Cosmological Constant Problem and Renormalization Group

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- Myth and Legends of Cosmological Constant.
- Cosmological Constant at classical level.
- Cosmological Constant (CC) Problem and trying to solve it.
- Renormalization Group Running of CC.
- Covariance and Physical Renormalization Group for CC.
- Applications to Cosmology and Astrophysics

Recommended reading:

- *S. Weinberg, The cosmological constant problem, Rev. Mod. Phys.* ***61*** *(1989) 1.*

- *I.Sh., J. Solà, Scaling behavior of the cosmological constant: Interface between quantum field theory and cosmology, JHEP* **02** *(2002) 006, hep-th/0012227;*
On the possible running of the cosmological "constant", Phys. Lett. **B682** *(2009) 105, arXiv:0910.4925.*

- *E. Bianchi & C. Rovelli, Why all these prejudices against a constant? arXiv:1002.3966 [astro-ph.CO].*

The history of the cosmological constant (CC) started when A. Einstein introduced a constant term into his equations.

Original purpose was to get a static cosmological solution.

Nowadays we know Universe is expanding according to the Hubble law. So, why do not we remove the CC from the scene?

Mathematically, the CC term comes to our mind first when we want to formulate covariant action for gravity

$$S_{grav} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,, \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G}\,.$$

So, what is the problem? Is there some?

● Myth and Legends of Cosmological Constant.

The greatest one: **The CC term can be calculated in the framework of Quantum Field Theory (QFT) or other Quantum Theory and, surprisingly, it has a strange value, 120 orders of magnitude larger than the one observed in cosmology.**

Real deal: In QFT we can not derive any independent massive (or massless) parameter from the first principles.

The values of all massive parameter are defined through a process which includes experimental measurement.

And CC is not an exception.

More precisely: naive calculation always provide an infinite value for a massive parameter, with both potential and logarithmic-type divergences. After infinity is subtracted, we have to fix the finite value. And this involves a measurement.

Not all those quantities which are calculated to be infinite, are in fact equal to zero.

W. Pauli

The famous "120 orders of magnitude" correspond to the Planck-scale cut-off of quartic divergence in the CC sector.

Taking this naive cut-off as a physical result leads to an absurd.

With similar logic masses of all particles should have Planck value. Since this is not the case, we are going to meet "m_e problem", "m_τ problem", "m_μ problem", "m_W problem", "m_Z problem", "m_H problem", "m_ν problem", etc.

In reality, there is no problem with neither one of them, since the corresponding values are fixed by renormalization conditions and eventually by measurements.

In case of CC, “measurement” means a full set of available observational data at the scale of the Universe. All of them (SN-Ia, CMB, LSS, ...) are apparently converging to the nonzero, positive value

$$\rho_\Lambda^0 \approx 0.7\, \rho_c^0 \,.$$

Definitely, at this level there is no problem with the CC term. We have an “observed” value. It CC is positive, and this is just fine.

So, where is the CC Problem?

The answer is: The Problem really exist, it is caused by finite huge contributions to the CC in the QFT framework.

- Λ **-term at the classical level. Is it a Constant?**

The action of renormalizable theory (e.g., SM) in curved space is

$$S_{total} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}\, (R + 2\Lambda) + S_{HD} + S_{matter}\,.$$

Higher derivative terms S_{HD} are necessary in quantum theory.

See, e.g., books: Birrell, Davies (1980); Buchbinder, Odintsov, & I.Sh. (1992).

In the low-energy domain, one can in principle disregard S_{HD} and the dynamical equations take on the Einstein form

$$R_\mu^\nu - \frac{1}{2} R\, \delta_\mu^\nu \;=\; 8\pi G\, T_\mu^\nu + \Lambda\, \delta_\mu^\nu\,.$$

For isotropic fluid in the locally co-moving frame

$$T_\mu^\nu \;=\; \mathrm{diag}\,(\rho,\, -p,\, -p,\, -p)\;. \qquad (1)$$

The Λ-dependent term has exactly the form (1), with

$$\rho_\Lambda^{vac} = \frac{\Lambda}{8\pi G} = -p_\Lambda^{vac}\,.$$

Definitely, it is a wrong idea to consider the Λ-term as a fluid with negative pressure, repulsive gravity and so on and so forth.

$$\rho_\Lambda^{vac} = \frac{\Lambda}{8\pi G} = -p_\Lambda^{vac}$$

is just a useful form to present the vacuum CC term.

The CC term is not a part of the action of matter, it is not a strange fluid. It is just the simplest possible covariant term.

Amazingly, it is not a constant term!

Without gravity the CC term is an irrelevant constant. However, it acquires dynamical significance through the Einstein equations.

Consider another parametrization of the metric

$$g_{\mu\nu} = \frac{\chi^2}{M_P^2}\,\bar{g}_{\mu\nu}\,,$$

where $\bar{g}_{\mu\nu}$ is some fiducial metric, for instance, it can be $\eta_{\mu\nu}$.

Furthermore, $\chi = \chi(x)$ is a new scalar field.

