Chiral limit of N=4 SYM and ABJM and integrable Feynman graphs

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based on work in progress with O. Gurdogan and V. Kazakov

Introduction

- Planar N=4 SYM and ABJM are all-loop integrable - extensively checked but never proven.

- Class of marginal deformations:

  - Exact Leigh-Strassler or β-deformation, preserves some SUSY
  - γ-deformation (three parameters), not exact in general, breaks SUSY.

All of these preserve **Integrability**!

New parametric family of higher dimensional integrable field theories

They depend on the coupling + twist parameters

Idea: play with these new parameters, to kill interacting terms in the Lagrangian

↓

Simpler field theories, with easier perturbation theory. Easier to prove integrability!

γ-deformation

[Frolov, Lunin, Maldacena]

- Involves 3 parameters γ_k ($\gamma_k = \beta$, β-deformation)
- Given two fields A,B:

$$[A, B] \to [A, B]_\gamma = e^{i\gamma_k Q_i^A Q_j^B \epsilon^{ijk}} AB - e^{-i\gamma_k Q_i^A Q_j^B \epsilon^{ijk}} BA$$

Cartan charges of the SU(4) R-symmetry group

R-symmetry is broken: $SU(4) \to U(1)^3$

Why is Integrability preserved?

At one-loop, in any compact subsector the untwisted dilatation operator,

$$\mathcal{D} \sim Id - P$$

$$\mathcal{D}_{i,i+1}\, \mathrm{tr}[\ldots ZZ\underset{i}{X}YZ\ldots] = \mathrm{tr}[\ldots ZZXYZ\ldots] - \mathrm{tr}[\ldots ZZYXZ\ldots]$$

For the twisted theory,

$$\tilde{\mathcal{D}} \sim Id - \tilde{P}$$

← twisted permutation involving the phases $e^{i\gamma_k}$

$\tilde{\mathcal{D}}$ is still integrable (ie. generated by an R-matrix satisfying the Yang-Baxter equation) [Beisert, Roiban'05]

Deformation based on conserved charges → all-loop integrability is equally preserved

γ-deformed Lagrangian

Lagrangian is now a function of g and γ_k

$$\mathcal{L}_{int} = N_c \mathrm{tr}\Big[\frac{g}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j$$
$$- e^{-\frac{i}{2}\gamma_j^-}\bar{\psi}_j\phi^j\bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\bar{\psi}_4\phi^j\bar{\psi}_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k\phi^i\psi^j$$
$$- e^{+\frac{i}{2}\gamma_j^-}\psi_4\phi_j^\dagger\psi_j + e^{-\frac{i}{2}\gamma_j^-}\psi_j\phi_j^\dagger\psi_4 + i\epsilon^{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar{\psi}_k\phi_i^\dagger\bar{\psi}\Big]$$

Double scaling limit [Gurdogan, Kazakov'15]

$$q_i \equiv e^{-i\frac{\gamma_i}{2}} \to \infty \qquad \text{with} \quad \xi_i \equiv g q_i \text{ finite}$$
$$g \to 0$$

Chiral limit: parameters are complex hence resulting theories will be non-unitary

Double-scaled Lagrangian from N=4 SYM

$$\begin{aligned}\mathcal{L}_{\rm int} \quad &= \quad N_c\,{\rm tr}\big[\xi_1^2\,\phi_2^\dagger\phi_3^\dagger\phi_2\phi_3 + \xi_2^2\,\phi_3^\dagger\phi_1^\dagger\phi_3\phi_1 + \xi_3^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2 \\ &+ \quad i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3\phi_1^\dagger\bar\psi_2) \\ &+ \quad i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1\phi_2^\dagger\bar\psi_3) \\ &+ \quad i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2\phi_3^\dagger\bar\psi_1)\,\big]\end{aligned}$$

Gauge fields decouple with the $g \to 0$ limit

$\xi_i = \xi$ strong β-deformation with $\mathcal{N} = 1$ SUSY

$\xi_1 = \xi_2 = 0\,, \quad \xi_3 = \xi$

$$\mathcal{L} = \frac{N_c}{2}{\rm tr}\big(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1 + \partial^\mu\phi_2^\dagger\partial_\mu\phi_2 + 2\xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\big)$$

