Exploring the Limits of the Standard Model
The role of low-energy particle physics

Hartmut Wittig

PRISMA Cluster of Excellence, Institute for Nuclear Physics and Helmholtz Institute Mainz

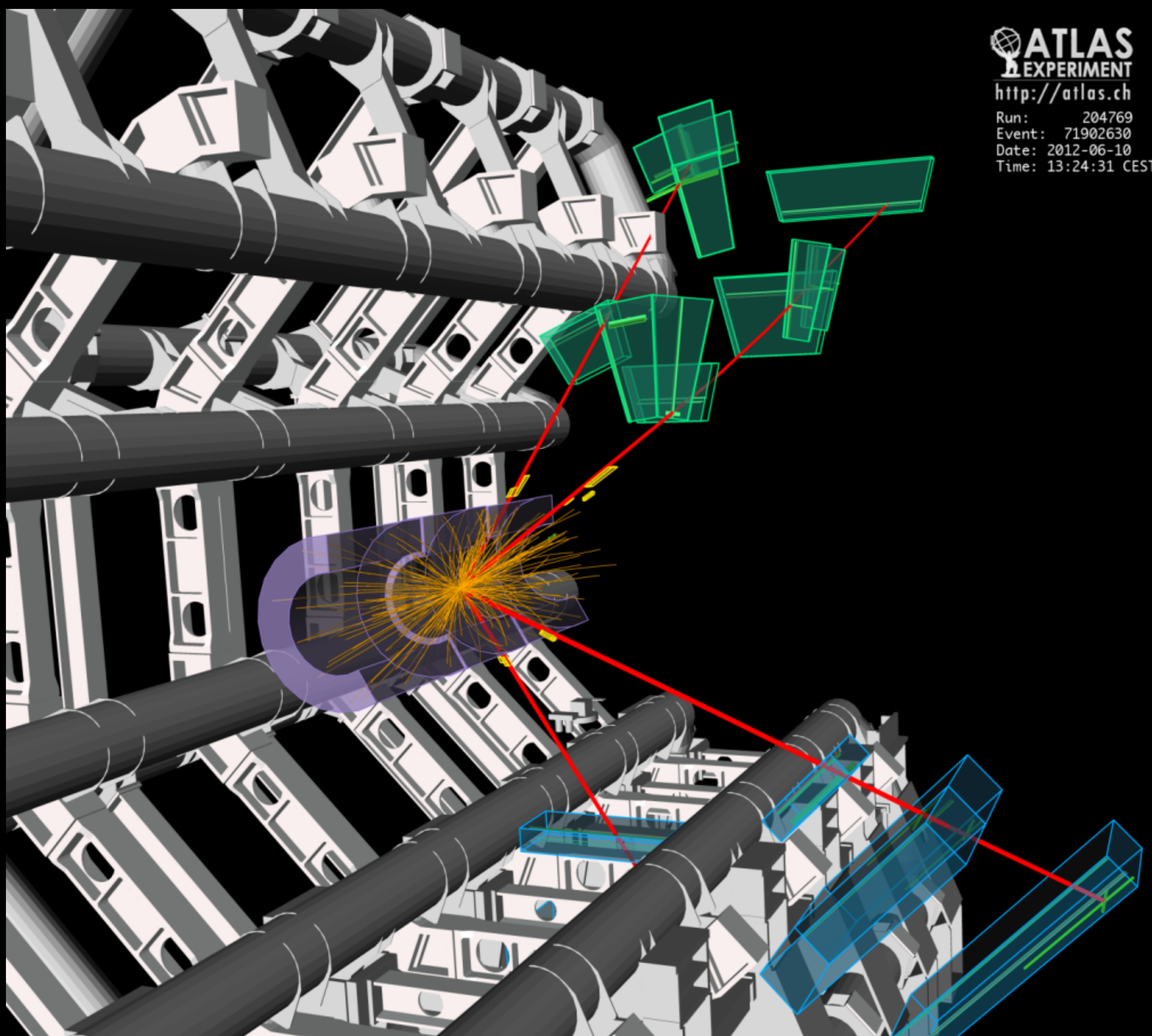
SHEP Friday Seminar
13 May 2016

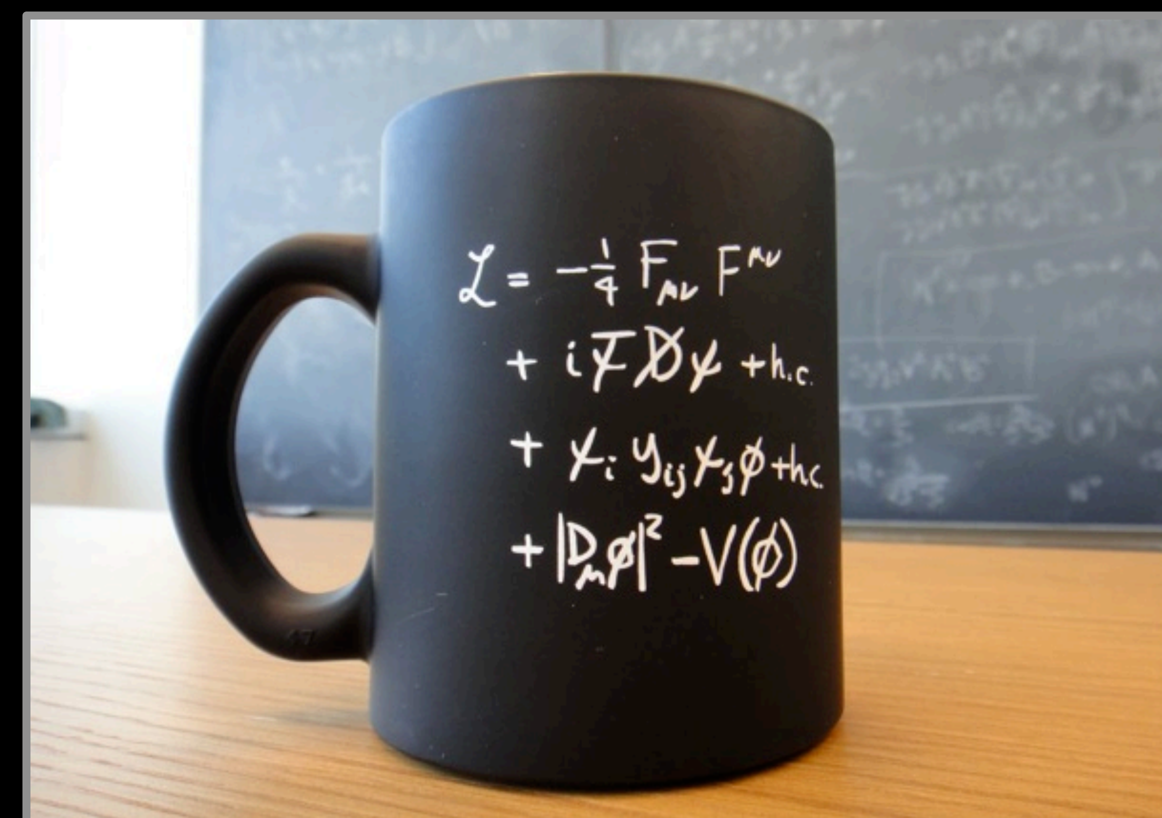
HELMHOLTZ ASSOCIATION
Helmholtz Institute Mainz

PRISMA

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

The Standard Model after the Higgs discovery



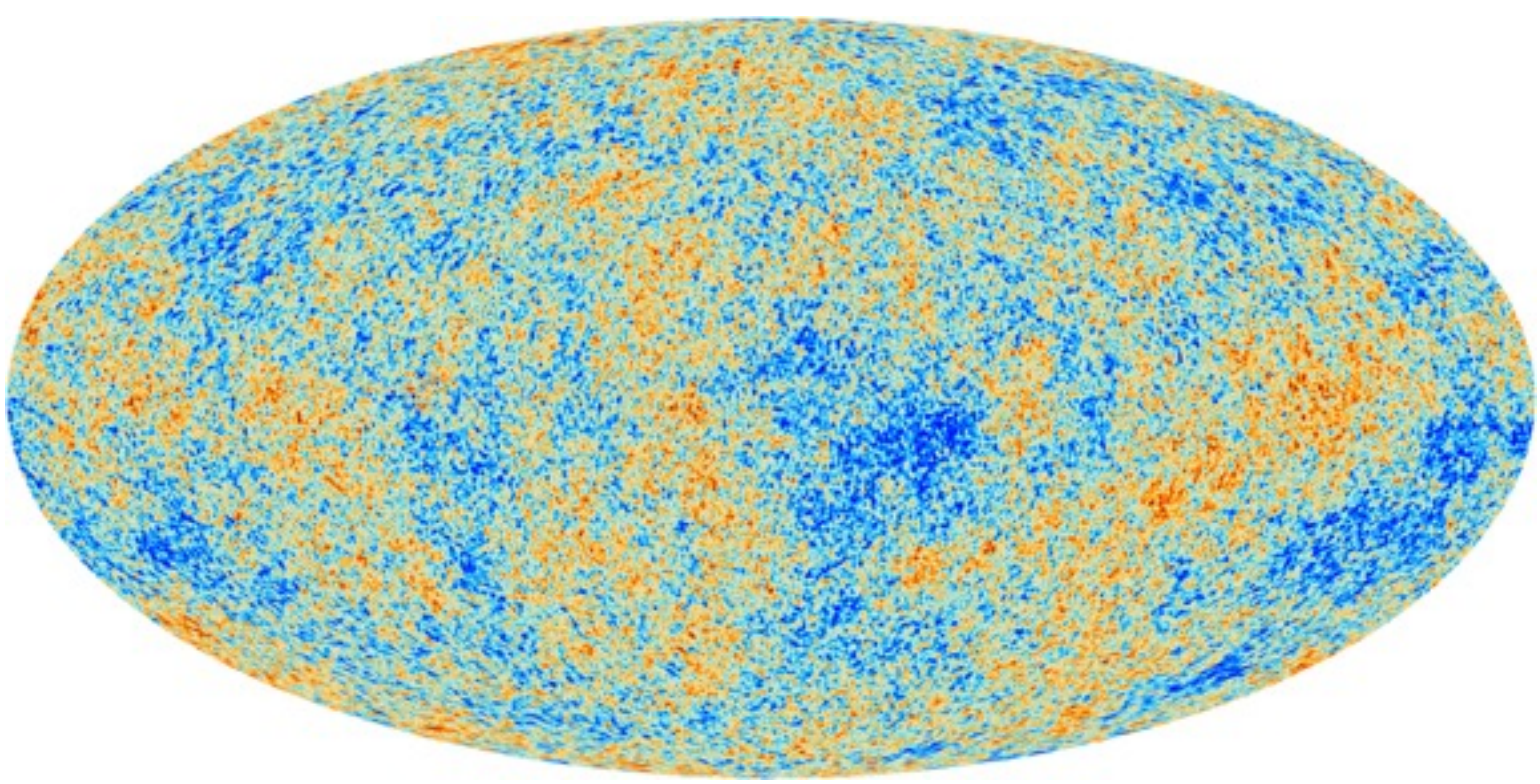


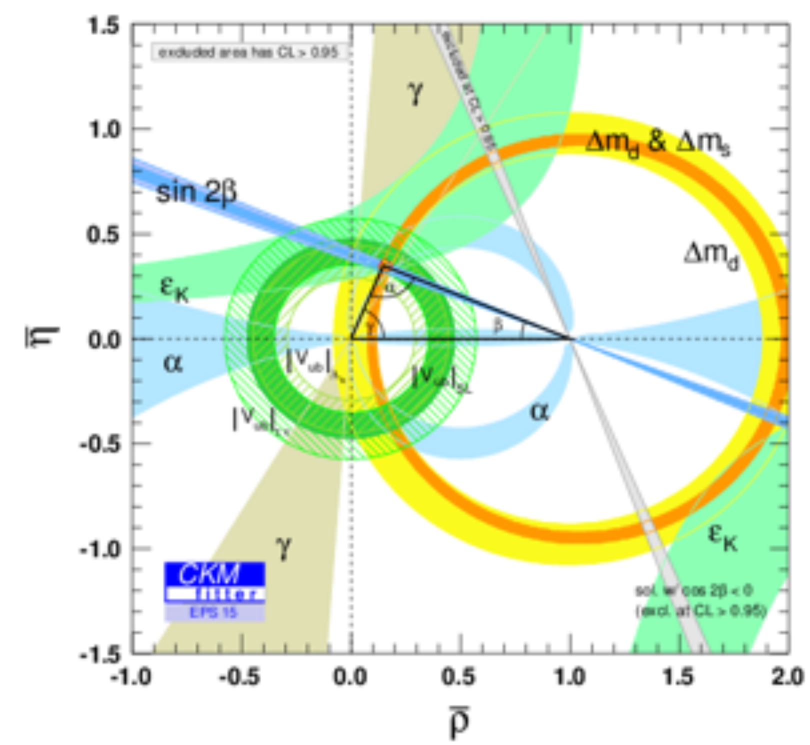
The Standard Model after the Higgs discovery

* Standard Model fully established but cannot account for:
    - Mass and scale hierarchies:

$$m_{\text{top}}/m_{\nu_e} > 10^{11} \qquad\qquad m_{\text{Higgs}} \ll m_{\text{Planck}}$$

    - Dark matter and dark energy
    - Amount of CP violation to sustain matter/antimatter asymmetry





The Standard Model after the Higgs discovery

* Standard Model fully established but cannot account for:
  - Mass and scale hierarchies:

    $$m_{\rm top}/m_{\nu_e} > 10^{11} \qquad\qquad m_{\rm Higgs} \ll m_{\rm Planck}$$

  - Dark matter and dark energy
  - Amount of CP violation to sustain matter/antimatter asymmetry
* Explore the limits of the Standard Model
  - Search for new particles and phenomena at higher energy
  - Search for enhancement of rare phenomena
  - Compare precision measurements to SM predictions

Control over **hadronic uncertainties**

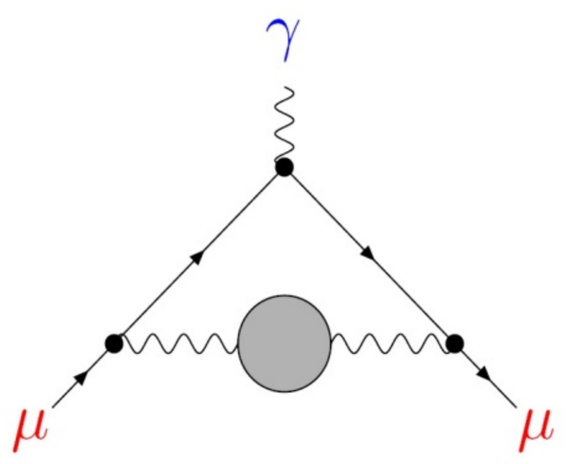
SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

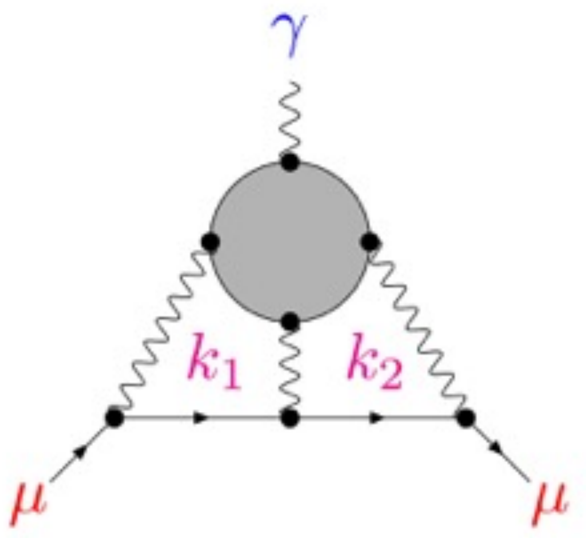
PRISMA

Precision Tests of the Standard Model

Anomalous magnetic moment of the muon: $a_\mu \equiv \frac{1}{2}(g-2)_\mu$

$$a_\mu = \begin{cases} 116\,592\,080(54)(33)\cdot 10^{-11} & \text{Experiment} \\ 116\,591\,802(2)(42)(26)\cdot 10^{-11} & \text{SM prediction} \end{cases}$$





