## Heavy Higgs Sector of the Two Higgs Doublet Model in the Natural Standard Model Alignment Limit

P. S. Bhupal Dev

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## Outline

- Introduction
- Natural Alignment
- Maximally Symmetric 2HDM
- Collider Phenomenology
- Conclusion


## A Higgs or the Higgs?



Unique opportunity for probing New Physics through the Higgs portal:

- Precision Higgs Study (Higgcision).
- Search for additional Higgses.


## Why more Higgses?

Several theoretical motivations to go beyond the SM Higgs sector.

- Electroweak Baryogenesis
- Additional sources of CP violation
- Strong first order phase transition
- Dark Matter
- Supersymmetry
- Why not?


## Two Higgs Doublets

- Any scalar sector in a local $S U(2) \times U(1)$ gauge theory must be consistent with $\rho_{\text {exp }}=1.0004_{-0.0004}^{+0.0003}$. [PDG '14]
- With $n$ Higgs multiplets $\Phi_{i}($ with $i=1,2, \ldots, n)$ :

$$
\rho_{\mathrm{tree}}=\frac{\sum_{i=1}^{n}\left[T_{i}\left(T_{i}+1\right)-Y_{i}^{2}\right] v_{i}}{2 \sum_{i=1}^{n} Y_{i}^{2} v_{i}}
$$

- Simplest choice: Add multiplets with $T(T+1)=3 Y^{2}$ so that $\rho_{\text {tree }}=1$.
- SM: One $S U(2)_{L}$ doublet $\Phi=\binom{\phi^{+}}{\phi^{0}}$ with $Y=\frac{1}{2}$.
- A simple extension: Two $S U(2)_{L}$ doublets $\Phi_{i}=\binom{\phi_{i}^{+}}{\phi_{i}^{0}}$ (with $\left.i=1,2\right)$.


## General 2HDM Potential

- Most general 2HDM potential in doublet field space $\Phi_{1,2}$ :

$$
\begin{aligned}
V= & -\mu_{1}^{2}\left(\Phi_{1}^{\dagger} \Phi_{1}\right)-\mu_{2}^{2}\left(\Phi_{2}^{\dagger} \Phi_{2}\right)-\left[m_{12}^{2}\left(\Phi_{1}^{\dagger} \Phi_{2}\right)+\text { H.c. }\right] \\
& +\lambda_{1}\left(\Phi_{1}^{\dagger} \Phi_{1}\right)^{2}+\lambda_{2}\left(\Phi_{2}^{\dagger} \Phi_{2}\right)^{2}+\lambda_{3}\left(\Phi_{1}^{\dagger} \Phi_{1}\right)\left(\Phi_{2}^{\dagger} \Phi_{2}\right)+\lambda_{4}\left(\Phi_{1}^{\dagger} \Phi_{2}\right)\left(\Phi_{2}^{\dagger} \Phi_{1}\right) \\
& +\left[\frac{1}{2} \lambda_{5}\left(\Phi_{1}^{\dagger} \Phi_{2}\right)^{2}+\lambda_{6}\left(\Phi_{1}^{\dagger} \Phi_{1}\right)\left(\Phi_{1}^{\dagger} \Phi_{2}\right)+\lambda_{7}\left(\Phi_{1}^{\dagger} \Phi_{2}\right)\left(\Phi_{2}^{\dagger} \Phi_{2}\right)+\text { H.c. }\right] .
\end{aligned}
$$

- Four real mass parameters $\mu_{1,2}^{2}, \operatorname{Re}\left(m_{12}^{2}\right), \operatorname{Im}\left(m_{12}^{2}\right)$.
- 10 real quartic couplings $\lambda_{1,2,3,4}, \operatorname{Re}\left(\lambda_{5,6,7}\right), \operatorname{Im}\left(\lambda_{5,6,7}\right)$.
- Rich vacuum structure. [Battye, Brawn, Pilatsis '11; Branco et al'12]


## Higgs Spectrum in a General 2HDM

- Consider normal vacua with real vevs $v_{1,2}$, where $\sqrt{v_{1}^{2}+v_{2}^{2}}=v_{\mathrm{SM}}$ and $\tan \beta=v_{2} / v_{1}$.
- Eight real scalar fields: $\phi_{j}=\binom{\phi_{j}^{+}}{\frac{1}{\sqrt{2}}\left(v_{j}+\rho_{j}+i \eta_{j}\right)} \quad$ (with $j=1,2$ ).
- After EWSB, 3 Goldstone bosons ( $G^{ \pm}, G^{0}$ ), eaten by $W^{ \pm}$and $Z$.
- Five physical scalar fields: two $C P$-even $(h, H)$, one $C P$-odd (a) and two charged ( $h^{ \pm}$).
- In the charged sector, $\binom{G^{ \pm}}{h^{ \pm}}=\left(\begin{array}{cc}\cos \beta & \sin \beta \\ -\sin \beta & \cos \beta\end{array}\right)\binom{\phi_{1}^{ \pm}}{\phi_{2}^{ \pm}}$
- In the CP-odd sector,



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$$
M_{h^{ \pm}}^{2}=\frac{1}{s_{\beta} c_{\beta}}\left[\operatorname{Re}\left(m_{12}^{2}\right)-\frac{1}{2}\left(\left\{\lambda_{4}+\operatorname{Re}\left(\lambda_{5}\right)\right\} s_{\beta} c_{\beta}+\operatorname{Re}\left(\lambda_{6}\right) c_{\beta}^{2}+\operatorname{Re}\left(\lambda_{7}\right) s_{\beta}^{2}\right)\right] .
$$

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$$

- In the $C P$-odd sector, $\binom{G^{0}}{a}=\left(\begin{array}{cc}\cos \beta & \sin \beta \\ -\sin \beta & \cos \beta\end{array}\right)\binom{\eta_{1}}{\eta_{2}}$.