The CC term looks rather different in these new variables:

$$S_\Lambda = -\int d^4x\sqrt{-g}\,\rho_\Lambda = -\int d^4x\sqrt{-\bar{g}}\,f\chi^4\,, \qquad f = \frac{\Lambda}{8\pi M_P^2}\,.$$

This is quartic term in the potential for the scalar interaction.

The same change of variables transforms $\int\sqrt{-g}R$ - term into the action of a scalar field χ with the negative kinetic term and conformal coupling to curvature.

• Main CC Problem (I) and attempts to solve it.

Why we can not remove the CC from the scene, set it zero?

The reason is that, from the theoretical side, there are many sources of the CC, and simply set it to zero is very difficult.

These sources are as follows:

1) CC is necessary for the consistent QFT in curved space;

2) Induced CC (vacuum energy) always comes from the SSB in the SM of particle physics;

3) Possible variation of the Λ-term due to quantum effects.

Observation about general structure of renormalization in curved space.

Starting from the first paper
R. Utiyama & B.S. DeWitt, J. Math. Phys. 3 (1962) 608.
we know that the divergences and counterterms in QFT in curved space-time satisfy two conditions:

- **They are covariant if the regularization is consistent with covariance.**

- **They are local functionals of the metric.**

See the book
I.Buchbinder, S. Odintsov & I.Sh., Effective Action in Quantum Gravity (IOPP, 1992).
for introduction and recent papers
I.Sh. Class.Quant.Grav. (2008 - Topical review). arXiv: 0801.0216.
P. Lavrov & I.Sh., Phys.Rev. D81 (2010) 044026.
for a more simple consideration and more rigid proof.

What may happen if we use a non-covariant regularization?

Example: cut-off regularization for the Energy-Momentum Tensor of vacuum.

B.S. DeWitt, Phys. Reports. (1975)

E.K. Akhmedov, arXiv: hep-th/0204048.

$$\rho_{vac} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m^2} \,,$$

$$p_{vac} = \frac{1}{6} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\sqrt{\vec{k}^2 + m^2}} \,,$$

For each mode we have, in the massless limit, EOS of radiation. Naturally, after integration with cut-off we will get the EOS for the radiation in the quartic divergences.

But, Lorentz invariance requires the EOS to be $p_{vac} = -\rho_{vac}$.

Indeed, this discrepancy only reflects the non-covariant nature of the momentum cut-off regularization.

Similarly, the quadratic divergences must have the EOS identical to the one of the Einstein tensor. But it can be, instead, any other EOS in a non-covariant regularization scheme.

Usually, only logarithmic divergences are stable even under non-covariant regularization.

In order to have the covariant cut-off, one has to choose, e.g, Schwinger-DeWitt proper-time representation with the cut-off on the lower limit of the integral.

New discussion of this issue:

M.Asorey, P.Lavrov, B.Ribeiro & I.Sh., Phys.Rev. **D85** *(2012) 104001.*

M. Maggiore, L. Hollenstein, M. Jaccard, and E. Mitsou, Phys. Lett. **B704** *(2011) 102.*

Reminder about QFT in curved space-time:

Renormalizable theory of matter fields on classical curved background requires classical action of vacuum

$$S_{vac} = S_{HE} + S_{HD} \,, \qquad S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda)\,.$$

Important remark: **Without independent vacuum parameter $\Lambda = \Lambda_{vac}$ the theory is inconsistent.**

Loops of massive particle give divergences of the Λ_{vac} -type.

If $\Lambda_{vac} \equiv 0$, these divergences can not be removed by renormalization, and we have a kind of theoretical disaster.

Of course the same is true for all other terms in S_{vac}, including Hilbert term and higher derivative terms.

RG equations for CC and G:

$$(4\pi)^2\,\mu\frac{d\,\rho_\Lambda^{vac}}{d\mu} = (4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{\Lambda_{vac}}{8\pi G_{vac}}\right) = \frac{N_s m_s^4}{2} - 2N_f m_f^4\,.$$

$$(4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{1}{16\pi G_{vac}}\right) = \frac{N_s m_s^2}{2}\left(\xi - \frac{1}{6}\right) + \frac{N_f m_f^2}{3}\,.$$

It is not clear how these equations can be used in cosmology, where the typical energies are very small.

However, even the UV running means the ρ_Λ^{vac} can not be much smaller then the fourth power of the typical mass of the theory.

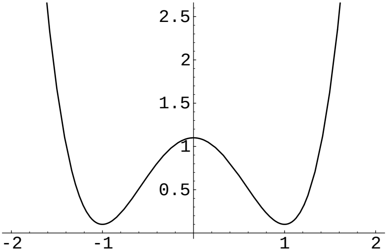
Consequence: the natural value from the MSM perspective is

$$\rho_\Lambda^{vac} \sim M_F^4 \sim 10^8\,GeV^4\,.$$

Induced CC from SSB in the Standard Model.

