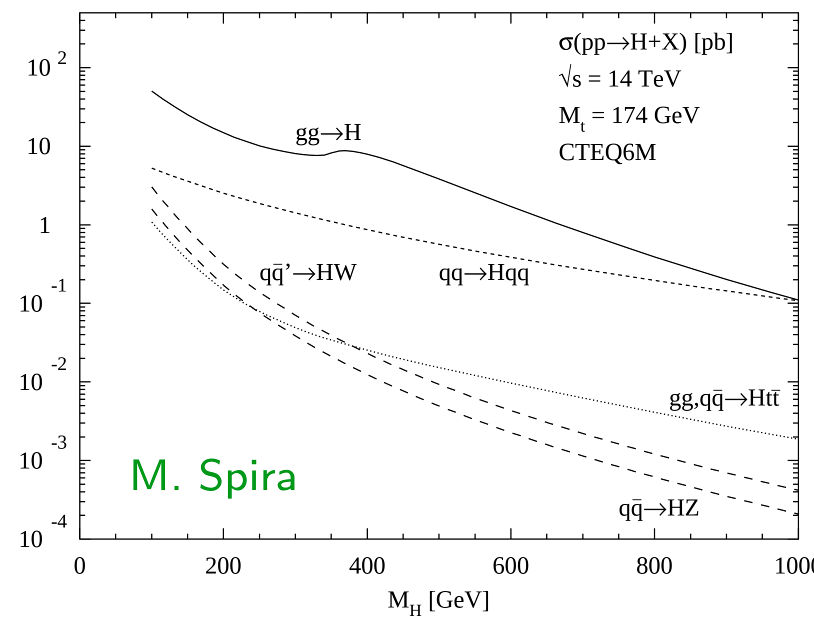

Backgrounds:

- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to V +$ up to 3 jets $\quad (V = \gamma, W, Z)$
- $PP \to VVV + 0, 1$ jet

Higgs boson + nothing...

After discovery of a Higgs like boson:

- measure Standard Model properties
- quantitative analysis of Higgs/Matter couplings
- Crucial: reliable background control
- not all backgrounds can be measured: theoretical input necessary!

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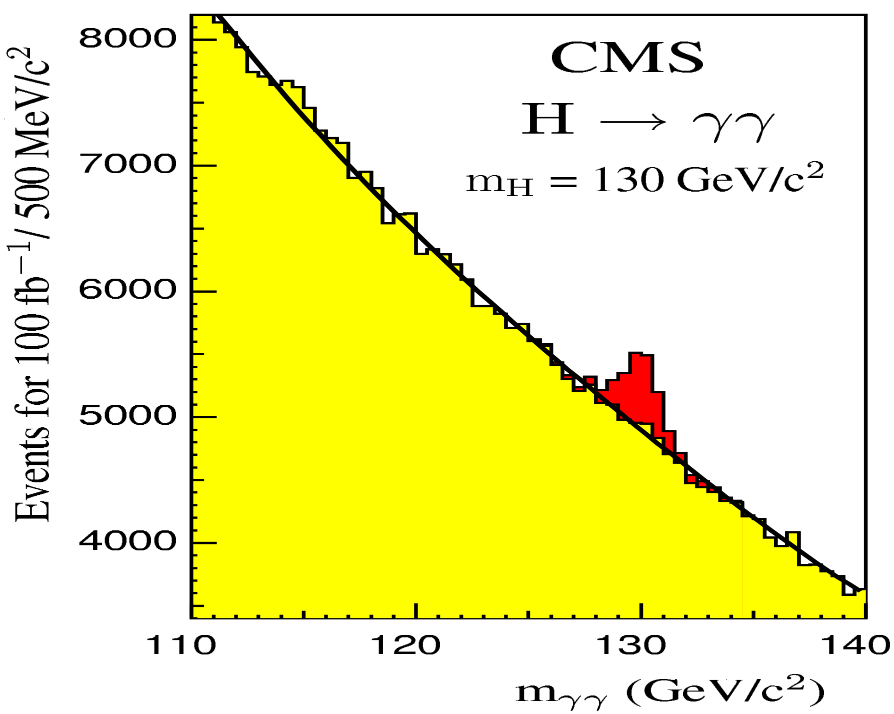
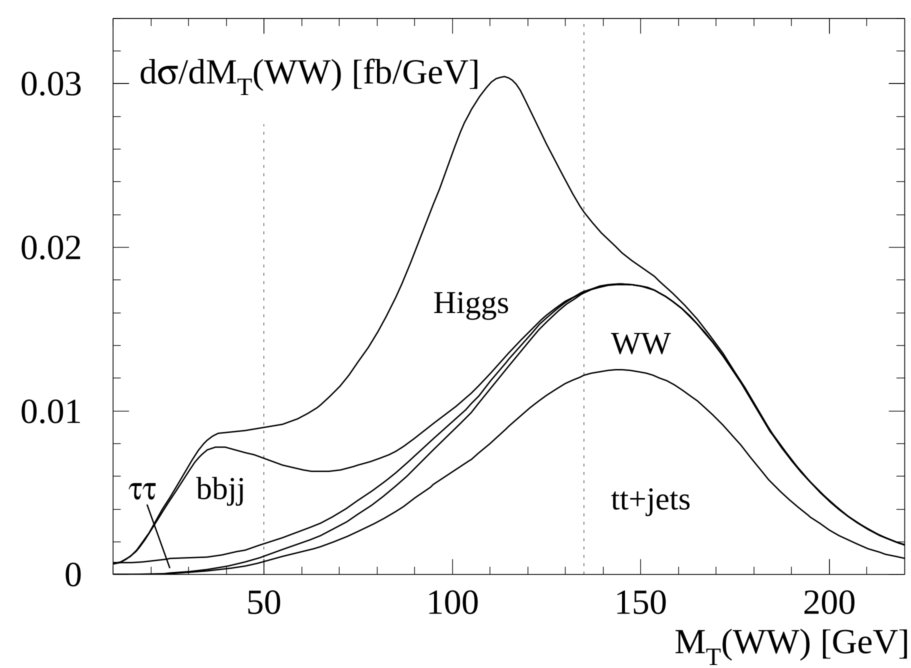
$PP \to H + X \to \gamma\gamma + X$



$H \to WW \to l^+l^- + \not{p}_T$

Kauer, Plehn, Rainwater, Zeppenfeld (2001)

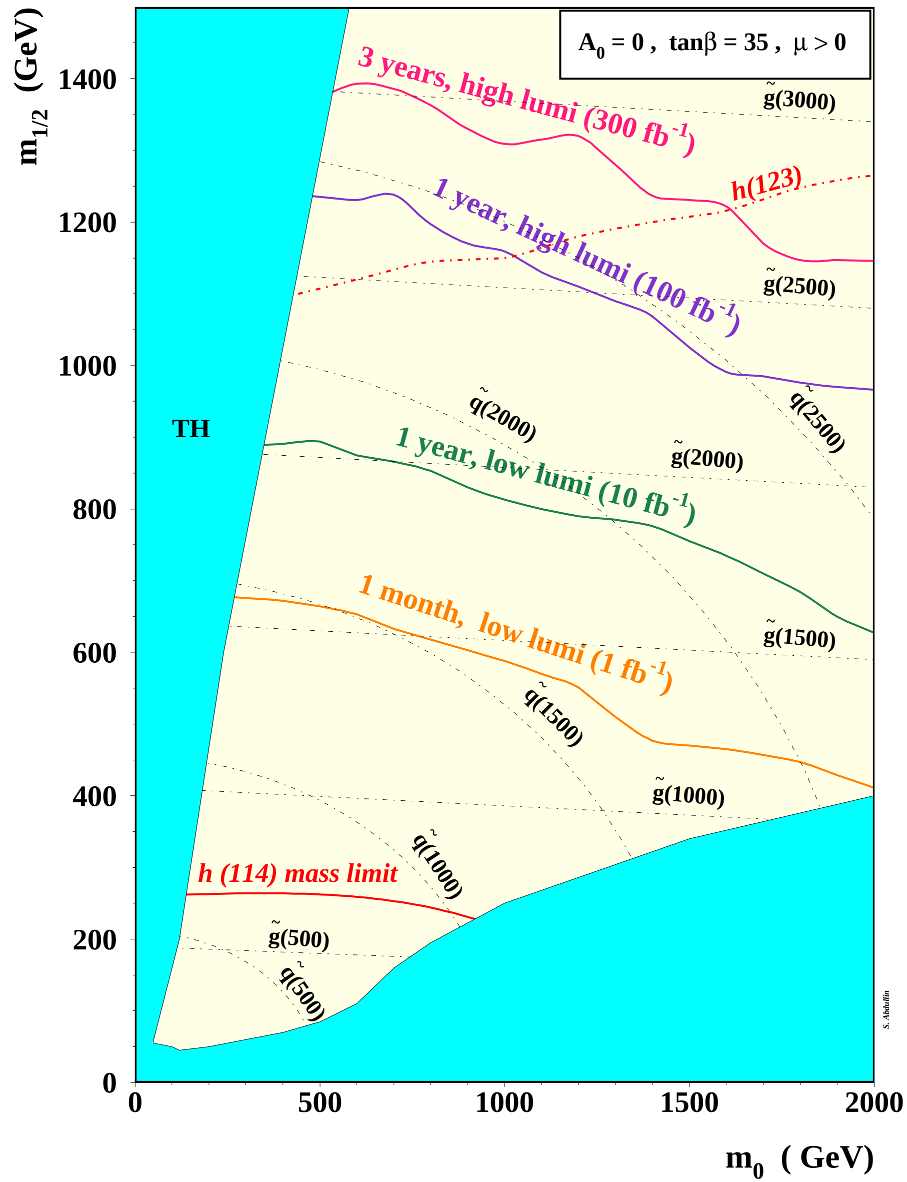


Higgs boson + something...

All Standard Model processes are background to new physics!

New physics signatures:

- Z' easy
- n jets + $\not{E}_T$
- multiparticle cascades



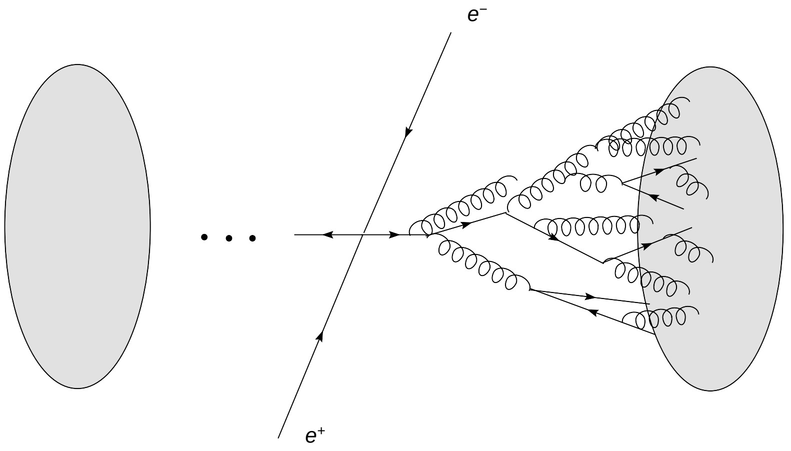
Tools for experimental analysis

Pythia

Herwig

Sherpa

- LO Matrixelements + parton shower + hadronization model



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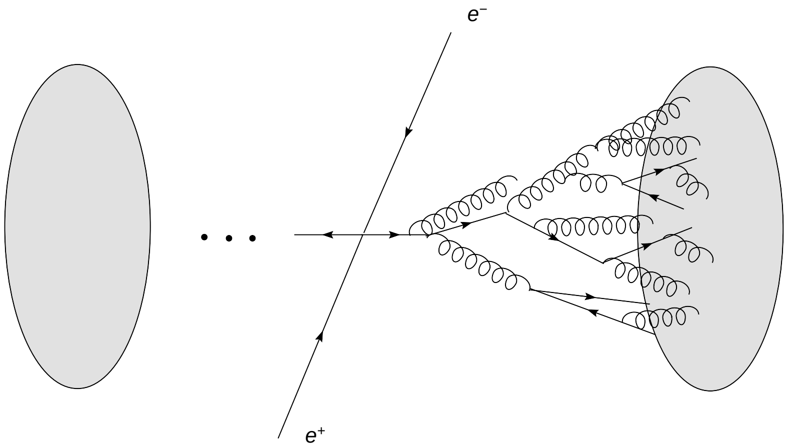
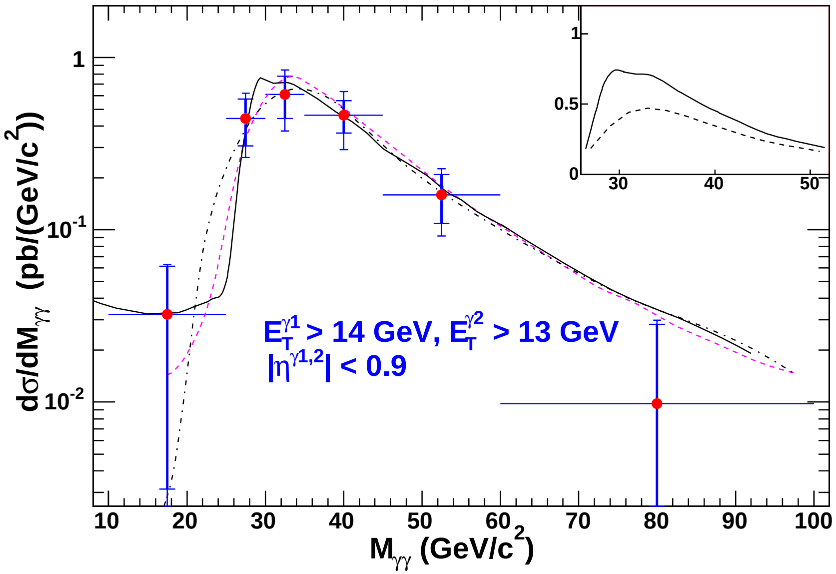


- $2 \to N$ Matrixelements: shapes, jet structure, well described after tuning
- LO absolute rates intrinsically unreliable!