$$
\begin{aligned}
M_{a}^{2} & =\frac{1}{s_{\beta} c_{\beta}}\left[\operatorname{Re}\left(m_{12}^{2}\right)-v^{2}\left(\operatorname{Re}\left(\lambda_{5}\right) s_{\beta} c_{\beta}+\frac{1}{2}\left\{\operatorname{Re}\left(\lambda_{6}\right) c_{\beta}^{2}+\operatorname{Re}\left(\lambda_{7}\right) s_{\beta}^{2}\right\}\right)\right] \\
& =M_{h^{ \pm}}^{2}+\frac{1}{2}\left[\lambda_{4}-\operatorname{Re}\left(\lambda_{5}\right)\right] v^{2}
\end{aligned}
$$

## Higgs Spectrum in a General 2HDM

- In the $C P$-even sector, $\binom{H}{h}=\left(\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right)\binom{\rho_{1}}{\rho_{2}}$

$$
\begin{aligned}
M_{S}^{2} \equiv & \left(\begin{array}{cc}
A & C \\
C & B
\end{array}\right) \\
= & M_{a}^{2}\left(\begin{array}{cc}
s_{\beta}^{2} & -s_{\beta} c_{\beta} \\
-s_{\beta} c_{\beta} & c_{\beta}^{2}
\end{array}\right) \\
& +v^{2}\left(\begin{array}{cc}
2 \lambda_{1} c_{\beta}^{2}+\operatorname{Re}\left(\lambda_{5}\right) s_{\beta}^{2}+2 \operatorname{Re}\left(\lambda_{6}\right) s_{\beta} c_{\beta} & \lambda_{34} s_{\beta} c_{\beta}+\operatorname{Re}\left(\lambda_{6}\right) c_{\beta}^{2}+\operatorname{Re}\left(\lambda_{7}\right) s_{\beta}^{2} \\
\lambda_{34} s_{\beta} c_{\beta}+\operatorname{Re}\left(\lambda_{6}\right) c_{\beta}^{2}+\operatorname{Re}\left(\lambda_{7}\right) s_{\beta}^{2} & 2 \lambda_{2} s_{\beta}^{2}+\operatorname{Re}\left(\lambda_{5}\right) c_{\beta}^{2}+2 \operatorname{Re}\left(\lambda_{7}\right) s_{\beta} c_{\beta}
\end{array}\right)
\end{aligned}
$$

with $\tan 2 \alpha=2 C /(A-B)$.

- The SM Higgs boson is given by
- SM alignment limit: $\alpha \rightarrow \beta$ (or $\beta-\pi / 2$ ).
- Usually attributed to either decoupling or accidental cancellations.
- Explore symmetries of the 2HDM potential to naturally justify the alignment limit.


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H_{\mathrm{SM}}=\rho_{1} \cos \beta+\rho_{2} \sin \beta=H \cos (\beta-\alpha)+h \sin (\beta-\alpha) .
$$

- SM alignment limit: $\alpha \rightarrow \beta$ (or $\beta-\pi / 2$ ).
- Usually attributed to either decoupling or accidental cancellations. [Gunion, Haber '03; Ginzburg, Krawczyk '05; Carena, Low, Shah, Wagner '13]
- Explore symmetries of the 2HDM potential to naturally justify the alignment limit.


## Natural Alignment Condition

- Rewrite $C P$-even mass matrix as

$$
\begin{aligned}
M_{S}^{2} & =\left(\begin{array}{cc}
c_{\beta} & -s_{\beta} \\
s_{\beta} & c_{\beta}
\end{array}\right)\left(\begin{array}{cc}
\widehat{A} v^{2} & \widehat{C} v^{2} \\
\widehat{C} v^{2} & M_{a}^{2}+\widehat{B} v^{2}
\end{array}\right)\left(\begin{array}{cc}
c_{\beta} & s_{\beta} \\
-s_{\beta} & c_{\beta}
\end{array}\right) \equiv O \widehat{M}_{S}^{2} O^{\top} \\
\widehat{A} & =2\left[c_{\beta}^{4} \lambda_{1}+s_{\beta}^{2} c_{\beta}^{2} \lambda_{345}+s_{\beta}^{4} \lambda_{2}+2 s_{\beta} c_{\beta}\left(c_{\beta}^{2} \lambda_{6}+s_{\beta}^{2} \lambda_{7}\right)\right] \\
\widehat{B} & =\lambda_{5}+2\left[s_{\beta}^{2} c_{\beta}^{2}\left(\lambda_{1}+\lambda_{2}-\lambda_{345}\right)-s_{\beta} c_{\beta}\left(c_{\beta}^{2}-s_{\beta}^{2}\right)\left(\lambda_{6}-\lambda_{7}\right)\right] \\
\widehat{C} & =s_{\beta}^{3} c_{\beta}\left(2 \lambda_{2}-\lambda_{345}\right)-c_{\beta}^{3} s_{\beta}\left(2 \lambda_{1}-\lambda_{345}\right)+c_{\beta}^{2}\left(1-4 s_{\beta}^{2}\right) \lambda_{6}+s_{\beta}^{2}\left(4 c_{\beta}^{2}-1\right) \lambda_{7}
\end{aligned}
$$

- Exact alignment $(\alpha=\beta)$ iff $\widehat{C}=0$, i.e.

$$
\lambda_{7} t_{\beta}^{4}-\left(2 \lambda_{2}-\lambda_{345}\right) t_{\beta}^{3}+3\left(\lambda_{6}-\lambda_{7}\right) t_{\beta}^{2}+\left(2 \lambda_{1}-\lambda_{345}\right) t_{\beta}-\lambda_{6}=0
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- Natural alignment if happens for any value of $\tan \beta$, independent of non-SM Higgs spectra:
- CP-even Higgs masses are given by


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$$
\lambda_{1}=\lambda_{2}=\lambda_{345} / 2, \quad \lambda_{6}=\lambda_{7}=0
$$

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$$
\begin{aligned}
M_{H}^{2} & =2 \lambda_{2} v^{2} \equiv \lambda_{\mathrm{SM}} v^{2} \\
M_{h}^{2} & =M_{a}^{2}+\lambda_{5} v^{2}
\end{aligned}
$$

## Higgs Couplings in a General 2HDM

- With respect to the SM Higgs couplings $H_{S M} V V\left(V=W^{ \pm}, Z\right)$,

$$
g_{h V V}=\sin (\beta-\alpha), \quad g_{H V V}=\cos (\beta-\alpha) .
$$



- Similar behavior for CP-even Higgs self-couplings:

$$
g_{h H H} \propto \sin (\beta-\alpha), \quad g_{H h h} \propto \cos (\beta-\alpha)
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- In the alignment limit $(\alpha \rightarrow \beta)$, one of the neutral Higgses $(h)$ is gaugephobic.
- Only effective probe is through its Yukawa coupling to SM fermions.