Double-scaled Lagrangian from ABJM

In ABJM, the twisted theory depend on the coupling $\lambda = \dfrac{N}{k}$ and 3 twist parameters.

$$q_i \equiv e^{-i\gamma_i} \to \infty \ \text{for } i = 1, 2\,, \quad q_3 \equiv e^{-i\gamma_3} \to 0 \ \text{ and } \ \lambda \to 0\,,$$

$$\xi_i \equiv q_i\, \lambda^{2/3} \ \text{for } i = 1, 2\,, \quad \xi_3 \equiv \frac{q_3}{\lambda^{2/3}} \quad \text{fixed}\,.$$

One gets a ϕ^6 type scalar Lagrangian

$$\mathcal{L} = N_c \,\mathrm{Tr}\Big[-\partial_\mu Y_1^\dagger \partial^\mu Y^1 - \partial_\mu Y_2^\dagger \partial^\mu Y^2 - \partial_\mu Y_4^\dagger \partial^\mu Y^4 + (4\pi)^2\, \frac{\xi_1 \xi_2}{\xi_3} Y^1 Y_4^\dagger Y^2 Y_1^\dagger Y^4 Y_2^\dagger \Big]$$

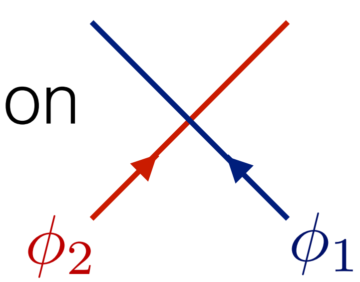
Let us focus on the 4D bi-scalar model

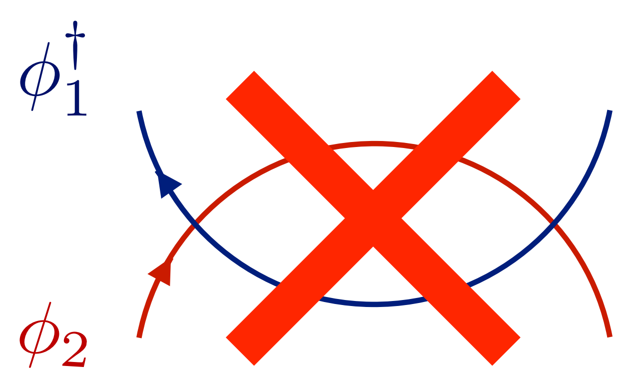
$$\mathcal{L} = \frac{N_c}{2}\mathrm{tr}\left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + 2\xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right)$$

- Non-unitary theory
- No gauge fields
- No supersymmetry

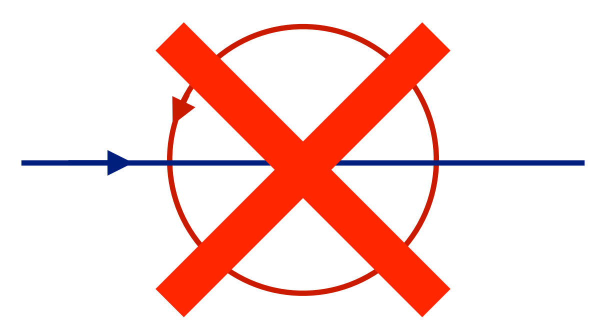
- What about conformal symmetry?

single interaction vertex



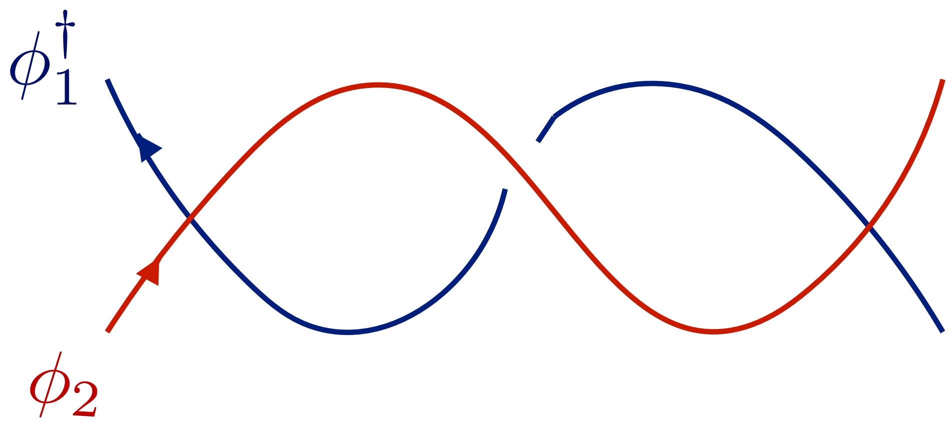


at large N,
no coupling renormalization



at large N,
no mass renormalization

At 1/N...