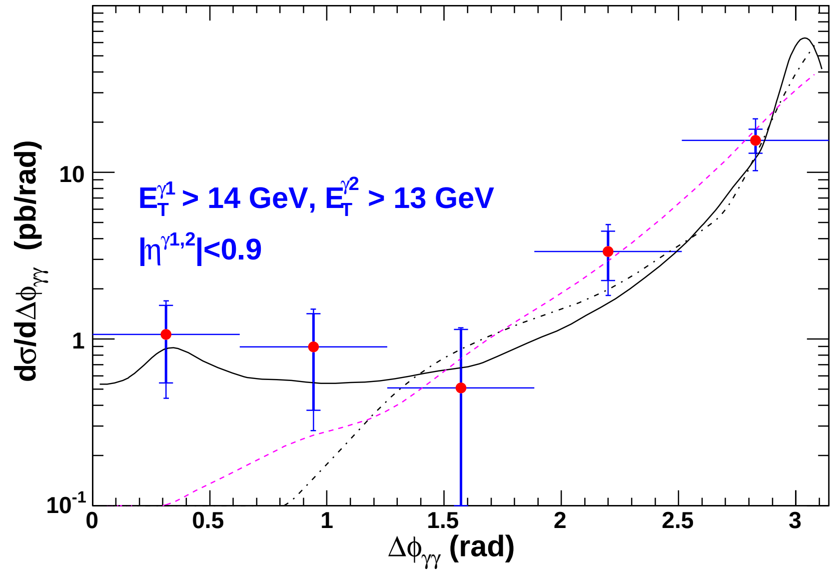
Example: $\gamma\gamma$ rate at Tevatron Run II [hep-ex/0412050]

- DIPHOX: NLO code for $\gamma\gamma$, $\gamma\pi^0$, $\pi^0\pi^0$ production (including fragmentation)
- http://lappweb.in2p3.fr/lapth/PHOX_FAMILY/diphox.html
  [T.B., J.P. Guillet, E. Pilon, M. Werlen]

$M_{\gamma\gamma}$ distribution

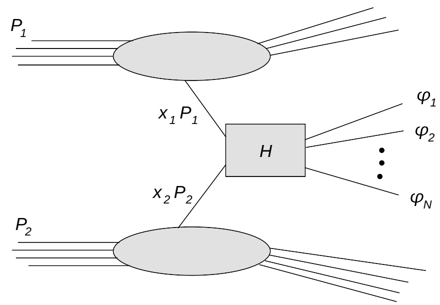


$\Delta\phi_{\gamma\gamma}$ distribution



DIPHOX (solid), RESBOS (dashed), PYTHIA$\times 2$!!! (dot-dashed)

Parton model and scale uncertainties



$$d\sigma(H_1H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\text{parton}_j(x_1P_1) + \text{parton}_l(x_2P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

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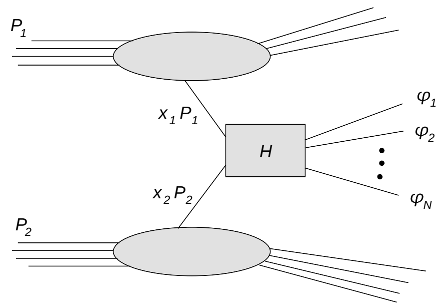

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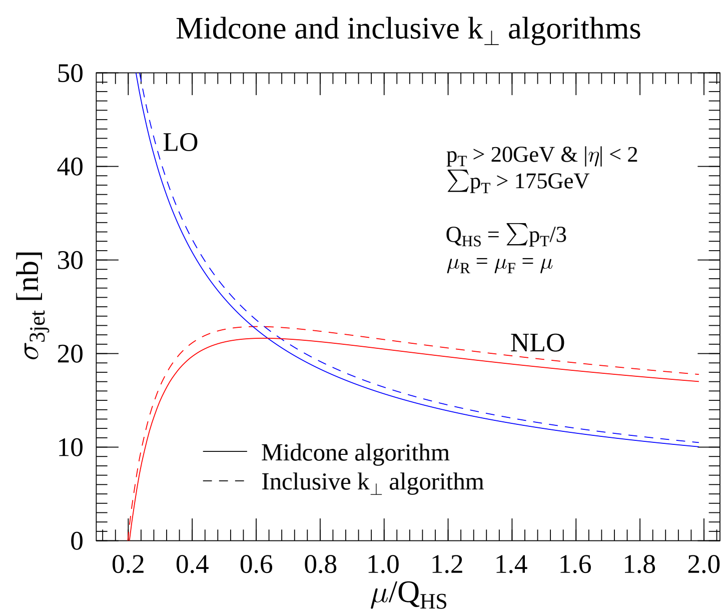
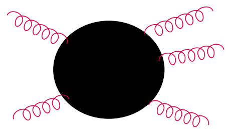
Example: 3 jet cross section at NLO

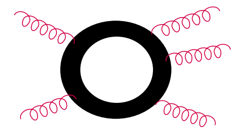
[Z. Nagy, Phys.Rev. D68 (2003)]

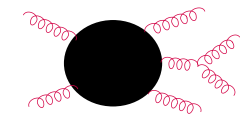


Higher order QCD calculations are mandatory to soften scale dependence !!!

Framework for NLO calculations

$\mathcal{M}_{\rm LO}$: 

$\mathcal{M}_{\rm NLO,virtual}$: 

$\mathcal{M}_{\rm NLO,real}$: 

$$\sigma \quad = \quad \int dPS_N \left(|\mathcal{M}_{\rm LO}|^2 + \alpha_s \left[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO,V} + \mathcal{M}^*_{\rm LO}\mathcal{M}_{\rm NLO,V} + \int dPS_1 \, |\mathcal{M}_{\rm NLO,R}|^2 \right] \right)$$

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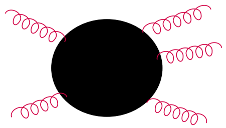
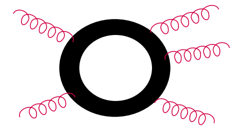
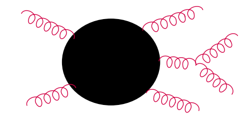
$\mathcal{M}_{\mathrm{LO}}$:  $\mathcal{M}_{\mathrm{NLO,virtual}}$:  $\mathcal{M}_{\mathrm{NLO,real}}$: 

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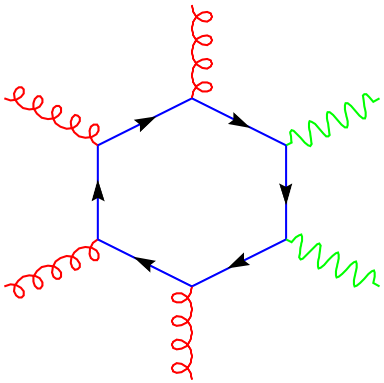
- $e^+e^- \rightarrow$ partons: IR divergences cancel
- $PP \rightarrow$ partons: remaining collinear divergences absorbed in "bare" pdfs
- treelevel LO, NLO contributions technically unproblematic
- treatment of IR divergences e.g. Dipolmethod à la Catani/Seymour
- Bottleneck: virtual corrections

Status QCD@NLO for LHC:

$2 \to 2:$ everything you want

$2 \to 3:$ $PP \to 3\,j,\ Vjj,\ \gamma\gamma j,\ Vb\bar{b},\ t\bar{t}H,\ b\bar{b}H,\ jjH,\ HHH, (t\bar{t}j)$

$2 \to 4:$ everything remains to be done !



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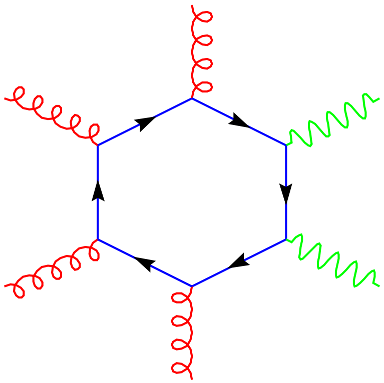
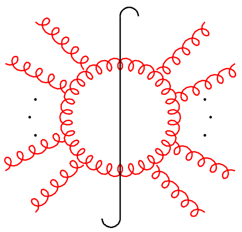


- LHC induces a lot of very recent activity !
- 4 partons @ NLO Ellis/Sexton, 1985
- 5 g @ NLO Bern/Dixon/Kosower, 1993
- Unitarity based and twistor space inspired methods
- "Modern" Algebraic/Seminumerical techniques
- 6 g @ NLO 2006

Unitarity based/Twistor space inspired approach:

- get loop amplitudes by sewing tree amplitudes using unitarity

$$\mathcal{A}_{1-\text{loop}} \sim \sum_C \int d\text{PS}_C$$



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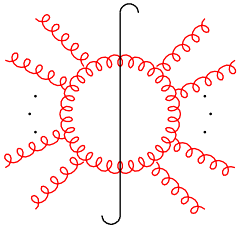

- tree amplitudes gauge invariant
- Bern,Dixon,Dunbar,Kosower-Theorem on cut-constructability:
  Sufficient condition for cut.-con. is that tensor integrals
  $\int d^D k k^R/(k^2 - M^2)^N$ obey $R \leq N - 2$
- successfully applied for $\mathcal{N} = 1$, $\mathcal{N} = 4$ susy amplitudes

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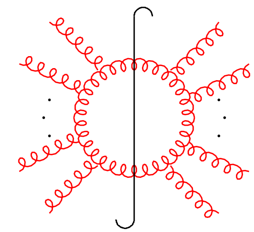

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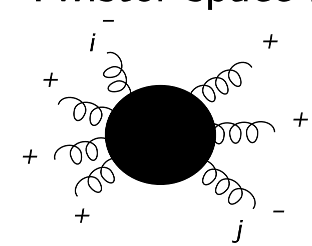
- successfully applied for $\mathcal{N} = 1$, $\mathcal{N} = 4$ susy amplitudes
- Revived by "Twistor space approach" [Cachazo, Svrcek, Witten (2004)]
- maximally helicity violating QCD tree amplitudes are lines in "Twistor space".

$$\mathcal{A}_{\text{MHV}} \sim i g^{N-2} \frac{\langle ij \rangle}{\langle 12 \rangle \langle 23 \rangle ... \langle n{-}1{,}n \rangle \langle n1 \rangle} \sim$$



- novel perturbative expansion: MHV-vertices + scalar propagators $\sim 1/P^2$

$\langle ij \rangle := \langle i^- | j^+ \rangle$, $[ij] := \langle i^+ | j^- \rangle$, $|j^+\rangle$ defined by $\not{p}_j |j^+\rangle = 0$, $|j^-\rangle = |j^+\rangle^C$

Unitarity based/Twistor space inspired approach:

- BDDK: $d = 4$ cuts do not fully determine one-loop amplitude

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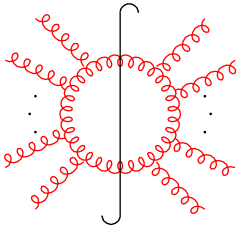

$$+ \ \mathcal{R}$$

- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]

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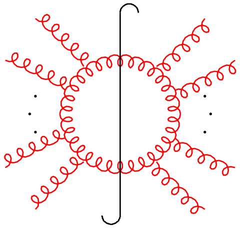

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- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]
- "bootstrap" approach exists to evaluate $\mathcal{R}$ by collinear limits and "auxiliary relations", done for $\mathcal{A}_{6-\text{gluon}}^{+++---}$ [Bern, Dixon, Kosower (2006)]
- d-dimensional cut techniques under investigation
- Feynman diagrammatic approach by Chinese group [Xiao, Yang, Zhu (2006)] $\mathcal{R}[\mathcal{A}_{6-\text{gluon}}^{\pm\cdots}]$ from tensor form factors.
- virtual part of 6g@NLO done!!!