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## $\mathrm{Z}_{2}$-symmetric 2 HDM

- General Yukawa Lagrangian

$$
-\mathcal{L}_{Y}=\bar{Q}_{L}\left(h_{1}^{u} \Phi_{1}+h_{2}^{u} \Phi_{2}\right) u_{R}+\bar{Q}_{L}\left(h_{1}^{d} \widetilde{\Phi}_{1}+h_{2}^{d} \widetilde{\Phi}_{2}\right) d_{R}+\bar{L}_{L}\left(h_{1}^{e} \widetilde{\Phi}_{1}+h_{2}^{e} \widetilde{\Phi}_{2}\right) e_{R}
$$

- Dangerous FCNC processes at tree-level.
- Can be naturally avoided by imposing a $Z_{2}$-symmetry. [Glashow, Weinberg '58]

|  |  | $Z_{2}$ charge |  |  |  |  |  | Coupling |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Phi_{1}$ | $\Phi_{2}$ | $Q_{L}$ | $L_{L}$ | $u_{R}$ | $d_{R}$ | $e_{R}$ | $u_{R}$ | $d_{R}$ | $e_{R}$ |
| Type-I | + | - | + | + | - | - | - | $\Phi_{2}$ | $\Phi_{2}$ | $\Phi_{2}$ |
| Type-II (MSSM-type) | + | - | + | + | - | + | + | $\Phi_{2}$ | $\Phi_{1}$ | $\Phi_{1}$ |
| Type-X (Lepton-specific) | + | - | + | + | - | - | + | $\Phi_{2}$ | $\Phi_{2}$ | $\Phi_{1}$ |
| Type-Y (Flipped) | + | - | + | + | - | + | - | $\Phi_{2}$ | $\Phi_{1}$ | $\Phi_{2}$ |

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## Symmetry Classifications of the 2HDM Potential

- Three classes of accidental symmetries of the 2HDM potential:
- Higgs Family (HF) Symmetries involving transformations of $\Phi_{1,2}$ only (but not $\Phi_{1,2}^{*}$ ), e.g. $\mathrm{Z}_{2}$ [Glashow, Weinberg '58], U(1) $\mathrm{PQ}^{[P e c c e i, ~ Q u i n n ~ ' 77], ~} \mathrm{SO}(3)_{\mathrm{HF}}$ [Deshpande, Ma '78; Ivanov '07; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
- CP Symmetries relating $\Phi_{1,2}$ to $\Phi_{1,2}^{*}$, e.g. $\Phi_{1(2)} \rightarrow \Phi_{1(2)}^{*}$ (CP1) [Lee '73; Branco '80], $\Phi_{1(2)} \rightarrow(-) \Phi_{2(1)}^{*}$ (CP2) [Davidson, Haber '05], CP1 combined with $\mathrm{SO}(2)_{\mathrm{HF}} / \mathrm{Z}_{2}$ (CP3) [Ivanov '07; Ferreira, Haber, Silva '09; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
- Additional mixed HF and CP symmetries that leave the gauge-kinetic terms of $\Phi_{1,2}$ invariant [Battye, Brawn, Pilaftsis '11].
- Includes all custodial symmetries of the 2HDM potential.
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- Maximum of 13 distinct accidental symmetries of the general 2HDM potential.
- Each of them imposes specific relations among the scalar parameters.


## Bilinear Formalism

- Introduce an 8-dimensional complex multiplet: [Battye, Brawn, Pilaftsis '11; Nishi '11; Pilaftsis '12]

$$
\boldsymbol{\Phi}=\left(\begin{array}{c}
\Phi_{1} \\
\Phi_{2} \\
i \sigma^{2} \Phi_{1}^{*} \\
i \sigma^{2} \Phi_{2}^{*}
\end{array}\right)
$$

- $\boldsymbol{\Phi}$ satisfies the Majorana condition: $\boldsymbol{\Phi}=\boldsymbol{C} \boldsymbol{\Phi}^{*}$, where $\boldsymbol{C}=\sigma^{2} \otimes \sigma^{0} \otimes \sigma^{2}=C^{-1}=C^{*}$.
- Define a null 6-dimensional Lorentz vector bilinear in $\Phi$ :
(with $A=0,1,2,3,4,5$ ), where



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- Define a null 6-dimensional Lorentz vector bilinear in $\Phi$ :

$$
R^{A}=\Phi^{\dagger} \Sigma^{A} \Phi
$$

(with $A=0,1,2,3,4,5$ ), where

$$
\begin{array}{lll}
\Sigma^{0}=\frac{1}{2} \sigma^{0} \otimes \sigma^{0} \otimes \sigma^{0} \equiv \frac{1}{2} \mathbf{1}_{8}, & \Sigma^{1}=\frac{1}{2} \sigma^{0} \otimes \sigma^{1} \otimes \sigma^{0}, & \Sigma^{2}=\frac{1}{2} \sigma^{3} \otimes \sigma^{2} \otimes \sigma^{0}, \\
\Sigma^{3}=\frac{1}{2} \sigma^{0} \otimes \sigma^{3} \otimes \sigma^{0}, & \Sigma^{4}=-\frac{1}{2} \sigma^{2} \otimes \sigma^{2} \otimes \sigma^{0}, & \Sigma^{5}=-\frac{1}{2} \sigma^{1} \otimes \sigma^{2} \otimes \sigma^{0} .
\end{array}
$$