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$ we meet $\Lambda_{ind} = \langle V \rangle \approx 10^8\, GeV^4$ – same order of magnitude as Λ_{vac} !

This is induced CC, similar to the one found by Zeldovich (1968).



The observed CC is a sum $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}$. Since ρ_Λ^{vac} is an independent parameter, the renormalization condition is

$$\rho_\Lambda^{vac}(\mu_c) = \rho_\Lambda^{obs} - \rho_\Lambda^{ind}(\mu_c)\,.$$

Here μ_c is the energy scale where ρ_Λ^{obs} is "measured".

Finally, the main CC relation is

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$$

The ρ_Λ^{obs} which is likely observed in SN-Ia, LSS and CMB is

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ and $\rho_\Lambda^{ind}(\mu_c)$ are a huge 55 orders of magnitude greater than the sum!

Obviously, these two huge terms do cancel.

"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

The origin of the problem is the difference between the M_F scale of ρ_Λ^{ind} and ρ_Λ^{vac} and the μ_c scale of ρ_Λ^{obs}.

Therefore, the CC Problem is nothing else but a hierarchy problem, perhaps the most difficult one.

There were attempts to fix the overall CC value to zero and replace it by quintessence, Chaplygin gas, k-essence etc.

Warning: The 5-th element looks nice only due to Milla Jovovich.

In reality we have to trade 55-orders fine-tuning to the ∞-orders fine-tuning, plus another 55 for quintessence.

Further aspects of the CC Problem are as follows:

1) The Universe is not static, hence both ρ_Λ^{vac} and ρ_Λ^{ind} can independently run, at least in the Early Universe.

2) Possible abrupt changes of the overall observed CC due to the phase transitions in the Early Universe.

3) Finally, it looks like our Universe was somehow "prepared", from the initial moment of its "creation", with a 55-order precision, such that after all that, today $\rho_\Lambda^{obs} \sim \rho_c$.

4) This fine-tuning, up to now, is impossible to explain.

5) The last observation on the CC Problem.

ρ_Λ^{vac} **is an independent parameter which has to be adjusted, with at least** 55 **orders of magnitude precision, to cancel** ρ_Λ^{ind} **.**

Therefore, a solution of the CC problem has to start by explaining the value of ρ_Λ^{ind} **from the first principles.**

However this quantity depends on the VEV of the Higgs field, on scalar coupling, on W **and** Z **masses, all other couplings, on the EW phase transition, on chiral phase transition and also on higher loop (up to 21 loops!!) corrections within MSM. And also on the details of possible physics beyond MSM, of course.**

55 **orders of precision require all this. So, we can see that "solving" the CC problem from the first principles requires, as a preliminary step, deriving the particle mass spectrum of the Standard Model (and its extensions) from the first principles.**

We are currently far from this level of knowledge in fundamental physics. For this reason it is right to call it the great CC Problem.

• Can symmetries help to solve the CC Problem?

There were many attempts to solve the CC problem introducing more symmetries. A remarkable example is SUSY.

However, the CC problem emerges at very low energies, where SUSY is broken.

Thus, SUSY may solve the problem, but only at high energies, where CC problem does not exist.

In (super)string theory, situation is even more complicated because the choice of a vacuum is not definite.

Furthermore, even if some string vacuum would "indicate" zero CC, it is unclear how this can affect the low-energy physics.

At low energies, we know that the appropriate theory is QFT (specifically the SM, with SSB etc) and not a string theory.

- **Auto-relaxation mechanisms**

There was a number of interesting attempts to create a sort of automatic mechanism for relaxing the CC

Dolgov; Peccei, Solà, Wetterich; Hawking; Ford et al.
Recently: *Štefančić; Grande, Solà, Štefančić.*

Weinberg (1989) discussed some of these approaches: they merely move fine tuning from CC to other parameter(s).

At the same time, it seems no comprehensive proof of this "no-go theorem" was given.

The only visible way to a solution:
Maybe one can modify SM or Einstein equations in such a way that gravity does not "feel" induced CC.

- **Antropic arguments.**

Weinberg, Garriga & Vilenkin, Donoghue, ...

This approach may be the most realistic, it also agrees with the QFT principles.

The idea is to study the limits on the CC and other parameters (e.g. neutrino mass) from the fact that the universe is compatible with the human life and civilization.

For example, negative CC does not let the cosmic structure form sufficiently fast, too large positive CC leads to other problems.

The "shortcoming" of this approach is that we never learn why the two counterparts of the CC do cancel.

- **Renormalization Group (RG) solutions.**

At low energies the quantum effects of some kind is supposed to produce an efficient screening of the observable CC.

Some realizations of this idea:

1) **IR effects of quantum gravity. Qualitative discussion -** *Polyakov, 1982, 2001.*

2) **Attempt to support this idea by direct calculations on fixed** dS **background** – *Tsamis & Woodard et al, 1995-2010.*

3) **More real thing: IR quantum effects of the conformal factor in** $4d$ *– Antoniadis and Mottola, 1992.*

4) **Using the assumed non-Gaussian UV fixed point in Quantum Gravity, assymptotic safety** *– Reuter, Percacci et al, from 2000.*

5) **Driving induced CC between the GUT scale** M_X **and the cosmic scale** μ_c **by the quantum effects of GUT's.** *– I.Sh., 1994; Jackiw et al, 2005.*

General Situation and effective approach to the CC Problem.

