The role of lattice QCD to test the Standard Model in the quark sector

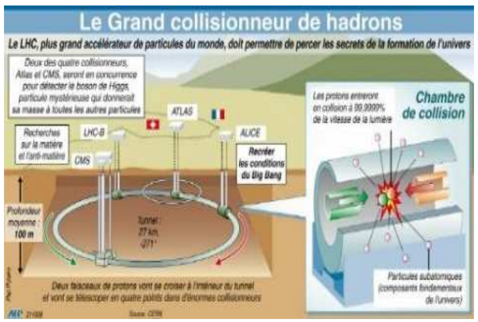
Benoît Blossier

LPT Orsay

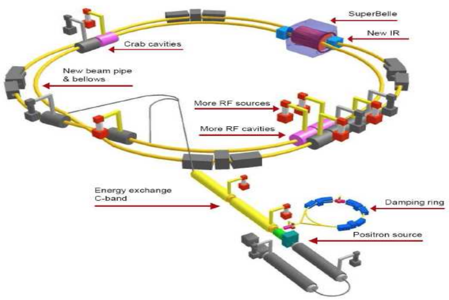
Southampton, $30^{\rm th}$ January 2015

Introductory remarks

LHC(CERN)

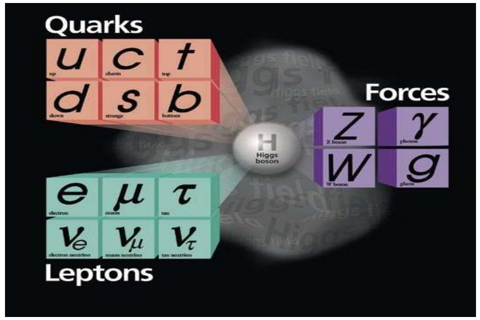


Super KEKB (KEK)



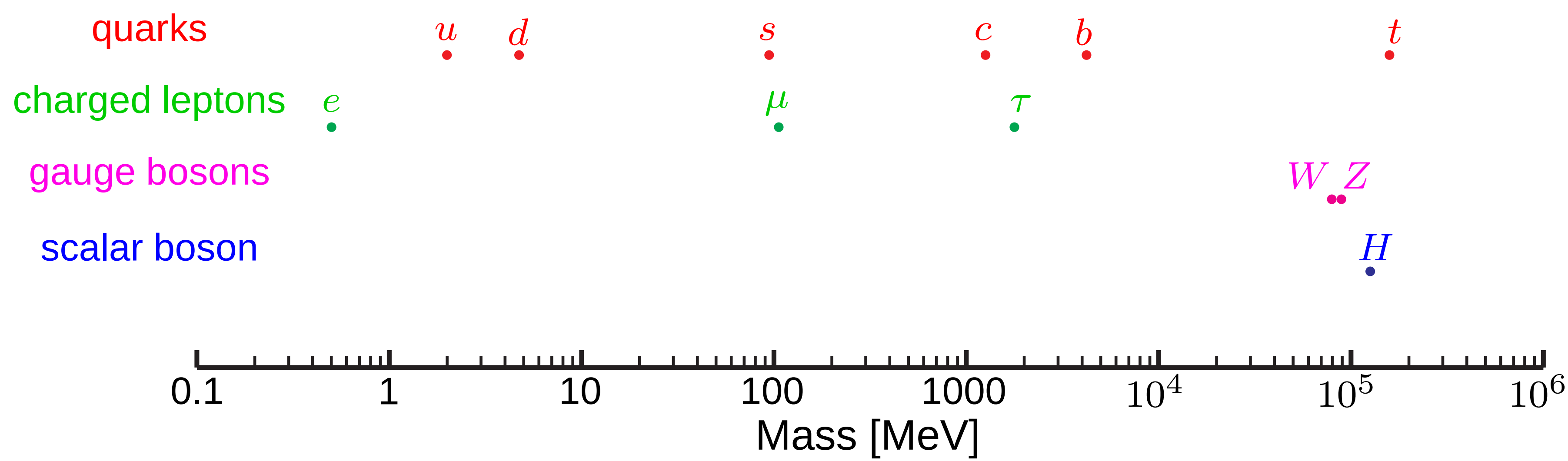
Large collisionners are working or are built to study the matter at scales smaller than 1 fm.

Elementary particles: a fascinating microscopic world.



Matter Particles	strong inter.	electromagnetic inter.	weak inter.
Quarks (u, d, s, c, b, t)	$\surd$	$\surd$	$\surd$
Charged leptons (e, μ, τ)	$\times$	$\surd$	$\surd$
Neutral leptons $(\nu_e, \nu_\mu, \nu_\tau)$	$\times$	$\times$	$\surd$
vectors boson of the interaction	gluon	photon	$W^\pm, Z^0$

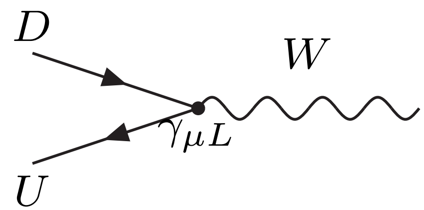
Matter particles are also sensitive to the gravitational interaction, maybe mediated by the graviton. They live in a sort of bath created by the Higgs boson which gives them a mass.



Standard Model in the quark sector

3 families of quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$; strong hierarchy among quark masses

Quarks are coupled to charged weak bosons by a left-handed current.



Quark flavour eigenstates $\neq$ quark weak eigenstates; the flavour mixing
is described by the Cabibbo-Kobayashi-Matrix mechanism, the only source of CP violation.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V_{ij} \sim \mathcal{O}(1)$

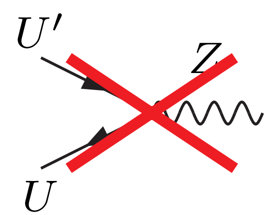
$V_{ij} \sim \mathcal{O}(\lambda)$

$V_{ij} \sim \mathcal{O}(\lambda^2)$

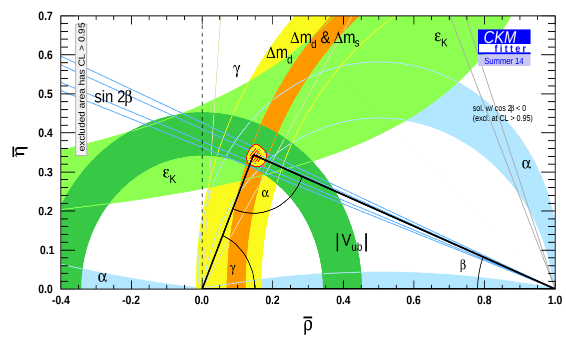
$V_{ij} \sim \mathcal{O}(\lambda^3)$

$\lambda \sim 0.22$

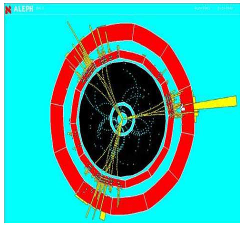
Unitarity of the CKM matrix: Glashow - Iliopoulos - Maiani mechanism,
no Flavour Changing Neutral Current at tree level.



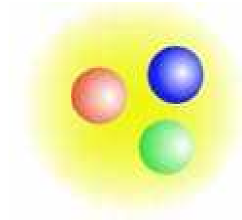
6 unitarity triangles: flavour physics constraints on sides and angles.



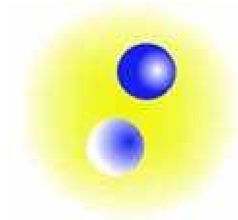
Quarks are not directly seen in experiments. After a collision one only detects jets of particles, whose a large number are composed of quarks: the hadrons.



jet quark+quark+gluon

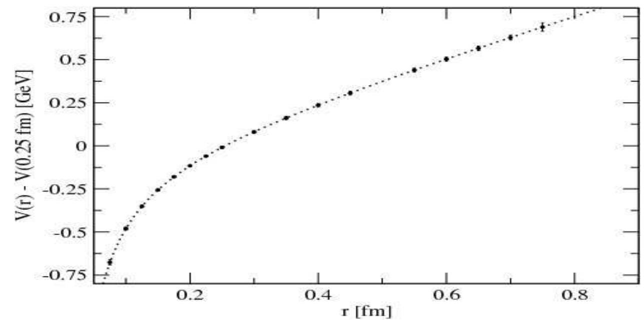


baryon (qqq)



meson ($q\bar{q}$)

The mechanism of the quark confinement within hadrons is one of the most mysterious questions for theoreticians.

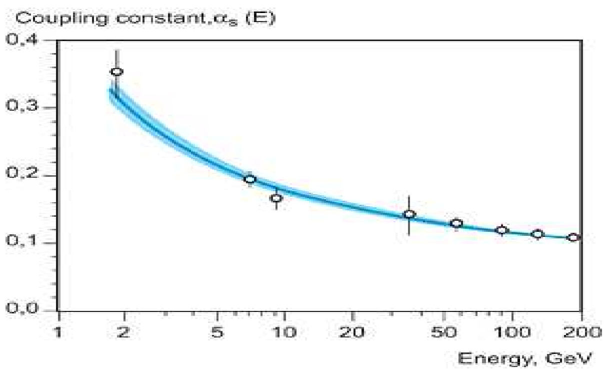


$$\Gamma = Q \times T$$

experimental measurement → Γ

QCD contribution → Q

parameters of the theory → T

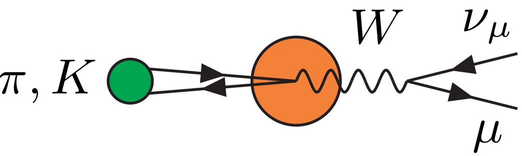


$Q = \sum_n q_n \alpha_s^n$: asymptotic series

Topics covered in this talk

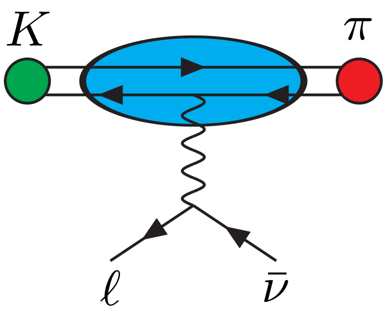
- Investigating the unitarity of the CKM matrix in the first and second flavour families

Leptonic decays



$$\underbrace{\frac{\Gamma(K \to \mu\nu_\mu(\gamma))}{\Gamma(\pi \to \mu\nu_\mu(\gamma))}}_{\text{exp. measurement}} = |V_{us}/V_{ud}|^2 \frac{m_K}{m_\pi} \left(\frac{f_K}{f_\pi}\right)^2 \frac{\left(1-\frac{m_\mu^2}{m_K^2}\right)^2}{\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2} \times \underbrace{0.9930(35)}_{\text{QED}}$$