## 2HDM Potential in Bilinear Field Space

- The general 2HDM potential takes a simple form:

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V=-\frac{1}{2} M_{A} R^{A}+\frac{1}{4} L_{A B} R^{A} R^{B} .
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\begin{aligned}
& V=-\frac{1}{2} M_{A} R^{A}+\frac{1}{4} L_{A B} R^{A} R^{B} . \\
& M=\left(\mu_{1}^{2}+\mu_{2}^{2}, \quad 2 \operatorname{Re}\left(m_{12}^{2}\right), \quad-2 \operatorname{Im}\left(m_{12}^{2}\right), \quad \mu_{1}^{2}-\mu_{2}^{2}, \quad 0, \quad 0\right), \\
& R=\left(\begin{array}{c}
\Phi_{1}^{\dagger} \Phi_{1}+\Phi_{2}^{\dagger} \Phi_{2} \\
\Phi_{1}^{\dagger} \Phi_{2}+\Phi_{2}^{\dagger} \Phi_{1} \\
-i\left(\Phi_{1}^{\dagger} \Phi_{2}-\Phi_{2}^{\dagger} \Phi_{1}\right) \\
\Phi_{1}^{\dagger} \Phi_{1}-\Phi_{2}^{\dagger} \Phi_{2} \\
\Phi_{1}^{\top} i \sigma^{2} \Phi_{2}-\Phi_{2}^{\dagger} i \sigma^{2} \Phi_{1}^{*} \\
-i\left(\Phi_{1}^{\top} i \sigma^{2} \Phi_{2}+\Phi_{2}^{\dagger} i \sigma^{2} \Phi_{1}^{*}\right)
\end{array}\right), \\
& L=\left(\begin{array}{cccccc}
\lambda_{1}+\lambda_{2}+\lambda_{3} & \operatorname{Re}\left(\lambda_{6}+\lambda_{7}\right) & -\operatorname{Im}\left(\lambda_{6}+\lambda_{7}\right) & \lambda_{1}-\lambda_{2} & 0 & 0 \\
\operatorname{Re}\left(\lambda_{6}+\lambda_{7}\right) & \lambda_{4}+\operatorname{Re}\left(\lambda_{5}\right) & -\operatorname{Im}\left(\lambda_{5}\right) & \operatorname{Re}\left(\lambda_{6}-\lambda_{7}\right) & 0 & 0 \\
-\operatorname{Im}\left(\lambda_{6}+\lambda_{7}\right) & -\operatorname{Im}\left(\lambda_{5}\right) & \lambda_{4}-\operatorname{Re}\left(\lambda_{5}\right) & -\operatorname{Im}\left(\lambda_{6}-\lambda_{7}\right) & 0 & 0 \\
\lambda_{1}-\lambda_{2} & \operatorname{Re}\left(\lambda_{6}-\lambda_{7}\right) & -\operatorname{Im}\left(\lambda_{6}-\lambda_{7}\right) & \lambda_{1}+\lambda_{2}-\lambda_{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{aligned}
$$

## 13 Symmetries of the 2HDM Potential

## [Pilaftsis '12]

Table 1
Parameter relations for the 13 accidental symmetries [1] related to the $\mathrm{U}(1)_{Y}$-invariant 2 HDM potential in the diagonally reduced basis, where $\operatorname{Im} \lambda_{5}=0$ and $\lambda_{6}=\lambda_{7}$. A dash signifies the absence of a constraint.

| No. | Symmetry | $\mu_{1}^{2}$ | $\mu_{2}^{2}$ | $m_{12}^{2}$ | $\lambda_{1}$ | $\lambda_{2}$ | $\lambda_{3}$ | $\lambda_{4}$ | $\operatorname{Re} \lambda_{5}$ | $\lambda_{6}=\lambda_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $Z_{2} \times \mathrm{O}(2)$ | - | - | Real | - | - | - | - | - | Real |
| 2 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(2)$ | - | - | 0 | - | - | - | - | - | 0 |
| 3 | $\left(Z_{2}\right)^{3} \times \mathrm{O}(2)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | - | - | 0 |
| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | - | - | 0 | - | - | - | - | 0 | 0 |
| 5 | $Z_{2} \times[\mathrm{O}(2)]^{2}$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | - | $2 \lambda_{1}-\lambda_{34}$ | 0 |
| 6 | $\mathrm{O}(3) \times \mathrm{O}(2)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | $2 \lambda_{1}-\lambda_{3}$ | 0 | 0 |
| 7 | SO(3) | - | - | Real | - | - | - | - | $\lambda .4$ | Real |
| 8 | $Z_{2} \times \mathrm{O}(3)$ | - | $\mu_{1}^{2}$ | Real | - | $\lambda_{1}$ | - | - | $\lambda_{4}$ | Real |
| 9 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(3)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | - | $\pm \lambda_{4}$ | 0 |
| 10 | $\mathrm{O}(2) \times \mathrm{O}(3)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | $2 \lambda_{1}$ | - | 0 | 0 |
| 11 | $\mathrm{SO}(4)$ | - | - | 0 | - | - | - | 0 | 0 | 0 |
| 12 | $Z_{2} \times \mathrm{O}(4)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | 0 | 0 | 0 |
| 13 | $\mathrm{SO}(5)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | $2 \lambda_{1}$ | 0 | 0 | 0 |

- Recall natural alignment condition $\lambda_{1}=\lambda_{2}=\lambda_{345} / 2, \lambda_{6}=\lambda_{7}=0$.
- Only three symmetries satisfy this.