Dispersion theory:

$$a_\mu^{\rm HVP} = (6923 \pm 42 \pm 3)\cdot 10^{-11}$$

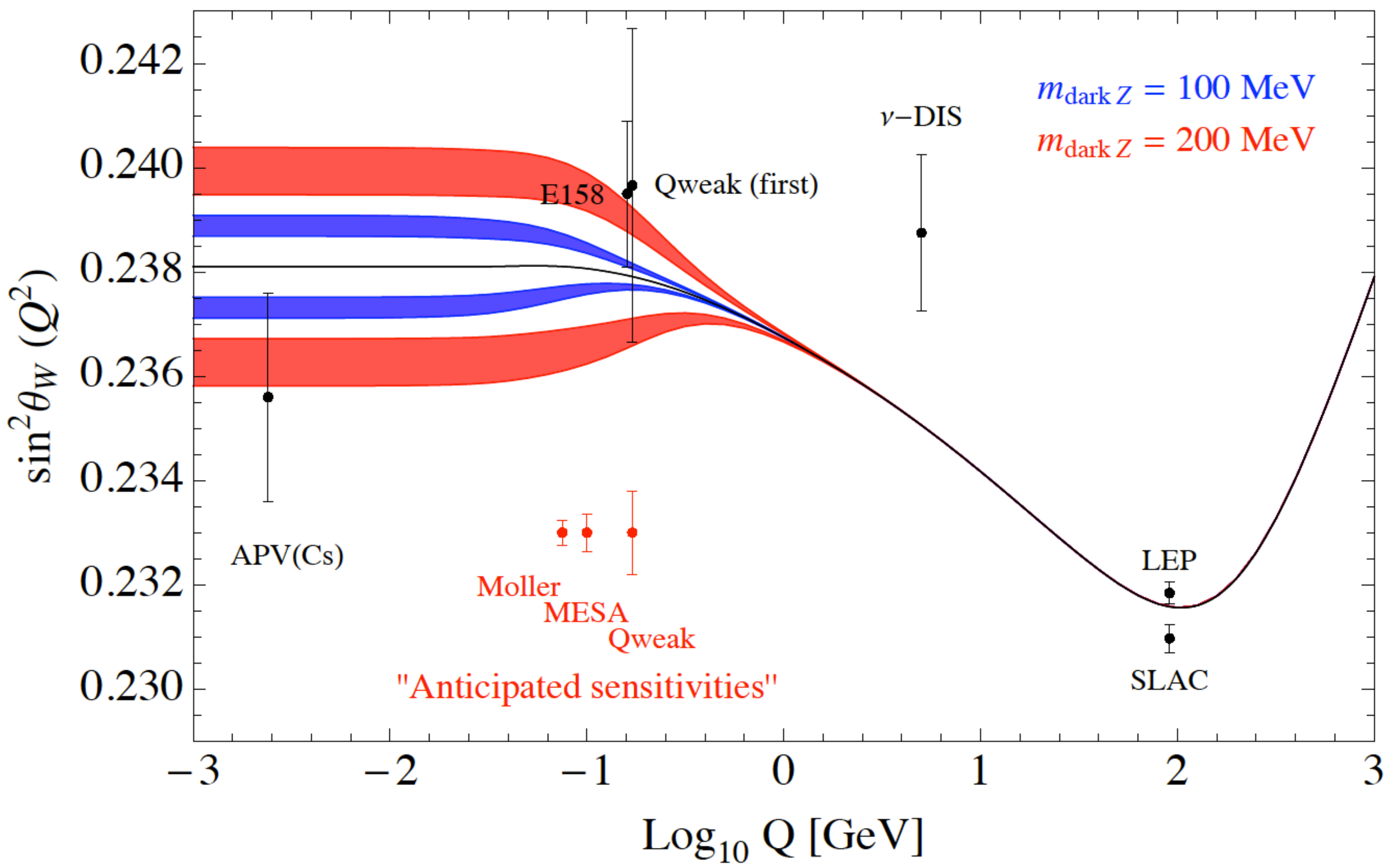
based on $R_{\rm exp}(e^+e^- \longrightarrow \text{hadrons})$

Model estimates:

$$a_\mu^{\rm HLbL} = \begin{cases} (105 \pm 26)\cdot 10^{-11} \\ (116 \pm 39)\cdot 10^{-11} \end{cases}$$

Precision Tests of the Standard Model

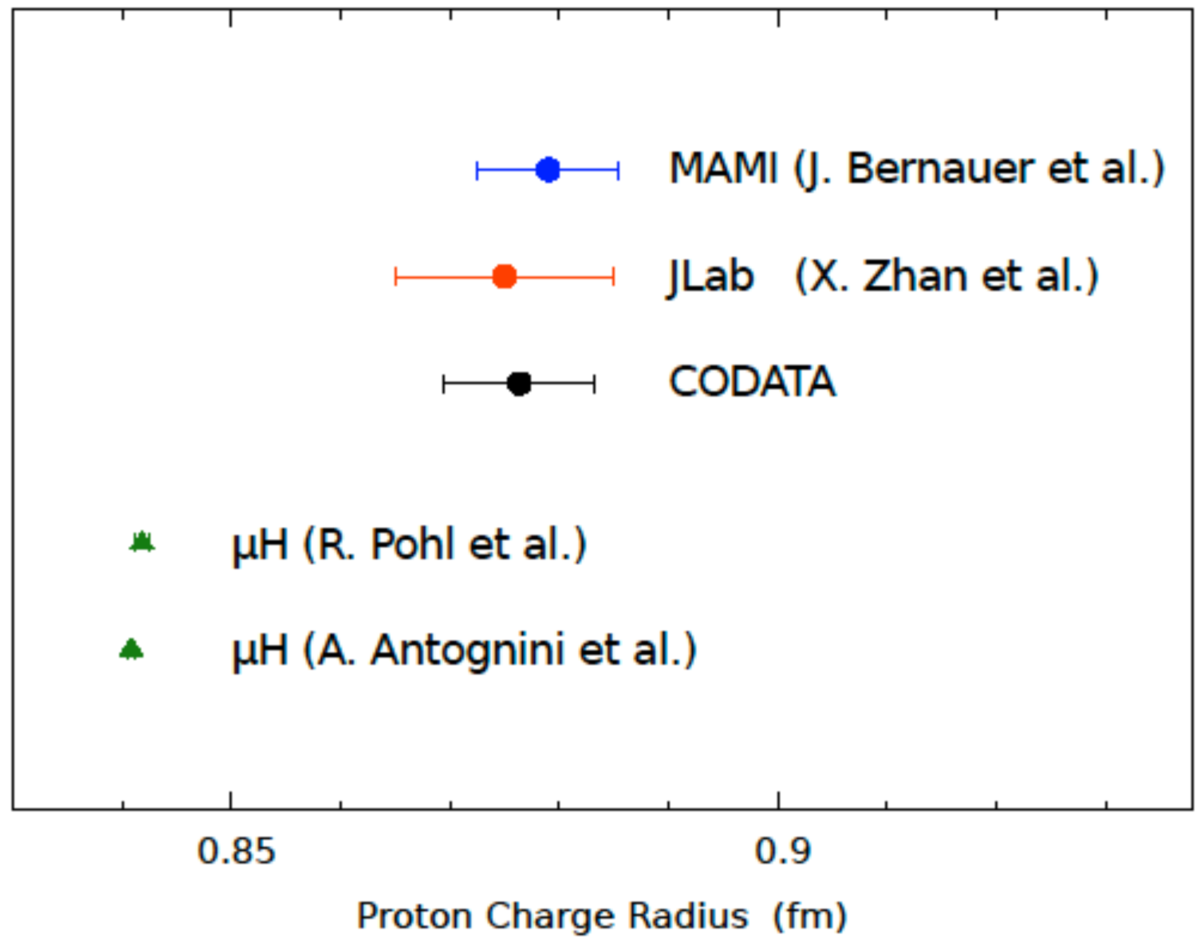
Running of electroweak mixing angle: $\sin^2\theta_W$



* Running of $\sin^2\theta_W$ at low energies discriminates between "New Physics" scenarios
* Challenge for theory: hadronic contributions

Precision Tests of the Standard Model

Proton Radius Puzzle



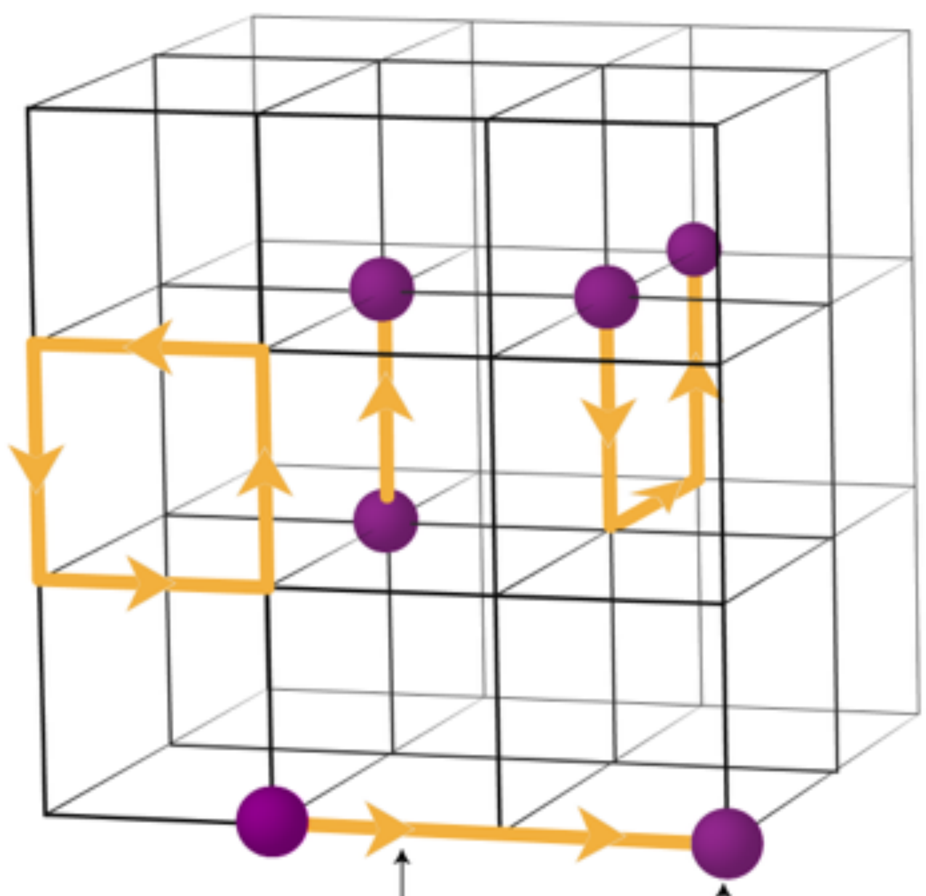
Muonic Hydrogen: $r_{\mathrm{E}} = 0.8409 \pm 0.0004\,\mathrm{fm}$ *[Antognini et al. 2013]*

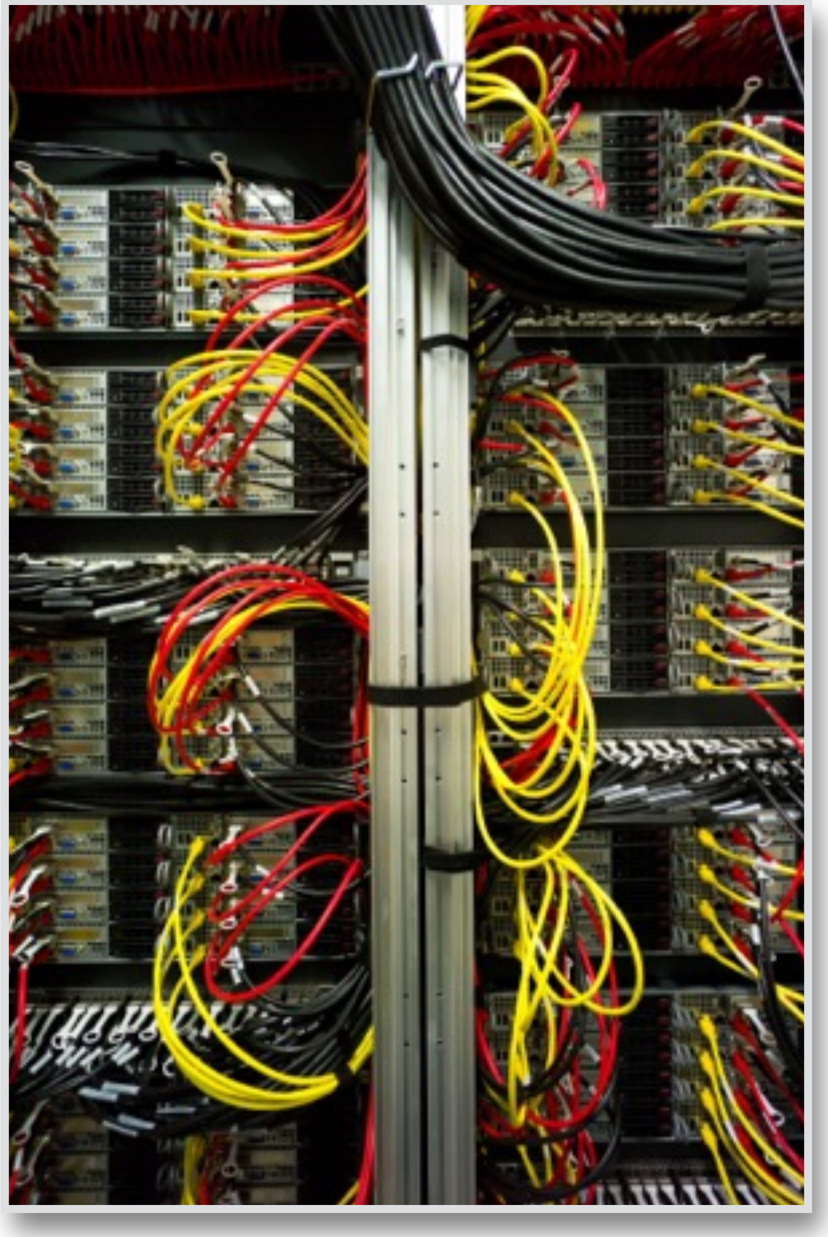
Electronic systems: $r_{\mathrm{E}} = 0.8775 \pm 0.0051\,\mathrm{fm}$ *[CODATA 2012]*

* Signal for “New Physics” or poorly understood hadronic effects?