Semileptonic decays



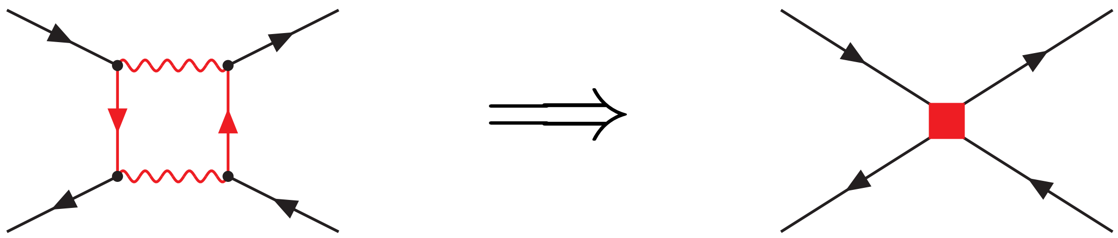
$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 \times \underbrace{1.0232(3)}_{\text{EW corr}} \times \left(|V_{us}|\, f_+^{K^0\to\pi^-}(0)\right)^2 \times I_{K\ell} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\ell})$$

$$\langle \pi(p')|\bar{s}\gamma_\mu u|K(p)\rangle = \left(p_\mu + p'_\mu - q_\mu \frac{m_K^2 - m_\pi^2}{q^2}\right) f_+^{K\to\pi}(q^2) + q_\mu \frac{m_K^2 - m_\pi^2}{q^2} f_0^{K\to\pi}(q^2)$$

Question: do we have $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}^2| = 1$?

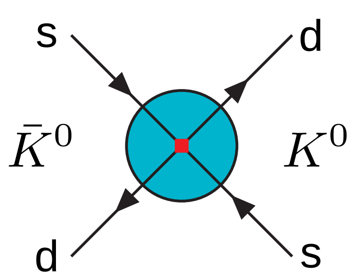
Topics covered in this talk

- Flavour Changing Neutral Currents: shed light on New Physics



$$\mathcal{L}_{\rm SM} \longrightarrow \mathcal{H}_{\rm eff} \;\alpha\; V_{\rm CKM} C(\mu) Q(\mu) + \underbrace{\mathcal{O}(m_b^2/M_W^2)}_{<1\%}$$

$K^0 - \bar{K}^0$ **mixing**



$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 m_W^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c) \tilde{\eta_1}(\mu) + \lambda_t^2 S_0(x_t) \tilde{\eta_2}(\mu) + 2\lambda_c \lambda_t S_0(x_c, x_t) \tilde{\eta_3}(\mu) \right]$$
$$\times \quad \langle \bar{K}^0 | Q_{LL}^{\Delta S=2} | K^0 \rangle (\mu)$$

BSM: $\mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_i C_i O_i$; $C_i = \frac{F_i L_i}{\Lambda^2}$ (F_i new coupling, L_i loop factor)
lower bounds on NP scale Λ.

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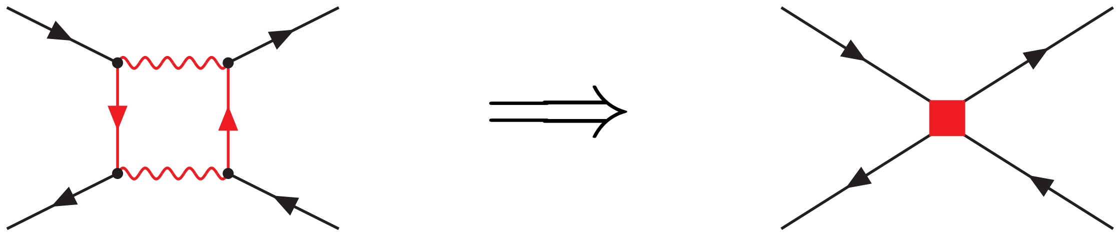
1-loop diagram in SM   diagram in effective theory



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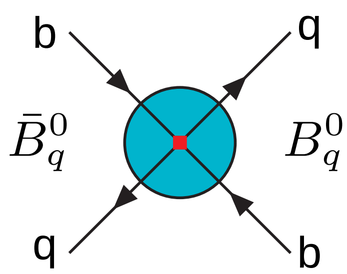
$B_q^0 - \bar{B}_q^0$ **mixing**



$$\langle \bar{B}_q^0 | \mathcal{H}_{\rm eff}^{\Delta B=2} | B_q^0 \rangle = \frac{G_F^2 m_W^2}{16\pi^2} \lambda_{tq}^2 S_0(x_t) \tilde{\eta}_{2B}(\mu) \langle \bar{B}_q^0 | Q_{LL}^{\Delta B=2} | B_q^0 \rangle (\mu)$$

Including BSM structures: lower bounds on NP scale as in the kaon sector

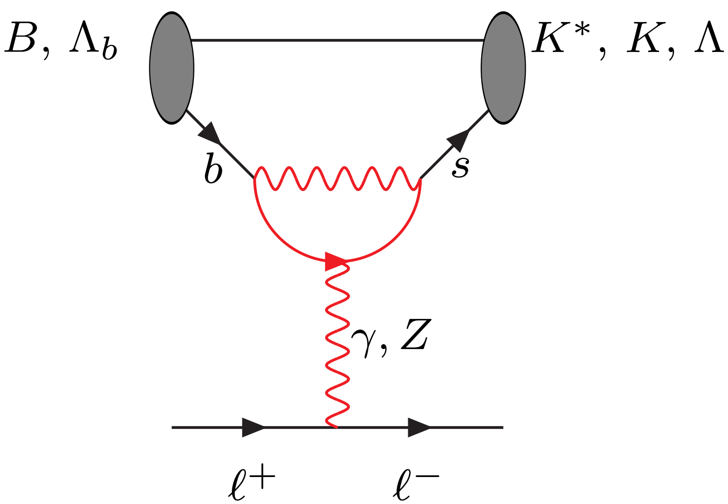
Question: do constraints from CP conserving and CP violating quantities match?

Topics covered in this talk

- Flavour Changing Neutral Currents: shed light on New Physics

Rare $b \to s$ transitions

Processes testing SM extensions: $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$



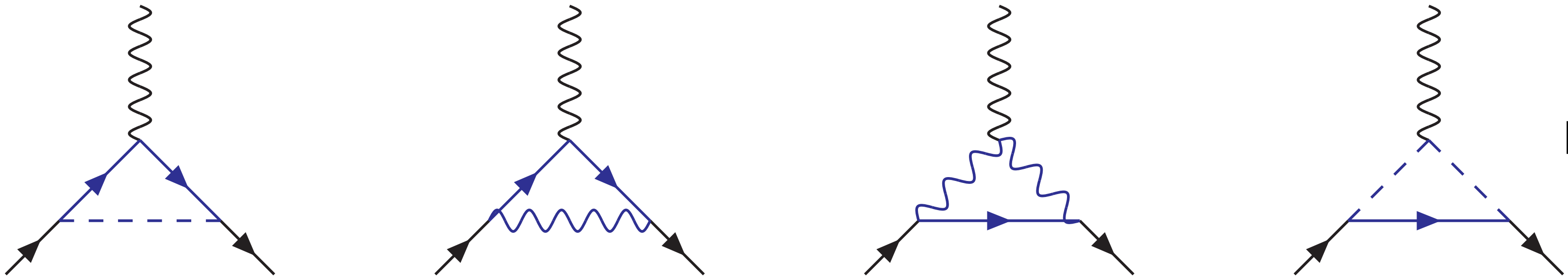
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=7,9,10,S,P} (C_i O_i + C_i' O_i')$$

Question: using lattice inputs, have we already observed effects of NP?

Topics covered in this talk

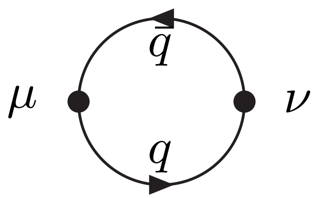
- A golden quantity to detect NP: anomalous moment of the muon

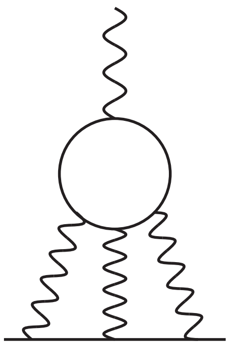
Extremely precise experimental measurement, theoretical computations say that there is room for BSM effects (3σ discrepancy)



BSM $\mathcal{O}(m_\mu^2/M^2)$

2 hadronic contributions are computed on the lattice: however, very difficult (strong dependence on Q^2 or complicated Green functions)





Question: is the error on SM prediction of $g_\mu - 2$ correctly estimated?

Topics covered in this talk

- The dominant error on the $H \to b\bar{b}$ coupling: b-quark mass

$$\Gamma(H \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} m_H m_b^2(\overline{\mathrm{MS}}, m_H) \left[1 + \underbrace{\Delta_{bb} + \Delta_H^2}_{\mathrm{QCD\ corr.}} \right]$$

Question: how much does the lattice improve the computation of m_b?