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| 2 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(2)$ | - | - | 0 | - | - | - | - | - | 0 |
| 3 | $\left(Z_{2}\right)^{3} \times \mathrm{O}(2)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | - | - | 0 |
| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | - | - | 0 | - | - | - | - | 0 | 0 |
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| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | - | - | 0 | - | - | - | - | 0 | 0 |
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| 7 | SO(3) | - | - | Real | - | - | - | - | $\lambda_{4}$ | Real |
| 8 | $Z_{2} \times 0$ (3) | - | $\mu_{1}^{2}$ | Real | - | $\lambda_{1}$ | - | - | $\lambda_{4}$ | Real |
| 9 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(3)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | - | $\pm \lambda_{4}$ | 0 |
| 10 | $\mathrm{O}(2) \times \mathrm{O}(3)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | $2 \lambda_{1}$ | - | 0 | 0 |
| 11 | SO(4) | - | - | 0 | - | - | - | 0 | 0 | 0 |
| 12 | $Z_{2} \times \mathrm{O}(4)$ | - | $\mu_{1}^{2}$ | 0 | - | $\lambda_{1}$ | - | 0 | 0 | 0 |
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## Maximal Symmetry Group

- Maximal symmetry group in the bilinear field space: $G_{2 \mathrm{HDM}}^{R}=\mathrm{SO}(5)$.
- In the original $\Phi$-field space, $\mathrm{G}_{2 \mathrm{HDM}}^{\boldsymbol{\Phi}}=\left(\mathrm{Sp}(4) / \mathrm{Z}_{2}\right) \otimes \mathrm{SU}(2)_{L}$.
- Conjecture: In a general $n H D M, G_{n H D M}^{\Phi}=\left(\operatorname{Sp}(2 n) / Z_{2}\right) \otimes \operatorname{SU}(2)_{L}$.
- For the SM , reproduces the well-known custodial symmetry $\mathrm{G}_{\mathrm{SM}}^{\Phi}=\left(\mathrm{SU}(2)_{C} / \mathrm{Z}_{2}\right) \otimes \mathrm{SU}(2)_{L}$. [Sikivie, Susskind, Voloshin, Zakharov '80].
- In 2HDM, 3 different realizations of custodial symmetry with [Pilaftsis '12; BD, Pilaftsis '14]

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- Conjecture: In a general $n H D M, G_{n H D M}^{\Phi}=\left(S p(2 n) / Z_{2}\right) \otimes \operatorname{SU}(2)_{L}$.
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- In 2HDM, 3 different realizations of custodial symmetry with [Pilaftsis '12; BD, Pilaftsis '14]

$$
\begin{align*}
& h_{1}^{u}=e^{i \theta} h_{1}^{d} \text { and } h_{2}^{u}=e^{i \theta} h_{2}^{d}  \tag{i}\\
& h_{1}^{u}=e^{i \theta} h_{1}^{d} \text { and } h_{2}^{u}=-e^{i \theta} h_{2}^{d}  \tag{ii}\\
& h_{1}^{u}=e^{i \theta} h_{2}^{d} \quad \text { and } h_{2}^{u}=e^{-i \theta} h_{1}^{d} \tag{iii}
\end{align*}
$$

- Equivalent only in the $\mathrm{SO}(5)$ limit.


## Maximally Symmetric 2HDM

- In the SO(5) limit, the 2HDM potential is very simple:

$$
V=-\mu^{2}\left(\left|\Phi_{1}\right|^{2}+\left|\Phi_{2}\right|^{2}\right)+\lambda\left(\left|\Phi_{1}\right|^{2}+\left|\Phi_{2}\right|^{2}\right)^{2}=-\frac{\mu^{2}}{2} \boldsymbol{\Phi}^{\dagger} \boldsymbol{\Phi}+\frac{\lambda}{4}\left(\boldsymbol{\Phi}^{\dagger} \boldsymbol{\Phi}\right)^{2}
$$

- More minimal than the MSSM scalar potential, which in the custodial limit $g^{\prime} \rightarrow 0$, has a smaller symmetry: $\mathrm{O}(2) \otimes \mathrm{O}(3) \subset \mathrm{SO}(5)$.
- After EWSB in the MS-2HDM, one massive Higgs boson $H$ with $M_{H}^{2}=2 \lambda_{2} v^{2}$, whilst remaining four ( $h, a$ and $h^{ \pm}$) are massless [Goldstone theorem].
- Natural SM alignment limit with $\alpha=\beta$. [Recall $\left.H_{\text {SM }}=H \cos (\beta-\alpha)+h \sin (\beta-\alpha)\right]$
- (Pseudo)-Goldstones in MS-2HDM acquire mass due to custodial symmetry-breaking $g^{\prime}$ and Yukawa coupling effects.


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- (Pseudo)-Goldstones in MS-2HDM acquire mass due to custodial symmetry-breaking $g^{\prime}$ and Yukawa coupling effects.


## $g^{\prime}$ and Yukawa Coupling Effects

- Custodial symmetry broken by non-zero $g^{\prime}$ and Yukawa couplings.

$$
\begin{aligned}
& \mathrm{SO}(5) \otimes \mathrm{SU}(2)_{L} \xrightarrow{g^{\prime} \neq 0} \mathrm{O}(3) \otimes \mathrm{O}(2) \otimes \mathrm{SU}(2)_{L} \sim \mathrm{O}(3) \otimes \mathrm{U}(1)_{Y} \otimes \mathrm{SU}(2)_{L} \\
& \xrightarrow{\text { Yukawa }} \mathrm{O}(2) \otimes \mathrm{U}(1)_{Y} \otimes \mathrm{SU}(2)_{L} \sim \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_{Y} \otimes \mathrm{SU}(2)_{L} \\
& \xrightarrow{\left\langle\Phi_{1,2}\right\rangle \neq 0} \\
& \mathrm{U}(1)_{\mathrm{em}} .
\end{aligned}
$$

- Assume $\mathrm{SO}(5)$-symmetry scale $\mu_{X} \gg v$.
- Use two-loop RGEs to find the mass spectrum at weak scale.