- **There are vacuum and induced contributions to CC. Both of them are ≥ 55 orders of magnitude greater than the observed sum. the vacuum part ρ_Λ^{vac} is unique independent part of CC.**

- **The main CC problem (I) is a hierarchy problem due to the conflict between particle physics scale $\sim 100\,GeV$ and the cosmic scale $\mu_c \sim 10^{-42}\,GeV$. That is why we need 55-order (at least!) fine-tuning.**

- **From the QFT viewpoint vanishing overall CC would be much worst thing. In this case we would need ∞-order fine-tuning.**

- **The coincidence problem (II) is: Why $\rho_\Lambda^{obs} \propto \rho_c$ at the present epoch. The two problems are closely related.**

We take a phenomenological point of view and don't try solving problems (I) & (II). Instead we consider problem (III): whether CC may vary due to IR quantum effects, e.g., of matter fields.

CC can vary due to the RG running?

At high energies scalar m_s and fermion m_f lead to RG equation

$$(4\pi)^2\,\mu\,\frac{d\rho_\Lambda}{d\mu} = \frac{m_s^4}{2} - 2m_f^4 + \dots . \qquad (1)$$

To use this RG in cosmology, we have to answer two questions:

- **What is μ?**

●● **At which energy scale Eq. (1) can be used?**

The answer to ● is almost obvious: in the late Universe $\mu \sim H$.

The answer to ●● is not that simple.

If applied to the late Universe, (1) results in too fast running of CC, breaking the standard cosmological model.

This does not happen, because in QFT there is a phenomenon called decoupling.

Decoupling at the classical level.

Consider propagator of massive field at very low energy

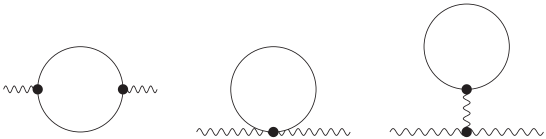
$$\frac{1}{k^2+m^2} = \frac{1}{m^2}\left(1 - \frac{k^2}{m^2} + \frac{k^4}{m^4} + ...\right) .$$

In case of $k^2 \ll m^2$ there is no propagation of particle.

What about quantum theory, loop corrections?

Formally, in loops integration goes over all values of momenta.

Is it true that the effects of heavy fields always become irrelevant at low energies?



For simplicity, consider the fermion loop effect in QED.

In the UV, the mass of quantum fermion is negligible, this simplifies the form factor, and we arrive at

$$\tilde{\beta}\, F^{\mu\nu}\, \ln\left(\frac{\Box}{\mu^2}\right) F_{\mu\nu}\,.$$

The momentum-subtraction β-function

$$\beta_e^1 \,=\, \frac{e^3}{6a^3\,(4\pi)^2}\,\left[20a^3 - 48a + 3(a^2-4)^2\,\ln\left(\frac{2+a}{2-a}\right)\right]\,,$$

$$a^2 = \frac{4\Box}{\Box - 4m^2}\,.$$

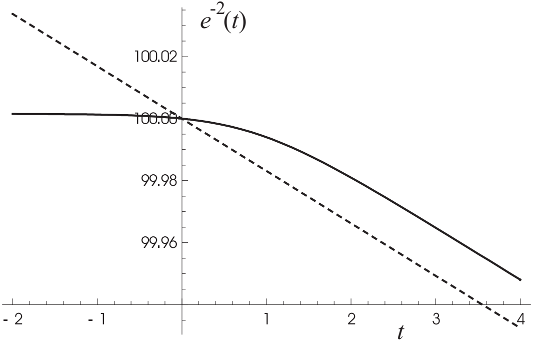
Special cases:

UV limit $\quad p^2 \gg m^2 \implies \beta_e^{1\;UV} = \dfrac{4\,e^3}{3\,(4\pi)^2} + \mathcal{O}\left(\dfrac{m^2}{p^2}\right).$

IR limit $\quad p^2 \ll m^2 \implies \beta_e^{1\;IR} = \dfrac{e^3}{(4\pi)^2}\cdot\,\dfrac{4\,p^2}{15\,m^2} \,+\, \mathcal{O}\left(\dfrac{p^4}{m^4}\right).$

This is the standard form of the Appelquist and Carazzone decoupling theorem (PRD, 1977).

One can obtain the general expression which interpolates between the UV and IR limits.



These plots show the effective electron charge as a function of $\log(\mu/\mu_0)$ **in the case of the MS-scheme,**
and for the momentum-subtraction scheme, with $\ln(p/\mu_0)$.

An interesting high-energy effect is a small apparent shift of the initial value of the effective charge.