The role of lattice QCD
to test the Standard Model in the quark sector

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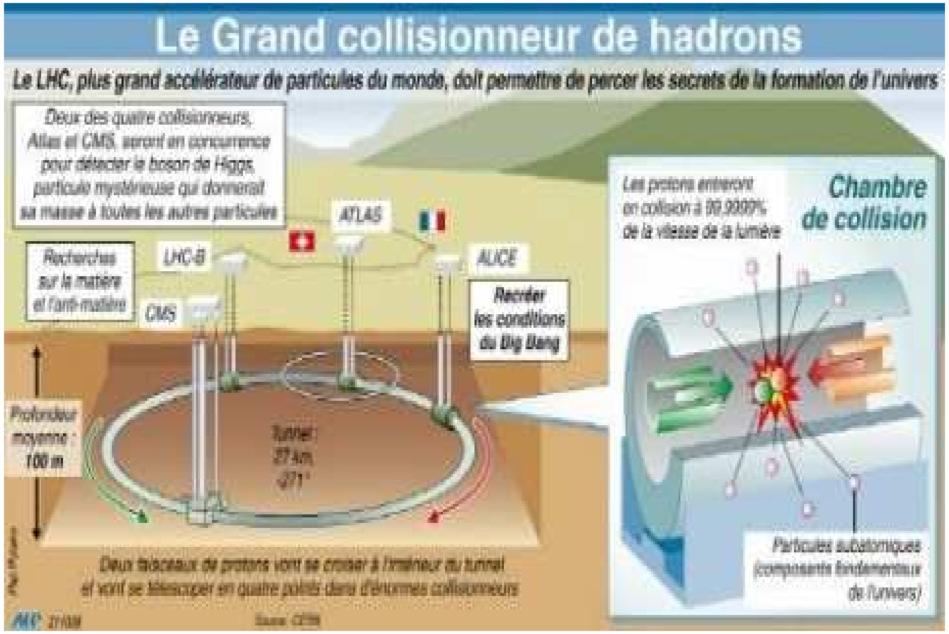
- Prerequisite
- Hints of lattice QCD
- Testing unitarity of the CKM matrix
- $\Delta F = 2$ processes
- Anomalous magnetic moment of the muon
- b coupling to the BEH boson
- Outlook

Prerequisite

LHC(CERN)



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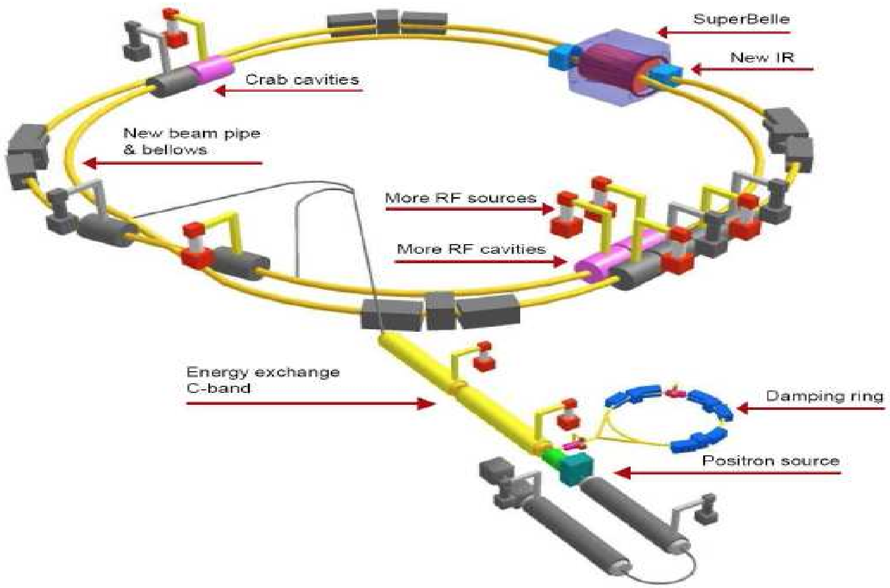
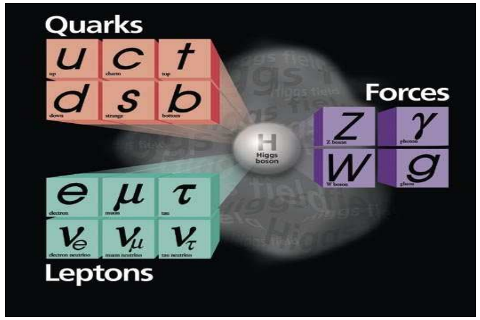


Large collisionners are working or are built to unveil the ultimate secrets of matter.

Elementary particles: a fascinating microscopic world.



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Quarks (u, d, s, c, b, t)	√	√	√
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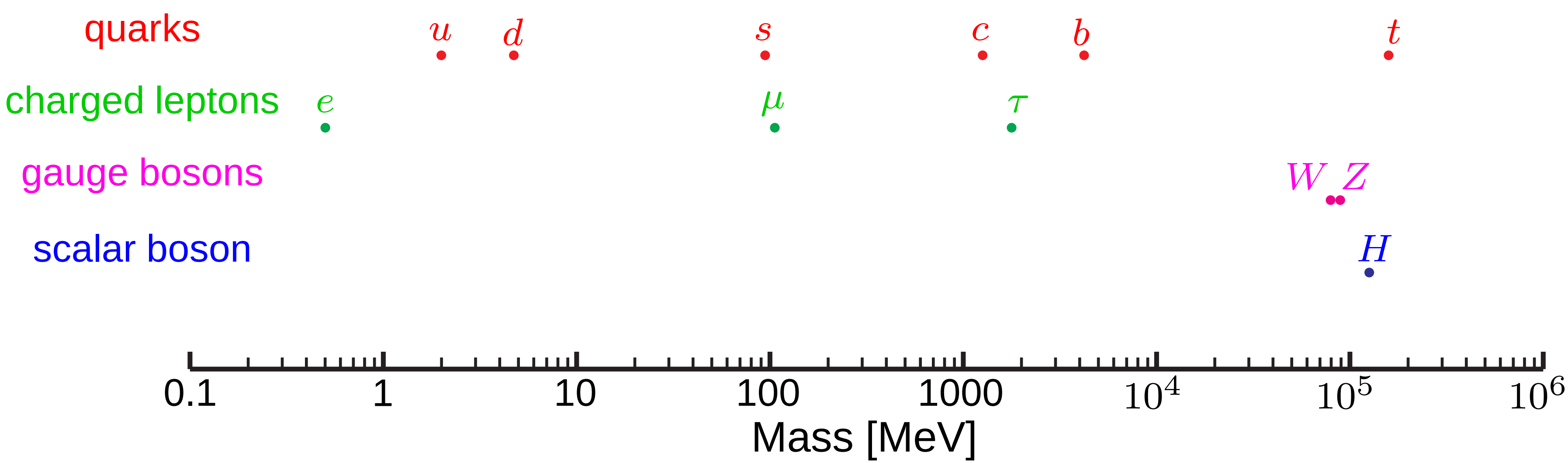

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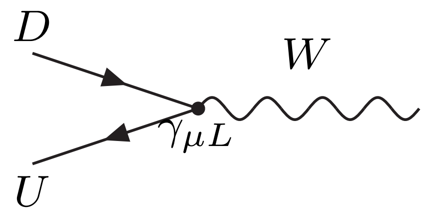

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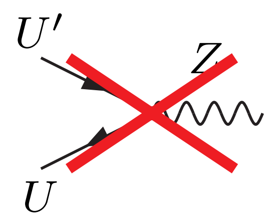

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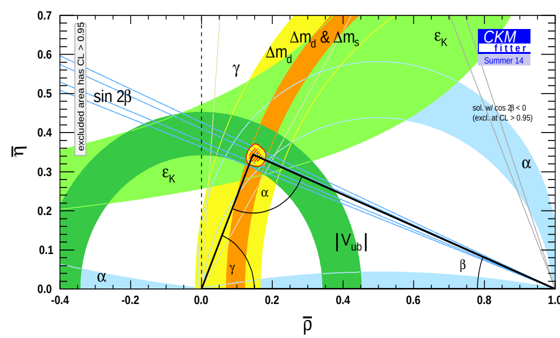

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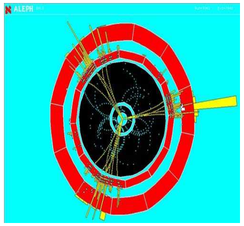
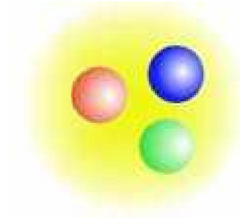
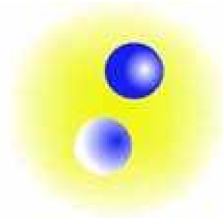

jet quark+quark+gluon baryon (qqq) meson ($q\bar{q}$)

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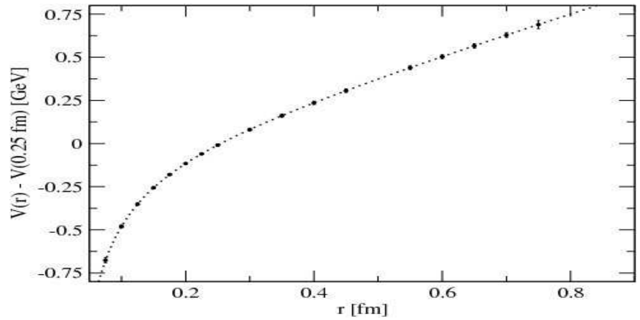
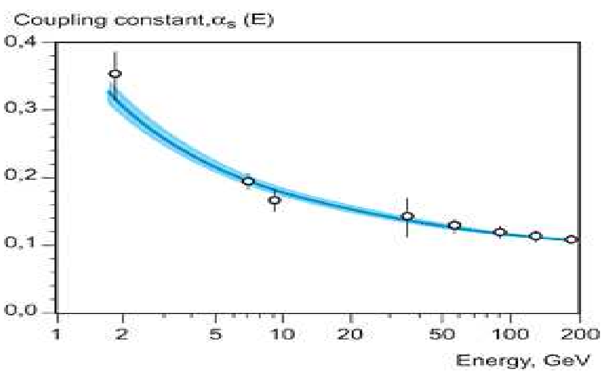


$\Gamma = Q \times T$

experimental measurement → Γ

QCD contribution → Q

parameters of the theory → T

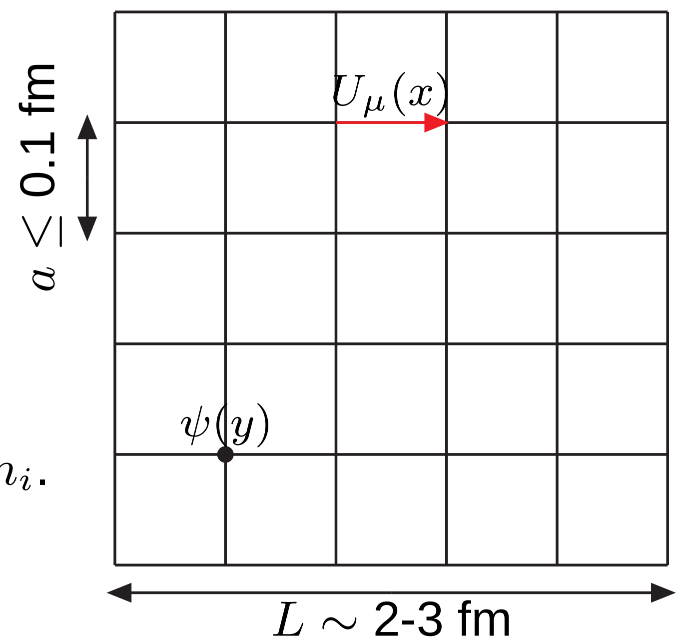


$Q = \sum_n q_n \alpha_s^n$: asymptotic series

Hints of lattice QCD

Discretisation of QCD in a finite volume of Euclidean space-time.