allowed, of order 1/N

—> New divergences that cannot be absorbed in the renormalization of operators. Need counterterms of the form trace-trace: [Fokken, Sieg, Wilhelm'13,16]

$$\frac{\eta_{ij}}{N}\mathrm{Tr}\phi_i\phi_j^\dagger\mathrm{Tr}\phi_i\phi_j^\dagger + \frac{\tilde{\eta}_{ij}}{N}\mathrm{Tr}\phi_i\phi_j\mathrm{Tr}\phi_i^\dagger\phi_j^\dagger + \ldots$$

$$\beta_\xi = \mathcal{O}(1/N)$$

$$\beta_\eta = \mathcal{O}(\xi)$$ ← run even at large N!

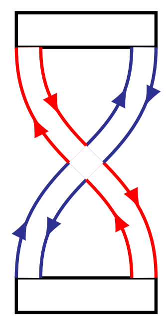
Conformal symmetry broken even in the planar limit!

How badly is it broken?

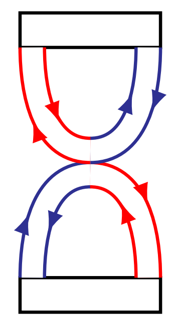
Two point functions of single traces with L> 2 ⟶ double trace couplings are subleading in N:

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle_L \sim (x^2)^{-L-\gamma}$$

But, for L=2, **double traces are important,**



and



same order in N

Two point functions for L=2 do not have the conformal behaviour.

These L=2 states play the role of **tachyons** in γ-twisted dual string theory. At strong coupling, these tachyons have been identified. [Rastelli, Pomoni'08]

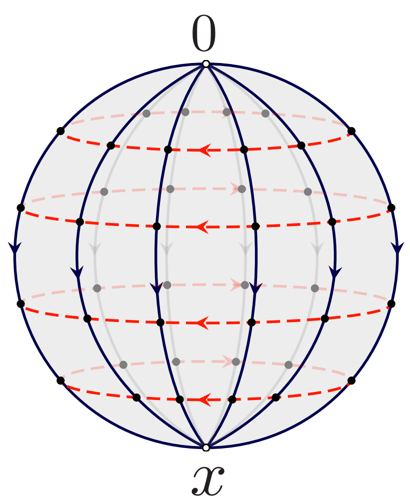
Diagrammatics of the bi-scalar model

Focus on two point functions of single trace operators in the **conformal sector** (L>2).

Simplest operator so-called BMN vacuum $\mathrm{Tr}[\phi_1^L]$

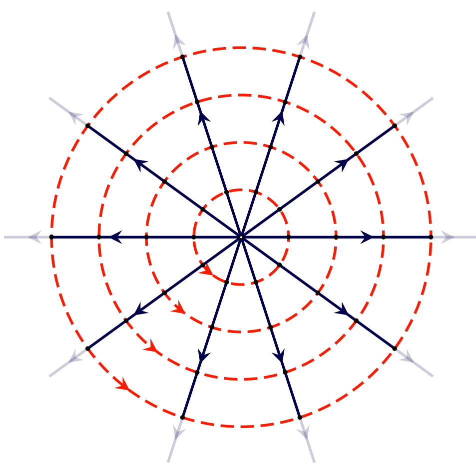
$$\langle \mathrm{Tr}[\phi_1^{\dagger\, L}](x)\, \mathrm{Tr}[\phi_1^L](0)\rangle \sim (x^2)^{-L-\gamma_{\mathrm{vac}}}$$

γ_{vac} is not zero, but rather given by wrapping corrections



amputation of one external operator →

wheel graphs



- 1 wheel [Broadhurst, '80]