In the gravitational sector we meet Appelquist and Carazzone - like decoupling, but only in the higher derivative sectors.
In the perturbative approach, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we do not see running for the cosmological and inverse Newton constants.
Why do we get $\beta_\Lambda = \beta_{1/G} = 0$?

Momentum subtraction running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ formfactors into effective action.

Say, in QED:

$$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

Similarly, one can insert formfactors into

$$C_{\mu\nu\alpha\beta}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,C_{\mu\nu\alpha\beta}\,.$$

However, such insertion is impossible for Λ and for $1/G$, because $\Box\Lambda \equiv 0$ and $\Box R$ is a full derivative.

Further discussion:
Ed. Gorbar & I.Sh., JHEP (2003); J. Solà & I.Sh., PLB (2010).

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Probably not. Perhaps the linearized gravity approach is simply not an appropriate tool for the CC and Einstein terms.

Let us use the covariance arguments. The EA can not include odd terms in metric derivatives. In the cosmological setting this means no $\mathcal{O}(H)$ **and also no** $\mathcal{O}(H^3)$ **terms, etc. Hence**

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{16\pi G(H)} = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\left(H^2 - H_0^2\right), \qquad \nu = const\,.$$

Then the conservation law for $G(H;\nu)$ **gives**

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}, \quad \textbf{where} \quad G(H_0) = G_0 = \frac{1}{M_P^2}\,.$$

Here we used the identification

$\mu \sim H$ **in the cosmological setting.**

The same $\rho_\Lambda(\mu)$ follows from the assumption of the Appelquist and Carazzone - like decoupling for CC.

A.Babic, B.Guberina, R.Horvat, H.Štefančić, PRD 65 (2002); I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB 574 (2003).

We know that for a single particle

$$\beta_\Lambda^{MS}(m) \sim m^4\,,$$

hence the quadratic decoupling gives

$$\beta_\Lambda^{IR}(m) = \frac{\mu^2}{m^2}\,\beta_\Lambda^{MS}(m) \sim \mu^2 m^2\,.$$

The total beta-function will be given by algebraic sum

$$\beta_\Lambda^{IR} = \sum k_i \mu^2 m_i^2 = \sigma M^2\,\mu^2 \propto \frac{3\nu}{8\pi}\,M_P^2\,H^2\,.$$

This leads to the same result in the cosmological setting,

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\,M_p^2\left(H^2 - H_0^2\right).$$

One can obtain the same $G(\mu)$ in one more independent way.

I.Sh., J. Solà, JHEP (2002); C. Farina, I.Sh. et al, PRD (2011).

Consider $\overline{\mathrm{MS}}$-based renormalization group equation for $G(\mu)$:

$$\mu\frac{dG^{-1}}{d\mu} = \sum_{particles} A_{ij}\, m_i\, m_j = 2\nu\, M_P^2 \,, \qquad G^{-1}(\mu_0) = G_0^{-1} = M_P^2 \,.$$

Here the coefficients A_{ij} depend on the coupling constants, m_i are masses of all particles. In particular, at one loop,

$$\sum_{particles} A_{ij}\, m_i\, m_j = \sum_{fermions} \frac{m_f^2}{3(4\pi)^2} - \sum_{scalars} \frac{m_s^2}{(4\pi)^2}\Big(\xi_s - \frac{1}{6}\Big)\,.$$

One can rewrite it as

$$\mu\frac{d(G/G_0)}{d\mu} = -2\nu\,(G/G_0)^2 \quad \Longrightarrow \quad G(\mu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}\,. \quad (*)$$

It is the same formula which results from covariance and/or from AC-like quadratic decoupling for the CC plus conservation law.
$(*)$ seems to be a unique possible form of a relevant $G(\mu)$.

All in all, it is not a surprise that the eq.

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)} \,.$$

emerges in different approaches to renorm. group in gravity:

- **Higher derivative quantum gravity.**

A. Salam & J. Strathdee, PRD (1978);
E.S. Fradkin & A. Tseytlin, NPB (1982).

- **Non-perturbative quantum gravity with (hipothetic) UV-stable fixed point.**

A. Bonanno & M. Reuter, PRD (2002).

- **Semiclassical gravity.**

B.L. Nelson & P. Panangaden, PRD (1982).

So, we arrived at the two relations:

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi} M_p^2 \left(\mu^2 - \mu_0^2\right) \qquad (1)$$

and

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)} \,. \qquad (2)$$

Remember the standard identification

$\mu \sim H$ **in the cosmological setting.**

A. Babic, B. Guberina, R. Horvat, H. Štefančić, PRD (2005).

Cosmological models based on the assumption of the standard AC-like decoupling for the cosmological constant:

- **Models with (1) and energy matter-vacuum exchange:**

I.Sh., J.Solà, Nucl.Phys. (PS), IRGA-2003;
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB (2003).