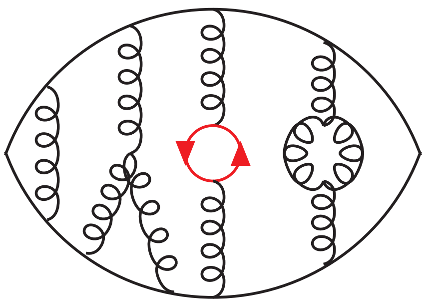
The lattice spacing a is a non perturbative UV cut-off of the theory.



Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0 A_\mu(x+\frac{a\hat{\mu}}{2})}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses m_i.

Computation of Green functions of the theory from first principles:

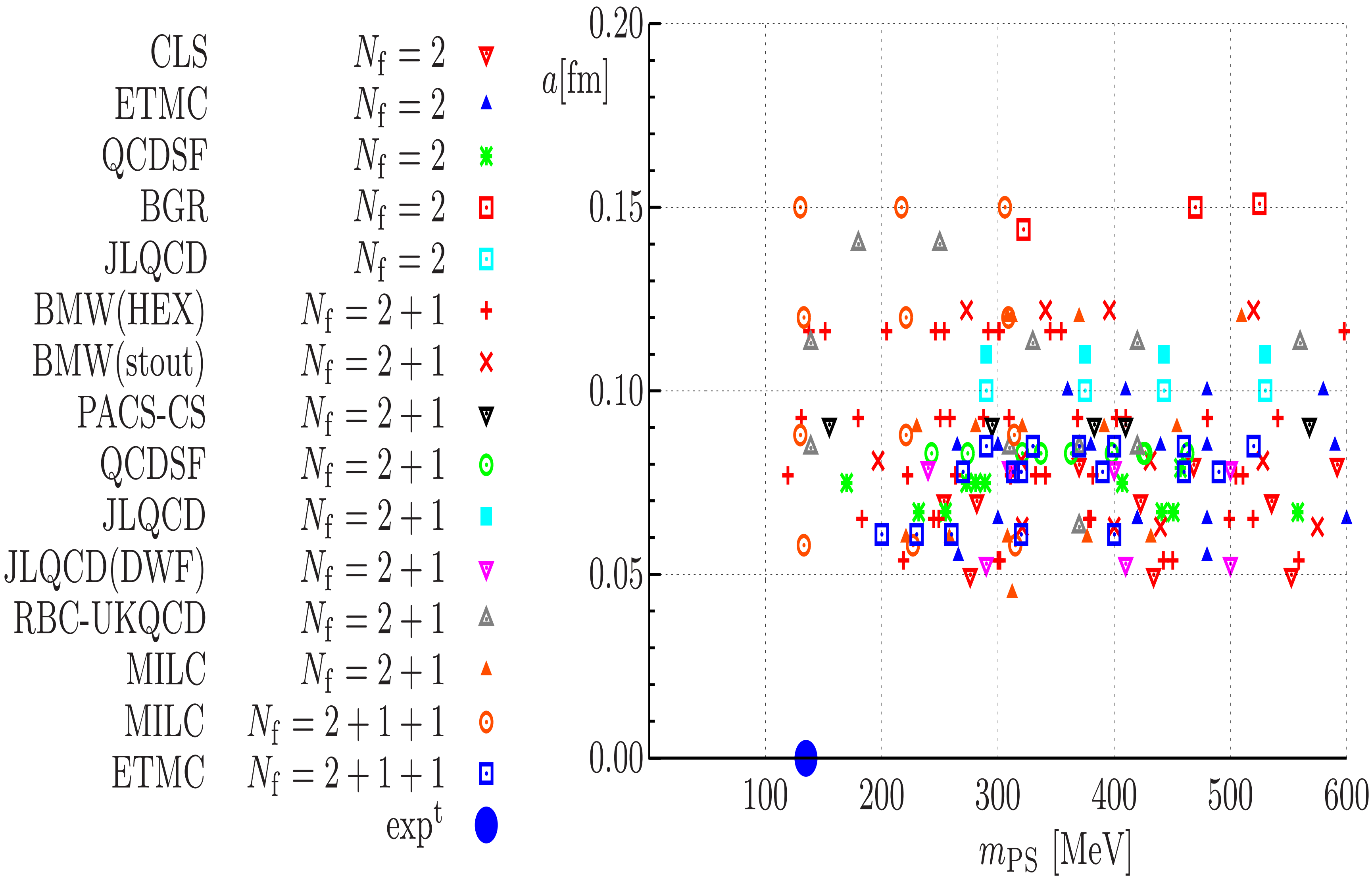


$$\langle O(U,\psi,\bar{\psi})\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,O(U,\psi,\bar{\psi})e^{-S(U,\psi,\bar{\psi})}$$
$$\mathcal{Z} = \int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}e^{-S(U,\psi,\bar{\psi})}$$
$$S(U,\psi,\bar{\psi}) = S^{\rm YM}(U) + \bar{\psi}^i_x M^{ij}_{xy}(U)\psi^j_y$$
$$\mathcal{Z} = \int \mathcal{D}U\,{\rm Det[M(U)]}e^{-S^{\rm YM}(U)} \equiv \int \mathcal{D}U e^{-S_{\rm eff}(U)}$$

Monte Carlo simulation: $\langle O\rangle \sim \frac{1}{\rm N_{conf}}\sum_i O(\{U\}_i)$: we have to build the statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\rm eff}}$. Incorporating the quark loop effects hidden in ${\rm Det}[M(U)]$ is particularly expensive in computer time.

Lattice simulations set up

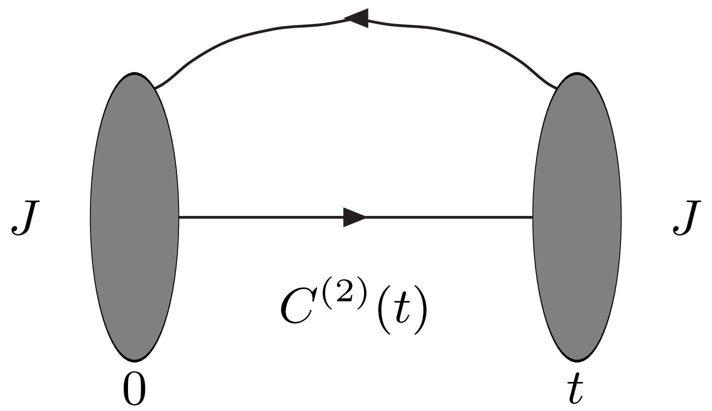
Nowadays, simulations are close to the physical point.



Isospin breaking and QED effects recently taken into account (BMW, MILC).

2-pt and 3-pt correlators

Extraction of masses and decay constants of bound states and hadronic matrix elements:

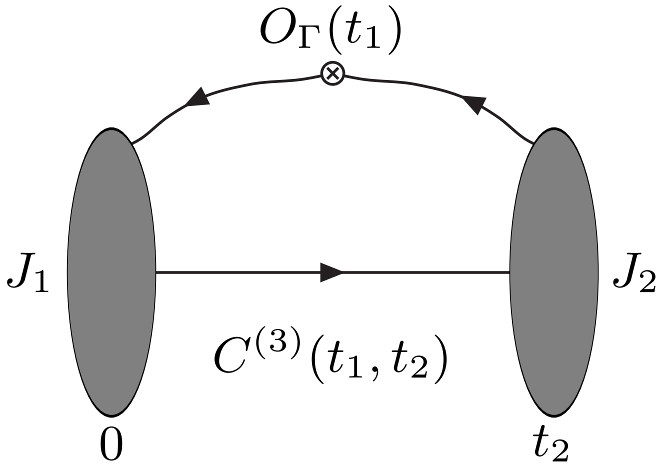


$$C^{(2)}_{JJ}(t) = \sum_{\vec{x}} \langle \Omega | \mathcal{T}[J(\vec{x},t) J^\dagger(0)] | \Omega \rangle$$

$$= \sum_n \frac{\mathcal{Z}_n^2 \, e^{-E_n t}}{2 E_n}$$

$$\mathcal{Z}_n = \langle \Omega | J | n \rangle \qquad \langle n | m \rangle = 2 E_n \delta_{mn}$$

$$C^{(2)}_{JJ}(t) \xrightarrow{(E_1 - E_0) t \gg 1} \frac{\mathcal{Z}_0^2 \, e^{-E_0 t}}{2 E_0}$$

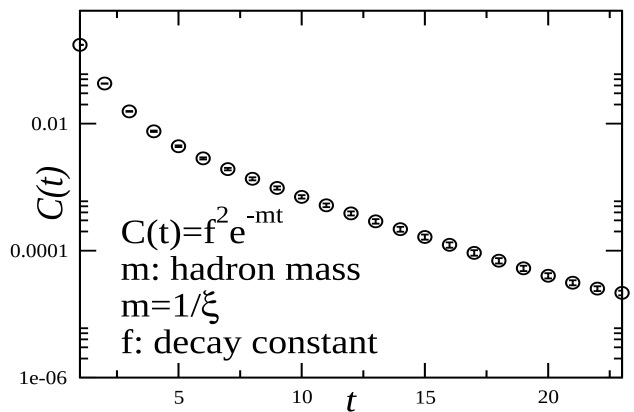


$$C^{(3)}_{J_1, J_2, O_\Gamma}(t_1, t_2) = \sum_{\vec{x}, \vec{y}} \langle \Omega | \mathcal{T}[J_2(\vec{y}, t_2) O_\Gamma(\vec{x}, t_1) J_1^\dagger(0)] | \Omega \rangle$$

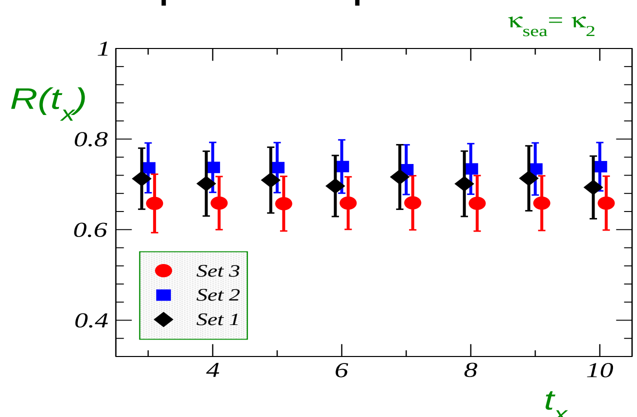
$$\xrightarrow{t_1, t_2 - t_1 \gg 0} \frac{\sqrt{\mathcal{Z}_{0,J_1}} \sqrt{\mathcal{Z}_{0,J_2}}}{2 E_{0,J_1} \, 2 E_{0,J_2}} e^{-E_{0,J_1} t_1} e^{-E_{0,J_2} (t_2 - t_1)}$$

$$\times \langle H_0^{J_2} | O_\Gamma | H_0^{J_1} \rangle$$

pion 2-pt correlator



ratio of 3-pt and 2-pt B correlators



Chiral fits and extrapolation to the continuum limit

Take under control the cut-off effects is nowadays mandatory to obtain a result included in global averages (e.g. by "Flavour Lattice Averaging Group" [itpwiki.unibe.ch]).