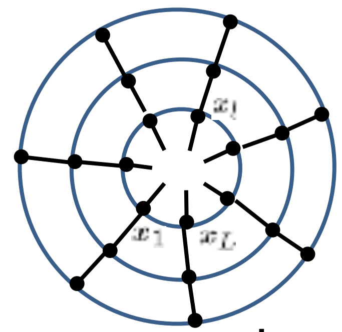
- 2 wheels from the integrability based solution of γ twisted

[Ahn et al., '11] [Gurdogan, Kazakov, '15]

A glimpse on the origin of Integrability

- 'Hamiltonian' generating the wheel graphs (fishnet lattice)

$$\hat{H}_L = \xi^{2L} \prod_{l=1}^{L} \frac{1}{(x_{l+1} - x_l)^2} \prod_{l=1}^{L} \Delta_{x_l}^{-1}$$



- Conformal symmetry easily checked, e.g. covariant under inversion
- Integrability: Hamiltonian commutes with conformal transfer matrix:

$$[\hat{H}_L, t(u)] = 0\,, \quad \text{with } \; t(u) = \mathrm{Tr}[R_1(u) \ldots R_L(u)]$$

Lax operator $R(u) = uI \otimes I + T(M_{ab}) \otimes M^{ab}$

physical space in principal series irreps of SL(4)

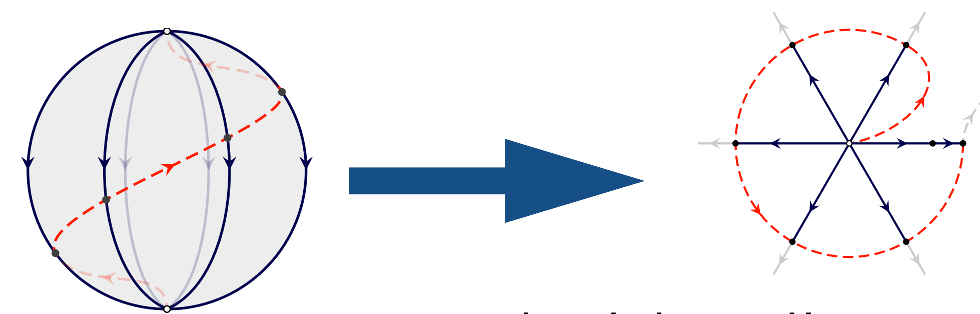
conformal generators

- The Hamiltonian stems from *t(u)* at a particular value of the spectral parameter! This proves integrability at least for this operator = fishnet lattice. [Zamolodchikov, '80]
- Other operators: modification of boundary conditions.
- Still fishnet in the bulk!

Excited states

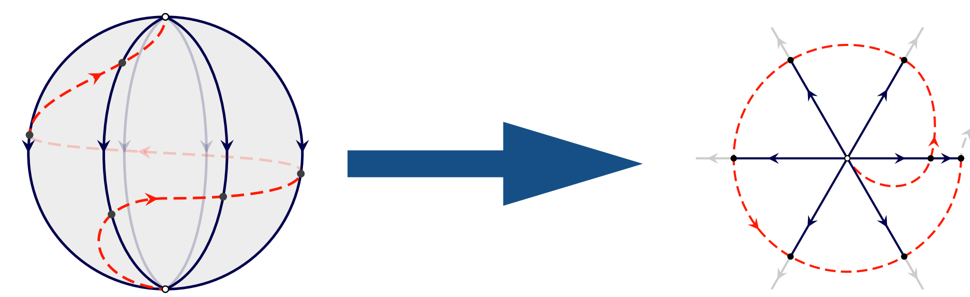
1 magnon states: $\mathrm{Tr}[\phi_1\phi_1 \ldots \phi_1 \phi_2 \phi_1 \ldots \phi_1]$

For L bigger than the loop order:



ladder diagrams

Including wrapping:



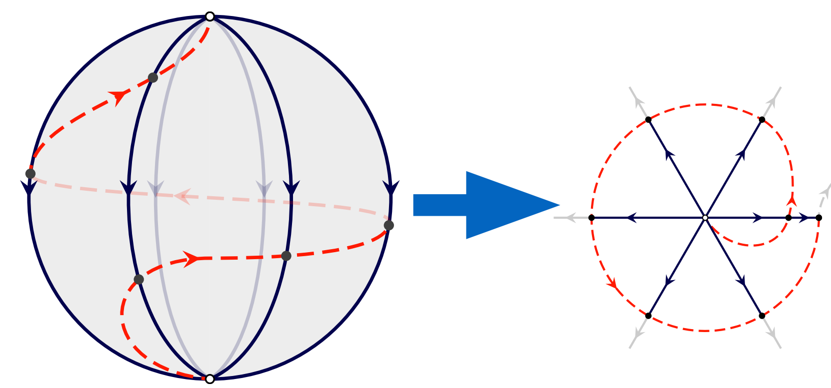
spiral diagrams

Asymptotic Bethe Ansatz: $e^{ipL} = q_3^{-2L}$

$$\gamma = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4g^2 \sin(p/2)^2} \to -\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\xi_3^2}$$

Resummation of ladder diagrams [Broadhurst, '93, Gross, Mikhailov, Roiban' 02]

Spiral graphs



For these operators, $\mathrm{Tr}[\phi_1\phi_1\ldots\phi_1{\color{red}\phi_2}\phi_1\ldots\phi_1]$

strongly β-deformed $=$ strongly γ-deformed

(They are given by the same graphs in both theories)

Anomalous dimension is exactly known
(including wrapping) in β-deformed theory by TBA:

[Gromov, Levkovich-Maslyuk,' 10]

$$(\delta\gamma)_{\mathrm{wrap}} = 4\xi^{2L}\sum_{k=3}^{L-1}\binom{2(-k+L[k/2]-1)}{L-k}\zeta(2(-k+L+[k/2]-1)+1)$$

We can easily predict this integral in dim-reg

wrapping integral = ladder integral up to $1/\epsilon$ (by exponentiation)

$+1/\epsilon$ term fixed by anomalous dimension from TBA

N-magnon states

$\mathrm{Tr}[\phi_1^{L-N}\phi_2^N]$ for L>>1 (asymptotic regime)

- So-called su(2) subsector of N=4 SYM
- In N=4 SYM, magnons are characterised by set of rapidities $\{u_1, u_2, \ldots, u_N\}$ satisfying Asymptotic Bethe Ansatz equations [Beisert, Eden, Staudacher,' 05]
- The anomalous dimension is then given by the sum of N dispersion relations evaluated on these solutions.

$$\gamma = ig\sum_j \left(\frac{1}{x^+(u_j)} + \frac{1}{x^-(u_j)}\right)$$

N-magnon states II

- In order to get the double scaling limit of the Asymptotic Bethe Ansatz, the rapidities are analytically continued to the mirror sheet.

$$\xi_3^{2L} = \left(u_k^2 + 1/4\right)^L \prod_{\substack{i=1 \\ i \neq l}}^{N} \frac{u_k - u_i + i}{u_k - u_i - i} \, \sigma_0^{m,m}(u_k, u_i)^2$$

- With the 'dressing phase' given by

$$\sigma_0^{m,m}(u,v)^2 = \frac{(4v^2+1)\,\Gamma\left(iu+\frac{1}{2}\right)\Gamma\left(iu+\frac{3}{2}\right)\Gamma\left(\frac{1}{2}-iv\right)\Gamma\left(\frac{3}{2}-iv\right)\Gamma(-iu+iv+1)^2}{(4u^2+1)\,\Gamma\left(\frac{1}{2}-iu\right)\Gamma\left(\frac{3}{2}-iu\right)\Gamma\left(iv+\frac{1}{2}\right)\Gamma\left(iv+\frac{3}{2}\right)\Gamma(iu-iv+1)^2}$$

- The anomalous dimension is: $\gamma = \sum_{k=1}^{N} i\left(u_k + \frac{i}{2}\right)$

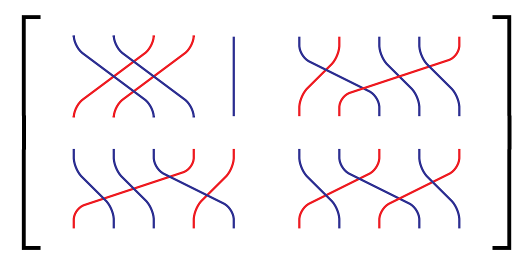
What can we say about integrals?