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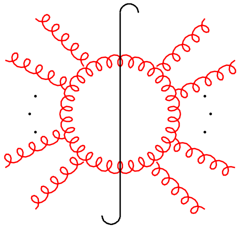

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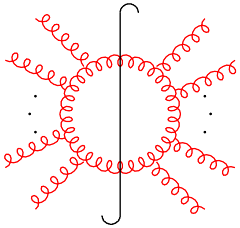
- cut constructible part of all 1-loop 6-gluon helicity amplitudes known [Feng, Britto, Mastrolia (2006)]
- “bootstrap” approach exists to evaluate $\mathcal{R}$ by collinear limits and “auxiliary relations”, done for $\mathcal{A}_{6-\text{gluon}}^{+++---}$ [Bern, Dixon, Kosower (2006)]
- d-dimensional cut techniques under investigation
- Feynman diagrammatic approach by Chinese group [Xiao, Yang, Zhu (2006)] $\mathcal{R}[\mathcal{A}_{6-\text{gluon}}^{\pm\cdots}]$ from tensor form factors.
- virtual part of 6g@NLO done!!!
- formalisms mainly for multi-gluon amplitudes
- external quarks, massive internal particles (?), big issue: automation

Unitarity based/Twistor space inspired approach:

- BDDK: $d = 4$ cuts do not fully determine one-loop amplitude

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_C \int d\mathrm{PS}_C$$



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Unitarity based/Twistor space inspired methods have good potential
further research necessary to establish a general method!!!

Feynman diagrammatic approach:

$$\Gamma^{c,\lambda}(p_j, m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{(q_1^2 - m_1^2)\dots(q_N^2 - m_N^2)} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\dots,\mu_R} \, I_N^{\mu_1\cdots\mu_R}(p_j, m_j)$$

$$I_N^{\mu_1\cdots\mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1}\dots k^{\mu_R}}{(q_1^2 - m_1^2)\dots(q_N^2 - m_N^2)} \quad , \quad q_j = k - r_j = k - p_1 \cdots - p_j$$

- Passarino-Veltman: momentum space reduction $\rightarrow 1/\det(G)^R, G_{ij} = 2\, r_i \cdot r_j$
- Davydychev representations separates Lorentz structure:

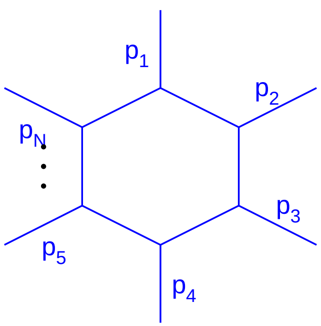
$$I_N^{\mu_1\cdots\mu_R} = \sum_{m=0}^{[R/2]} \left(-\frac{1}{2}\right)^m \sum_{j_1,\dots,j_{R-2m}=1}^{N-1} \left[g^{\cdot\cdot}_{(m)} r^{\cdot}_{j_1} \dots r^{\cdot}_{j_{R-2m}}\right]^{\{\mu_1\cdots\mu_R\}} I_N^{n+2m}(j_1,\dots,j_{R-2m})$$

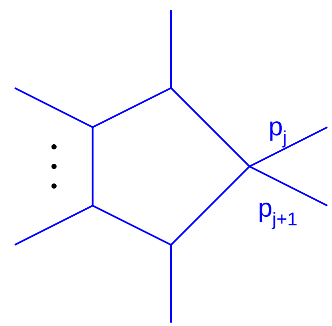
$$I_N^D(j_1,\dots,j_R) = (-1)^N \Gamma(N-D/2) \int_0^\infty d^N z \, \delta(1-\sum_{l=1}^N z_l) \frac{z_{j_1}\dots z_{j_R}}{(z\cdot S\cdot z)^{N-D/2}}$$

Reduction of scalar integrals with trivial numerator

[T.B., J.P. Guillet, G. Heinrich, (2000)]

$$
\begin{aligned}
I_N^n &= (-1)^N \Gamma(N-\frac{n}{2}) \int_0^1 d^N z \frac{\delta(1-\sum_{j=1}^N z_j)}{\left(-\frac{1}{2}\sum_{i,j=1}^N S_{ij}\, x_i\, x_j\right)^{N-\frac{n}{2}}} \\
S_{ij} &= (r_i - r_j)^2 - m_i^2 - m_j^2 \\
G_{ij} &= 2\, r_i \cdot r_j \quad , \quad r_j = p_1 + \cdots + p_j \quad , \quad n = 4 - 2\,\epsilon
\end{aligned}
$$

$I_N^n =$ 

$= \sum_{j=1}^{N} b_j$ 

$+ \begin{cases} -(1+2\epsilon)\frac{\det(G)}{\det(S)}\, I_N^{n+2} & , N=4 \\ \mathcal{O}(\epsilon) & , N=5 \\ 0 & , N \geq 6 \end{cases}$

$$
\sum_{j=1}^{N} S_{ij} b_j = 1 \quad \Leftrightarrow \quad b_i = \sum_{j=1}^{N} S_{ij}^{-1}
$$

Any N point integral can be represented by n–dimensional triangle functions and (n+2) dimensional box functions. The latter are infrared finite.

Five and six point functions

- Hexagons/pentagons are expressed by the functions I_3^n and I_4^{n+2}
- The isolation of infrared singularities is straight forward!!!

$$
\begin{aligned}
I_5^n &= (b_1 b_{12} + b_2 b_{21}) I_{3,12}^n + (b_1 b_{13} + b_3 b_{31}) I_{3,13}^n + b_1 (b_{12} + b_{13} + b_{14} + b_{15}) I_{4,1}^{n+2} \\
&+ (b_2 b_{23} + b_3 b_{32}) I_{3,23}^n + (b_2 b_{24} + b_4 b_{42}) I_{3,24}^n + b_2 (b_{21} + b_{23} + b_{24} + b_{25}) I_{4,2}^{n+2} \\
&+ (b_3 b_{34} + b_4 b_{43}) I_{3,34}^n + (b_3 b_{35} + b_5 b_{53}) I_{3,35}^n + b_3 (b_{31} + b_{32} + b_{34} + b_{35}) I_{4,3}^{n+2} \\
&+ (b_4 b_{45} + b_5 b_{54}) I_{3,45}^n + (b_4 b_{41} + b_1 b_{14}) I_{3,14}^n + b_4 (b_{41} + b_{42} + b_{43} + b_{45}) I_{4,4}^{n+2} \\
&+ (b_5 b_{51} + b_1 b_{15}) I_{3,15}^n + (b_5 b_{52} + b_2 b_{25}) I_{3,25}^n + b_5 (b_{51} + b_{52} + b_{53} + b_{54}) I_{4,5}^{n+2} \\
I_6^n &= \left\{ [b_1 (b_{12} b_{123} + b_{13} b_{132}) + b_2 (b_{21} b_{123} + b_{23} b_{231}) + b_3 (b_{31} b_{132} + b_{32} b_{231})] \, I_{3,123}^n + 5 \text{ c.p.} \right\} \\
&+ \left\{ [b_1 (b_{12} b_{124} + b_{14} b_{142}) + b_2 (b_{21} b_{214} + b_{24} b_{241}) + b_4 (b_{41} b_{412} + b_{42} b_{421})] \, I_{3,124}^n + 5 \text{ c.p.} \right\} \\
&+ \left\{ [b_1 (b_{13} b_{134} + b_{14} b_{143}) + b_3 (b_{31} b_{314} + b_{34} b_{341}) + b_4 (b_{41} b_{413} + b_{43} b_{431})] \, I_{3,134}^n + 5 \text{ c.p.} \right\} \\
&+ \left\{ [b_1 (b_{13} b_{135} + b_{15} b_{153}) + b_3 (b_{31} b_{315} + b_{35} b_{351}) + b_5 (b_{51} b_{513} + b_{53} b_{531})] \, I_{3,135}^n + 1 \text{ c.p.} \right\} \\
&+ \left\{ (b_1 b_{12} + b_2 b_{21})(b_{123} + b_{124} + b_{125} + b_{126}) I_{4,12}^{n+2} + 5 \text{ c.p.} \right\} \\
&+ \left\{ (b_1 b_{13} + b_3 b_{31})(b_{132} + b_{134} + b_{135} + b_{136}) I_{4,13}^{n+2} + 5 \text{ c.p.} \right\} \\
&+ \left\{ (b_1 b_{14} + b_4 b_{41})(b_{142} + b_{143} + b_{145} + b_{146}) I_{4,14}^{n+2} + 2 \text{ c.p.} \right\}
\end{aligned}
$$

Reduction of scalar integrals with non-trivial numerators

$$I_N^n(l_1,\dots,l_R) = (-1)^N \Gamma(N-\frac{n}{2}) \int_0^1 d^N z\, \delta(1-\sum_{j=1}^N z_j) \frac{z_{l_1}\dots z_{l_R}}{\left(\frac{1}{2}\sum_{i,j=1}^N S_{ij}\, x_i\, x_j\right)^{N-\frac{n}{2}}}$$

$$I_N^n(l_0,\dots,l_R) = \sum_{k=1}^R S^{-1}_{l_0 l_k} I_N^{n+2}(l_1,\dots,l_{k-1},l_{k+1},\dots,l_R)$$

$$+\sum_{j=1}^N S^{-1}_{j l_0}(N-n-R-1) I_N^{n+2}(l_1,\dots,l_p) - \sum_{j=1}^N S^{-1}_{j l_0} I^n_{N-1,j}(l_1,\dots,l_R)$$

Each N–point integral with a non–trivial numerator can be represented by scalar integrals with shifted dimensions.

- $I_{N=5,6}^{n+2m}$ drop out.
- $I_N^{n+2m} \to (I_N^{n+2m-2}, I_{N-1}^{n+2m-2})$ by scalar integral reduction $\to 1/\det(G)$.

Each N–point integral with non–trivial numerator can be represented by scalar integrals $I_1^n, I_2^n, I_3^n, I_4^{n+2}$. But $1/\det(G)$ unavoidable!

The GOLEM project

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Automated evaluation of one-loop amplitudes
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ flexibility to switch between algebraic/numeric representations

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- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, F. Mahmoudi, E. Pilon, T. Reiter, C. Schubert

The GOLEM algorithm

Step 1: Amplitude organization

- Split amplitude into gauge invariant subamplitudes
  $\longrightarrow$ No compensations between subamplitudes

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_I \mathcal{A}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

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Step 2: Graph generation

- generate Feynman diagrams
- project onto gauge invariant structures defined in step 1

$$\begin{aligned} \mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) &= \sum_G \mathcal{G}_G(|p_j\rangle, \epsilon_j^\lambda, \ldots) \\ &= \sum_I \sum_G \mathcal{C}_{IG}(s_{jk})\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots) \end{aligned}$$

$(s_{jk} = (p_j + p_k)^2)$

The GOLEM algorithm

Step 3: Reduction to integral basis

- Choose integral basis $\{I_B\}$ (see below)
- apply algebraic or semi-numerical reduction methods to map onto $\{I_B\}$
- semi-numerical reduction done with Fortran/C code

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \ldots) = \sum_B \sum_I \sum_G \mathcal{C}_{BIG}(s_{jk}, \ldots)\, I_B\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \ldots)$$

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Step 4: Export/manipulate coefficients C_{BIG}'s (optional)

- Denominator structure and size of C_{BIG}'s critical for numerical evaluation
- Export C_{BIG} to MAPLE/MATHEMATICA $\rightarrow$ simplification/factorization
- Export C_{BIG} to Fortran/C code $\rightarrow$ produce optimized output

$$\mathcal{A}(|p_j\rangle, \epsilon_j^\lambda, \dots) = \sum_B \sum_I \sum_G \mathrm{simplify}[\mathcal{C}_{BIG}(s_{jk}, \dots)]\, I_B\, \mathcal{T}_I(|p_j\rangle, \epsilon_j^\lambda, \dots)$$