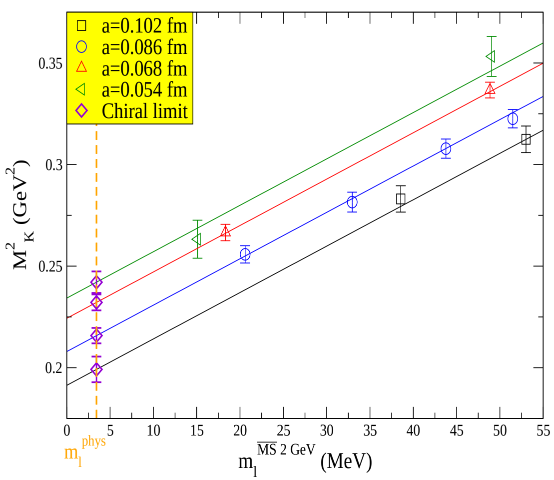
Data coming from several lattice spacings are put together, after a proper rescaling (through the Sommer parameter r_0 for example). χ PT is used as a guide in the extrapolation to the physical point.

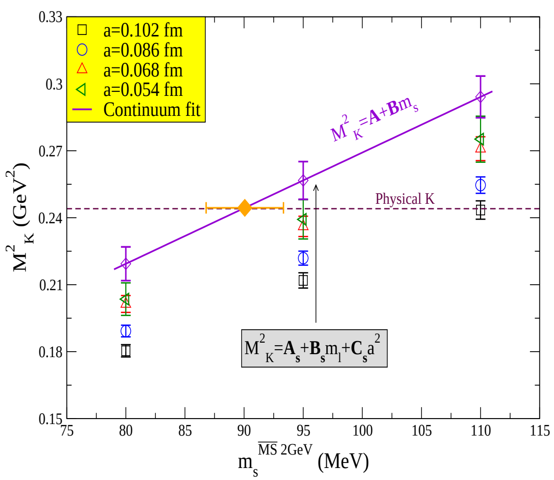
Measurement of m_l and m_s [ETMC, '10]

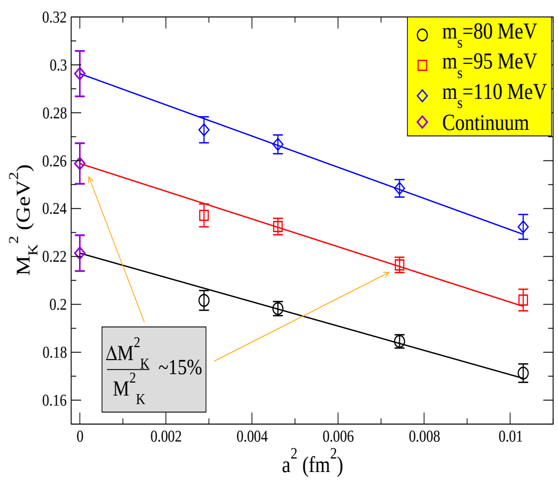
m_K analysed with 2 fits: SU(2) χPT [C. Allton et al, '08] and SU(3) χPT [S. Sharpe, '97]

$$\begin{aligned}(m_K^2)_{\mathrm{SU2}} &= Q_1(m_s) + Q_2(m_s)m_l + Q_3(m_s)a^2 \\ (m_K^2)_{\mathrm{SU3}} &= 2B_0\frac{m_l+m_s}{2}\left[1+\frac{2B_0m_s}{(4\pi f_0)^2}\ln\left(\frac{2B_0m_s}{(4\pi f_0)^2}\right)+Q_4m_s\right. \\ &\qquad \left.+Q_5m_l+Q_6m_s^2+Q_7a^2+Q_8a^2m_s\right]\end{aligned}$$

NNLO terms hardly visible in the fit of m_K^2; discretisation effects are present, as expected







Flavour Lattice Averaging Group (FLAG) [http://itpwiki.unibe.ch/flag/]

The lattice community is doing an effort in providing to phenomenologists a collection of useful results after a careful survey of the world-wide work.

Quantities under study:

- u, d and s quark masses
- V_{ud} and V_{us}
- Low Energy Constants
- Kaon mixing bag parameter B_K
- Strong coupling constant α_s
- $B_{(s)}$ and $D_{(s)}$ meson decay constants
- B mixing bag parameter B_B
- form factors of $B_{(s)}$ and D semileptonic decays

A lot of technicalities and issues about systematics, difficult to present outside our community in a pedagogical way, are thus often hidden. FLAG is performing global averages of results, after a selection according to several quality criteria:

– continuum limit extrapolation

★ 3 or more lattice spacings, $a_{\max}^2/a_{\min}^2 \geq 2$, $D(a_{\min}) \leq 2\%$, $\delta(a_{\min}) \leq 1$

○ 2 or more lattice spacings, $a_{\max}^2/a_{\min}^2 \geq 1.4$, $D(a_{\min}) \leq 10\%$, $\delta(a_{\min}) \leq 2$

■ otherwise

$$D(a) = \frac{Q(a)-Q(0)}{Q(a)} \quad \delta(a) = \frac{Q(a)-Q(0)}{\sigma_Q^{\text{cont}}}$$

– renormalization and matching:

★ absolutely renormalized or non-pertubative

○ 1-loop perturbation theory or higher with an estimate of truncation error

■ otherwise

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– finite-volume

★ $m_\pi L \gtrsim 3.7$ or 2 volumes at fixed parameters of the simulation

○ $m_\pi L \gtrsim 3$

■ otherwise

– chiral extrapolation

★ $m_{\pi\,\mathrm{min}} \lesssim 200$ MeV

○ 200 MeV $\lesssim m_{\pi\,\mathrm{min}} \lesssim 400$ MeV

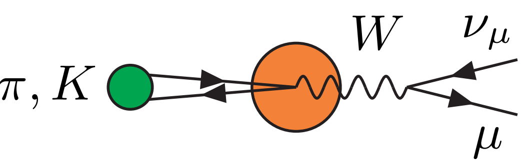
■ otherwise

⚠ Results with tiny errors must be taken with care, unfortunately they sometimes dominate too much the averages.

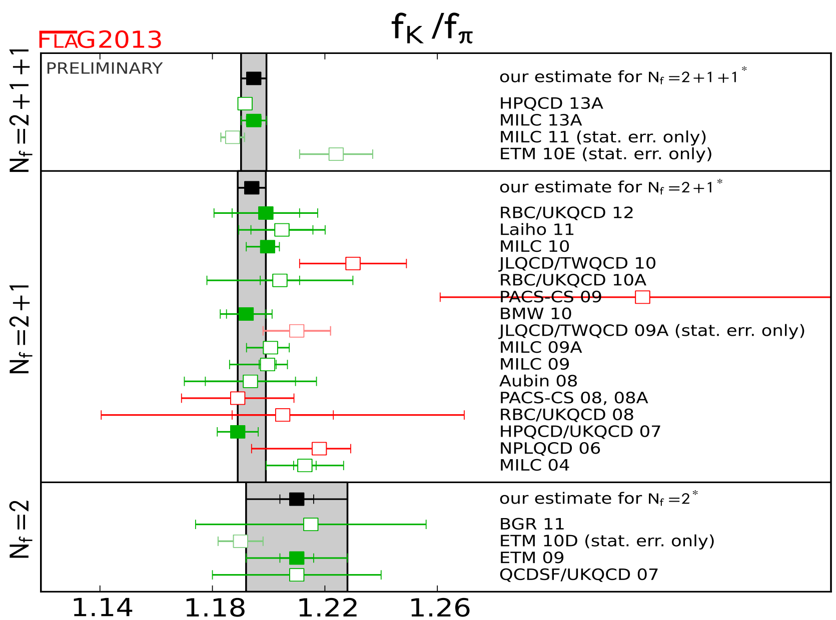
Testing unitarity of the CKM matrix

It is done by looking at kaon, pion and neutron decays

Leptonic decays



$$\underbrace{\frac{\Gamma(K\to\mu\nu_\mu(\gamma))}{\Gamma(\pi\to\mu\nu_\mu(\gamma))}}_{\text{exp. measurement}} = \left|V_{us}/V_{ud}\right|^2 \frac{m_K}{m_\pi}\left(\frac{f_K}{f_\pi}\right)^2 \frac{\left(1-\frac{m_\mu^2}{m_K^2}\right)^2}{\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2} \times \underbrace{0.9930(35)}_{\text{QED}}$$



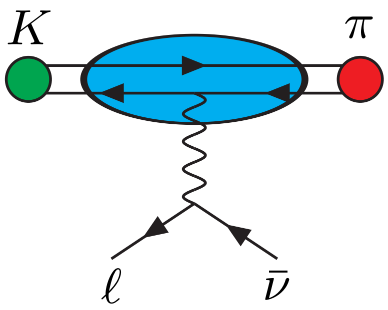
An update is expected after new results obtained in 2014:

$$f_K/f_\pi(\mathrm{N_f}=2+1,\ \mathrm{RBC/UKQCD}) = 1.1945(45)$$

$$f_K/f_\pi(\mathrm{N_f}=2+1+1,\mathrm{ETMC}) = 1.188(15)$$

$$f_K/f_\pi(\mathrm{N_f}=2+1+1,\mathrm{MILC}) = 1.1956(10)(^{+26}_{-18}),$$

Semileptonic decays



$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 \times \underbrace{1.0232(3)}_{\text{EW corr}} \times \left(|V_{us}| \, f_+^{K^0 \to \pi^-}(0) \right)^2$$