Precision Tests of the Standard Model

* Accuracy of Standard Model tests limited by hadronic contributions
* Employ "ab initio" approach: Lattice QCD





"Clover" @ Mainz

Outline

I. Low-energy precision experiments at Mainz

II. The muon $(g-2)$ in Lattice QCD

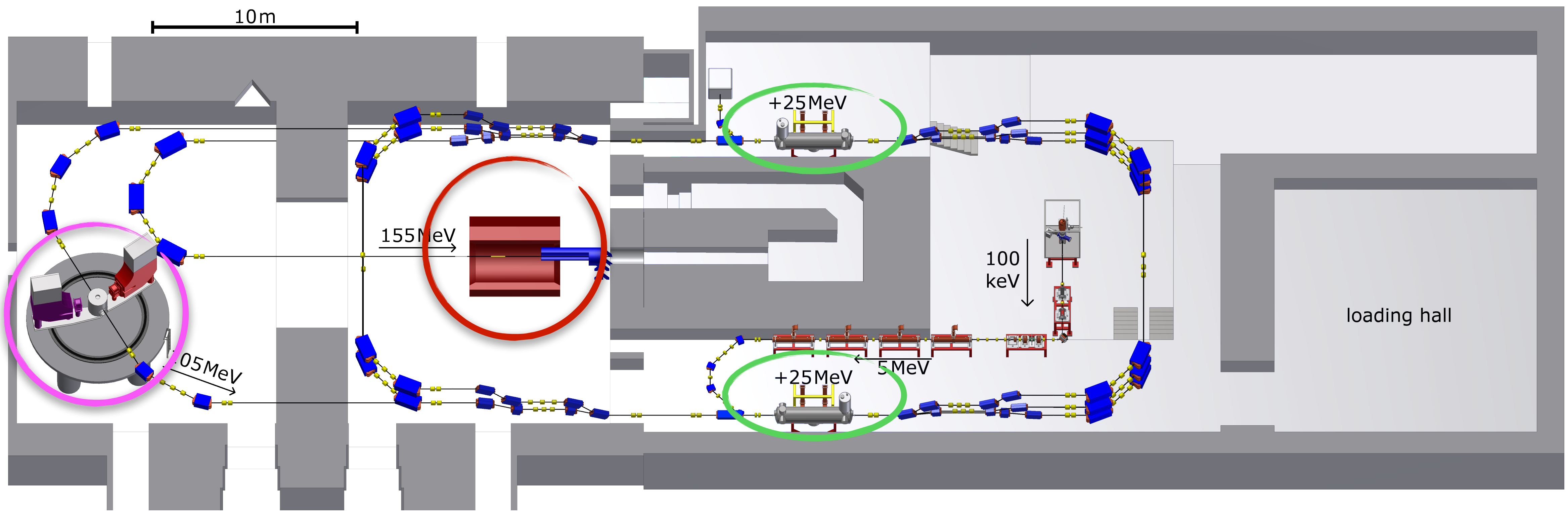
III. Running of electroweak couplings

IV. The charge radius of the nucleon

V. Summary & Outlook

Low-energy precision experiments at Mainz

* MESA — "**M**ainz **E**nergy-Recovering **S**uperconducting **A**ccelerator



MAGIX P2 Superconducting cavities

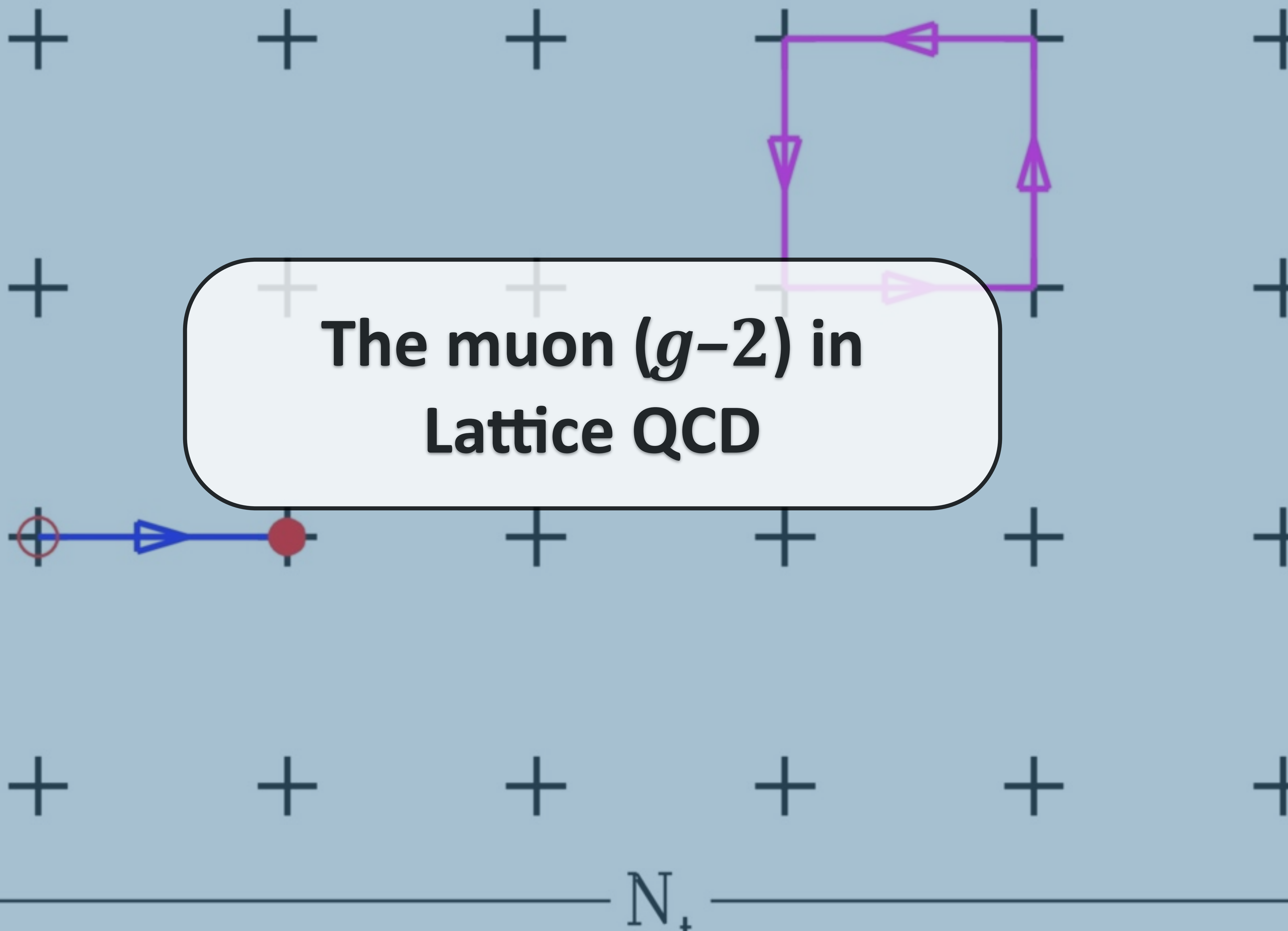
PRiSMA

"Energy Recovery" vs. "Extracted Beam" modes

Beam energy: 105 MeV / 155 MeV Current: 1–2 mA

Luminosity: up to 10^{39} $cm^{-2}s^{-1}$

The muon (g–2) in Lattice QCD



The Mainz $(g-2)_\mu$ project

Collaborators:

N. Asmussen, A. Gérardin, J. Green, O. Gryniuk, G. von Hippel, H. Horch, H. Meyer, A. Nyffeler, V. Pascalutsa, A. Risch, HW

M. Della Morte, A. Francis, B. Jäger, V. Gülpers, G. Herdoíza

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

Topics:

* Hadronic vacuum polarisation
* Light-by-light scattering
* Running of $\alpha_{\rm em}$ and $\sin^2\theta_{\rm W}$
* Determination of $\alpha_{\rm s}$ from vacuum polarisation function

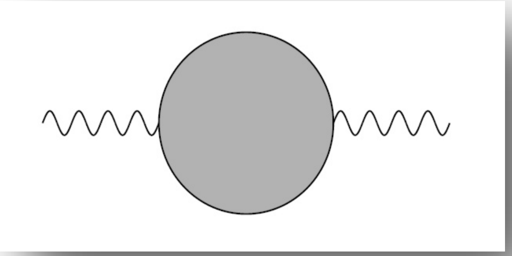
Lattice QCD approach to HVP

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\, \{\Pi(Q^2) - \Pi(0)\}$$

$$\Pi_{\mu\nu}(Q) = \int d^4x\, \mathrm{e}^{iQ\cdot(x-y)}\, \langle J_\mu(x) J_\nu(y)\rangle \equiv (Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi(Q^2)$$

$$J_\mu = \tfrac{2}{3}\overline{u}\gamma_\mu u - \tfrac{1}{3}\overline{d}\gamma_\mu d - \tfrac{1}{3}\overline{s}\gamma_\mu s + \ldots$$



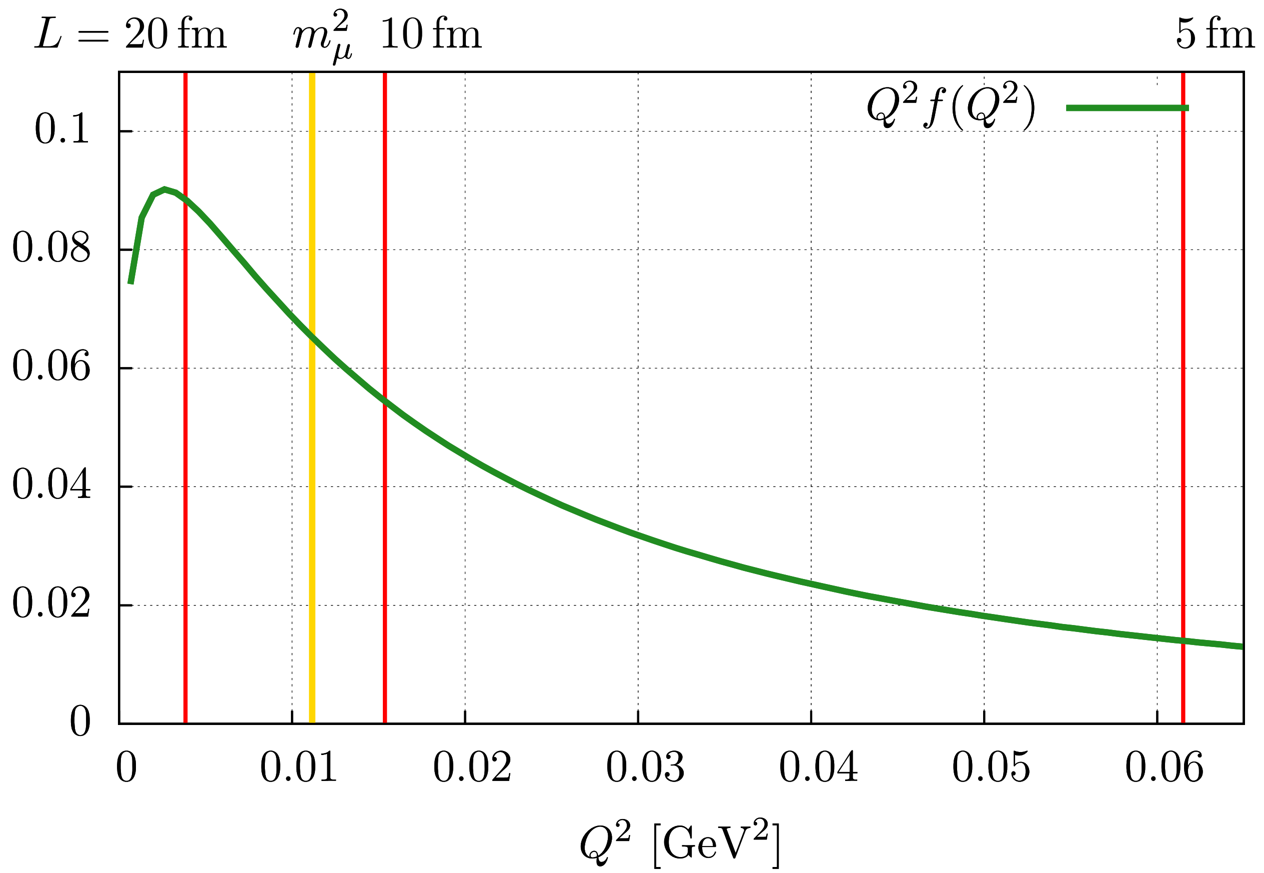
* Lattice momenta are quantised: $Q_\mu = \dfrac{2\pi}{L_\mu}$
* Determine VPF $\Pi(Q^2)$ and additive renormalisation $\Pi(0)$
* Statistical accuracy of $\Pi(Q^2)$ deteriorates as $Q \longrightarrow 0$

Lattice QCD approach to HVP

- Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

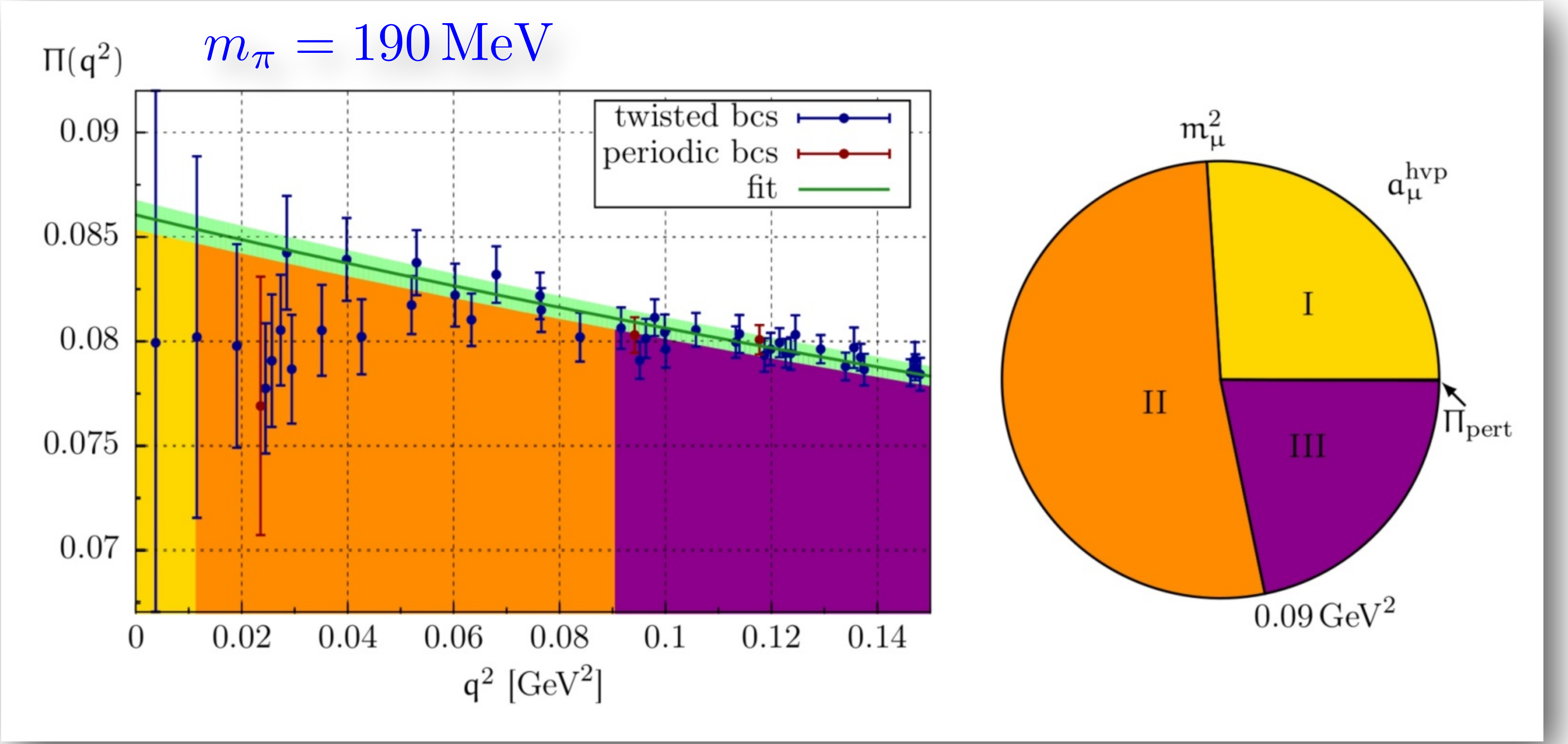
$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\, \{\Pi(Q^2) - \Pi(0)\}$$

- Integrand peaked near $Q^2 \approx (\sqrt{5}-2)m_\mu^2$



Accurate determination requires large statistics on large volumes!