Integral reduction

- Lorentz Tensor Integrals → Formfactor representation à la Davydychev, Tarasov

$$
\begin{aligned}
I_N^{\mu_1\cdots\mu_R} &= \sum \tau^{\mu_1\cdots\mu_R}(r_{j_1},\ldots,r_{j_r},g^m)\, I_N^{n+2m}(j_1,\ldots,j_r) \\
I_N^D(j_1,\ldots,j_r) &= (-1)^N\Gamma(N-\frac{D}{2})\int_0^\infty d^N z\,\delta(1-\sum_{l=1}^N z_l)\,\frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2}z\cdot\mathcal{S}\cdot z)^{N-D/2}} \\
\mathcal{S}_{ij} &= (r_i-r_j)^2-m_i^2-m_j^2\ ,\ r_j=p_1+\cdots+p_j \\
G_{ij} &= 2\,r_i\cdot r_j \qquad \text{“Gram matrix”}
\end{aligned}
$$

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\end{aligned}
$$

Differentiation by parts in parameter space and $d=4$ kinematical identities $\Rightarrow$

- get rid of higher dimensional integrals for $N>4$ (always possible!)
- reduce N-point scalar integrals for $N>4$ algebraically to I_3^n, I_4^{n+2}
- extraction of IR divergences trivial
- integral basis without inverse Gram determinants: GOLEM basis

N=5 rank 2 tensor form factors

$$I_5^{n,\,\mu_1\mu_2}(S=\{1,2,3,4,5\}) = g^{\mu_1\,\mu_2}\,B^{5,2}(S) + \sum_{l_1,l_2\in S} r_{l_1}^{\mu_1}\,r_{l_2}^{\mu_2}\,A_{l_1\,l_2}^{5,2}(S)$$

$$B^{5,2}(S) = -\frac{1}{2}\sum_{j\in S} b_j\,I_4^{n+2}(S\setminus\{j\})$$

$$A_{l_1 l_2}^{5,2}(S) =$$

$$\sum_{j\in S}\left(\mathcal{S}^{-1}{}_{j\,l_1}\,b_{l_2} + \mathcal{S}^{-1}{}_{j\,l_2}\,b_{l_1} - 2\,\mathcal{S}^{-1}{}_{l_1\,l_2}\,b_j + b_j\,\mathcal{S}^{\{j\}-1}{}_{l_1\,l_2}\right) I_4^{n+2}(S\setminus\{j\})$$

$$+\frac{1}{2}\sum_{j\in S}\sum_{k\in S\setminus\{j\}}\left[\mathcal{S}^{-1}{}_{j\,l_2}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_1} + \mathcal{S}^{-1}{}_{j\,l_1}\,\mathcal{S}^{\{j\}-1}{}_{k\,l_2}\right] I_3^n(S\setminus\{j,k\})$$

- relatively compact formulae
- Algebraic separation of IR poles always possible
- massive/massless internal propagators
- 5-point case most complicated

GOLEM Basis integrals

$$I_3^n(j_1,\dots,j_r) = -\Gamma\left(3-\frac{n}{2}\right)\int_0^1 \prod_{i=1}^{3} dz_i\,\delta(1-\sum_{l=1}^{3} z_l)\,\frac{z_{j_1}\dots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$

$$I_3^{n+2}(j_1) = -\Gamma\left(2-\frac{n}{2}\right)\int_0^1 \prod_{i=1}^{3} dz_i\,\delta(1-\sum_{l=1}^{3} z_l)\,\frac{z_{j_1}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{2-n/2}}$$

$$I_4^{n+2}(j_1,\dots,j_r) = \Gamma\left(3-\frac{n}{2}\right)\int_0^1 \prod_{i=1}^{4} dz_i\,\delta(1-\sum_{l=1}^{4} z_l)\,\frac{z_{j_1}\dots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$

$$I_4^{n+4}(j_1) = \Gamma\left(2-\frac{n}{2}\right)\int_0^1 \prod_{i=1}^{4} dz_i\,\delta(1-\sum_{l=1}^{4} z_l)\,\frac{z_{j_1}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{2-n/2}}$$

$(r_{\max} = 3)$ and scalar integrals I_2^n ,I_3^n, I_3^{n+2}, I_4^{n+2}.

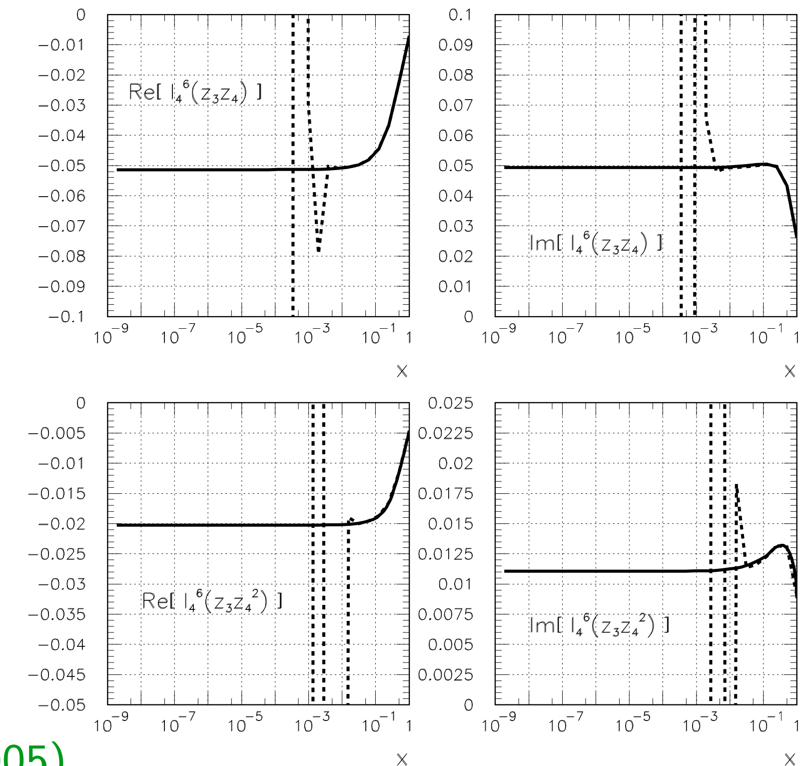
Three alternatives for evaluation:

1. algebraic reduction to “standard” basis I_2^n ,I_3^n, I_4^{n+2} (“Master integrals”)
2. semi-numerical reduction to scalar integrals
   $[1.\&2. \rightarrow$ Gram determinants $\sim 1/\det(G)^r]$
3. direct numerical evaluation

Numerical evaluation of basis integrals

1. Contour deformation in parameter space:
   3-dimensional integral representations for box integrals → robust but slow
2. One analytical integration → Cauchy integration done, real/imag. part separated
   2-dimensional integral representations for box integrals → fast

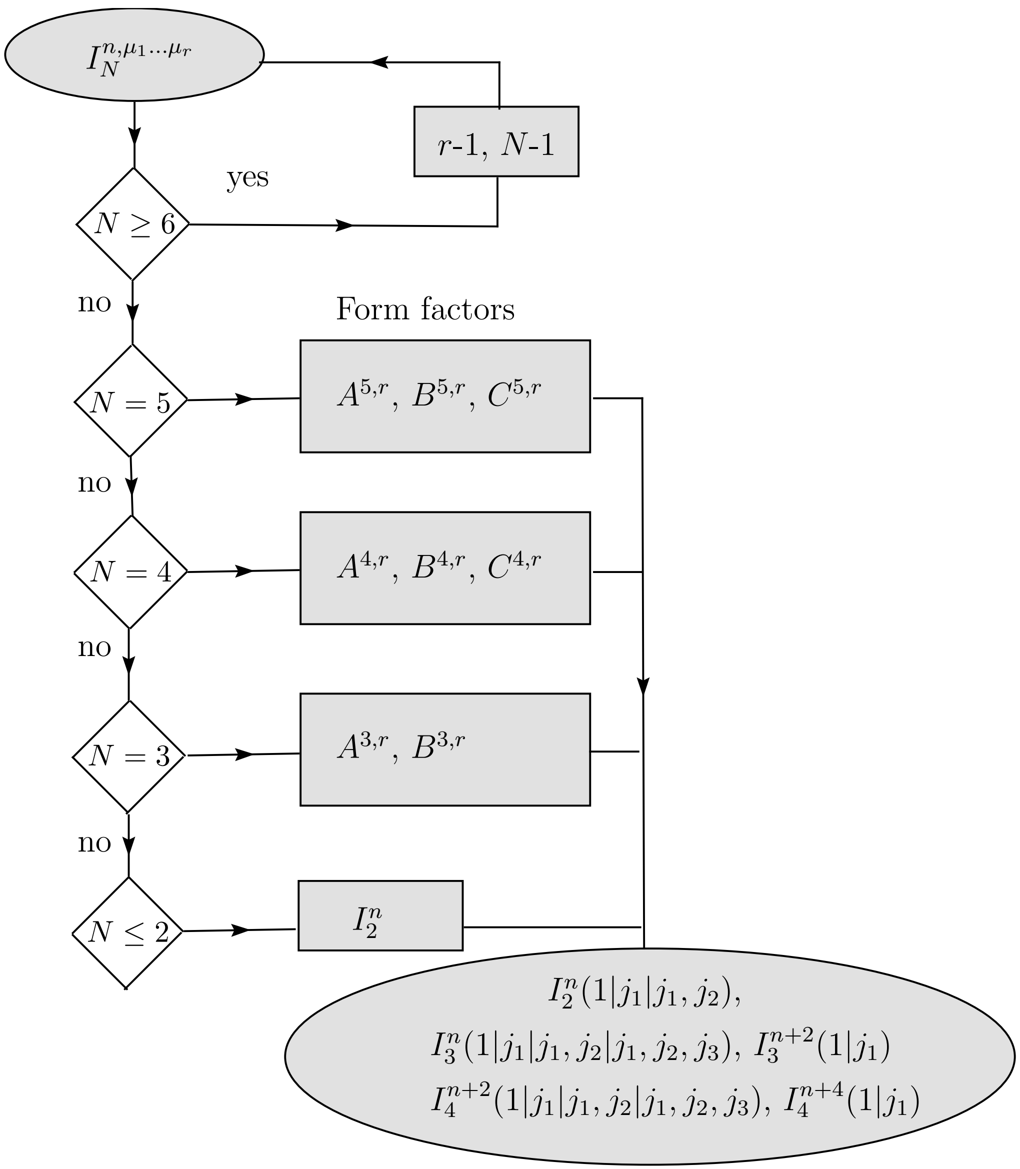
- both methods cross checked
- basis integrals tested for large Monte Carlo sample of LHC phase space points



1.) T.B., Guillet, Heinrich, Pilon, Schubert (2005)
2.) T.B., Heinrich, Kauer (2002)
see also: Soper (2000); Ferroglia, Passera, Passarino, Uccirati (2002);
Y. Kurihara, T. Kaneko, (2005); Anastasiou, Daleo (2005); Soper, Nagy (2006).