Consider N=2 states and their (bare) 2-point function

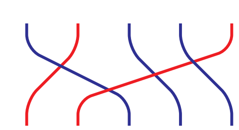
$$\mathcal{G}_{\alpha\beta} = \langle \mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)\rangle$$

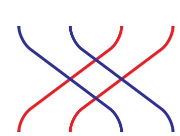
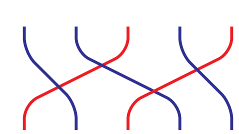
Only 1 diagram for each pair (α, β) at a given loop order

Say at four loops for L=5:

$$\mathcal{G}_{\alpha\beta}\Big|_{\xi^8} =$$ 

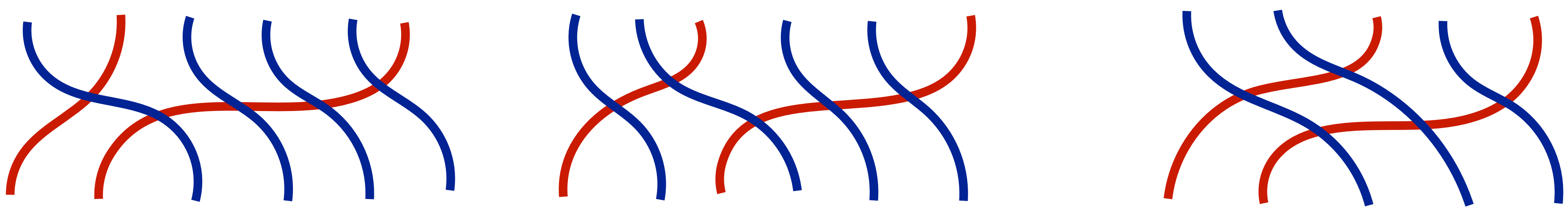
Spectrum provides relations between integrals

Spectrum + input of lower loop integrals + 

fixes completely  and  up to $1/\epsilon$ term

Predictions at five-loops

Spectrum provides relations between these 5-loop integrals and lower loop ones



I_a I_b I_c

Spectrum + input lower loop integrals fixes everything up to a constant. The nontrivial part are $1/\epsilon$ terms:

$$I_b\big|_{1/\epsilon} = -I_a\big|_{1/\epsilon} - \frac{160\zeta(3)}{9} + \frac{53\pi^4}{72} - \frac{187}{5} - \frac{25\pi^2}{12}$$

$$I_c\big|_{1/\epsilon} = I_a\big|_{1/\epsilon} + \frac{418\zeta(3)}{45} + \frac{121\pi^4}{360} + \frac{2\pi^2}{9} - \frac{112}{5}$$

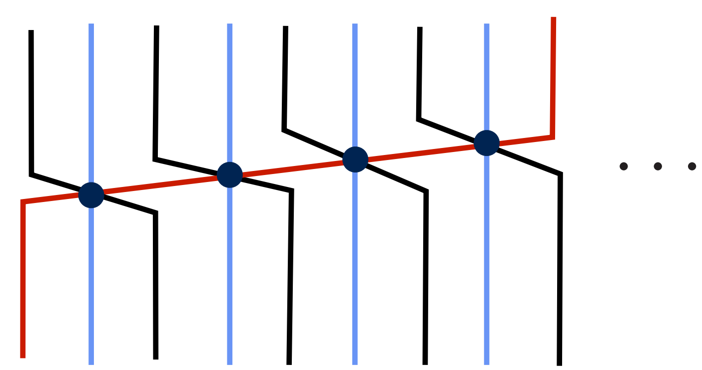
About ABJM

Same set of ideas apply for DS limit in ABJM

In particular for single excited states $\mathrm{Tr}[(Y^1 Y_4^\dagger)^{L-1}(Y^2 Y_4^\dagger)]$