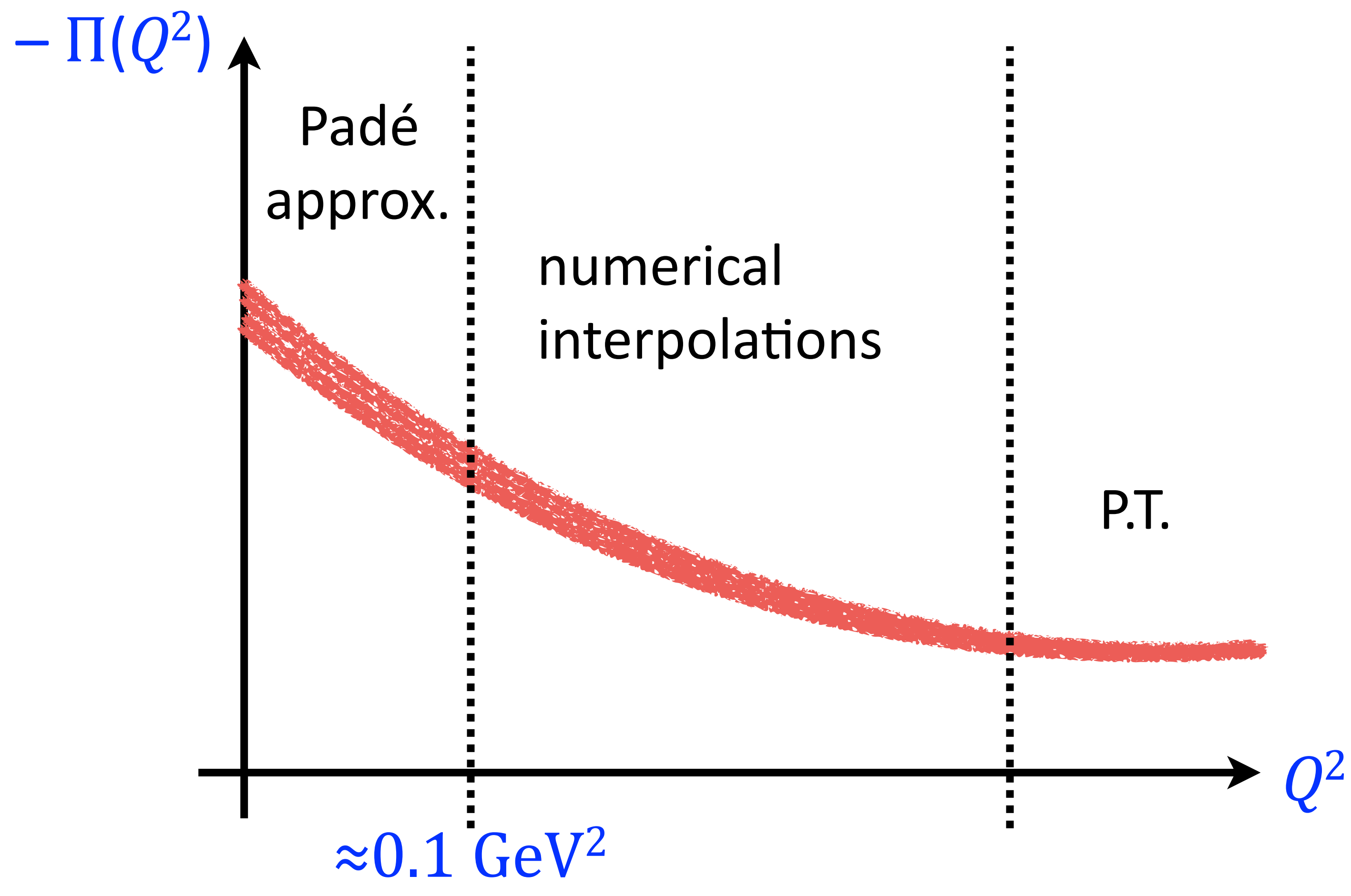
Lattice QCD approach to HVP



- Model-independent fits compromised when applied to $Q^2 \gg m_\mu^2$
- Determination of $\Pi(0)$ may be biased by more accurate data at large Q^2

Lattice QCD approach to HVP

* "Hybrid" method: *[Golterman, Maltman & Peris, Phys Rev D90 (2014) 074508]*



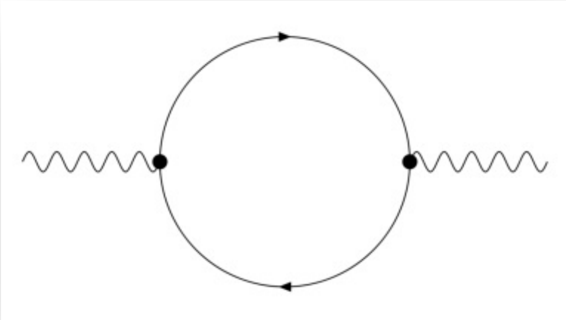
* Determine $\Pi(0)$ from Padé approximation in small-momentum region

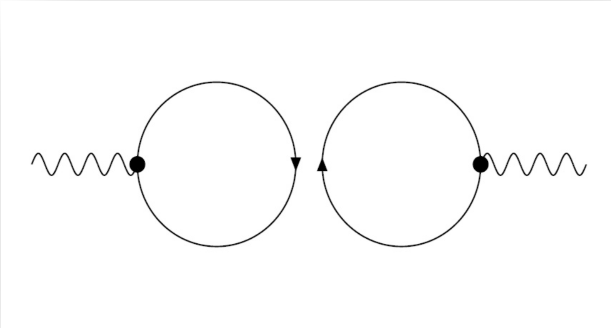
* Requires sub-percent accuracy in u,d-part for $Q^2 = \mathrm{O}(0.1\ \mathrm{GeV}^2)$

Lattice QCD approach to HVP

Main issues:

- Statistical accuracy at the sub-percent level required
- Comprehensive study of finite-volume effects
- Reduce systematic uncertainty associated with region of small Q^2
  $\Leftrightarrow$ accurate determination of $\Pi(0)$
- Include quark-disconnected diagrams





- Include isospin breaking: $m_u \neq m_d$, QED corrections

Low-momentum region: Time moments

* Expansion of VPF at low-Q^2: $$\Pi(Q^2) = \Pi_0 + \sum_{j=1}^{\infty} Q^{2j}\Pi_j$$

* Vacuum polarisation for $Q = (\omega, \vec{0})$:

$$\Pi_{kk}(\omega) = a^4 \sum_{x_0} \mathrm{e}^{i\omega x_0} \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

* Spatially summed vector correlator: $G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$

* Time moments: *[Chakraborty et al., Phys Rev D89 (2014) 114501]*

$$G_{2n} \equiv a \sum_{x_0} x_0^{2n} G(x_0) = (-1)^n \frac{\partial^{2n}}{\partial \omega^{2n}} \left\{ \omega^2 \hat{\Pi}(\omega^2) \right\}_{\omega^2=0}$$

* Expansion coefficients: $\Pi(0) \equiv \Pi_0 = \frac{1}{2} G_2, \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$

Time-Momentum Representation

* Integral representation of subtracted VPF $\hat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$

$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2}\int_0^\infty dx_0\, G(x_0)\left[Q^2 x_0^2 - 4\sin^2\left(\tfrac{1}{2}Qx_0\right)\right]$$

$$G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0)\rangle$$

[Bernecker & Meyer, Eur Phys J A47 (2011) 148]

[Francis et al. 2013; Feng et al. 2013; Lehner & Izubuchi 2014, Del Debbio & Portelli 2015,…]

* Q^2 is a tuneable parameter

* No extrapolation to $Q^2 = 0$ required; related to time-moments

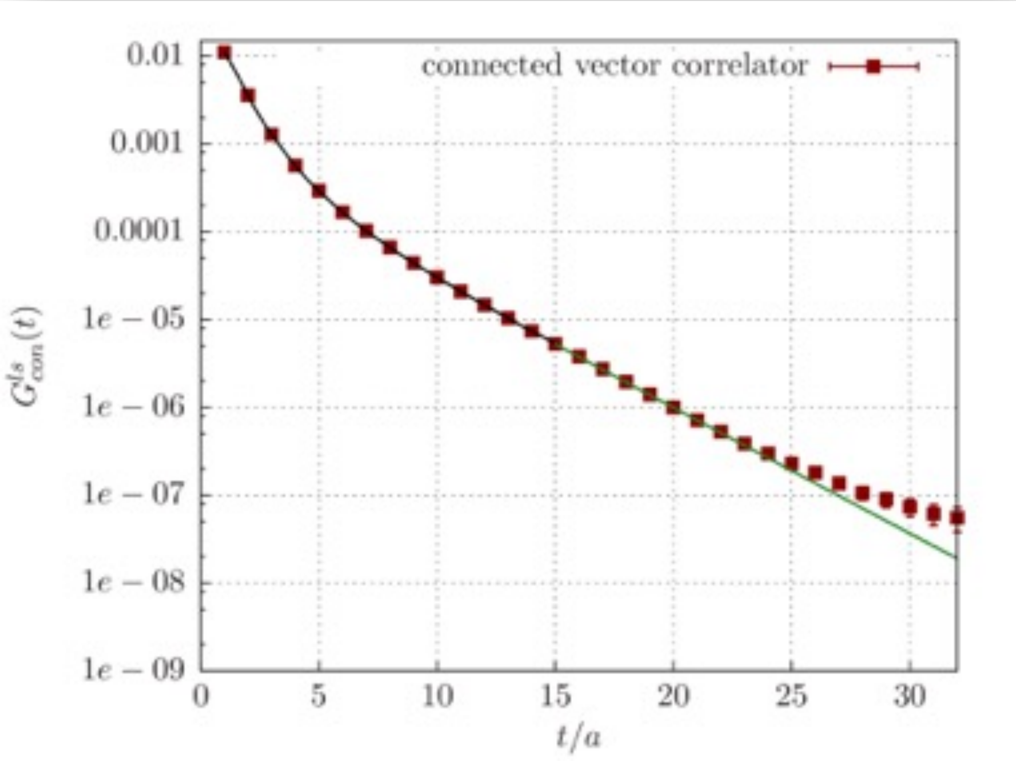
* Must determine $I = 1$ vector correlator $G(x_0)$ for $x_0 \to \infty$

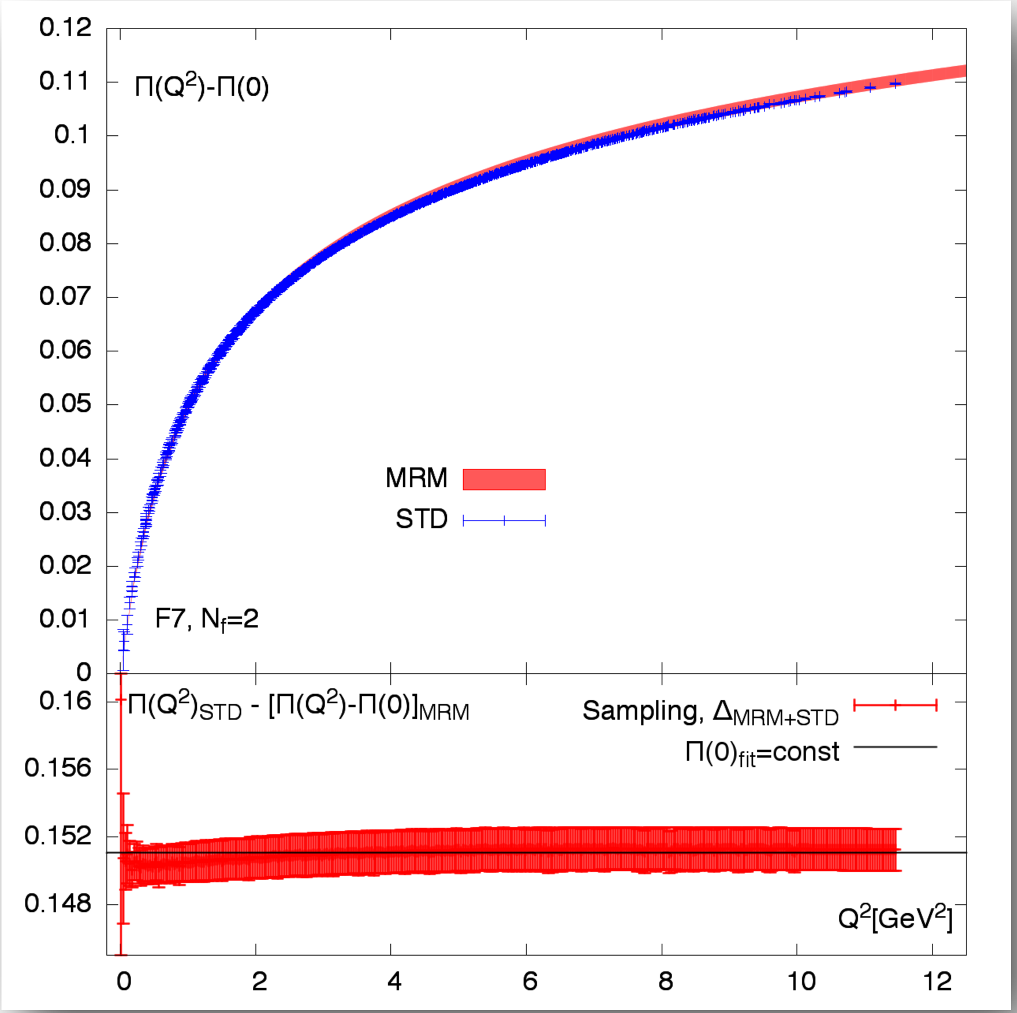
  $\longrightarrow$ Include two-pion states to capture long-distance behaviour

Time-Momentum Representation

$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2}\int_0^\infty dx_0\, G(x_0)\left[Q^2x_0^2 - 4\sin^2\left(\tfrac{1}{2}Qx_0\right)\right]$$

$$G(x_0) = -a^3\sum_{\vec{x}}\langle J_k(x)J_k(0)\rangle$$





[Gülpers et al., arXiv:1411.7592; Francis et al., arXiv:1410.7491]

Current data sets and statistics

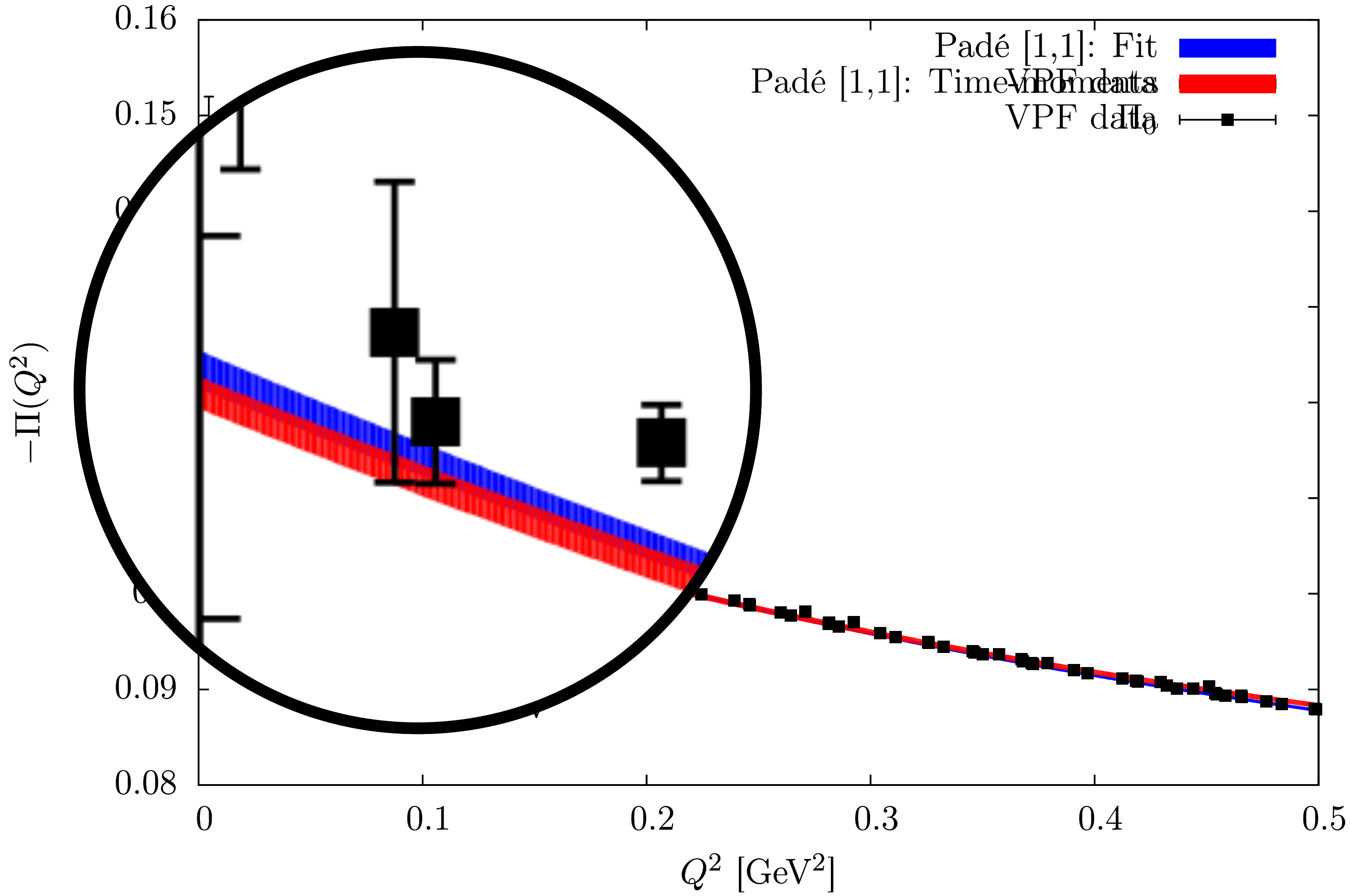
* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions
* Three values of the lattice spacing: $a = 0.076, 0.066, 0.049$ fm
* Pion masses and volumes: $m_\pi^{\min} = 185\,\mathrm{MeV}, \quad m_\pi L > 4$
* 1000–4000 measurements per ensemble