- - **Models with (1), (2) and without matter-vacuum exchange:**

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- **Models with constant $G \equiv G_0$ and permitted energy exchange between vacuum and matter sectors.**

For the equation of state $P = \alpha\rho$ the solution is analytical,

$$\rho(z;\nu) = \rho_0\,(1+z)^r\,,$$

$$\rho^\Lambda(z;\nu) = \rho_0^\Lambda + \frac{\nu}{1-\nu}\,[\,\rho(z;\nu) - \rho_0\,]\,,$$

The limits from density perturbations / LSS data: $|\nu| < 10^{-6}$.

Analog models:

R.Opher & A.Pelinson, PRD (2004);
P. Wand & X.H. Meng, Cl.Q.Gr. 22 (2005).

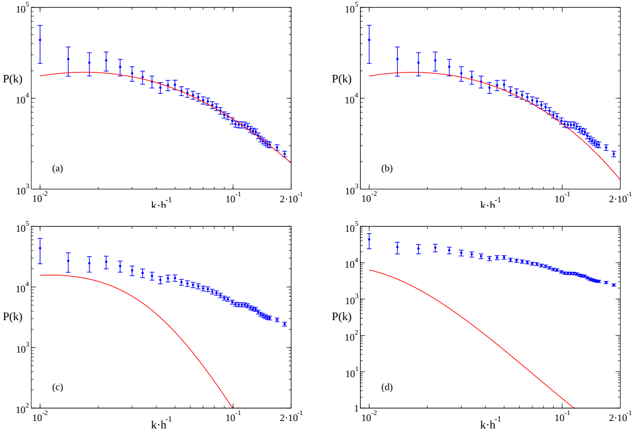
Direct analysis of cosmic perturbations:

J. Fabris, I.Sh., J. Solà, JCAP 0702 (2007).

For the Harrison-Zeldovich initial spectrum, the power spectrum today is obtained by integrating the eqs. for perturbations.

Initial data based on $w(z)$ **from** *J.M. Bardeen et al, Astr.J. (1986).*

Results of numerical analysis for the ● model:



The ν-dependent power spectrum vs the LSS data from the 2dfFGRS. The ordinate axis represents $P(k) = |\delta_m(k)|^2$ where $\delta_m(k)$ is the solution at $z = 0$. $\quad \nu = 10^{-8},\ 10^{-6},\ 10^{-4},\ 10^{-3}$.
In all cases $\Omega_B^0, \Omega_{DM}^0, \Omega_\Lambda^0 = 0.04, 0.21, 0.75$.

●● **Models with variable $G = G(H)$ but without energy exchange between vacuum and matter sectors.**

Theoretically this looks much better!

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2 \left(H^2 - H_0^2\right).$$

By using the energy-momentum tensor conservation we find

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}\,, \quad \textbf{where} \quad G(H_0) = \frac{1}{M_P^2}\,.$$

These relations exactly correspond to the RG approach discussed above, with $\mu = H$.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

The limits on ν from density perturbations, etc.

J.Grande, J.Solà, J.Fabris & I.Sh., Cl. Q. Grav. 27 (2010) .

An important general result is: In the models with variable Λ and G in which matter is covariantly conserved, the solutions of perturbation equations *do not* depend on the wavenumber k.

As a consequence we meet relatively weak modifications of the spectrum compared to ΛCDM.

The bound $\nu < 10^{-3}$ comes just from the "F-test". It is related only to the modification of the function $H(z)$.

R. Opher & A. Pelinson, astro-ph/0703779.
J.Grande, R.Opher, A.Pelinson, J.Solà, JCAP 0712 (2007).

One can obtain the same restriction for ν also from the primordial nucleosynthesis (BBN).

Can we apply the running $G(\mu)$ to other physical problems?

In the renormalization group framework the relation

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)}\,, \quad \textbf{where} \quad \mu = H$$

in the cosmological setting.

What could be an interpretation of μ in astrophysics?

Consider the rotation curves of galaxies. The simplest assumption is $\mu \propto 1/r$.

Applications for the point-like model of galaxy:

J.T.Goldman, J.Perez-Mercader, F.Cooper & M.M.Nieto, PLB (1992).
O. Bertolami, J.M. Mourao & J. Perez-Mercader, PLB 311 (1993).
M. Reuter & H. Weyer, PRD **70** *(2004); JCAP* **0412** *(2004).*
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

We can safely restrict the consideration by a weakly varying G,

$$G = G_0 + \delta G = G_0(1+\kappa)\,, \qquad |\kappa| \ll 1\,.$$

The value of ν is small, the same should be with $\kappa = \delta G/G_0$.
Perform a conformal transformation

$$\bar{g}_{\mu\nu} = \frac{G_0}{G}\,g_{\mu\nu} = (1-\kappa)g_{\mu\nu}\,.$$

In $\mathcal{O}(\kappa)$, metric $\bar{g}_{\mu\nu}$ obeys Einstein equations with $G_0 = const$.