$$\gamma = -\frac{1}{2} + \frac{1}{2}\sqrt{1-4\xi^2}$$

Resums 3D ladders

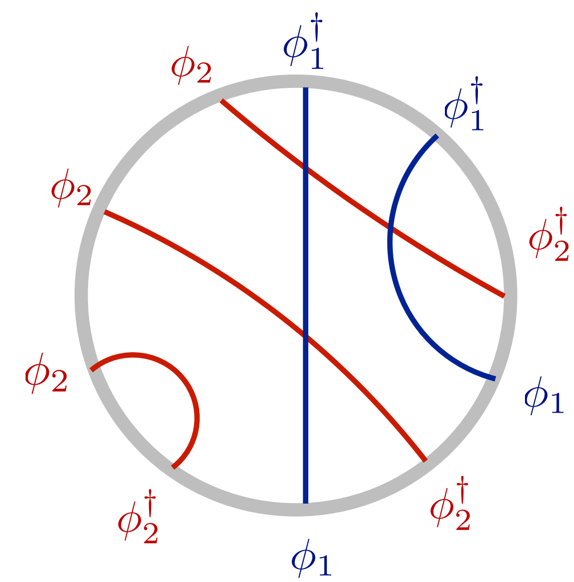


Also Bethe Ansatz for the states $\mathrm{Tr}[(Y^1 Y_4^\dagger)^{L-N}(Y^2 Y_4^\dagger)^N]$

$$\xi^{-L}\,(u_j^2+1/4)^L \quad = \quad \prod_{j\neq k}^{N}\left[\frac{u_k-u_j+i}{u_k-u_j-i}\,\sigma_0^{m,m}(u_k,u_j)\right]$$

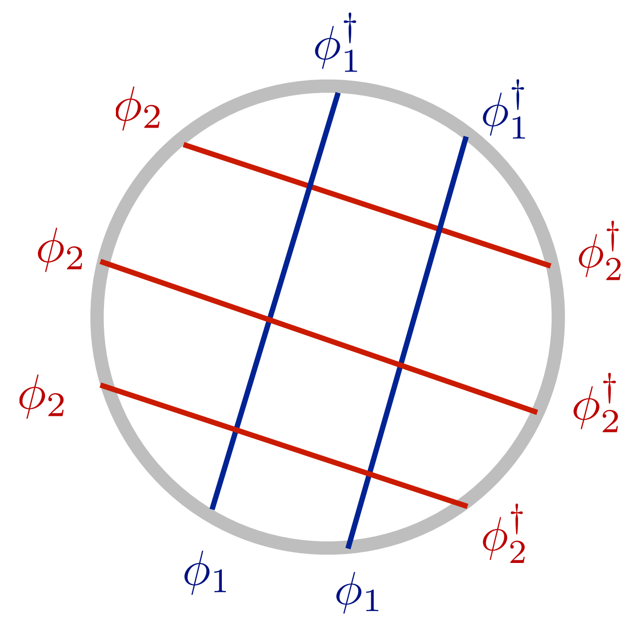
Can be used for 3D Feynman integrals

Other observables

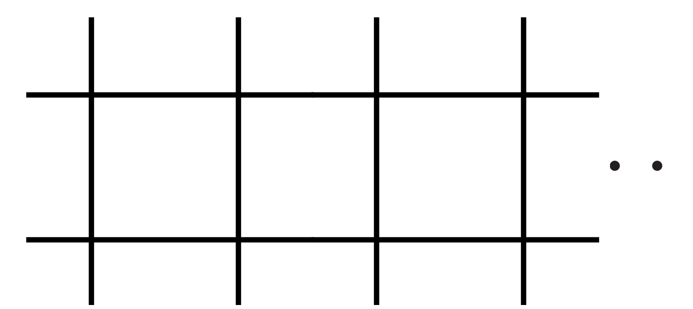


Scattering amplitudes: for a given ordering of external fields in the disk -> single Feynman diagram

Whenever there are loops:



Only boxes are allowed



Amplitudes are **finite** and manifestly **dual conformal invariant!**

Three point functions: simple perturbation theory. Hexagon program for bi-scalar model? [Basso, Komatsu, Vieira,' 15]

Conclusions

- New 4D and 3D integrable field theories.
- Shed light on the origins of Integrability. First explicit demonstration of all-loop integrability in a corner of N=4 SYM - related to fishnet graphs.
- How to take the DS limit at the level of the Quantum Spectral Curve?
- Can we derive from first principles Thermodynamic Bethe Ansatz and/or Quantum Spectral Curve?
- Structure constants, scattering amplitudes, four-point functions?
- Can integrability further help in computing more integrals?
- Integrability to compute beta function?
- Dual string theory?

Thank you!