## Soft Breaking Effects



- In the $S O(5)$ limit for quartic couplings,

$$
M_{H}^{2}=2 \lambda_{2} v^{2}, \quad M_{h}^{2}=M_{a}^{2}=M_{h \pm}^{2}=\frac{\operatorname{Re}\left(m_{12}^{2}\right)}{s_{\beta} c_{\beta}}
$$

- Still preserves natural alignment, irrespective of other 2HDM parameters.
- Predicts a quasi-degenerate heavy Higgs sector.


## Quartic Coupling Unification



## Constraints from Global Fit

- Electroweak precision observables.
- LHC signal strengths of the light $C P$-even Higgs boson.
- Limits on heavy $C P$-even scalar from $H \rightarrow W W, Z Z, \tau \tau$ searches.
- Flavor observables such as $B_{s}$ mixing and $B \rightarrow X_{s} \gamma$.
- Stability of the potential:

$$
\lambda_{1,2}>0, \quad \lambda_{3}+\sqrt{\lambda_{1} \lambda_{2}}>0, \quad \lambda_{3}+\lambda_{4}+\sqrt{\lambda_{1} \lambda_{2}}-\operatorname{Re}\left(\lambda_{5}\right)>0 .
$$

- Perturbativity of the Higgs self-couplings: $\left\|S_{\Phi \Phi \rightarrow \Phi \Phi}\right\|<\frac{1}{8}$.




## Misalignment Predictions in MS-2HDM



## Misalignment Predictions in MS-2HDM



## Misalignment Predictions in MS-2HDM



## Misalignment Predictions in MS-2HDM



## Lower Limit on Charged Higgs Mass



## Lower and Upper Limits on Charged Higgs Mass



## Implications for the LHC Searches

- Recall that $g_{h V V}=\sin (\beta-\alpha), \quad g_{H V V}=\cos (\beta-\alpha)$.
- In the alignment limit $\alpha \rightarrow \beta, H$ is SM-like and the heavy Higgs $h$ is gaugephobic.
- Dominant production modes at the LHC: ggF and associated production with $t \bar{t}$.

Higgs production processes:


## Branching Fractions



## Branching Fractions



## Branching Fractions



## $\tan \beta$ Dependance



## LHC Searches so far

- Existing collider limits on the heavy Higgs sector derived from $W W$ and $Z Z$ modes are not applicable in the alignment limit.
- Limits from $g g \rightarrow h \rightarrow \tau^{+} \tau^{-}$and $g g \rightarrow b \bar{b} h \rightarrow b \bar{b} \tau^{+} \tau^{-}$are easily satisfied.
- Similarly for $h \rightarrow H H \rightarrow \gamma \gamma b b$.
- In the charged-Higgs sector, most of the searches focus on the low-mass regime $\left(M_{h^{ \pm}}<M_{t}\right): p p \rightarrow t t \rightarrow W b b h^{+}, h^{+} \rightarrow c s$.
- Recently, the search was extended beyond the top-threshold: [CMS-PAS-HIG-13-026]

$$
g g \rightarrow h^{+} t b \rightarrow(\ell \nu b b)\left(\ell^{\prime} \nu b\right) b
$$



## Predictions in the MS-2HDM



## Simulations for $\sqrt{s}=14$ TeV LHC

- Used MadGraph5_aMC@NLO.
- Event reconstruction using the CMS cuts:

$$
\begin{array}{ll}
p_{T}^{\ell}>20 \mathrm{GeV}, & \left|\eta^{\ell}\right|<2.5, \quad \Delta R^{\ell \ell}>0.4 \\
M_{\ell \ell}>12 \mathrm{GeV}, & \left|M_{\ell \ell}-M_{Z}\right|>10 \mathrm{GeV} \\
p_{T}^{j}>30 \mathrm{GeV}, & \left|\eta^{j}\right|<2.4, \quad \not \mathbb{E}_{T}>40 \mathrm{GeV}
\end{array}
$$

- For charged Higgs mass reconstruction, used 'stransverse mass' variable [Lester, Summers '99]

$$
M_{T 2}=\min _{\left\{\boldsymbol{p}_{T_{1}}+\boldsymbol{p}_{T_{2}}=\boldsymbol{p}_{T}\right\}}\left[\max \left\{m_{T_{1}}, m_{T_{2}}\right\}\right] .
$$



## Mass Reconstruction using $M_{T 2}$



## Reach at 14 TeV LHC



## New Signal in the Neutral Higgs Sector

$$
g g \rightarrow t \bar{t} h \rightarrow t \bar{t} t \bar{t}
$$

- Existing $95 \%$ CL experimental upper limit on $\sigma_{t \bar{t} t \bar{t}}$ is 32 fb (CMS).
- SM prediction for $\sigma(p p \rightarrow t \bar{t} t \bar{t}+X) \simeq 10-15 \mathrm{fb}$ at NLO. [Bevilacqua, Worek '12]
- Still lot of room for BSM contribution.




## Towards a Full Analysis of the $t \bar{t} t \bar{t}$ Signal

35 final states, grouped into five channels:

- Fully hadronic: 12 jets, with $4 b$-jets.
- Mostly hadronic: 6 light jets, $4 b$-jets, one charged lepton and $\mathbb{E}_{T}$.
- Semi-leptonic/hadronic: 4 light jets, $4 b$-jets, 2 charged leptons and $\mathbb{E}_{T}$.
- Mostly leptonic: 2 light jets, $4 b$-jets, 3 charged leptons and $\mathbb{E}_{T}$.
- Fully leptonic: $4 b$-jets, 4 charged leptons and $\mathbb{E}_{T}$.


Figure 1.4: Branching fractions for the different decays of the four top quarks, depending on whether the $W$ boson decays hadronically $(h)$ or leptonically $(\ell)$.