The nonrelativistic limits of the two metrics

$$g_{00} = -1 - \frac{2\Phi}{c^2} \quad \text{and} \quad \bar{g}_{00} = -1 - \frac{2\Phi_{\text{Newt}}}{c^2}\,,$$

Φ_{Newt} being Newton potential and Φ is a modified potential.

$$g_{00} = -1 - \frac{2\Phi}{c^2} \approx -1 - \frac{2\Phi_{\text{Newt}}}{c^2} - \kappa \implies \Phi = \Phi_{\text{Newt}} + \frac{c^2\,\delta G}{2\,G_0}\,.$$

For the nonrelativistic limit of the modified gravitaty we obtain

$$-\Phi^{,i} = -\Phi_{\text{Newt}}^{,i} - \frac{c^2\,G^{,i}}{2\,G_0}\,, \quad \text{where we used} \quad G^{,i} = (\delta G)^{,i}\,.$$

The last formula $-\Phi^{,i} = -\Phi^{,i}_{\text{Newt}} - \dfrac{c^2\, G^{,i}}{2\, G_0}$ **is very instructive.**

- **Quantum correction comes with the factor of c^2 $\Longrightarrow$ can make real effect at the typical galaxy scale.**

E.g., for a point-like model of galaxy and $\mu \propto 1/r$ it is sufficient to have $\nu \approx 10^{-6}$ to provide flat rotation curves.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

●● $\mu \propto 1/r$ **is, obviously, not a really good choice for a non-point-like model of the galaxy.**

The reason is that this identification produces the "quantum-gravitational" force even if there is no mass at all !!

What would be the "right" identification of μ ?

Let us come back to QFT, which offers a good hint:
μ must be $\sim$ energy of the external gravitational line in the Feynman diagram in the almost-Newtonian regime.

The phenomenologically good choice is

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\text{Newt}}}{\Phi_0}\right)^{\alpha},$$

where α is a phenomenological parameter We have found that α is generally growing with the mass of the galaxy.

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010).

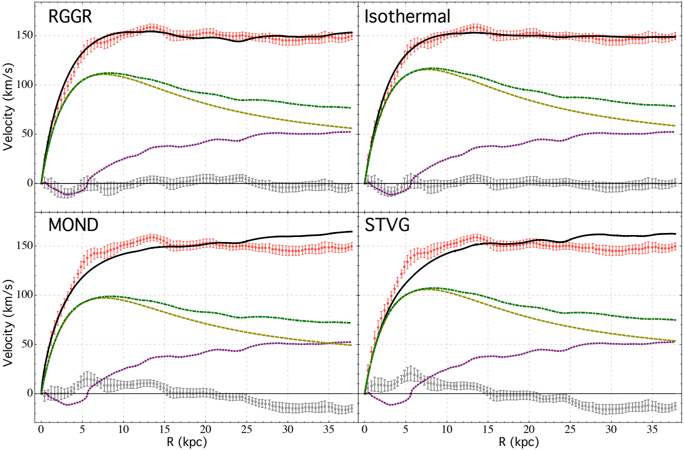
QFT viewpoint: α reflects $\mu \sim \Phi_{\text{Newt}}$ is not an ultimate choice.

With greater mass of the galaxy the "error" in identification becomes greater too, hence we need a greater α to correct this. α must be quite small at the scale of the Solar system.

Regular scale-setting procedure gives the same result:
S. Domazet & H. Štefančić, PLB (2011).

Last, but not least, the astro-ph application is impressively successful

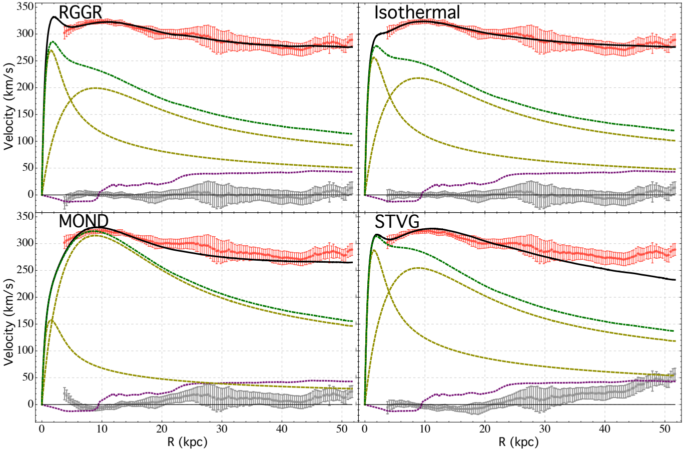
D. Rodrigues, P. Letelier & I.Sh., JCAP (2010). *(9 samples)*



Rotation curve of the spiral galaxy *NGC 3198.* $\alpha\nu = 1.7 \times 10^{-7}$.
[Collaboration THINGS (2008)].