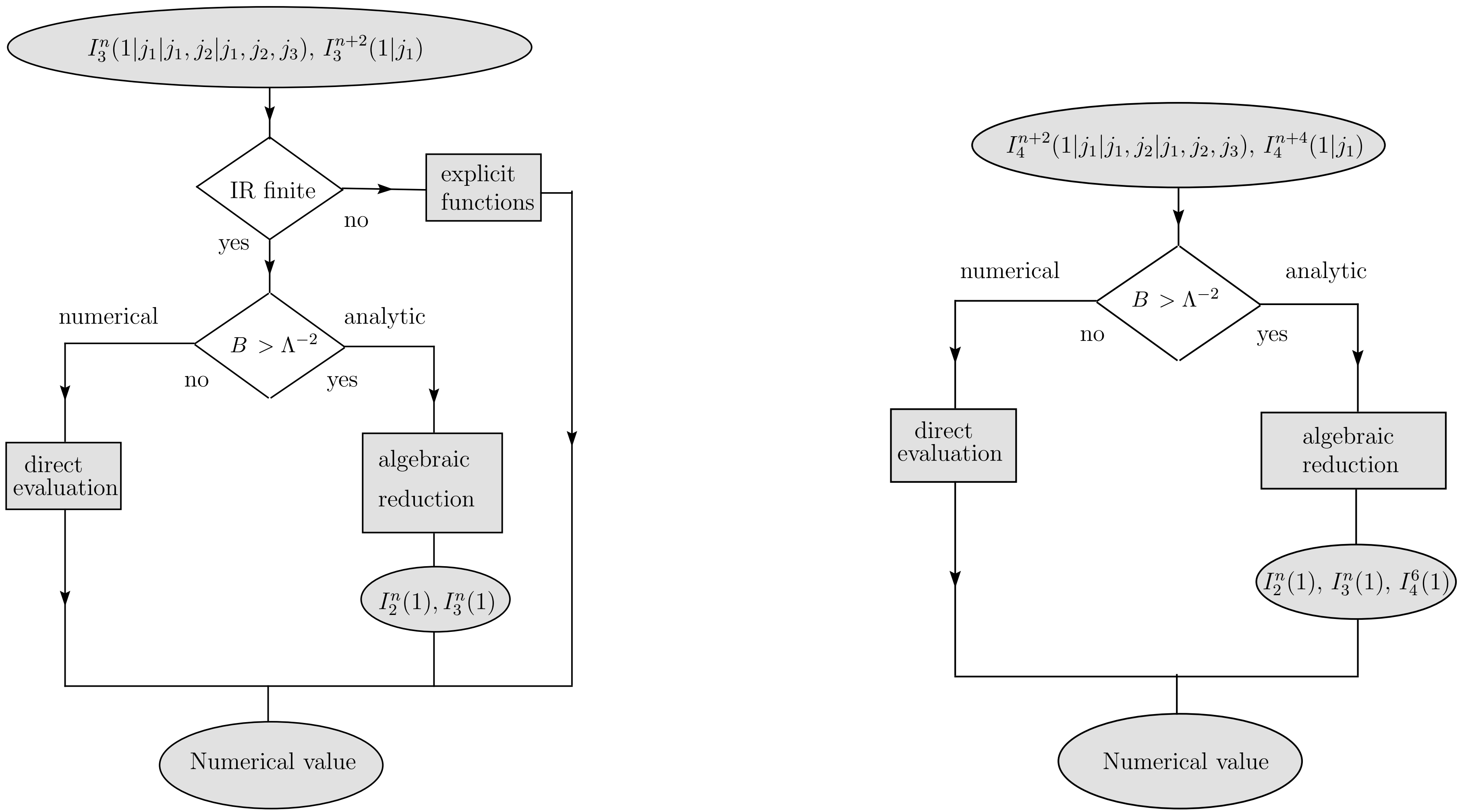
$$x \sim \sqrt{\det(G)}$$

Schematic overview of N-point tensor integral reduction



Treatment of basis integrals:

$B = |\det(G)/\det(\mathcal{S})|$



The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

- missing background for $gg \to H \to W^*W^*$
  [T.B., Ciccolini, Kauer, Krämer, 2005/2006. $m_q \neq 0$, W's offshell.]
  [Dührssen, Jacobs, Marquard, van der Bij, 2005. $m_q \neq 0$, W's onshell.]
- On-shell amplitude known since a long time
  [N. Glover, J.J. van der Bij (1989) $m_q = 0$; C. Kao, D. A. Dicus (1991) $m_q \neq 0$]

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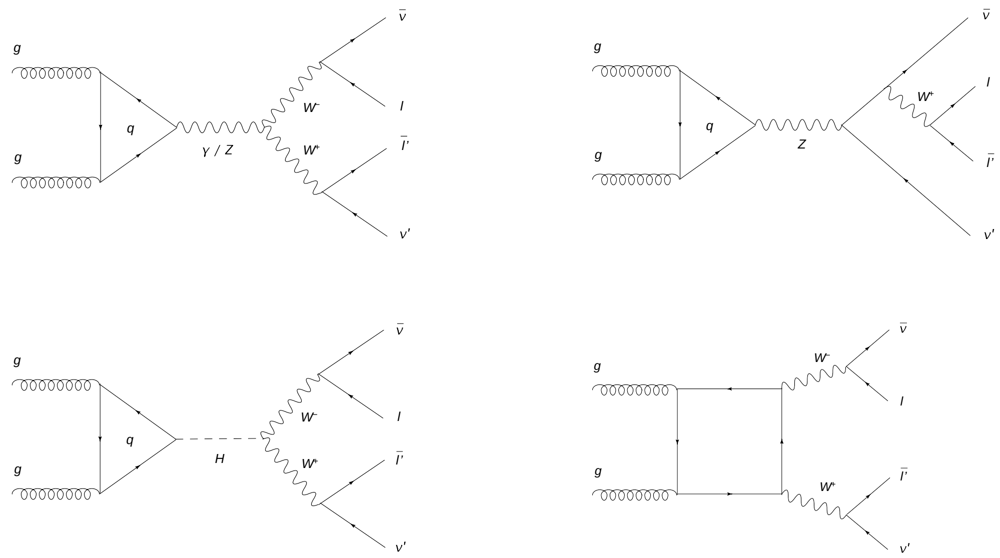

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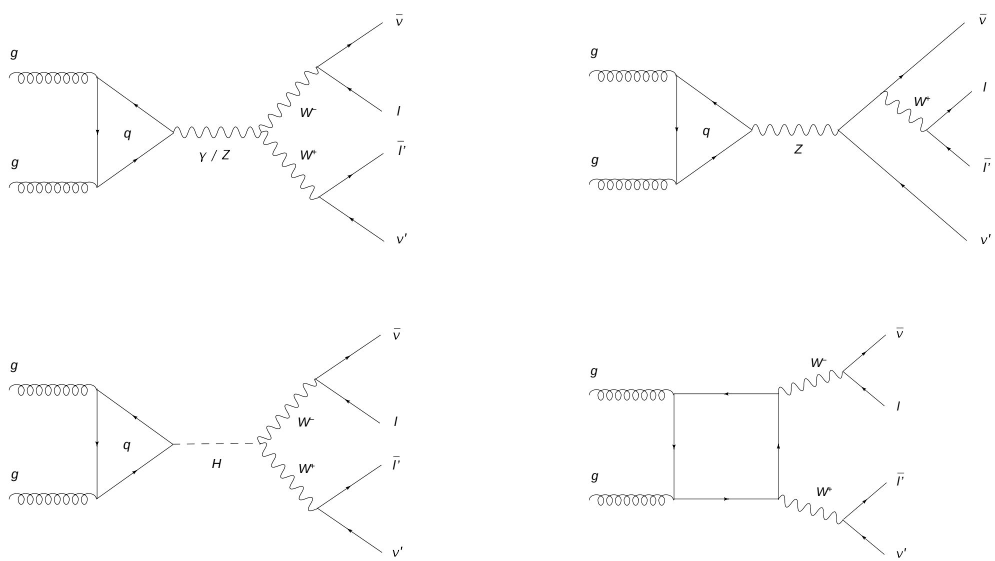
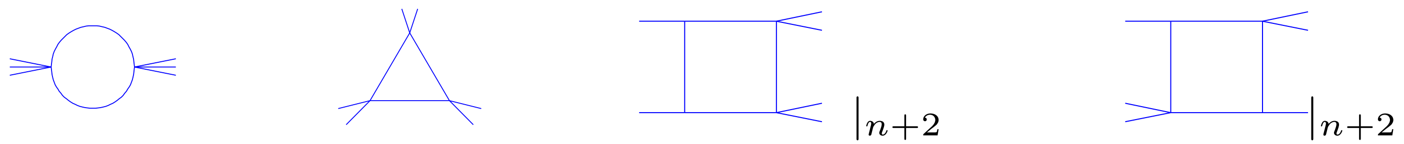


- single resonant graphs add to zero
- interference between Higgs signal and background also below WW threshold

The $gg \to W^*W^* \to l\nu l'\nu'$ amplitude

Helicity amplitudes Γ^{++}, Γ^{+-}, off-shell W's, $m_q \neq 0$, S/B interference

Fully algebraic reduction:



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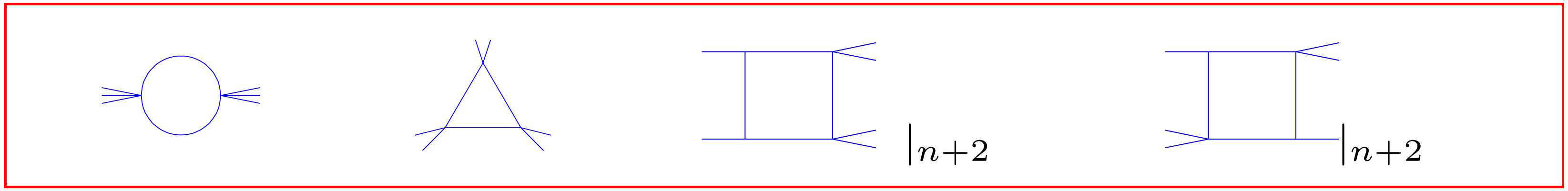


- box/triangle topologies $\to$ 27 Basis functions: $I_4^{d=6}$, $I_3^{d=4}$, $I_2^{d=n}$, 1.
- Decomposition of amplitude by gauge invariant structures (9 independent)
- Coefficients at most $1/\det(G)$, 6 scales $(s, t, s_3, s_4, M_b^2, M_t^2)$
- Instability region: $p_T^2(W) = \det(G)/s^2 < 0.01$ GeV^2, $|s_{3,4} - M_W^2| \gg M_W \Gamma_W$.
- Code available: http://hepsource.sf.net/GG2WW for $m_q = 0$, $m_q \neq 0$
- Shortly: All $gg \to VV$ $(V = \gamma, Z, W)$ box processes

Results: 2 Massless Generations, 3 Generations

LHC (pp, $\sqrt{s} = 14$ TeV)

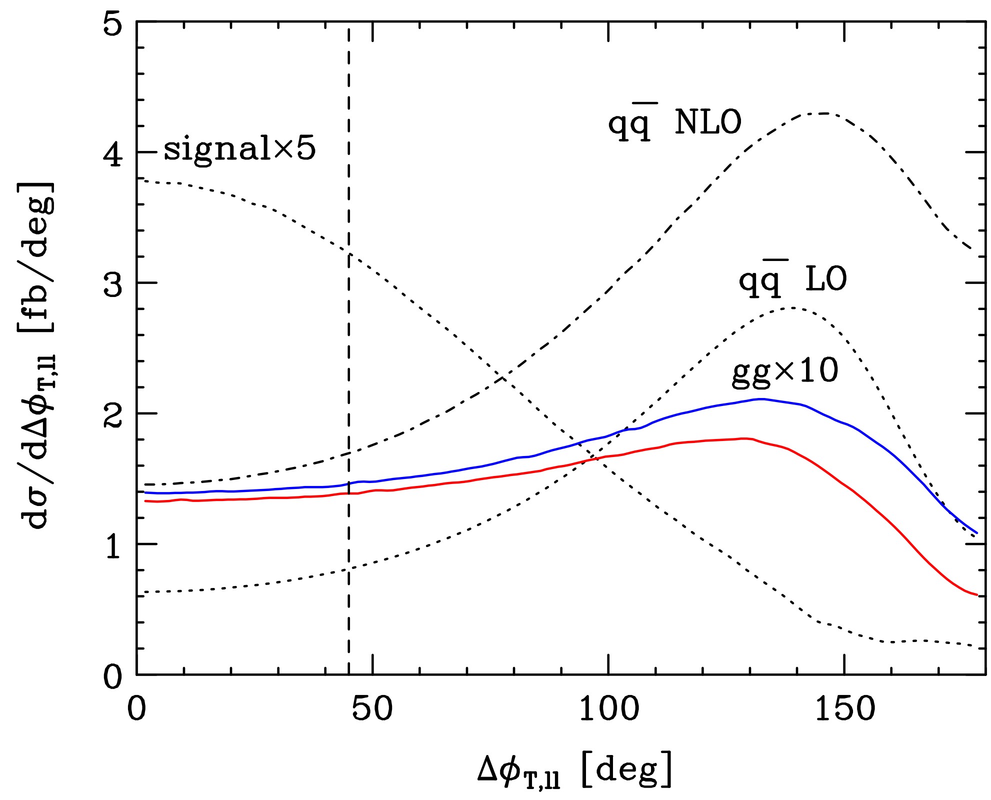
	$\sigma(pp \to W^*W^* \to \ell\bar{\nu}\bar{\ell}'\nu')$ [fb]					
	gg	$\frac{\sigma_{gg,3gen}}{\sigma_{gg,2gen}}$	$q\bar{q}$ LO	$q\bar{q}$ NLO	$\frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LO}}}$	$\frac{\sigma_{\mathrm{NLO}+gg}}{\sigma_{\mathrm{NLO}}}$
σ_{tot}	$\frac{60.12(7)}{53.61(2)^{+14.0}_{-10.8}}$	1.12	$875.8(1)^{+54.9}_{-67.5}$	$1373(1)^{+71}_{-79}$	1.57	$\frac{1.04}{1.04}$
σ_{std}	$\frac{29.79(2)}{25.89(1)^{+6.85}_{-5.29}}$	1.15	$270.5(1)^{+20.0}_{-23.8}$	$491.8(1)^{+27.5}_{-32.7}$	1.82	$\frac{1.06}{1.05}$
σ_{bkg}	$\frac{1.416(3)}{1.385(1)^{+0.40}_{-0.31}}$	1.02	$4.583(2)^{+0.42}_{-0.48}$	$4.79(3)^{+0.01}_{-0.13}$	1.05	$\frac{1.30}{1.29}$

$M_W/2 \leq \mu_{\mathrm{ren,fac}} \leq 2M_W$ ($q\bar{q} \to WW$ from MCFM by J. Campbell, R.K. Ellis)

standard cuts: $p_{T,\ell} > 20$ GeV, $|\eta_\ell| < 2.5$, $\not{p}_T > 25$ GeV

search cuts: $\Delta\phi_{T,\ell\ell} < 45°$, $M_{\ell\ell} < 35$ GeV, 25 GeV $< p_{T,\min}$, 35 GeV $< p_{T,\max} < 50$ GeV
jet veto removes jets with: $p_{T,\mathrm{jet}} > 20$ GeV, $|\eta_{\mathrm{jet}}| < 3$

The $gg \to W^*W^* \to l\nu l'\nu'$ cross section



- $\Rightarrow$ severe Higgs search cuts amplify $ggWW$ contribution $\sim 30\%$!