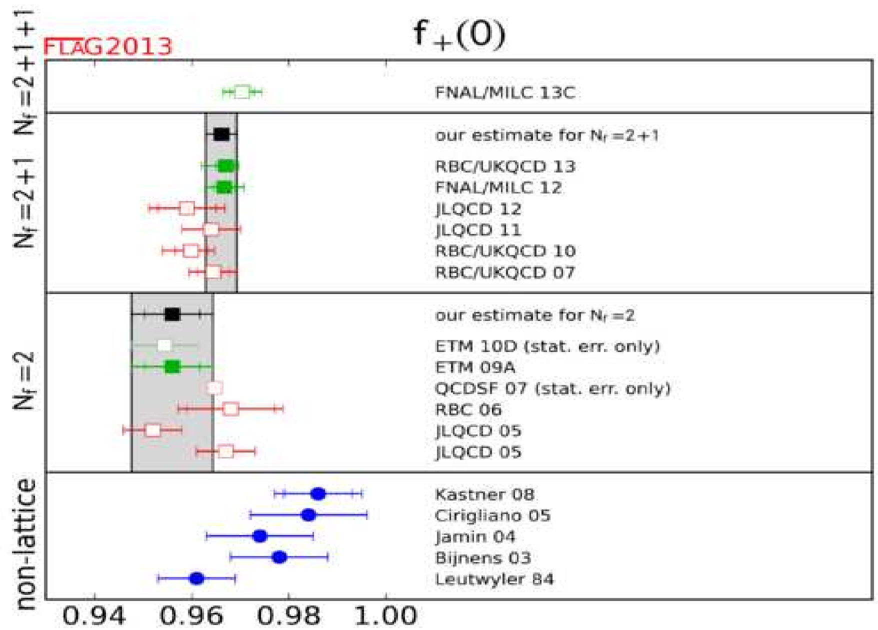
$$\times I_{K\ell} \times (1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\ell})$$

$$\langle \pi(p') | \bar{s}\gamma_\mu u | K(p) \rangle = \left(p_\mu + p'_\mu - q_\mu \frac{m_K^2 - m_\pi^2}{q^2} \right) f_+^{K\to\pi}(q^2)$$

$$+ q_\mu \frac{m_K^2 - m_\pi^2}{q^2} f_0^{K\to\pi}(q^2)$$

The null plane $q^2 = 0$ is particularly interesting: it remains only $f_+^{K\to\pi}(0)$

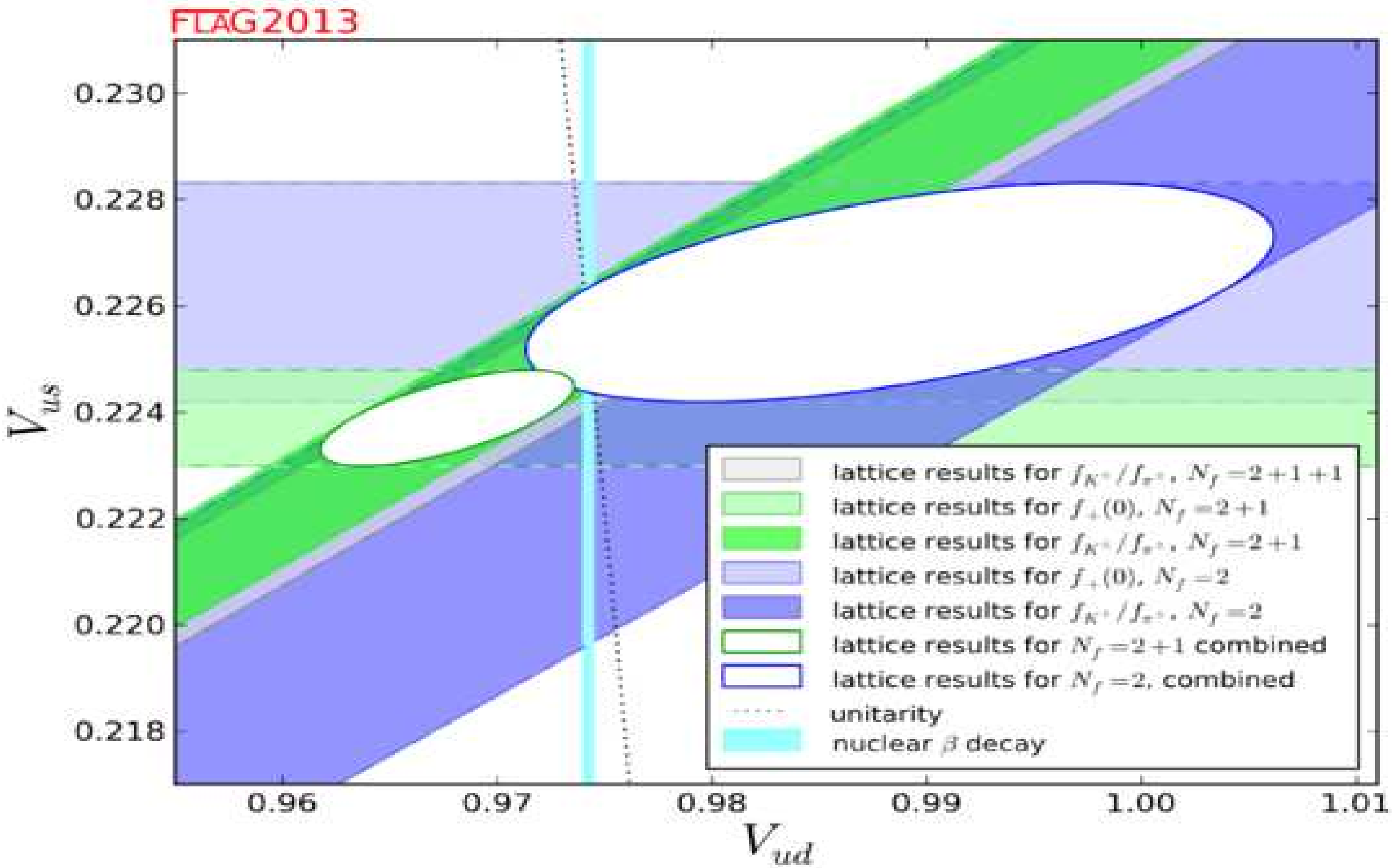
at NLO of χPT, $f_+^{K\to\pi}(0) - 1$ depends only on m_π, m_K, m_η and f_π [H. Leutwyler and M. Roos, ’84].



Update in 2014: $f_+(0)(\mathrm{N_f} = 2+1+1, \mathrm{ETMC}) = 0.9683(65)$.

Conclusion on V_{ud} and V_{us}

$$V_u^2 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \quad |V_{ub}| = 4.13(49) \times 10^{-3}$$ [PDG '14]



$\mathrm{N_f} = 2+1$ lattice data only: $V_u^2 = 0.987(10)$

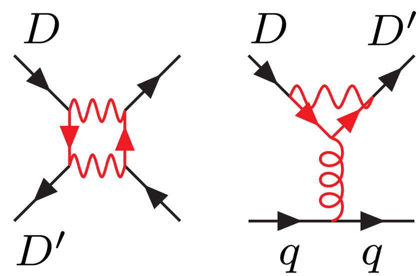
$\mathrm{N_f} = 2$ lattice data only: $V_u^2 = 1.029(35)$

Using V_{ud} from the β neutron decay, $V_u^2([f_+(0)]^{\mathrm{N_f}=2+1}) = 0.9993(5)$ and $V_u^2([f_{K^\pm}/f_{\pi^\pm}]^{\mathrm{N_f}=2+1}) = 1.0000(6)$

Lattice data confirm the unitarity of the CKM matrix within the SM

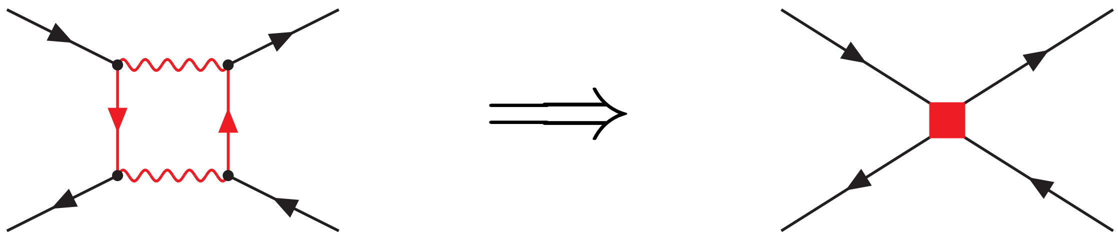
$\Delta F = 2$ processes

In the SM FCNC processes are forbidden at tree level.
They are mediated by quantum loops: box and penguin diagrams



Heavy degrees of freedom (W and Z bosons, top quark) running in loops are integrated out; derivation of an effective Hamiltonian in the Operator Product Expansion framework.

1-loop diagram in SM | diagram in effective theory



$$\mathcal{L}_{\rm SM} \longrightarrow \mathcal{H}_{\rm eff} \; \alpha \; V_{\rm CKM} C(\mu) Q(\mu) + \underbrace{\mathcal{O}(m_b^2/M_W^2)}_{<1\%}$$

– $C(\mu_b)$: term computed perturbatively and integrating the short-distance physics from the electroweak scale to m_b

– $\langle H_f|Q|H_i\rangle$ contains all the information about long-distance physics of QCD: it must be calculated non perturbatively

– Physics beyond the Standard Model allows exotic particles to run in the quantum loops and couplings with different chiral structures: Wilson coefficients $C(\mu)$ and effective operators $Q(\mu)$ contain useful information

$K^0 - \bar{K}^0$ **mixing**

K^0 and $\bar{K}^0$ are a mixture of the CP eigenstates K_L and K_S. $\epsilon_K \equiv \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$ is a very important phenomenological quantity.