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## Mass Reconstruction using $M_{T 2}$



## $p p \rightarrow t \bar{t} h \rightarrow t \bar{t} t \bar{t}$ Signal



## Conclusions

- Examined the SM alignment limit of the 2HDM potential.
- Listed the symmetries leading to natural alignment.
- Analyzed the simplest one, namely, the Maximally Symmetric 2HDM potential with SO (5) symmetry.
- Deviations from alignment are induced naturally by RG effects due to $g^{\prime}$ and Yukawa couplings, and due to soft-breaking mass parameter.
- Predicts a quasi-degenerate and 'gaugephobic' heavy Higgs sector.
- Using the alignment constraints, we predict lower limits on the heavy Higgs spectrum, which prevail the present limits in a wide range of parameter space.
- Depending on the $S O(5)$-breaking scale, we also obtain an upper limit on the heavy Higgs masses, which could be completely probed during LHC run-II.
- Initiated study on a new collider signal with four ton quarks in the final state, which can become a valuable observational tool to directly probe the heavy Higgs sector in the alignment limit.


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## ps: BABAR Result




- Improved measurements of the ratios

$$
\mathcal{R}(D)=\frac{\operatorname{BR}\left(\bar{B} \rightarrow D \tau^{-} \bar{\nu}_{\tau}\right)}{\operatorname{BR}\left(\bar{B} \rightarrow D \ell^{-} \bar{\nu}_{\ell}\right)}, \quad \mathcal{R}\left(D^{*}\right)=\frac{\operatorname{BR}\left(\bar{B} \rightarrow D^{*} \tau^{-} \bar{\nu}_{\tau}\right)}{\operatorname{BR}\left(\bar{B} \rightarrow D^{*} \ell^{-} \bar{\nu}_{\ell}\right)},
$$

- Excess w.r.t. the SM predictions at $3.4 \sigma$. Charged Higgs interpretation?
- Disfavored in Type-II 2HDM, but by the same token, SM is also excluded !!


## Symmetry Generators

Table 2
Symmetry generators [cf. (10), (14)] and discrete group elements [cf. (17)] for the 13 accidental symmetries of the $\mathrm{U}(1)_{Y}$-invariant 2 HDM potential. For each symmetry, the maximally broken $S O(5)$ generators compatible with a neutral vacuum are displayed, along with the pseudo-Goldstone bosons (given in parentheses) that result from the Goldstone theorem.

| No. | Symmetry | Generators $T^{a} \leftrightarrow K^{a}$ | Discrete group elements | Maximally broken $\mathrm{SO}(5)$ generators | Number of pseudo-Goldstone bosons |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $Z_{2} \times \mathrm{O}(2)$ | $T^{0}$ | $D_{\text {CP1 }}$ | - | 0 |
| 2 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(2)$ | $T^{0}$ | $D_{Z_{2}}$ | - | 0 |
| 3 | $\left(Z_{2}\right)^{3} \times \mathrm{O}(2)$ | $T^{0}$ | $D_{\text {CP2 }}$ | - | 0 |
| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | $T^{3}, T^{0}$ | - | $T^{3}$ | 1 (a) |
| 5 | $Z_{2} \times[\mathrm{O}(2)]^{2}$ | $T^{2}, T^{0}$ | $D_{\text {CP1 }}$ | $T^{2}$ | 1 (h) |
| 6 | $\mathrm{O}(3) \times \mathrm{O}(2)$ | $T^{1,2,3}, T^{0}$ | - | $T^{1,2}$ | 2 (h,a) |
| 7 | SO(3) | $T^{0,4,6}$ | - | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 8 | $Z_{2} \times \mathrm{O}$ (3) | $T^{0,4,6}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 9 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(3)$ | $T^{0,5,7}$ | $D_{\text {CP1 }} \cdot D_{\text {CP2 }}$ | $T^{5,7}$ | $2\left(h^{ \pm}\right)$ |
| 10 | $\mathrm{O}(2) \times \mathrm{O}(3)$ | $T^{3}, T^{0,8,9}$ | - | $T^{3}$ | 1 (a) |
| 11 | SO(4) | $T^{0,3,4,5,6,7}$ | - | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 12 | $Z_{2} \times \mathrm{O}(4)$ | $T^{0,3,4,5,6,7}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 13 | $\mathrm{SO}(5)$ | $T^{0,1,2, \ldots, 9}$ | - | $T^{1,2,8,9}$ | $4\left(h, a, h^{ \pm}\right)$ |

- $T^{a}$ and $K^{a}$ are the generators of $S O(5)$ and $S p(4)$ respectively $(a=0, \ldots, 9)$.
- $T^{0}$ is the hypercharge generator in $R$-space, which is equivalent to the electromagnetic generator $Q_{\mathrm{em}}=\frac{1}{2} \sigma^{0} \otimes \sigma^{0} \otimes \sigma^{3}+K^{0}$ in $\Phi$-space.
- $S p(4)$ contains the custodial symmetry group $S U(2)_{C}$.
- Three independent realizations of custodial symmetry induced by (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$.


## Symmetry Generators

## Table 2

Symmetry generators [cf. (10), (14)] and discrete group elements [cf. (17)] for the 13 accidental symmetries of the $\mathrm{U}(1) \mathrm{y}$-invariant 2 HDM potential. For each symmetry, the maximally broken SO(5) generators compatible with a neutral vacuum are displayed, along with the pseudo-Goldstone bosons (given in parentheses) that result from the Goldstone theorem.