The $\gamma\gamma \rightarrow ggg$ amplitude

[T.B., J.-Ph. Guillet, F. Mahmoudi, (2004)]

- Relevant for $\gamma\gamma +$ jet background for Higgs+jet production
  [D. de Florian, Z. Kunszt, (1999)]
- Amplitude indirectly known from $gg \rightarrow ggg$
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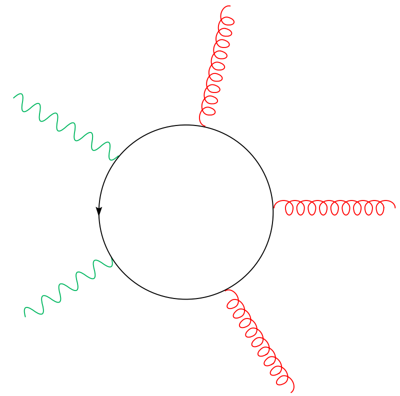

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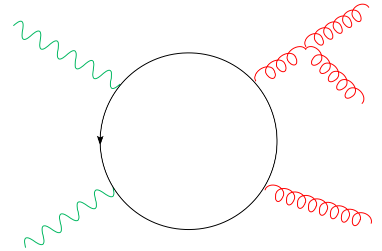
Independent helicity structures:
$\Gamma^{+++++}, \Gamma^{++++-}, \Gamma^{+++--}, \Gamma^{+-+++}, \Gamma^{+-++-}, \Gamma^{--+++}$
All helicity amplitudes calculated by algebraic reduction



- Box, pentagon topologies, 5 scales
- One colour structure: $\sim f^{abc}$
- Sorted by scalar integrals and gauge independent structures

The $gg \to HH, HHH$ amplitude

- Cross sections for multi-Higgs production by gluon fusion
  [T.B., S. Karg, N. Kauer]
- $gg \to HH$ and effective amplitudes $M_T \to \infty$ known since a long time
  [N. Glover, J.J. van der Bij (1987/1988)]
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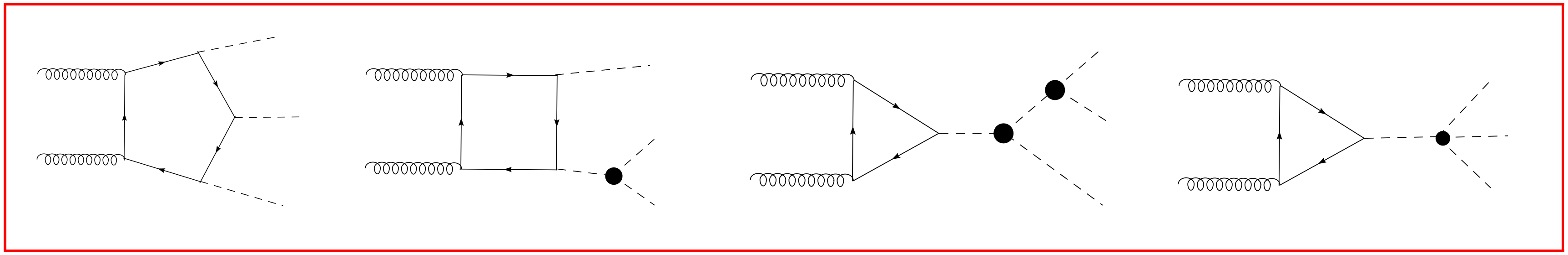

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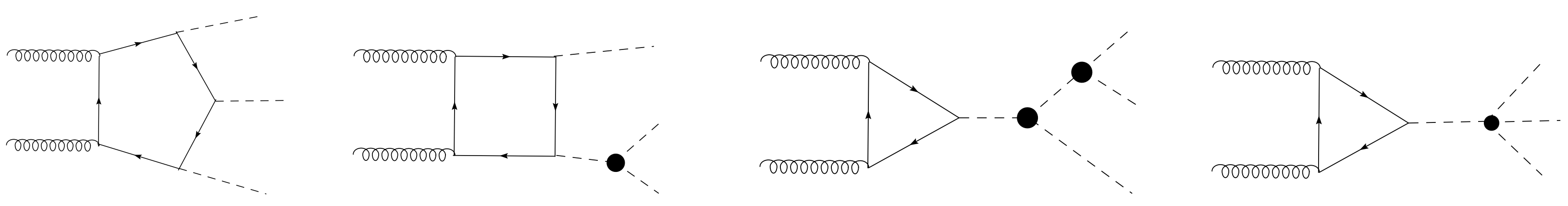
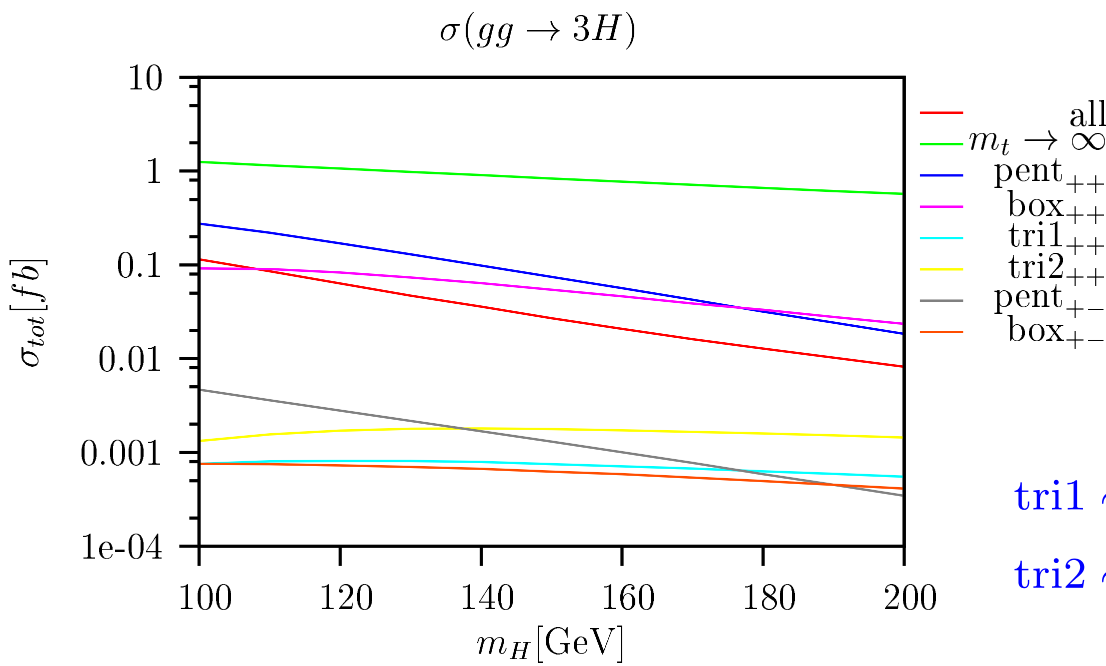


- box/triangle/pentagon topologies, 7 scales $(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, M_H^2, M_t^2)$
- Gauge invariant structures: tr$(\mathcal{F}_1\mathcal{F}_2)$, $p_2.\mathcal{F}_1.p_i\ p_1.\mathcal{F}_2.p_j$, $\mathcal{F}_j^{\mu\nu} = p_j^\mu \varepsilon_j^\nu - p_j^\nu \varepsilon_j^\mu$
- Basis functions: $I_4^{d=6}$, $I_3^{d=4}$, $I_2^{d=n}$, 1. Coefficients at most $1/\det(G)$

The $gg \to HHH$ cross section

- perfect agreement with Plehn/Rauch
- Numerically stable result
- CPU time: 1 h for inclusive cross section on pentium 4 PC (2.8 GHz)



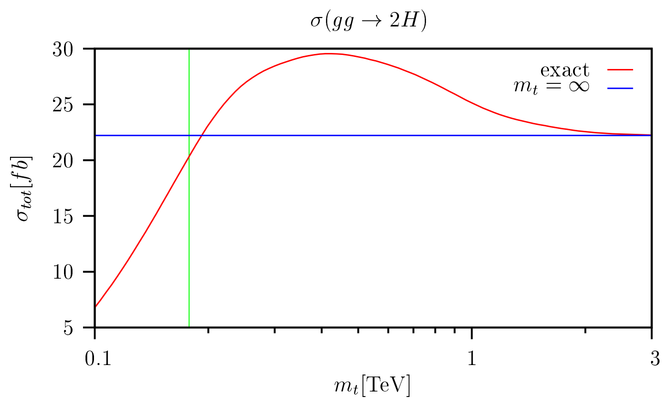
tri1 $\sim \lambda_{4H}$

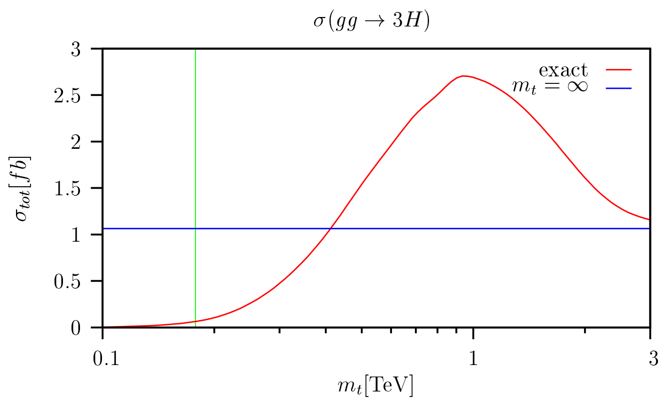
tri2 $\sim \lambda_{3H}^2$

- $\Rightarrow$ quartic Higgs coupling can not be tested at the LHC

The $gg \to HH, HHH$ cross section

- $L_{M_T \to \infty} = \frac{\alpha_s}{12\pi} \mathcal{F}^a_{\mu\nu} \mathcal{F}^{\mu\nu\ a} \log(1 + H/v) \Rightarrow gg + nH$ effective vertices
- effective vertices not a good description at LHC for $n \geq 2$