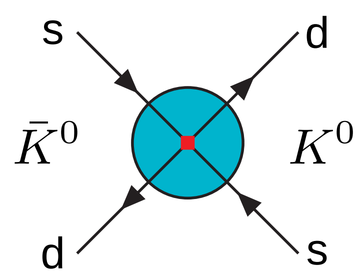
$$\epsilon_K = e^{i\phi_c}\sin(\phi_c)\left(\frac{\mathrm{Im}(\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle}{\Delta m_K} - \underbrace{(4\pm 2)\%}_{\text{long dist}}\right) \qquad |\epsilon_K|^{\exp} = 2.228(11)\times 10^{-3}$$

$$\phi_c = \arctan\left(\frac{\Delta m_K}{\Delta\Gamma_K/2}\right) \sim \pi/4 \qquad \Delta m(\Gamma)_K = m(\Gamma)_{K_{S(L)}} - m(\Gamma)_{K_{L(S)}}$$

$$\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle = \frac{G_F^2 m_W^2}{16\pi^2}\left[\lambda_c^2 S_0(x_c)\tilde{\eta}_1(\mu) + \lambda_t^2 S_0(x_t)\tilde{\eta}_2(\mu) + 2\lambda_c\lambda_t S_0(x_c,x_t)\tilde{\eta}_3(\mu)\right]$$
$$\times \quad \langle \bar{K}^0|Q_{LL}^{\Delta S=2}|K^0\rangle(\mu)$$

$\lambda_a = V_{as}^* V_{ad}$, S_0 is an Inami-Lim function, $\tilde{\eta}_i$ are Wilson coefficients and $Q_{LL}^{\Delta S=2} = [\bar{s}\gamma_{\mu\,L} d]\,[\bar{s}\gamma_{\mu\,L} d]$

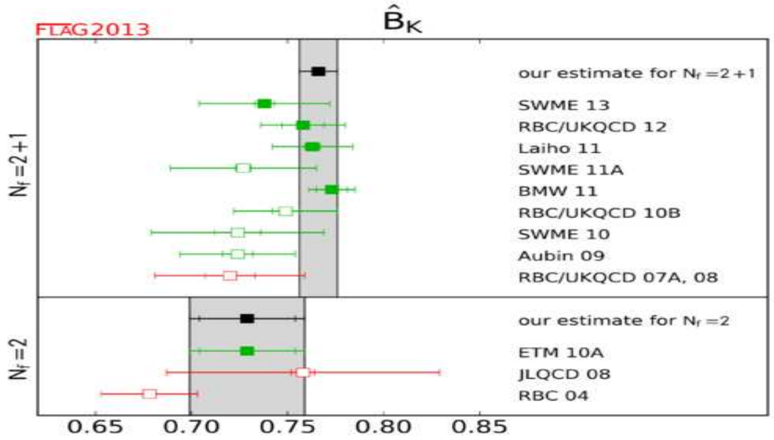
In the SM the dominant term of $\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle$ is $\propto |V_{cb}|^4$

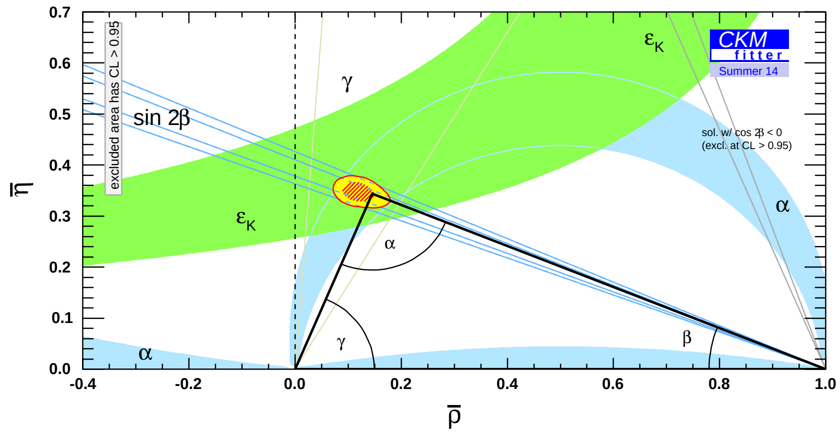


Usual parametrization: $\langle \bar{K}^0|Q_{LL}^{\Delta S=2}|K^0\rangle(\mu) = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$

$(\bar{\rho},\bar{\eta})$ plane: $|\epsilon_K| = \bar{\eta} A^2 \hat{B}_K [1.11(5) A^2 (1-\bar{\rho}) + 0.31(5)]$, $A \sim V_{cb}/\lambda^2$,
$\hat{B}_K$ is the RGI B_K parameter

constraints from CP violating quantities





Update in 2014: $\hat{B}_K(\mathrm{N_f} = 2+1, \mathrm{RBC/UKQCD}) = 0.7499 \pm 0.0014 \pm 0.0150$

The uncertainty on V_{cb} is now the main limiting factor on the ϵ_K constraint.

BSM: $\mathcal{H}_{\mathrm{eff}}^{\Delta S=2} = \sum_i C_i O_i$; computing the associated bag parameters B_i on the lattice and writing $C_i = \frac{F_i L_i}{\Lambda^2}$ (F_i new coupling, L_i loop factor), one obtains lower bounds on NP scale Λ.

[N. Carrasco *et al*, '12]

$$R_i = \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle}$$

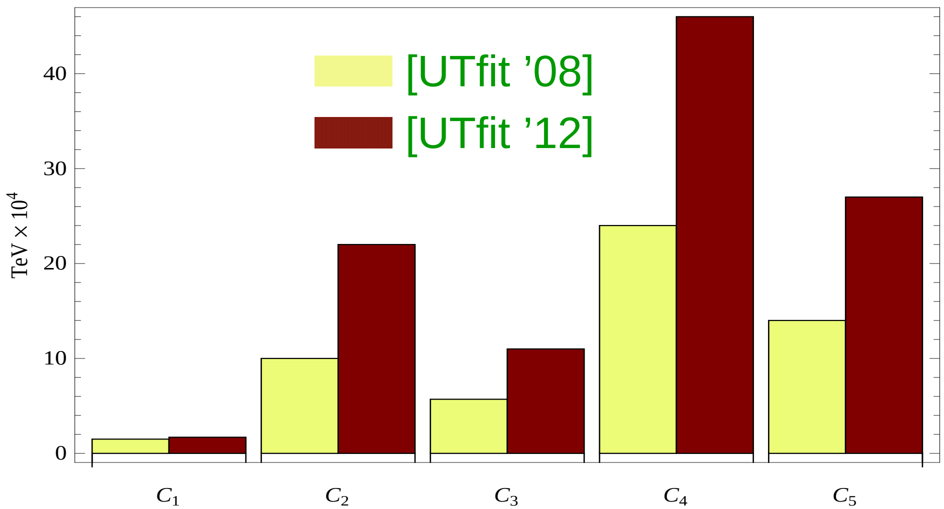
$$O_1 = [\bar{s}\gamma_{\mu\,L} d]\,[\bar{s}\gamma_{\mu\,L} d]$$

$$O_2 = [\bar{s}P_L d]\,[\bar{s}P_L d] \qquad O_3 = [\bar{s}^\alpha P_L d^\beta]\,[\bar{s}^\beta P_L d^\alpha]$$

$$O_4 = [\bar{s}P_L d]\,[\bar{s}P_R d] \qquad O_5 = [\bar{s}^\alpha P_L d^\beta]\,[\bar{s}^\beta P_R d^\alpha]$$

$\overline{\mathrm{MS}}$ scheme at 2 GeV

R_1	R_2	R_3	R_4	R_5
1	-14.0(5)	4.8(3)	24.2(8)	5.9(4)



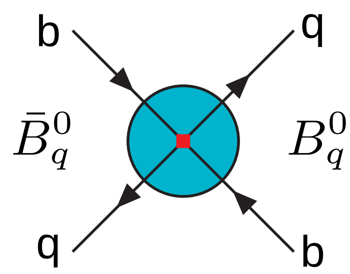
$B_q^0 - \bar{B}_q^0$ mixing

B_q^0 and $\bar{B}_q^0$ are a mixture of the CP eigenstates $B_{L\,q}$ and $B_{S\,q}$

$$\langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle = \frac{G_F^2 m_W^2}{16\pi^2} \lambda_{tq}^2 S_0(x_t) \tilde{\eta}_{2B}(\mu) \langle \bar{B}_q^0 | Q_{LL}^{\Delta B=2} | B_q^0 \rangle (\mu)$$

$\lambda_{tq} = V_{tq}^* V_{tb}$ S_0: Inami-Lim function $\tilde{\eta}_{2B}$: Wilson coefficient $Q_{LL}^{\Delta B=2} = [\bar{b}\gamma_{\mu\,L} q]\,[\bar{b}\gamma_{\mu\,L} q]$

Usual parametrization: $\langle \bar{B}_q^0 | Q_{LL}^{\Delta B=2} | B_q^0 \rangle (\mu) = \frac{8}{3} f_{B_q}^2 m_{B_q}^2 B_{B_q}(\mu)$

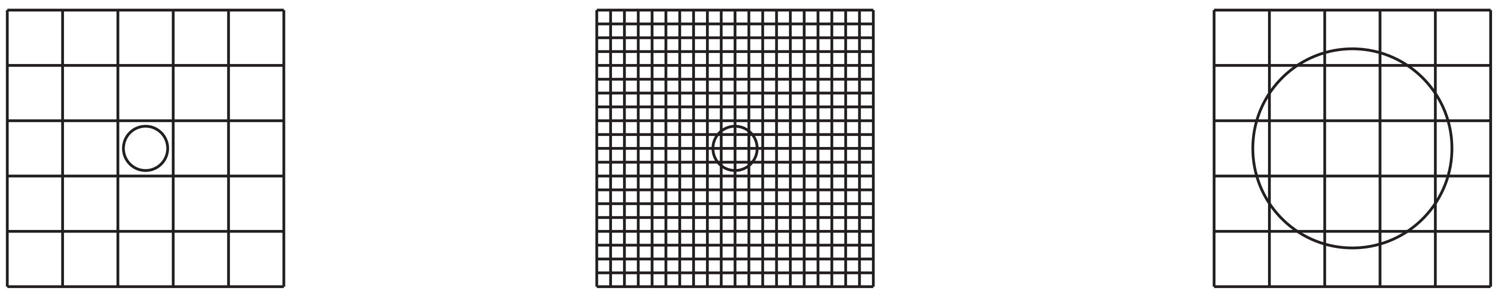


Mass difference: $\Delta m_q = \frac{G_F^2 m_W^2 m_{B_q}}{16\pi^2} |\lambda_{tq}^2|^2 S_0(x_t) \eta_{2B} f_{B_q}^2 \hat{B}_{B_q}$

$\hat{B}_{B_q}$ is the RGI B_{B_q} parameter

SU(3) breaking ratio: $\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_q}}}$

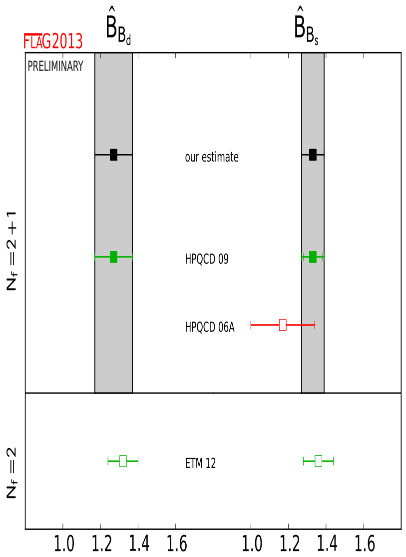
Issue for B-physics on the lattice: systematics coming from large discretisation effects ($\Lambda_{\text{Compt}} \sim 1/m_Q$).