To be processed:

* $N_f = 2+1$ flavours of $O(a)$ improved Wilson fermions; tree-level Symanzik gauge action; open boundary conditions
* Five values of the lattice spacing; physical pion mass

Comparison: Fits versus Time moments

* Construct Padé approximants either from fits or time moments



Fit Padé [1,1] for

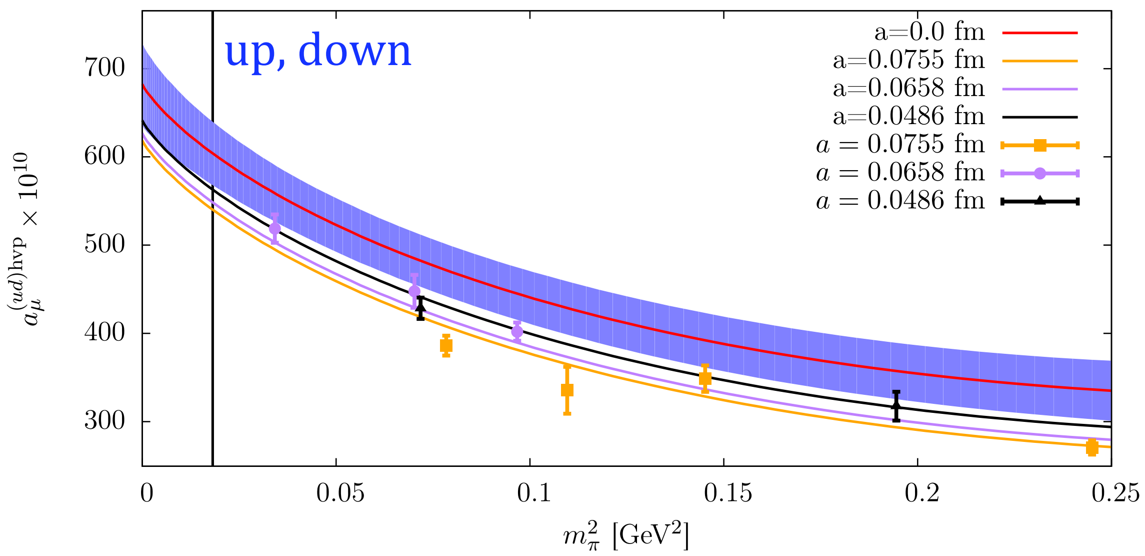
$Q^2_{\rm low} \lesssim 0.5\,\mathrm{GeV}^2$

* Low-order Padé approximants consistent for $Q^2 < 0.5$ GeV^2

* Apply trapezoidal rule to evaluate convolution integral for $Q^2 \geq 0.5$ GeV^2

Chiral and continuum extrapolations

* Use collection of different functional forms, e.g.



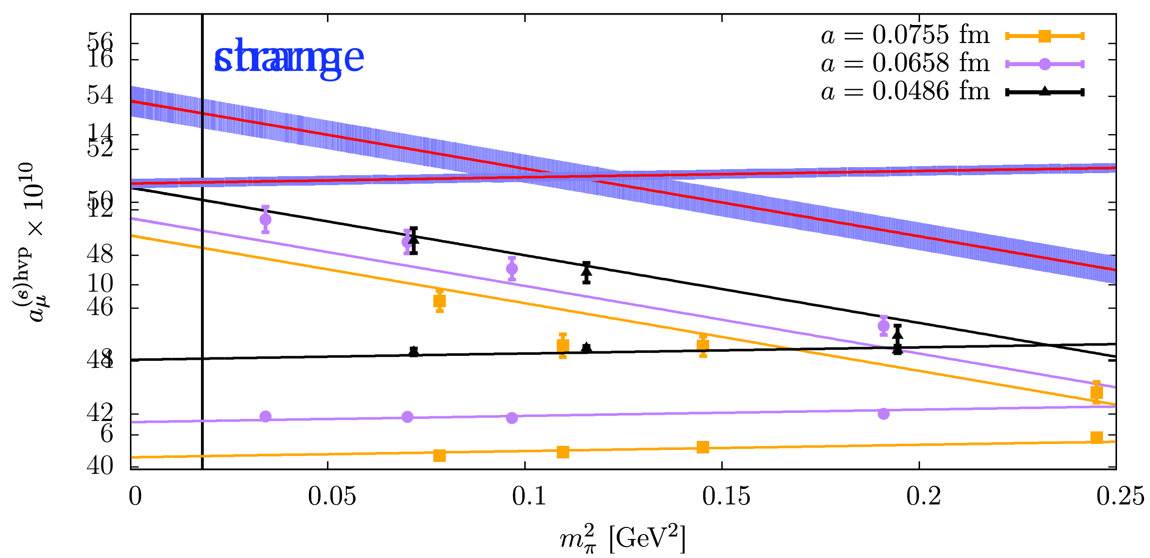
Fit A:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^2 \ln(m_\pi^2) + b_3 a$$

Fit B:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^4 + b_3 a$$

.....



* Perform cuts in pion mass and lattice spacing

* Lattice spacing effects clearly resolved for larger quark masses

Disconnected Contributions

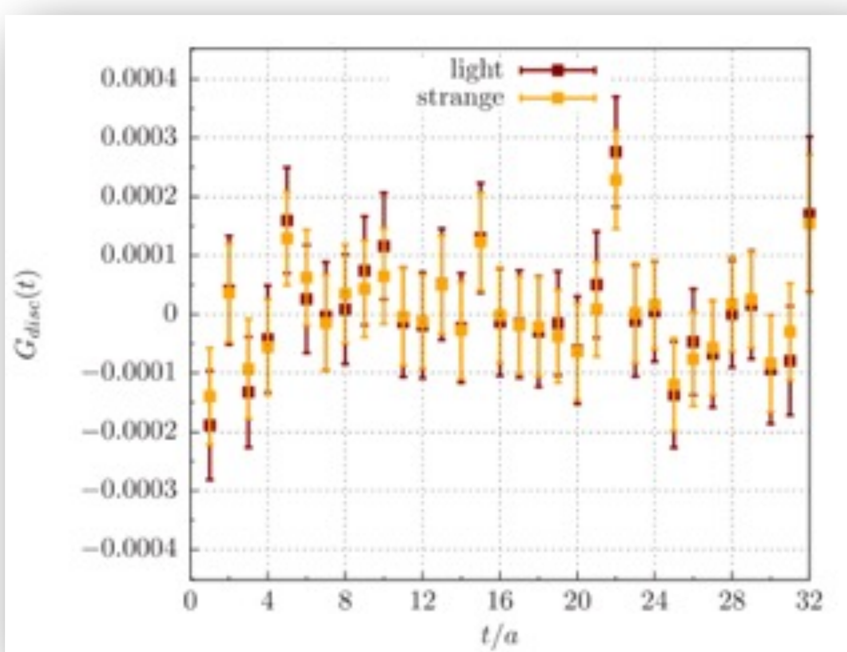
* Electromagnetic current correlator with *u, d, s* quarks:

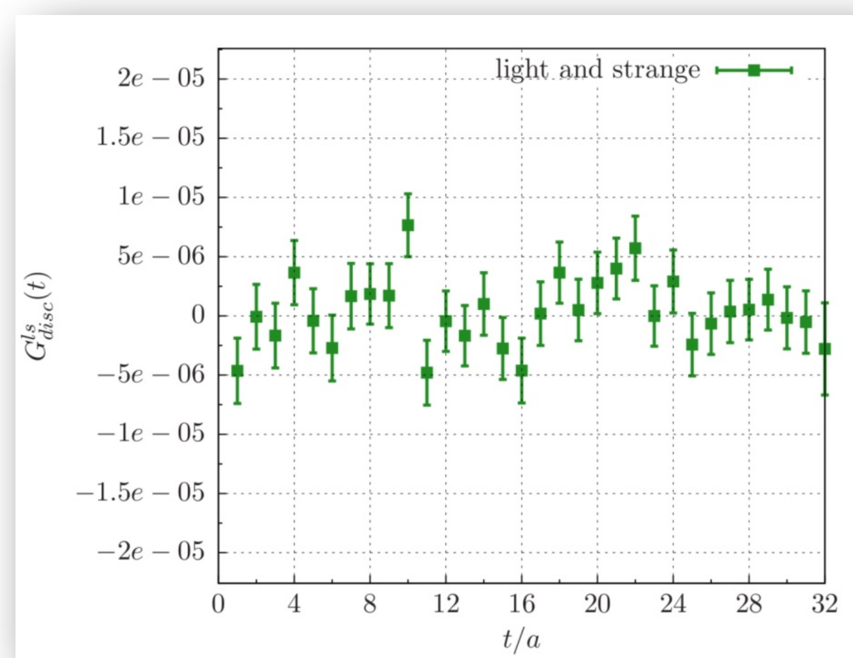
$$G^{\ell s}(x_0) := -\int d^3x\, \langle J_k^{\ell s}(x) J_k^{\ell s}(0)\rangle\,, \qquad J_\mu^{\ell s} = \tfrac{2}{3}\overline{u}\gamma_k u - \tfrac{1}{3}\overline{d}\gamma_k d - \tfrac{1}{3}\overline{s}\gamma_k s$$

* Identify connected and disconnected contributions:

$$G^{\ell s}(x_0) = \tfrac{5}{9} G^{\ell}_{\rm con}(x_0) + \tfrac{1}{9} G^{s}_{\rm con}(x_0) - \tfrac{1}{9} G^{\ell s}_{\rm disc}(x_0)$$

$$G^{\ell s}_{\rm disc}(x_0) = \int d^3x\, \left\{ {\rm Tr}\left[S^\ell(x,x)\gamma_k\right] - {\rm Tr}\left[S^s(x,x)\gamma_k\right]\right\} \times \{x \to 0\}$$

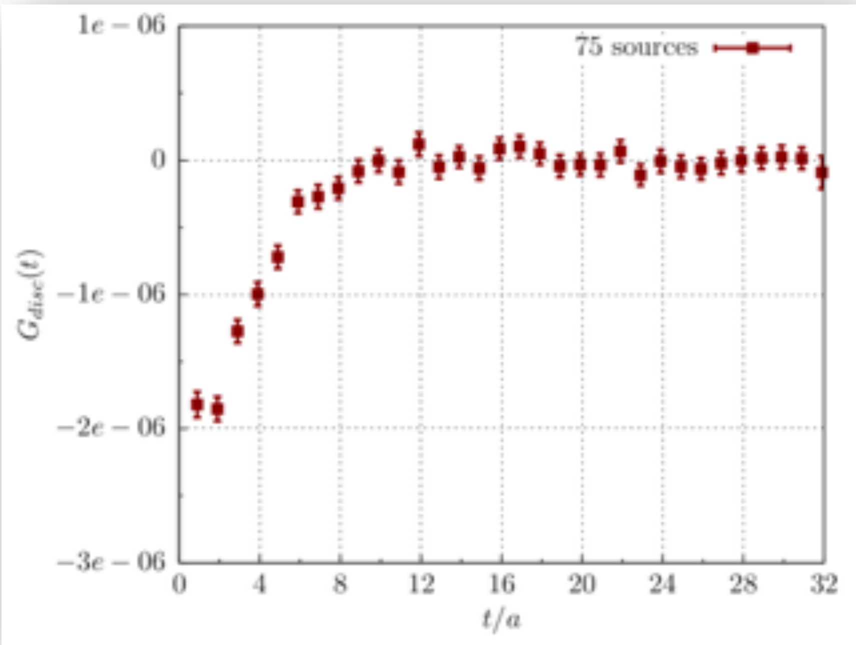


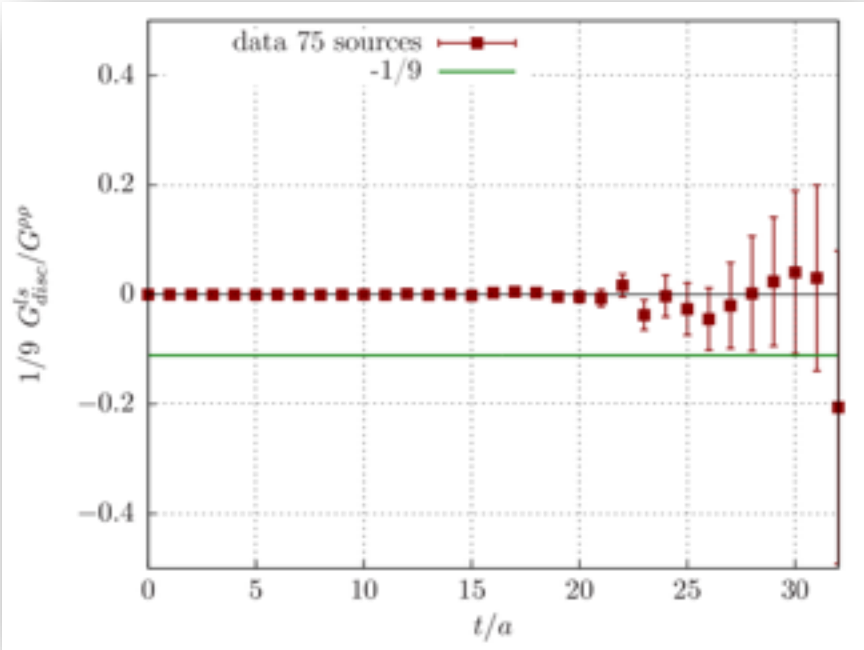


[Gülpers et al., arXiv:1411.7592; V. Gülpers, PhD Thesis 2015]

Disconnected Contributions

* Non-zero disconnected contribution can be resolved:





* Disconnected contribution for $x_0 \longrightarrow \infty$:

$$-\frac{1}{9}\frac{G^{\ell s}_{\rm disc}}{G^{\rho\rho}} = \frac{G^{\ell s}-G^{\rho\rho}}{G^{\rho\rho}} - \frac{1}{9}\left(1-\frac{2G^{s}_{\rm con}}{G^{\ell}_{\rm con}}\right) \xrightarrow{x_0\to\infty} -\frac{1}{9}$$

* Dominates accuracy of $G(x_0)$ for $x_0 \gtrsim 1.6\ \mathrm{fm}$

* Disconnected diagrams contribute less than 1% to $a_\mu^{\rm hvp}$

Running of electroweak couplings

Running of α — phenomenological approach

- Fine structure constant: $$\alpha(Q^2) = \frac{\alpha}{1-\Delta\alpha(Q^2)}$$

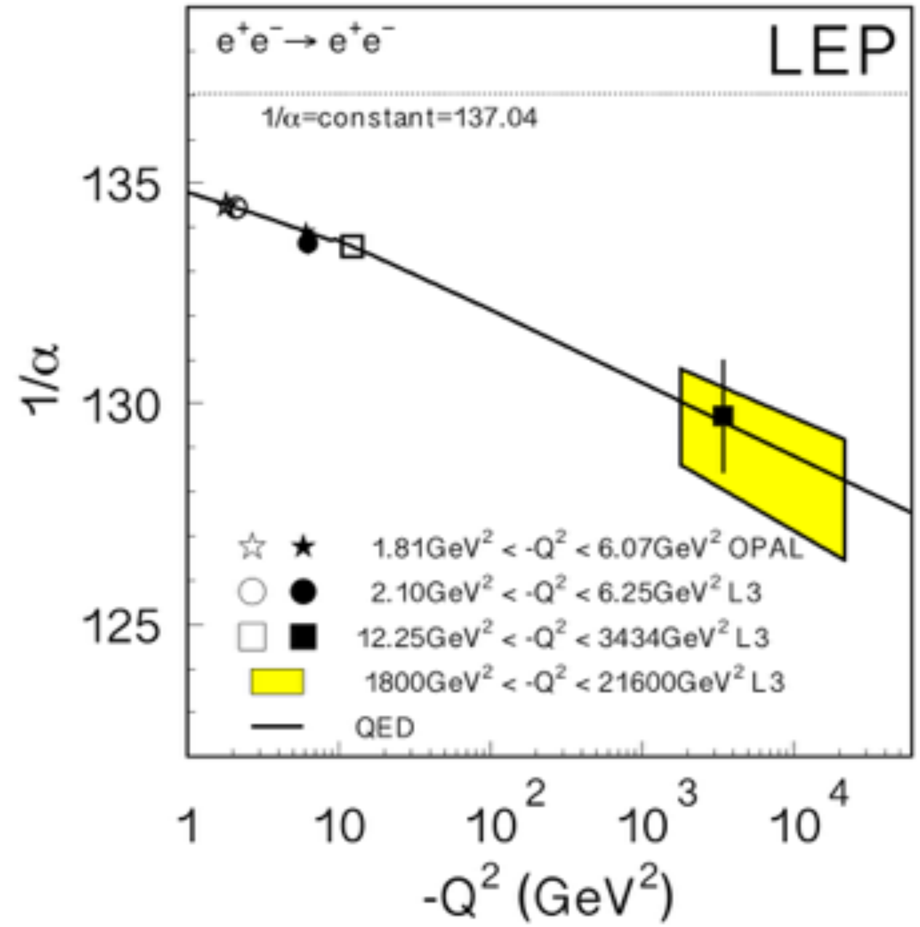
- Hadronic contributions — phenomenological approach:

$$\Delta\alpha_{\rm had}(Q^2) = -\frac{\alpha Q^2}{3\pi}\int_{4m_\pi^2}^{\infty} ds\, \frac{R_{\rm had}(s)}{s(s-Q^2)}$$

c.f.