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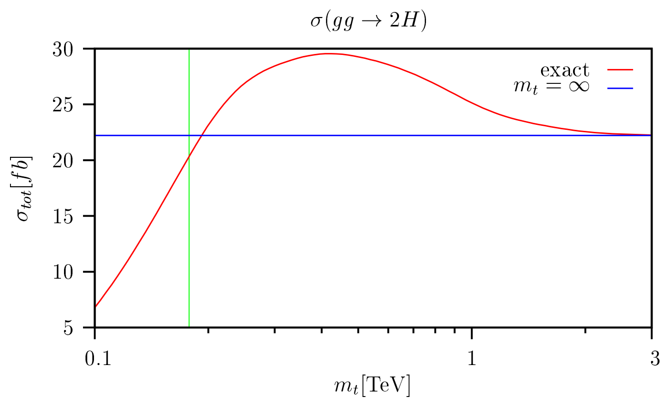
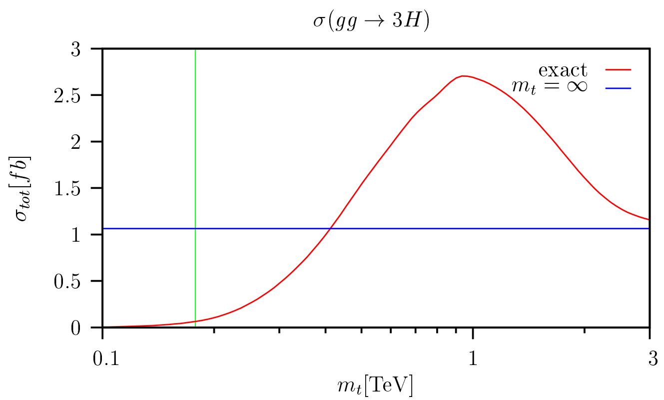
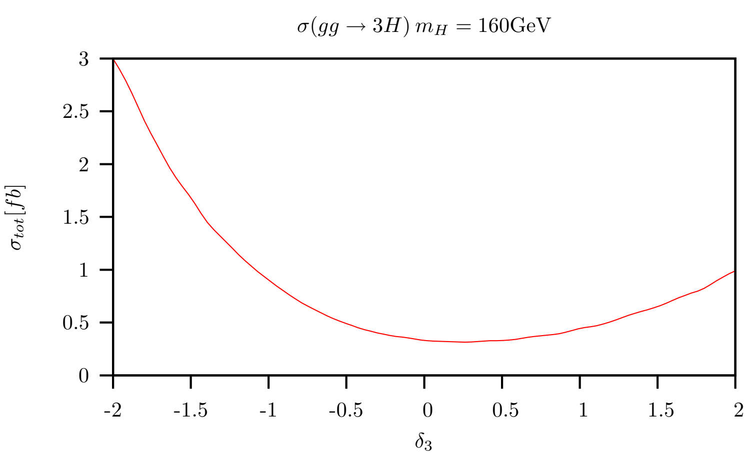


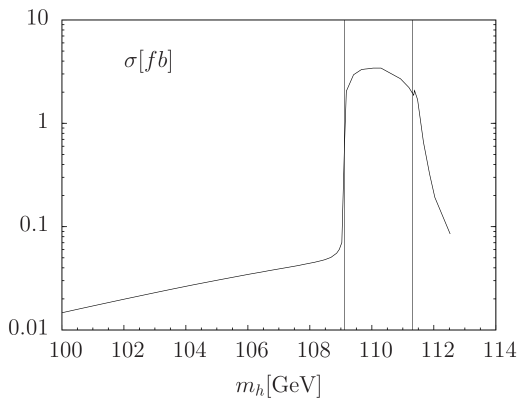


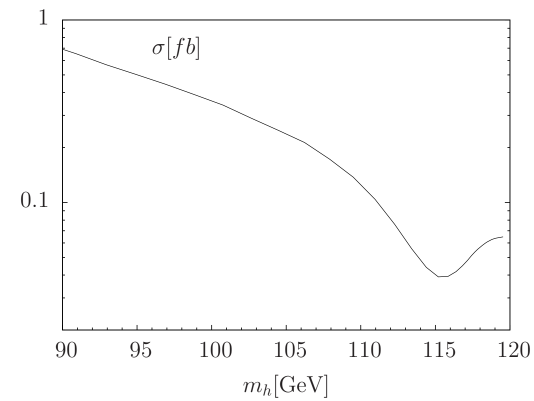
- cross section enhanced by BSM physics, $\delta_3 = (\lambda_{3H,BSM} - \lambda_{3H,SM})/\lambda_{3H,SM}$
- trilinear Higgs coupling not uniquely fixed at LHC (if at all)



The $gg \to HH, HHH$ cross section

- amplification possible in two Higgs doublet models
- resonant amplification does, $\tan\beta$ amplification does not help





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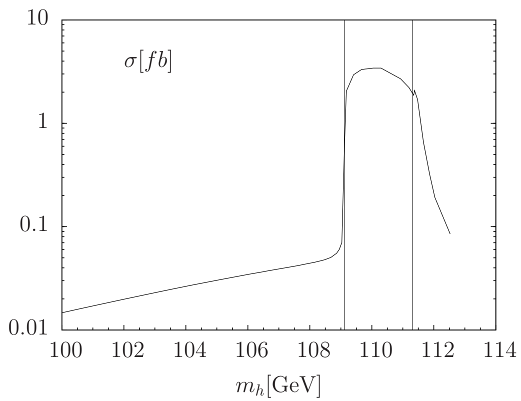
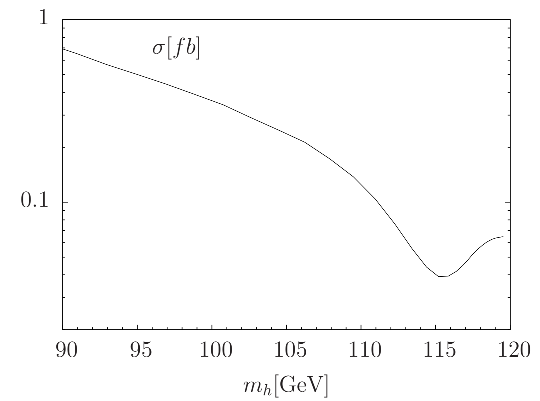
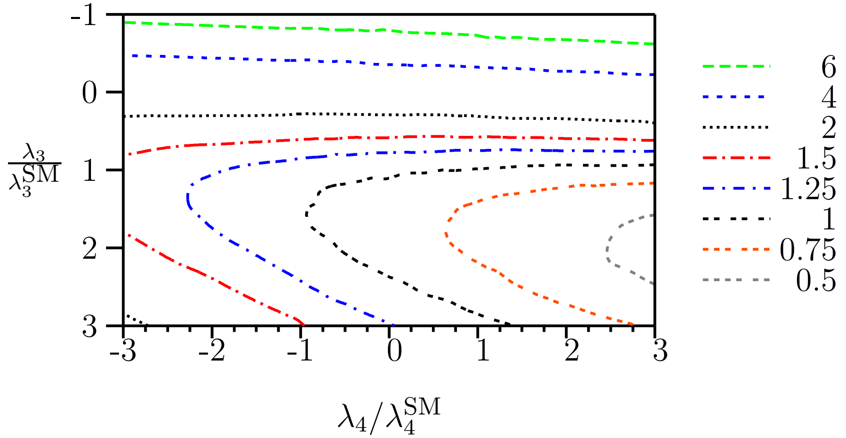




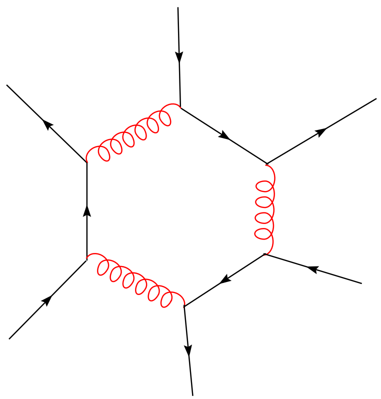
- Higher dimensional operators $\Rightarrow \lambda_{3H}, \lambda_{4H}$ free parameters

$$V = \sum_{k=1}^{\infty} \frac{g_k}{\Lambda^{2k}} \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^{2+k}$$



The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude (in progress)

- Contribution of $PP \to 4$ jets, $bbbb$ at NLO [$\sigma \sim \mathcal{O}$(nb) at LHC!]
- Two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- Other partonic contributions: $gg \to gggg$, $gg \to q\bar{q}gg$, $gg \to q\bar{q}q\bar{q}$ plus crossings
  $\to$ accessible with twistor space inspired/unitarity based methods (?!)



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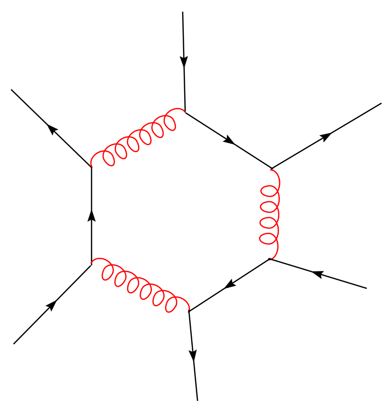


- algebraic reduction done $\rightarrow$ Masterintegrals
- semi-numerical reduction $\rightarrow$ Golem basis with Fortran 90 code "golem90 v0.2"
- Amplitude evaluation $\mathcal{O}(s)$, rank 3 6-point form factor ~ 40 ms
  (Pentium4, 1.6 GHz)
- Evaluation time of virtual corrections small compared to real emission corrections

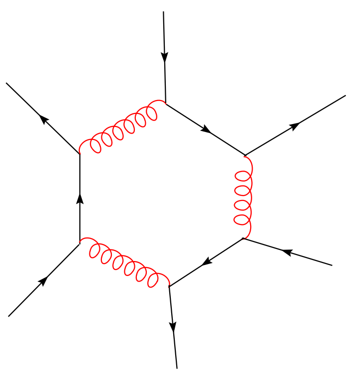
The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude

Numerical results of hexagon diagram of helicity Amplitude A^{++++++}:

$$A^{++++++}(k_1,\ldots,k_6) = \frac{g_s^6}{(4\pi)^{n/2}}\frac{1}{s}[\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C + \mathcal{O}(\epsilon)]$$

Spinor lines closed by multiplying $1 = \frac{\langle 1^+|4|2^+\rangle}{\sqrt{s_{14}s_{24}}}\frac{\langle 4^+|1|3^+\rangle}{\sqrt{s_{14}s_{13}}}\frac{\langle 6^+|1|5^+\rangle}{\sqrt{s_{15}s_{16}}}e^{i\Phi}$

Kinemtical point:



k $=($	$k^0,$	$k^1,$	$k^2,$	$k^4)$
$k_1 = ($	0.5,	0.,	0.,	0.5)
$k_2 = ($	0.5,	0.,	0.,	-0.5)
$k_3 = ($	0.1917819,	0.1274118,	0.08262477,	0.1171311)
$k_4 = ($	0.3366271,	-0.06648281,	-0.3189379,	-0.08471424)
$k_5 = ($	0.2160481,	-0.2036314,	0.04415762,	0.05710657)
$k_6 = ($	0.2555428,	0.1427024,	0.1921555,	-0.08952338)

Up to phase/color factor:

$\mathrm{Re}(A)$	$\mathrm{Im}(A)$	$\mathrm{Re}(B)$	$\mathrm{Im}(B)$	$\mathrm{Re}(C)$	$\mathrm{Im}(C)$
-5.313592	-1.245007	-23.74344	-23.54086	-14.37056	-96.23081

Evaluation of rational terms à la GOLEM

[T.B., J.Ph. Guillet, G. Heinrich, hep-ph/0609054]

- Unitarity and Twistor space inspired methods very successful for extracting all $D = 4$ information from (massless) amplitudes, i.e. logs, dilogs.
- Scattering amplitudes sensitive to ultraviolet behaviour $\mathcal{O}(\epsilon/\epsilon) \Rightarrow$ "rational polynomials".
- The GOLEM algebra packages contain all this information!

Definition of “rational parts”

- Amplitude Γ can be written a la Davydychev:

$$\Gamma \quad = \quad \sum C(n, \{j_l\}, \{s_{ij}, m_k\}) I_N^{n+2m}(\{j_l\}; \{s_{ij}, m_k\})$$

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- Define rational part of the amplitude Γ (assume IR finite) as:

$$\mathcal{R}[\Gamma] = \sum C(4, \{j_l\}, \{s_{ij}, m_k\}) \mathcal{R}[I_N^{n+2m}(\{j_l, \ldots\}; \{s_{ij}, m_k\})]$$
$$+(n-4) \sum C'(4, \{j_l\}, \{s_{ij}, m_k\}) \, \mathcal{P}[I_N^{n+2m}(\{j_l, \ldots\}; \{s_{ij}, m_k\})]$$
$$C'(4, \{j_l\}, \{s_{ij}, m_k\}) = \frac{d}{dn} C(n, \{j_l\}, \{s_{ij}, m_k\}) \Big|_{n=4} \, .$$

$\mathcal{P}$ is projector onto $1/\epsilon$ pole, C' depends on renormalization scheme.

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$\mathcal{P}$ is projector onto $1/\epsilon$ pole, C' depends on renormalization scheme.