| No. | Symmetry | Generators $T^{a} \leftrightarrow K^{a}$ | Discrete group elements | Maximally broken SO(5) generators | Number of pseudo-Goldstone bosons |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $Z_{2} \times 0$ (2) | $T^{0}$ | $D_{\text {CP1 }}$ | - | 0 |
| 2 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(2)$ | $T^{0}$ | $D_{Z_{2}}$ | - | 0 |
| 3 | $\left(Z_{2}\right)^{3} \times \mathrm{O}(2)$ | $T^{0}$ | $D_{\text {CP2 }}$ | - | 0 |
| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | $T^{3}, T^{0}$ | - | $T^{3}$ | 1 (a) |
| 5 | $Z_{2} \times[\mathrm{O}(2)]^{2}$ | $T^{2}, T^{0}$ | $D_{\text {CP1 }}$ | $T^{2}$ | 1 (h) |
| 6 | $\mathrm{O}(3) \times \mathrm{O}(2)$ | $T^{1,2,3}, T^{0}$ | - | $T^{1,2}$ | 2 (h,a) |
| 7 | SO(3) | $T^{0,4,6}$ | - | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 8 | $Z_{2} \times 0$ (3) | $T^{0,4,6}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 9 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(3)$ | $T^{0,5,7}$ | $D_{\text {CP1 }} \cdot D_{\text {CP2 }}$ | $T^{5,7}$ | $2\left(h^{ \pm}\right)$ |
| 10 | $\mathrm{O}(2) \times \mathrm{O}(3)$ | $T^{3}, T^{0,8,9}$ | - | $T^{3}$ | 1 (a) |
| 11 | SO(4) | $T^{0,3,4,5,6,7}$ | - | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 12 | $Z_{2} \times 0$ (4) | $T^{0,3,4,5,6,7}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 13 | SO(5) | $T^{0,1,2, \ldots, 9}$ | - | $T^{1,2,8,9}$ | $4\left(h, a, h^{ \pm}\right)$ |

[Pilaftsis '12]

- $T^{a}$ and $K^{a}$ are the generators of $S O(5)$ and $S p(4)$ respectively $(a=0, \ldots, 9)$.
- $T^{0}$ is the hypercharge generator in $R$-space, which is equivalent to the electromagnetic generator $Q_{\mathrm{em}}=\frac{1}{2} \sigma^{0} \otimes \sigma^{0} \otimes \sigma^{3}+K^{0}$ in $\Phi$-space.


## Quark Yukawa Couplings

- By convention, choose $h_{1}^{u}=0$. For Type-I (Type-II) 2HDM, $h_{1}^{d}\left(h_{2}^{d}\right)=0$.
- Quark yukawa couplings w.r.t. the SM are given by

| Coupling | Type-I | Type-II |
| :---: | :---: | :---: |
| $g_{h t \bar{t}}$ | $\cos \alpha / \sin \beta$ | $\cos \alpha / \sin \beta$ |
| $g_{h b \bar{b}}$ | $\cos \alpha / \sin \beta$ | $-\sin \alpha / \cos \beta$ |
| $g_{H t \bar{t}}$ | $\sin \alpha / \sin \beta$ | $\sin \alpha / \sin \beta$ |
| $g_{H b \bar{b}}$ | $\sin \alpha / \sin \beta$ | $\cos \alpha / \cos \beta$ |
| $g_{a t \bar{t}}$ | $\cot \beta$ | $\cot \beta$ |
| $g_{a b \bar{b}}$ | $-\cot \beta$ | $\tan \beta$ |

## $g^{\prime}$ Effect




| No. | Symmetry | Generators $T^{a} \leftrightarrow K^{a}$ | Discrete group elements | Maximally broken SO(5) generators | Number of pseudo-Goldstone bosons |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $Z_{2} \times \mathrm{O}(2)$ | $T^{0}$ | $D_{\text {CP1 }}$ | - | 0 |
| 2 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(2)$ | $T^{0}$ | $D_{Z_{2}}$ | - | 0 |
| 3 | $\left(Z_{2}\right)^{3} \times O(2)$ | $T^{0}$ | $D_{\text {CP2 }}$ | - | 0 |
| 4 | $\mathrm{O}(2) \times \mathrm{O}(2)$ | $T^{3}, T^{0}$ | - | $T^{3}$ | 1 (a) |
| 5 | $Z_{2} \times[0(2)]^{2}$ | $T^{2}, T^{0}$ | $D_{\text {CP1 }}$ | $T^{2}$ | 1 (h) |
| 6 | $\mathrm{O}(3) \times \mathrm{O}(2)$ | $T^{1,2,3}, T^{0}$ | - | $T^{1,2}$ | $2(h, a)$ |
| 7 | SO(3) | $T^{0,4,6}$ | - | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 8 | $Z_{2} \times 0$ (3) | $T^{0,4,6}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{4,6}$ | $2\left(h^{ \pm}\right)$ |
| 9 | $\left(Z_{2}\right)^{2} \times \mathrm{SO}(3)$ | $T^{0,5,7}$ | $D_{\text {CP1 }} \cdot D_{\text {CP2 }}$ | $T^{5,7}$ | $2\left(h^{ \pm}\right)$ |
| 10 | $\mathrm{O}(2) \times \mathrm{O}(3)$ | $T^{3}, T^{0,8,9}$ | - | $T^{3}$ | 1 (a) |
| 11 | SO(4) | $T^{0,3,4,5,6,7}$ | - | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 12 | $Z_{2} \times \mathrm{O}(4)$ | $T^{0,3,4,5,6,7}$ | $D_{Z_{2}} \cdot D_{\text {CP2 }}$ | $T^{3,5,7}$ | $3\left(a, h^{ \pm}\right)$ |
| 13 | SO(5) | $T^{0,1,2, \ldots, 9}$ | - | $T^{1,2,8,9}$ | $4\left(h, a, h^{ \pm}\right)$ |

## Yukawa Coupling Effects



$\left.\begin{array}{clllll}\hline \text { No. } & \text { Symmetry } & \begin{array}{l}\text { Generators } \\ T^{a} \leftrightarrow K^{a}\end{array} & \begin{array}{l}\text { Discrete group } \\ \text { elements }\end{array} & \begin{array}{l}\text { Maximally broken } \\ \text { SO(5) generators }\end{array} \\ \hline 1 & Z_{2} \times \mathrm{O}(2) & T^{0} & D_{\mathrm{CP} 1} & - \\ 2 & \left(Z_{2}\right)^{2} \times \mathrm{SO}(2) & T^{0} & D_{Z_{2}} & - \\ 3 & T^{0} & D_{\mathrm{CP} 2} & - & 0 \\ \text { pseudo-Goldstone bosons }\end{array}\right]$

## Production of 4 tops in the SM


(a)

(b)

## Production of 4 tops in BSM