$$a_\mu^{\rm HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.26 \pm 1.38)\cdot 10^{-4}$$

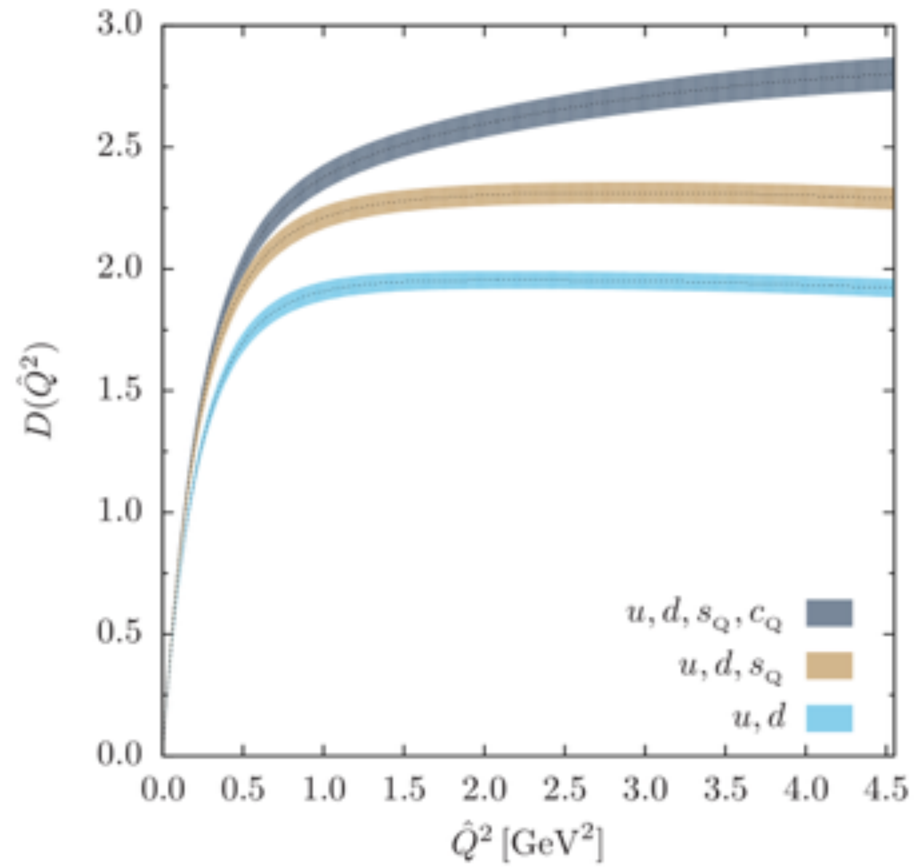


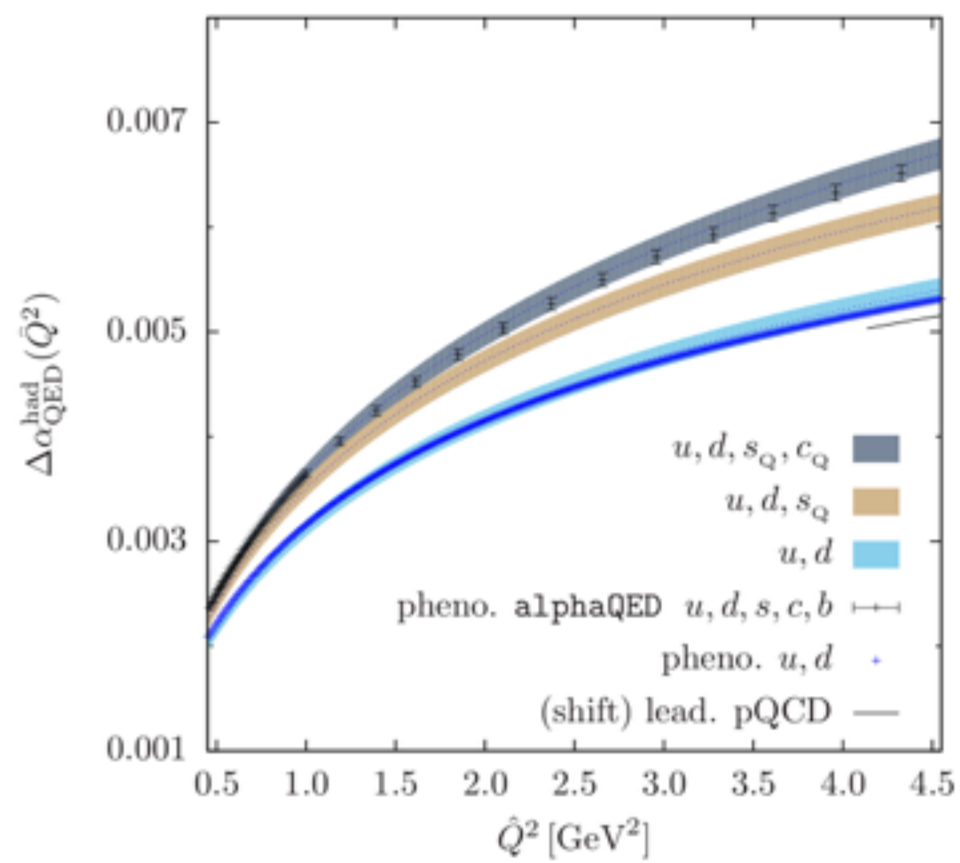
- Error on $\Delta\alpha_{\rm had}$ limits accuracy of Standard Model tests

Running of α — Euclidean approach

* Vacuum polarisation function: $\Delta\alpha_{\rm had}(Q^2) = 4\pi\alpha\Big(\Pi(Q^2) - \Pi(0)\Big)$

* Adler function: $D(Q^2) = -12\pi^2 \dfrac{d\,\Pi(Q^2)}{d\ln Q^2} = \dfrac{3\pi}{\alpha}\dfrac{d}{d\ln Q^2}\Delta\alpha_{\rm had}(Q^2)$



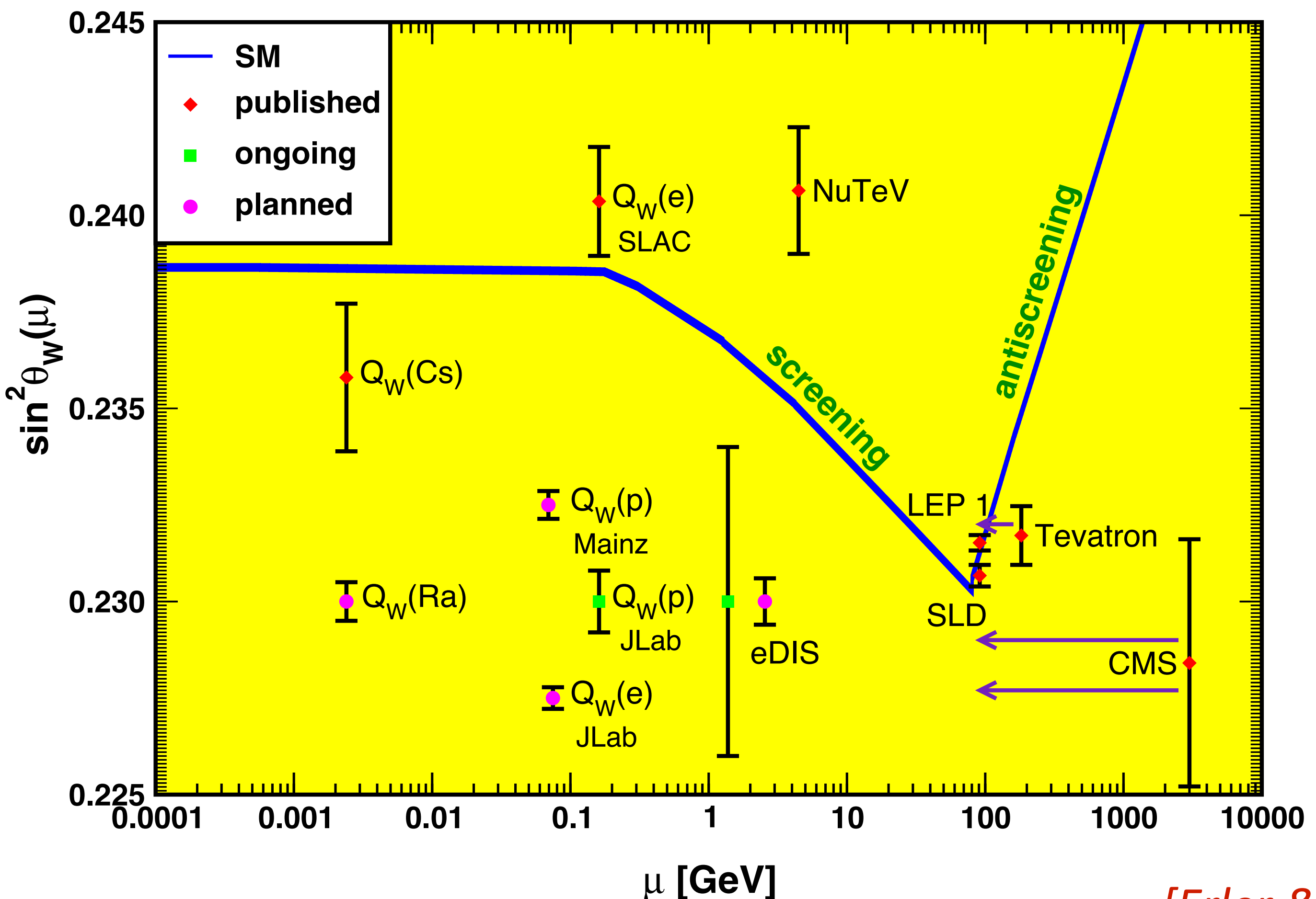


[H. Horch, G. Herdoíza @ Lattice 2015]

* Lattice QCD: similar accuracy as phenomenological approach

Running of $\sin^2\theta_W$

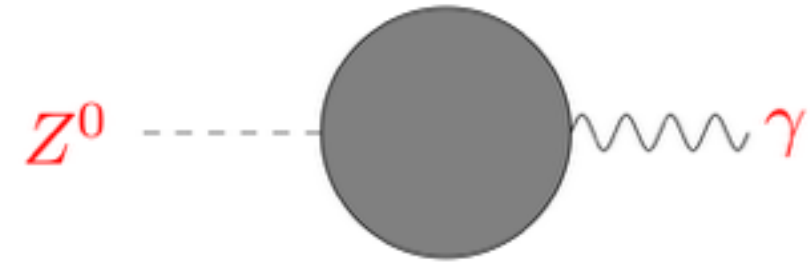
* Definition: $$\sin^2\theta_W(Q^2) = \underbrace{\sin^2\theta(0)}_{0.23864}\Big(1 - \Delta\sin^2\theta_W(Q^2)\Big)$$



[Erler & Su, PPNP 71 (2013) 119]

* Dispersive approach requires separation of contributions from up/down-type quarks

Running of $\sin^2\theta_W$



- Euclidean approach:

$$\Pi_{\mu\nu}^{\gamma Z}(Q) = \int d^x\, \mathrm{e}^{iQ\cdot x}\,\langle V_\mu^Z(x) J_\nu^\gamma(0)\rangle$$

$$V_\mu^Z = V_\mu^3 - \sin^2\theta_\mathrm{W}\, J_\mu^\gamma$$

$$V_\mu^3 = \tfrac{1}{4}\left(\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d - \overline{s}\gamma_\mu s + \overline{c}\gamma_\mu c + \ldots\right)$$

$$\Pi^{\gamma Z}(Q^2) = \Pi^{\gamma 3}(Q^2) - \sin^2\theta_\mathrm{W}\Pi^{\gamma\gamma}(Q^2)$$

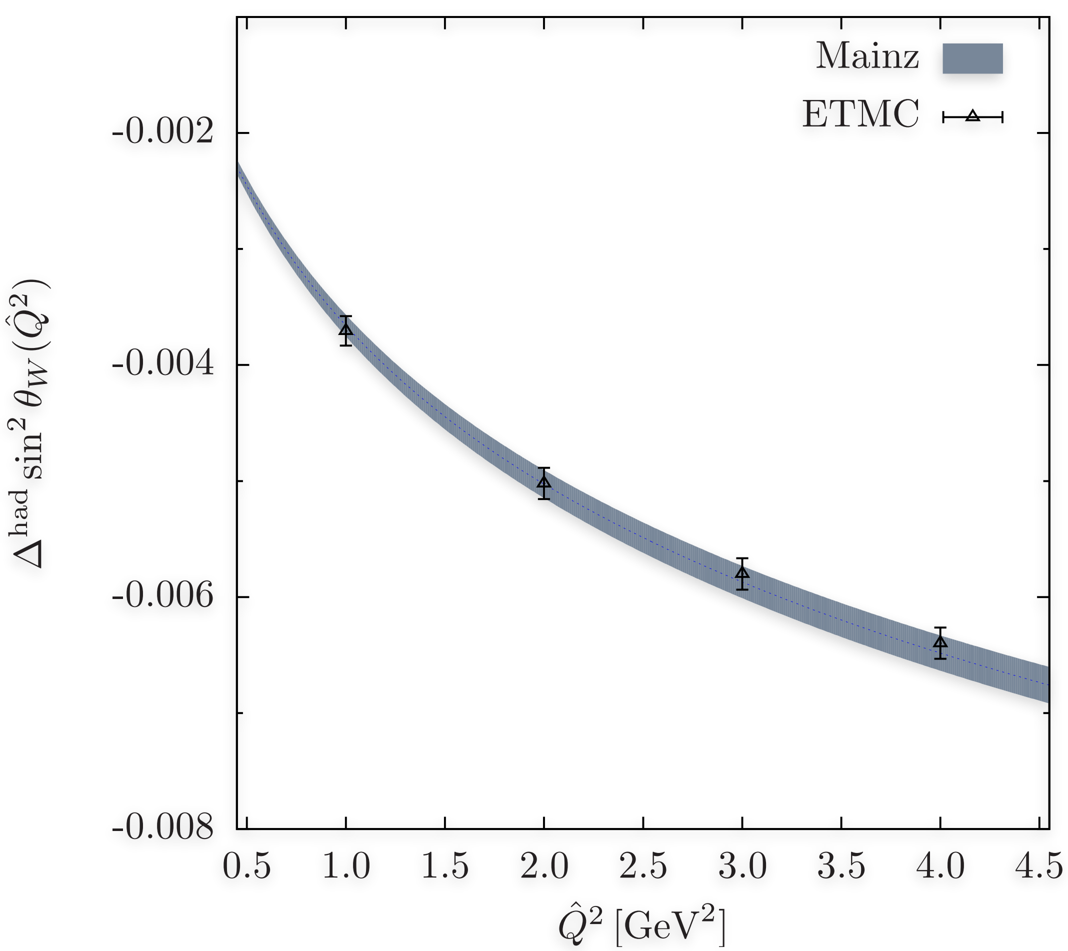
$$\Delta_\mathrm{had}\sin^2\theta_\mathrm{W}(Q^2) = \frac{e^2}{\sin^2\theta_0}\Big(\Pi^{\gamma Z}(Q^2) - \Pi^{\gamma Z}(0)\Big)$$