- Reduction to master integrals using algebraic GOLEM codes:

$$I_N^{n+2m}(\{j_l\}) \quad = \quad \sum c_1(n)\, I_1^n + \sum c_2(n)\, I_2^n + \sum c_3(n)\, I_3^n + \sum c_4(n)\, I_4^{n+2}$$
$$\mathcal{R}[c(n) I_N] \quad = \quad c(4)\, \mathcal{R}[I_N] + (n-4)\, c'(4) \mathcal{P}[I_N]$$

Definition of “rational parts”

- Need pole/rational terms of scalar integral basis:

$$\mathcal{P}[I_4^{n+2}] = 0 \quad , \quad \mathcal{P}[I_3^{n}] = 0$$
$$\mathcal{R}[I_4^{n+2}] = 0 \quad , \quad \mathcal{R}[I_3^{n}] = 0$$

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- For one and two point functions use:

$$\begin{aligned} I_1^n(m^2) &= m^2 \frac{\Gamma(1+\epsilon)}{(1-\epsilon)\epsilon} - m^2 \log(m^2) \\ I_2^n(s, m_1^2, m_2^2) &= \frac{\Gamma(1+\epsilon)}{\epsilon} - \int_0^1 dx \log(-sx(1-x) + xm_1^2 + (1-x)m_2^2) \end{aligned}$$

$$\mathcal{P}[I_1^n(m^2)] = \frac{m^2}{\epsilon} \quad , \quad \mathcal{R}[I_1^n(m^2)] = m^2 \quad , \quad \mathcal{P}[I_2^n] = \frac{1}{\epsilon} \quad , \quad \mathcal{R}[I_2^n] = 0$$

Indirect definition of "cut-constructibility" for general amplitudes

- To avoid infrared trickery define cut-constructible part of Γ:

$$\mathcal{C}[\Gamma] = (1 - \lim_{\text{on-shell}} \mathcal{R})[\Gamma_{\text{off-shell}}])$$

- Need only to know pole and rational parts of off-shell tensor form factors (easy!)
- on-shell limit of these terms exist

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- Need only to know pole and rational parts of off-shell tensor form factors (easy!)
- on-shell limit of these terms exist
- All rational parts of 1,2,3,4,5-point tensor integrals for massless $2 \to 4$ kinematics evaluated.
- Compact formulas, valid for general massive/massless kinematics
- Result compared to Xiao, Yang, Zhu.
- $\mathcal{R}[I_2^{\mu_1}, I_3^{\mu_1}, I_4^{\mu_1,\mu_1\mu_2}, I_5^{\mu_1,\mu_1\mu_2,\mu_1\mu_2\mu_3}] = 0$
  $\Rightarrow$ Amplitudes with at most rank N–2 N–point functions have no rational terms !
  $\Rightarrow$ true for susy amplitudes ! (BDDK-theorem)

Example 1: Gluon fusion $gg \to H$

Amplitude:

$$\begin{aligned}
\mathcal{M} &= -\delta^{ab} \frac{m_t}{v} \frac{g_s^2}{(4\pi)^{n/2}} \int \frac{d^n k}{i\,\pi^{n/2}} \frac{\mathrm{tr}(\varepsilon_1(q_1+m_t)(q_2+m_t)\varepsilon_2(k+m_t))}{(q_1^2-m_t^2)(q_2^2-m_t^2)(k-m_t^2)} \\
&= I_3^n(r_1,r_2,0,m_t^2,m_t^2,m_t^2)\,(-\varepsilon_1\cdot\varepsilon_2 + 2\,\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_1 + 2\,m_t^2\,\varepsilon_1\cdot\varepsilon_2) \\
&\quad + I_3^{n\,\mu\nu}(r_1,r_2,0,m_t^2,m_t^2,m_t^2)\,(8\,\varepsilon_{1\,\mu}\varepsilon_{2\,\nu} - 2\,\varepsilon_1\cdot\varepsilon_2\,g_{\mu\nu})
\end{aligned}$$

Form factors:

$$(\mathcal{P}+\mathcal{R})[B^{3,2}] = \frac{1+\epsilon}{4\,\epsilon}$$

$$(\mathcal{P}+\mathcal{R})[A_{11}^{3,2}] = -(\mathcal{P}+\mathcal{R})[A_{12}^{3,2}] = (\mathcal{P}+\mathcal{R})[A_{22}^{3,2}] = -\frac{1}{2\,s}$$

Rational part of gluon fusion amplitude ($\mathcal{F}_j^{\mu\nu} = p_j^\mu \varepsilon_j^\nu - \varepsilon_j^\mu p_j^\nu$):

$$\mathcal{R}[\mathcal{M}] = \delta^{ab}\,\frac{\alpha_s}{\pi}\,\frac{m_t^2}{v}\,\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_2)}{s}$$

$\Rightarrow$ Method works for massive amplitudes !

Example 2: Scattering of light-by-light $\gamma\gamma \to \gamma\gamma$

Amplitude:

$$\mathcal{M} = \frac{e^4}{(4\pi)^{n/2}} \sum_{\sigma \in S_4/Z_4} \mathcal{G}(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$$

$$\mathcal{G}(1,2,3,4) = -\int \frac{d^n k}{i\,\pi^{n/2}} \frac{\mathrm{tr}(\varepsilon_1(q_1 + m_e)\varepsilon_2(q_2 + m_e)\varepsilon_3(q_3 + m_e)\varepsilon_4(k + m_e))}{(q_1^2 - m_e^2)(q_2^2 - m_e^2)(q_3^2 - m_e^2)(k - m_e^2)}$$

Form factors, $\mathcal{U} = \mathcal{P} + \mathcal{R}$: (only rank 4 shown)

$$\mathcal{U}[C^{4,4}] = \frac{1}{24}\frac{1}{\epsilon} + \frac{5}{72}$$

$$\mathcal{U}[B_{11}^{4,4}] = \mathcal{U}[B_{13}^{4,4}] = \mathcal{U}[B_{22}^{4,4}] = \mathcal{U}[B_{33}^{4,4}] = -\mathcal{U}[B_{12}^{4,4}] = -\mathcal{U}[B_{23}^{4,4}] = -\frac{1}{12\,u}$$

$$\mathcal{U}[A_{1111}^{4,4}] = \mathcal{U}[A_{3333}^{4,4}] = \frac{1}{s\,t} - \frac{1}{s\,u} + \frac{1}{2\,u^2} \quad , \quad \mathcal{U}[A_{1112}^{4,4}] = \mathcal{U}[A_{2333}^{4,4}] = \frac{1}{2\,s\,u} - \frac{1}{2\,u^2}$$

$$\mathcal{U}[A_{1113}^{4,4}] = \mathcal{U}[A_{1333}^{4,4}] = -\frac{1}{2\,s\,t} - \frac{1}{2\,s\,u} + \frac{1}{2\,u^2} \quad , \quad \mathcal{U}[A_{1122}^{4,4}] = \mathcal{U}[A_{2233}^{4,4}] = -\frac{1}{6\,s\,t} + \frac{1}{2\,u^2}$$

$$\mathcal{U}[A_{1123}^{4,4}] = \mathcal{U}[A_{1233}^{4,4}] = \frac{1}{6\,s\,t} + \frac{1}{6\,s\,u} - \frac{1}{2\,u^2} \quad , \quad \mathcal{U}[A_{1133}^{4,4}] = -\frac{1}{3\,s\,t} - \frac{1}{3\,s\,u} + \frac{1}{2\,u^2}$$

$$\mathcal{U}[A_{1222}^{4,4}] = \mathcal{U}[A_{2223}^{4,4}] = -\frac{1}{2\,s\,t} - \frac{1}{2\,s\,u} - \frac{1}{2\,u^2} \quad , \quad \mathcal{U}[A_{1223}^{4,4}] = \frac{1}{6\,s\,t} + \frac{1}{6\,s\,u} + \frac{1}{2\,u^2}$$

Rational term for $\gamma\gamma \to \gamma\gamma$

$\mathcal{R}[\mathcal{M}_{\gamma\gamma\to\gamma\gamma}] \sim$

$$\frac{8}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_2)}{s}\frac{\mathrm{tr}(\mathcal{F}_3\mathcal{F}_4)}{s} + \frac{8}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_3)}{u}\frac{\mathrm{tr}(\mathcal{F}_2\mathcal{F}_4)}{u} + \frac{8}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_4)}{t}\frac{\mathrm{tr}(\mathcal{F}_2\mathcal{F}_3)}{t}$$
$$+\frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_2)}{s}\frac{p_4\cdot\mathcal{F}_3\cdot p_1\;\;p_3\cdot\mathcal{F}_4\cdot p_1}{s\,t\,u} - \frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_3)}{u}\frac{p_1\cdot\mathcal{F}_2\cdot p_3\;\;p_3\cdot\mathcal{F}_4\cdot p_1}{s\,t\,u}$$
$$+\frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_1\mathcal{F}_4)}{t}\frac{p_1\cdot\mathcal{F}_2\cdot p_3\;\;p_4\cdot\mathcal{F}_3\cdot p_1}{s\,t\,u} + \frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_2\mathcal{F}_3)}{t}\frac{p_2\cdot\mathcal{F}_1\cdot p_3\;\;p_3\cdot\mathcal{F}_4\cdot p_1}{s\,t\,u}$$
$$-\frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_2\mathcal{F}_4)}{u}\frac{p_2\cdot\mathcal{F}_1\cdot p_3\;\;p_4\cdot\mathcal{F}_3\cdot p_1}{s\,t\,u} + \frac{64}{3}\frac{\mathrm{tr}(\mathcal{F}_3\mathcal{F}_4)}{s}\frac{p_2\cdot\mathcal{F}_1\cdot p_3\;\;p_1\cdot\mathcal{F}_2\cdot p_3}{s\,t\,u}$$
$$+1024\,\frac{p_2\cdot\mathcal{F}_1\cdot p_3\;\;p_1\cdot\mathcal{F}_2\cdot p_3}{s\,t\,u}\frac{p_4\cdot\mathcal{F}_3\cdot p_1\;\;p_3\cdot\mathcal{F}_4\cdot p_1}{s\,t\,u}$$

Up to irrelevant phases:

$$\mathcal{R}[\mathcal{M}^{++++}] \sim \mathcal{R}[\mathcal{M}^{+++-}] \sim \mathcal{R}[\mathcal{M}^{++--}] \sim \quad 8\,\alpha^2$$

Rational term for $\gamma\gamma \to ggg$

$$\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \frac{Q_q^2 g_s^3}{i\pi^2} f^{c_3 c_4 c_5} \mathcal{A}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5}$$

$$\mathcal{R}[\mathcal{A}^{+++++}] = \mathcal{A}^{+++++} = -\frac{\mathrm{tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\mathrm{tr}(\mathcal{F}_3^+\mathcal{F}_4^+\mathcal{F}_5^+)}{2\, s_{34}s_{45}s_{35}}$$

$$\begin{aligned} \mathcal{A}^{-++++} &= \frac{\mathrm{tr}(\mathcal{F}_2^+\mathcal{F}_3^+)\mathrm{tr}(\mathcal{F}_4^+\mathcal{F}_5^+)}{s_{23}^2 s_{45}^2}\Big[C^{-++++}\, p_2\cdot\mathcal{F}_1^-\cdot p_4 - (4\leftrightarrow 5)\Big] \\ C^{-++++} &= -\frac{s_{15}s_{12}}{s_{24}s_{35}} - \frac{s_{15}}{s_{35}} + \frac{s_{23}}{s_{24}} - \frac{s_{15}}{s_{34}} \end{aligned}$$

- Also evaluated $\mathcal{A}^{++++-}$, $\mathcal{R}[\mathcal{A}^{--+++}]$, $\mathcal{R}[\mathcal{A}^{+++--}]$, $\mathcal{R}[\mathcal{A}^{-+++-}]$ perfect agreement with rational terms of full computation presented by T.B., J.-Ph. Guillet, F. Mahmoudi, (2004)
- tests five-point tensor form factors

Rational term for $\gamma\gamma \to \gamma\gamma\gamma\gamma$

- Four independent helicity amplitudes: $+++++\pm$, $++++--$, $+++---$
- Mahlon: $\mathcal{M}^{+++++\pm} = 0$, $R[\mathcal{M}^{++++--}] = 0$
- We find by standard Feynman diagrammatic approach:

$$
\begin{aligned}
\mathcal{R}[\mathcal{M}^{++++++}] &= 0 \\
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\mathcal{R}[\mathcal{M}^{+++---}] &= 0
\end{aligned}
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\end{aligned}
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- All necessary rational form factors evaluated for $2 \to 4$ processes
- Rational parts evaluation by-product of GOLEM project
- Problem of automated evaluation of rational terms solved !

Summary

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  - massive and massless particles
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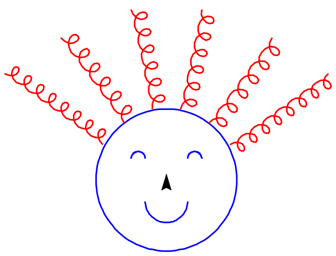

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- Automated evaluation of rational parts of amplitudes:
  - Tested for $gg \to H$, $\gamma\gamma \to \gamma\gamma$, $\gamma\gamma \to ggg$, $\gamma\gamma \to \gamma\gamma\gamma\gamma$
  - Complementary to Twistor space inspired/unitarity based methods !!!