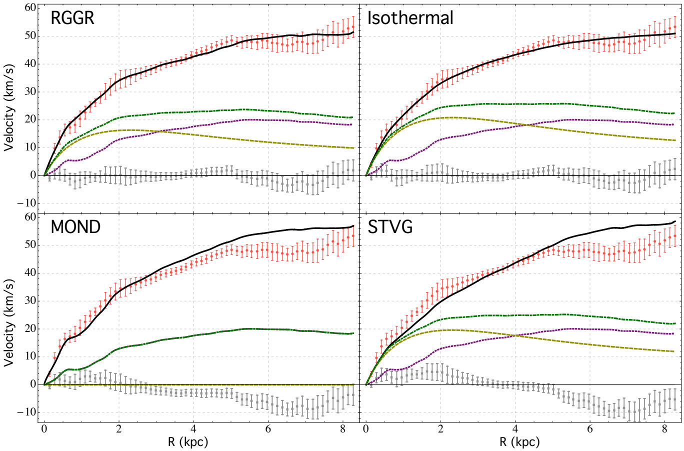
One more example, this time with descendent rotation curve.

$\alpha\nu = 6.7 \times 10^{-7}$.



Rotation curve of the galaxy *NGC 2841*. **RGGR is based on hypothetical covariant quantum corrections without DM.**

One more example: low-surface brightness galaxy with ascendent rotation curve. $\alpha\nu = 0.2 \times 10^{-7}$.



Rotation curve of the galaxy *DDO 154*. RGGR is based on hypothetical covariant quantum corrections without DM.

What about the Solar System?

C. Farina, W. Kort-Kamp, S. Mauro & I.Sh., PRD 83 (2011).

We used the dynamics of the Laplace-Runge-Lenz vector in the $G(\mu) = G_0/(1 + \mu \log(\mu/\mu_0))$ **- corrected Newton gravity.**

Upper bound for the Solar System: $\alpha\nu \leq 10^{-17}$.

One of the works now on track: extending the galaxies sample.

P. Louzada, D. Rodrigues, J. Fabris, ..., in work: **50+ disk galaxies.**

Davi Rodrigues, ..., in progress: **elliptical galaxies.**

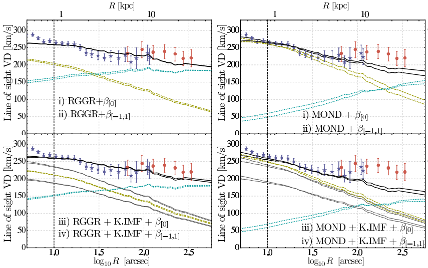
The general tendency which we observe so far is greater α **needed to for larger mass of the astrophysical object: from Solar System (upper bound) to biggest tested galaxies.**

Recent developments of RGGR model by Davi Rodrigues et al:

D. C. Rodrigues, JCAP 1209, 031 (2012), 1203.2286.
D. C. Rodrigues, P. L. de Oliveira, J. C. Fabris and G. Gentile, MNRAS (2014), 1409.7524.

Last publication:

P.L.C. de Oliveira, J.A. de Freitas Pacheco, G. Reinisch, Testing two alternatives theories to dark matter with the Milky Way dynamics. arXive: 1501.0108.



Rotation curve of the giant elliptic galaxy NGC 4374: RGGR vs MOND. $\alpha\nu = 17 \times 10^{-7}$.

It looks like we do not need CDM **to explain the rotation curves of the galaxies. However, does it really mean that we can really go on with one less dark component?**

Maybe not, but it is worthwhile to check it. The requests for the DM **come from the fitting of the LSS, CMB, BAO, lenthing etc. However there is certain hope to relpace, e.g.,** ΛCDM **by a** ΛWDM **(e.g. sterile neutrino) with much smaller** Ω_{DM}**.**

The idea to trade $0.04, 0.23, 0.73 \quad \Longrightarrow \quad 0.04,\ 0.0x,\ 0.9(1\text{-}x)$

Such a new concordance model would have less coincidence problem, and this is interesting to verify.

First move: *J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275; PRD (2012).*

We are using "our" Reduced Relativistic Gas model.
G. de Berredo-Peixoto, I.Sh., F. Sobreira, Mod.Ph.Lett. A (2005);
J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

Earlier: *A.D. Sakharov, Soviet Physics JETP, 49 (1965) 345.*

In the recent paper

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275 [astro-ph.CO]; PRD-2012

we have used RRG without quantum effects to fit
Supernova type Ia (Union2 sample), $H(z)$, CMB (R factor), BAO, LSS (2dfGRS data)
In this way we confirm that ΛCDM is the most favored model.

However, for the LSS data alone we met the possibility of an alternative model with a small quantity of a WDM.

This output is potentially relevant due to the fact the LSS is the test which is not affected by the possible quantum RG running in the low-energy gravitational action.

Such a model almost has no issue with the coincidence CC problem (II), because $\Omega_\Lambda^0 \simeq 0.95$.

Conclusions

- **CC term is a natural and necessary concept, which should be separated from myths and legends.**

- **It looks like there is no real chance to solve the great CC problem from the "first principles", especially because we do not have a real knowledge of these principles.**

- **We can learn a lot by thinking about the CC problem, such thinking is definitely not "forbidden".**

- **The question of whether CC can be variable is, to some extent, reduced to existing-nonexisting paradigm.**

- **In the positive case we arrive at the very rich cosmological and astrophysical model with one free parameter ν plus certain freedom of scale identification.**