- Spin-off of calculation of running of $\Delta\alpha_\mathrm{had}$

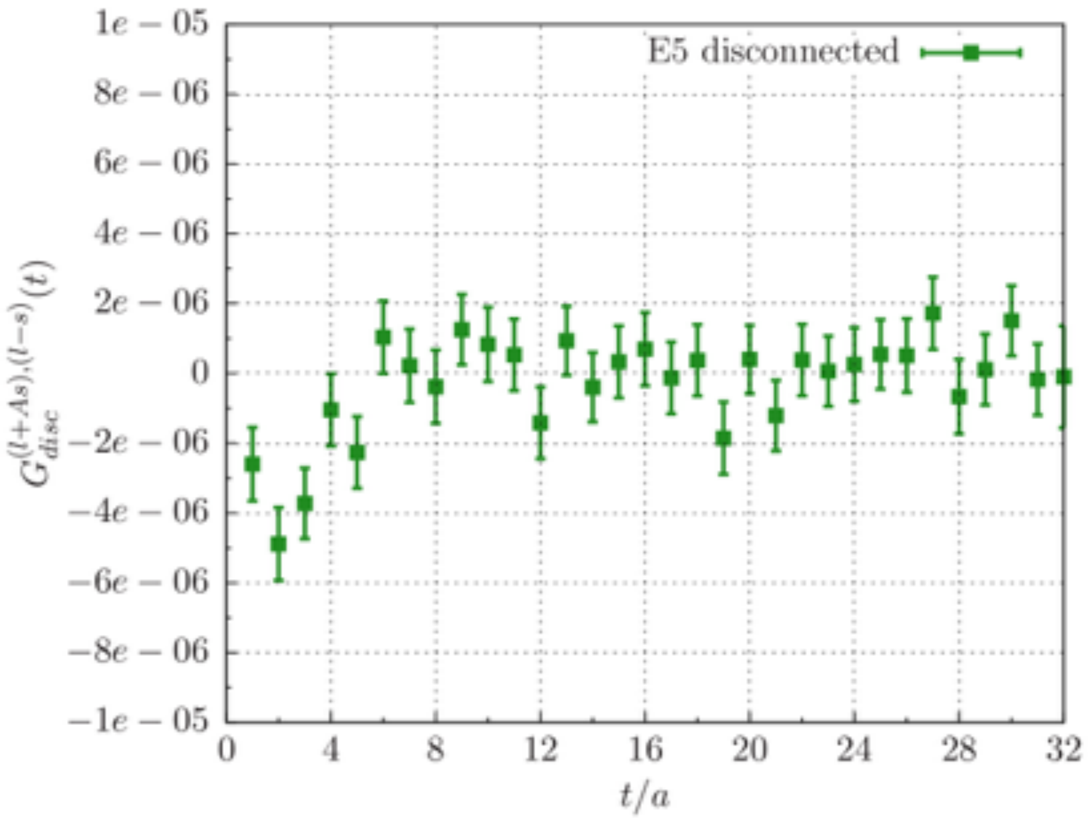
Running of $\sin^2\theta_W$

* Preliminary results:

Connected contributions:



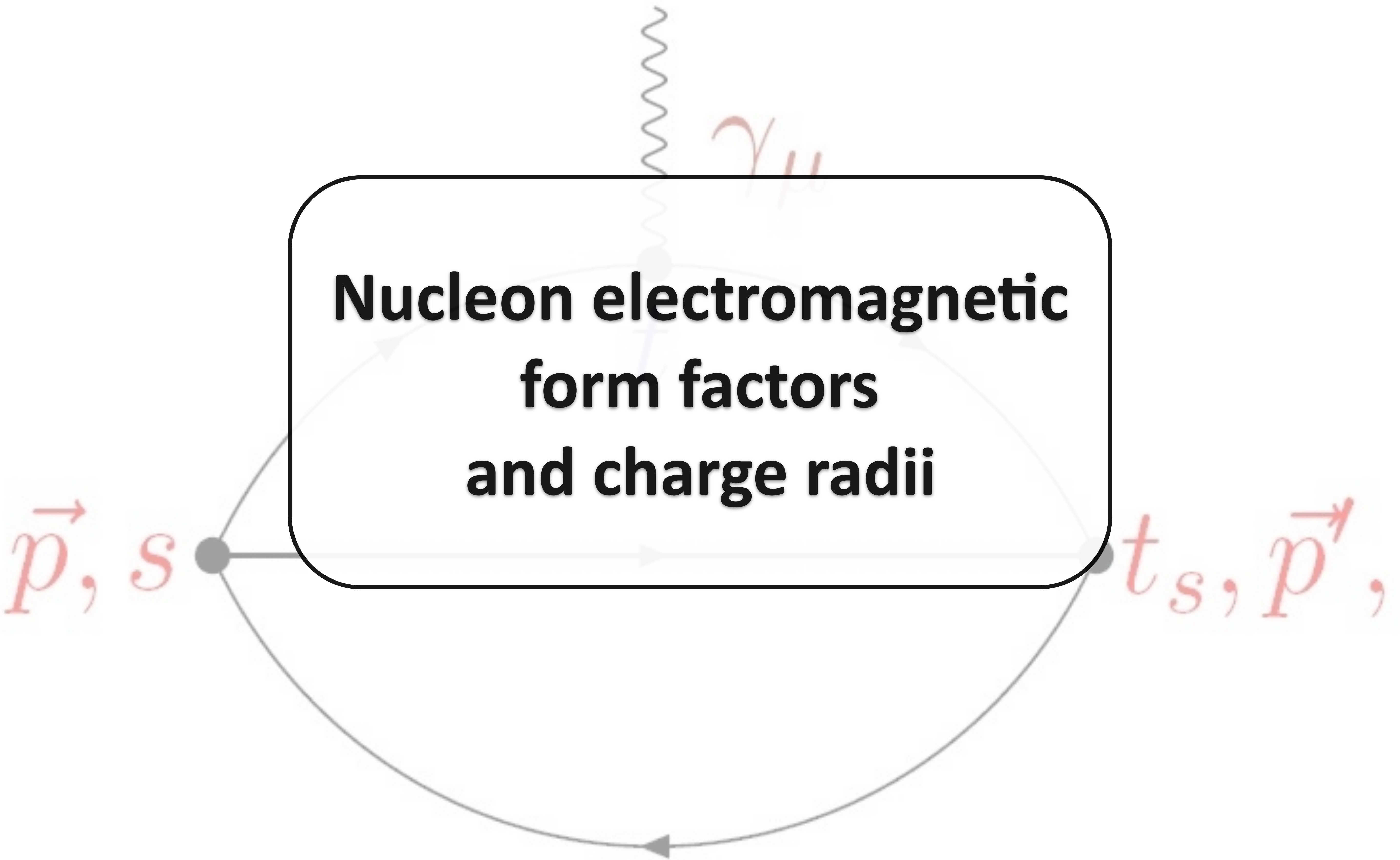
Disconnected contributions:



* Long-distance behaviour of total correlator limited by accuracy of disconnected contribution ⟶ systematic error estimate

[H. Horch, G. Herdoíza, V. Gülpers @ Lattice 2015]

Nucleon electromagnetic form factors and charge radii



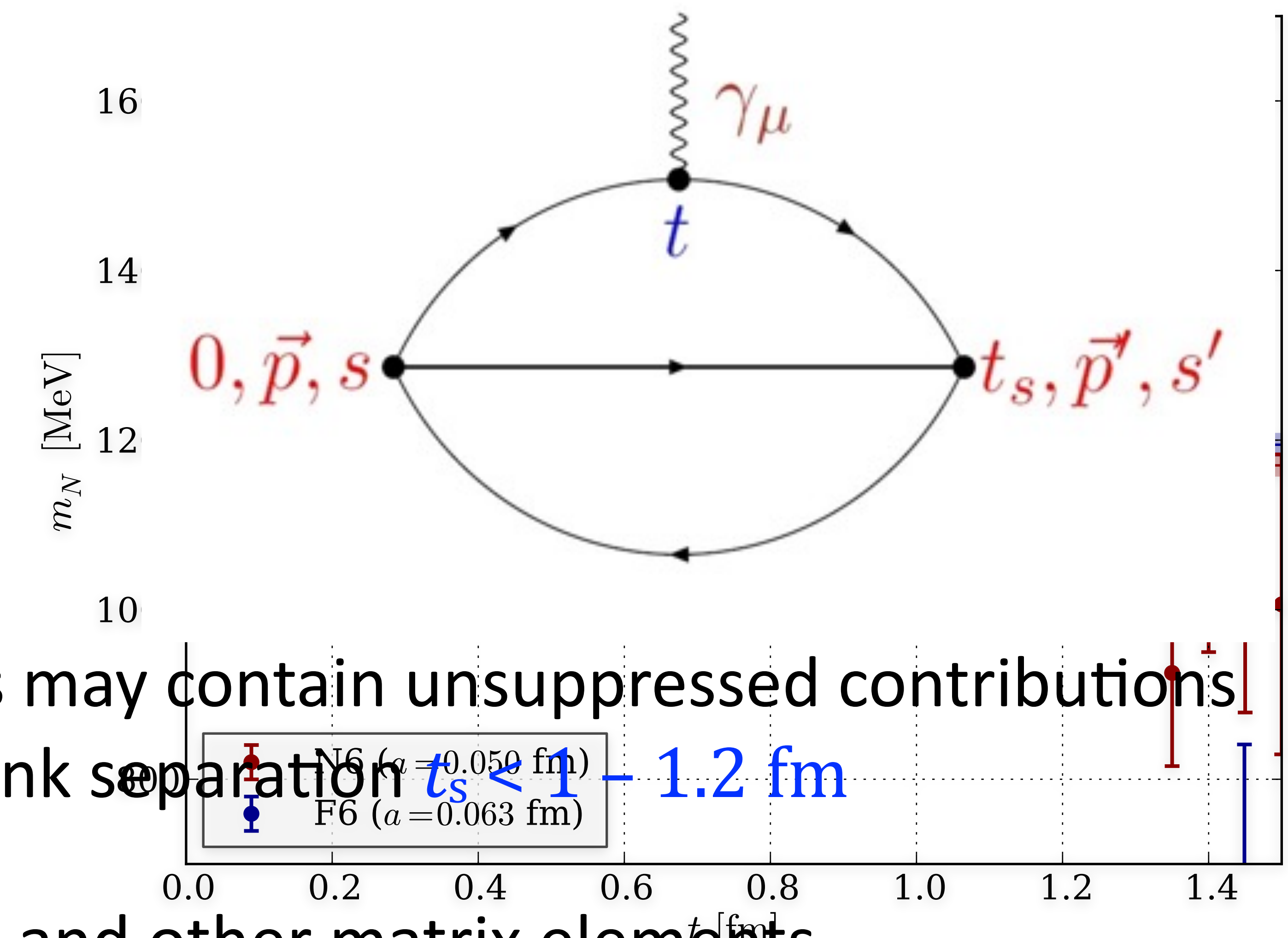
The noise problem of baryonic correlators

* Exponentially increasing noise-to-signal ratio:

$$R_{\rm NS}(x_0) \propto \exp\left\{(m_{\rm N} - \tfrac{3}{2}m_\pi)x_0\right\}$$

* Excited-state contributions die out slowly

* Ground state dominates for $x_0 \geq 0.6$ fm

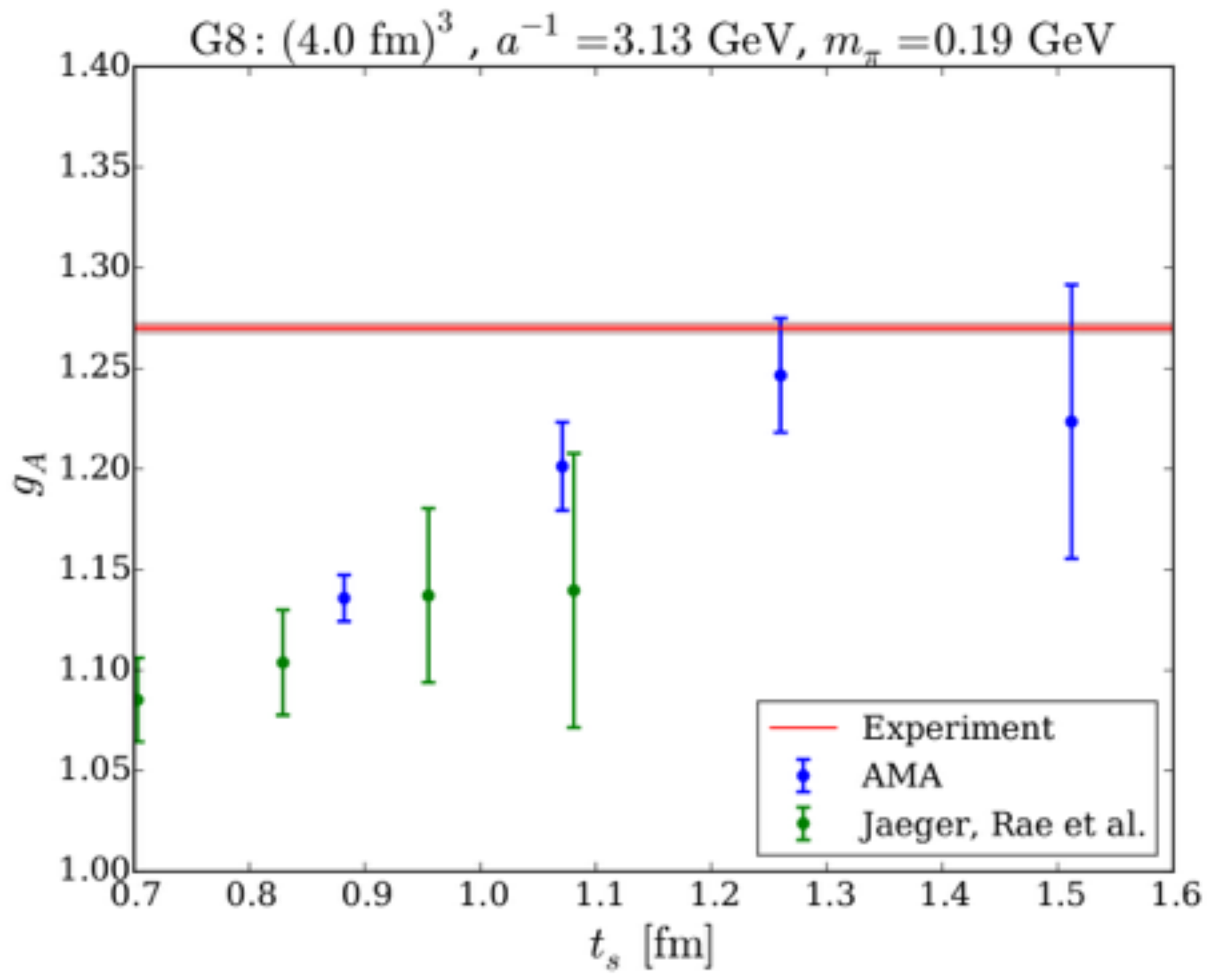


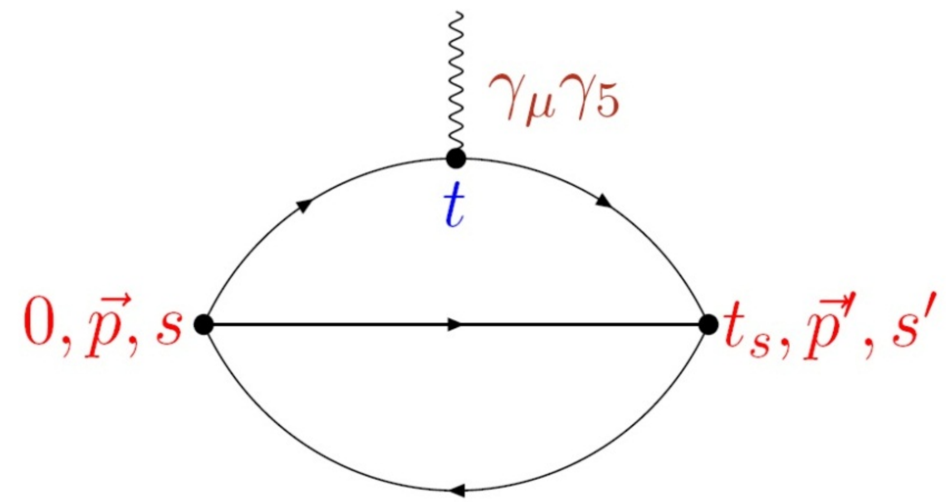
* Baryonic three-point functions may contain unsuppressed contributions from excited states if source-sink separation $t_s < 1 - 1.2$ fm

➢ Systematic bias in form factors and other matrix elements

The noise problem of baryonic correlators

- Example: lattice calculation of nucleon axial charge:





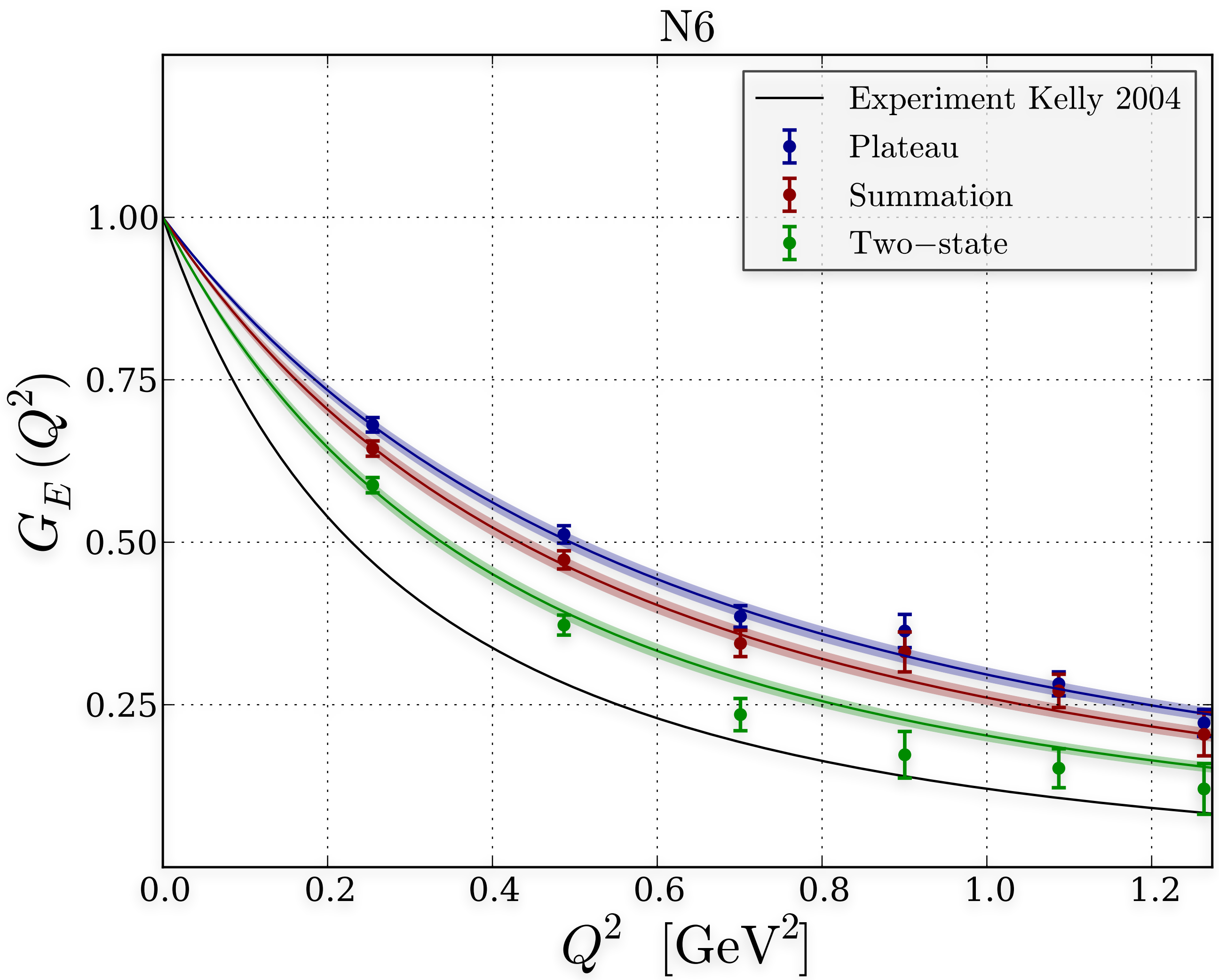
[von Hippel, Rae, Shintani, HW, arXiv:1605.00564]

- Systematic trend in the data as source-sink separation is increased

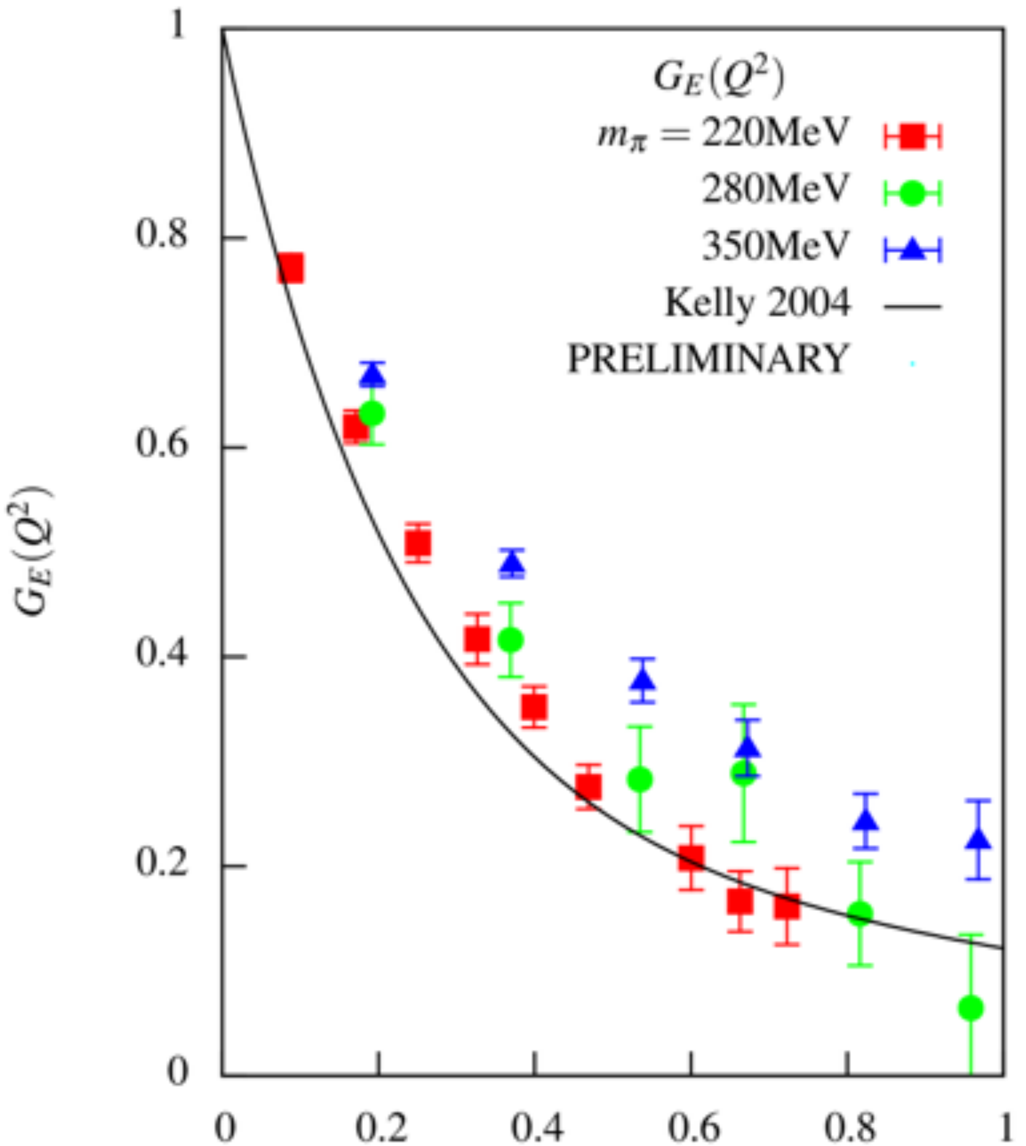
- Must employ noise reduction methods, e.g. “all-mode-averaging”
  [Blum et al, Phys Rev D88 (2013) 094503]

Controlling excited state contributions

* Mainz approach: use complementary methods to determine nucleon form factors



[Capitani et al., Phys Rev D92 (2015) 054511]

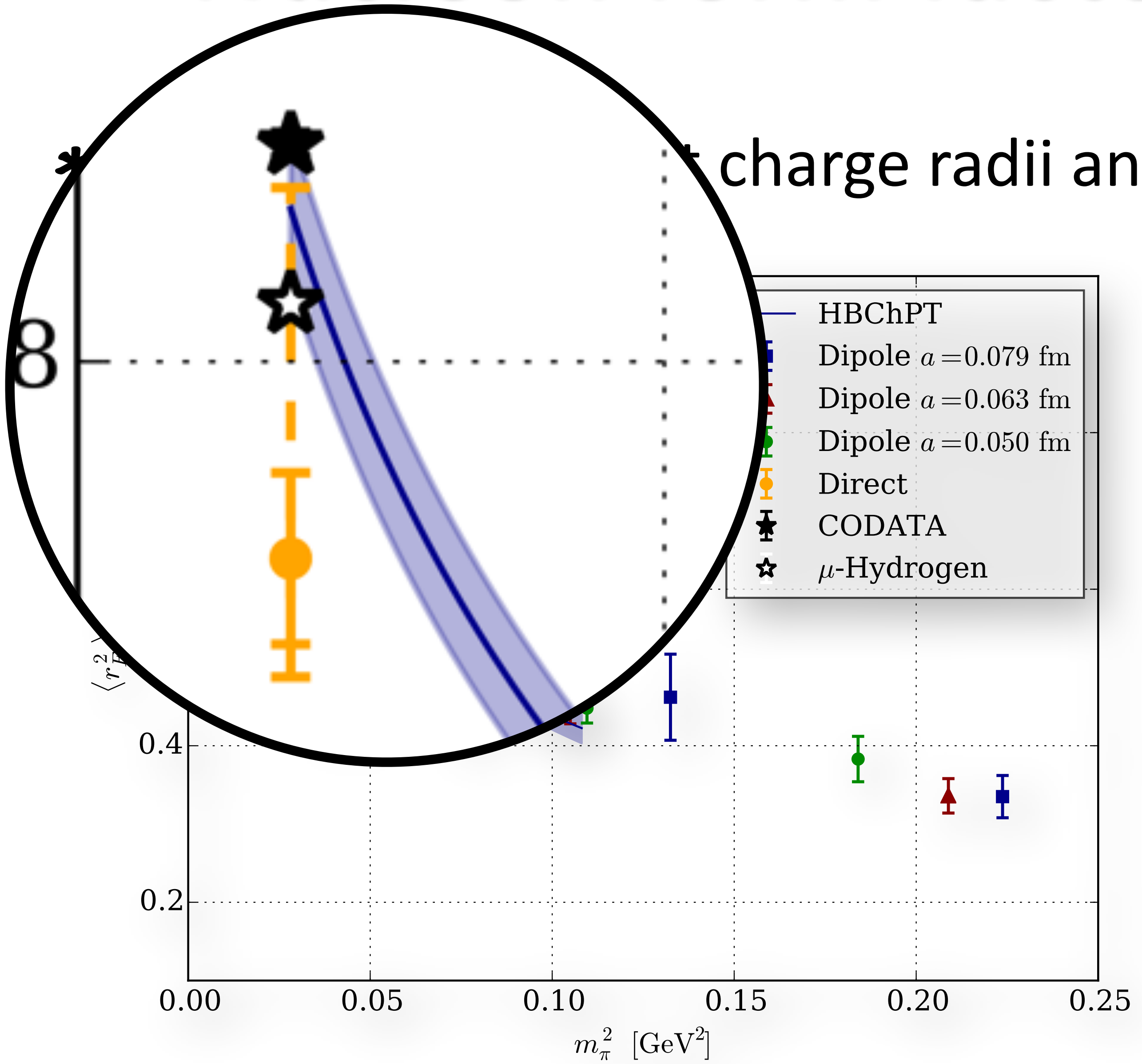


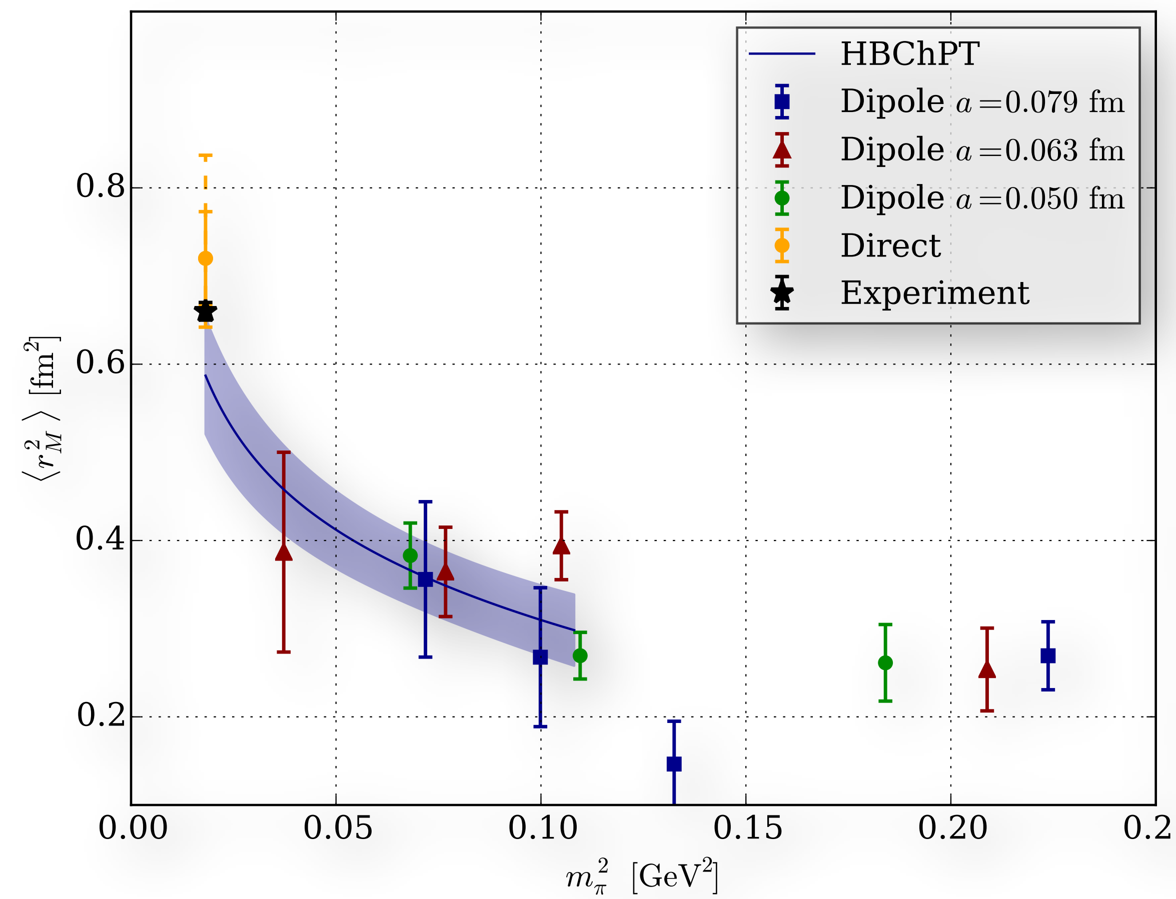
[T. Harris @ Lattice 2015]

* Chiral trend towards phenomenological parameterisation

Nucleon form factors and charge radii

charge radii and magnetic moment





$$\langle r_{\rm E}^2 \rangle = 0.722 \pm 0.034\,^{+0.088}_{-0.013}\ \mathrm{fm}^2 \qquad \langle r_{\rm M}^2 \rangle = 0.720 \pm 0.053\,^{+0.045}_{-0.025}\ \mathrm{fm}^2$$

* Full error budget — sub-percent accuracy required to match ep-scattering

$\kappa = 3.33 \pm 0.35^{+0.28}_{-0.43}$

[Capitani et al., Phys Rev D92 (2015) 054511]

Summary

Sub-percent accuracy required to have an impact on SM precision tests

Technical challenges:

- ✱ Large noise-to-signal ratio in baryonic correlation functions
- ✱ Quark-disconnected diagrams

$(g-2)_\mu$ and running of electroweak couplings:

- ✱ sub-percent accuracy achievable for $O(500k)$ lattice “measurements”

Proton radius and nucleon matrix elements:

- ✱ large statistics necessary to eliminate bias from excited states