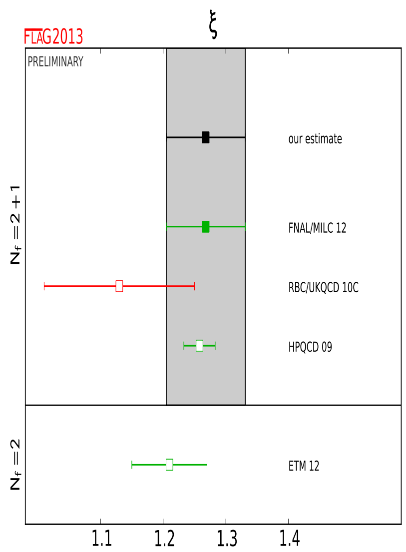


Cut-off Effects cut-off effects cut-off effects

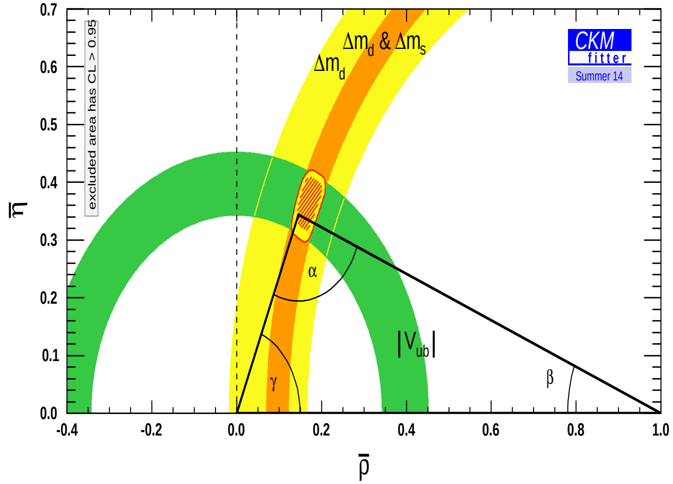
Several strategies are proposed in the literature to deal with those cut-off effects:

- Use NRQCD to describe the heavy quark [P. Lepage and B. Thacker, '91]; though, no continuum limit when the theory is regularised on the lattice
- Define an action with counterterms that are tuned to get $\mathcal{O}(a)$, $\mathcal{O}(am_Q)$ and $\mathcal{O}(\alpha_s(am_Q)^n)$ improvements [A El Khadra *et al*, '96; N. Christ *et al*, '06]
- Computation within Heavy Quark Effective Theory, the effective couplings are determined non perturbatively by imposing matching conditions between QCD and HQET [J. Heitger and R. Sommer, '03]
- Computation within QCD: use of the HQET scaling laws to interpolate easily a quantity between the charm region and the (exactly known) infinite heavy mass limit [B. B. *et al*, '09]



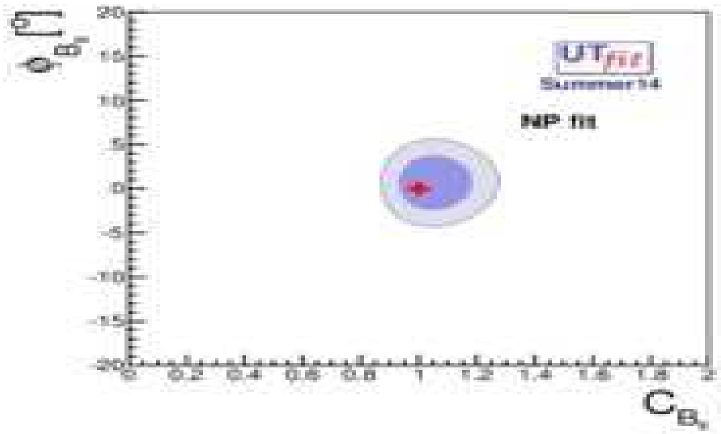


constraints from CP conserving quantities



It is remarkable that constraints from CP violating and CP conserving quantities are fully consistent: great success of the Standard Model!

Thanks to the experimental and theoretical improvements, precision tests can be realised to discover New Physics effects, especially in the B_s sector. With $\Delta m_s^{\exp} = C_{B_s} \Delta m_s^{\rm SM}$ and $\phi_s^{\exp} = \beta_s^{\rm SM} - \phi_{B_s}$:



Global fits are consistent with the SM. As in the K sector, lower bounds on NP scale can be put using $B - \bar{B}$ mixing [N. Carrasco *et al*, '13].

$$R_i^{(q)} = \frac{\langle \bar{B}_q^0 | O_i^q | B_q^0 \rangle}{\langle \bar{B}_q^0 | O_1^q | B_q^0 \rangle}$$

$$O_1^q = [\bar{b}\gamma_{\mu\ L} q]\,[\bar{b}\gamma_{\mu\ L} q]$$

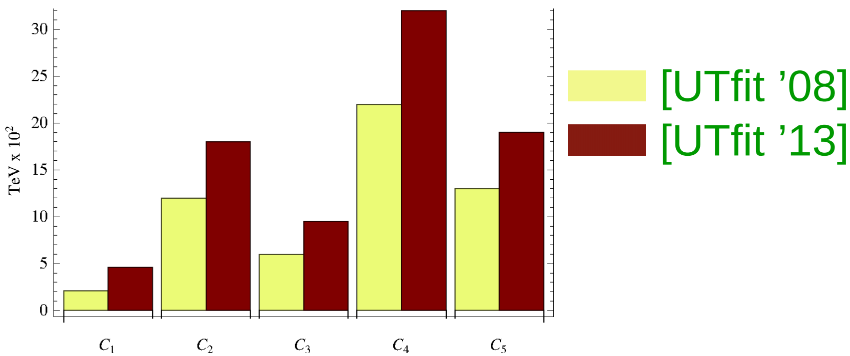
$$O_2^q = [\bar{b}P_L q]\,[\bar{b}P_L q] \qquad O_3^q = [\bar{b}^\alpha P_L q^\beta]\,[\bar{b}^\beta P_L q^\alpha]$$

$$O_4^q = [\bar{b}P_L q]\,[\bar{b}P_R q] \qquad O_5^q = [\bar{b}^\alpha P_L q^\beta]\,[\bar{b}^\beta P_R q^\alpha]$$

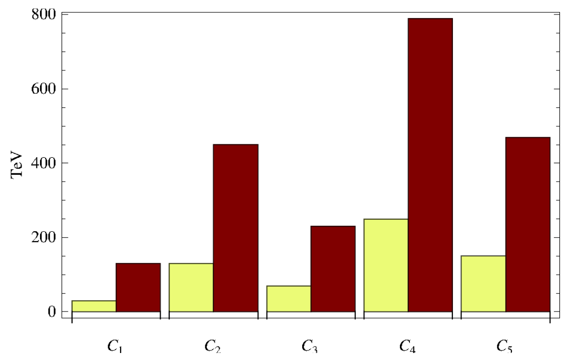
$\overline{\rm MS}$ scheme at m_b

R_1^d	R_2^d	R_3^d	R_4^d	R_5^d
1	0.85(5)	1.04(3)	1.12(8)	1.73(4)
R_1^s	R_2^s	R_3^s	R_4^s	R_5^s
1	0.85(5)	1.03(3)	1.08(8)	1.83(4)

NP scale based on B_d sector constraints

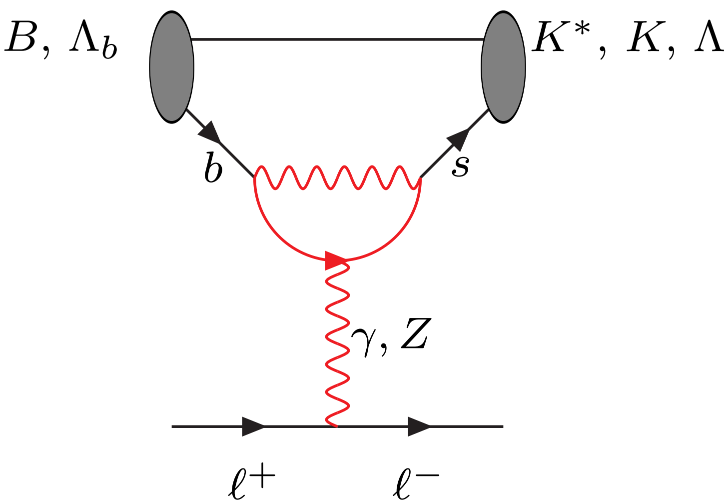


NP scale based on B_s sector constraints



$b \to s$ transitions

Those processes are among the most important to test SM extensions. $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$ rare events offer a rich set of constraints on New Physics scenarios.



$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=7,9,10,S,P} (C_i O_i + C_i' O_i')$$

$$O_7^{(\prime)} = \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu} P_{L(R)} b\, F^{\mu\nu}$$
$$O_9^{(\prime)} = \frac{\alpha}{4\pi} \bar{s}\gamma_{\mu\ L(R)} b\, \bar{l}\gamma^\mu l$$
$$O_{10}^{(\prime)} = \frac{\alpha}{4\pi} \bar{s}\gamma_{\mu\ L(R)} b\, \bar{l}\gamma^\mu\gamma^5 l$$
$$O_S^{(\prime)} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l}l$$
$$O_P^{(\prime)} = \frac{\alpha}{4\pi} m_b \bar{s} P_{R(L)} b\, \bar{l}\gamma^5 l$$

– 3 form factors $T_{1,2,3}(q^2)$ associated to $\langle K^*(\epsilon_{(\lambda)}, k)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle$

– 2 form factors $f_{+,0}(q^2)$ associated to $\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle$

– 1 form factor $f_0(q^2)$ associated to $\langle K(k)|\bar{s}b|B(p)\rangle$

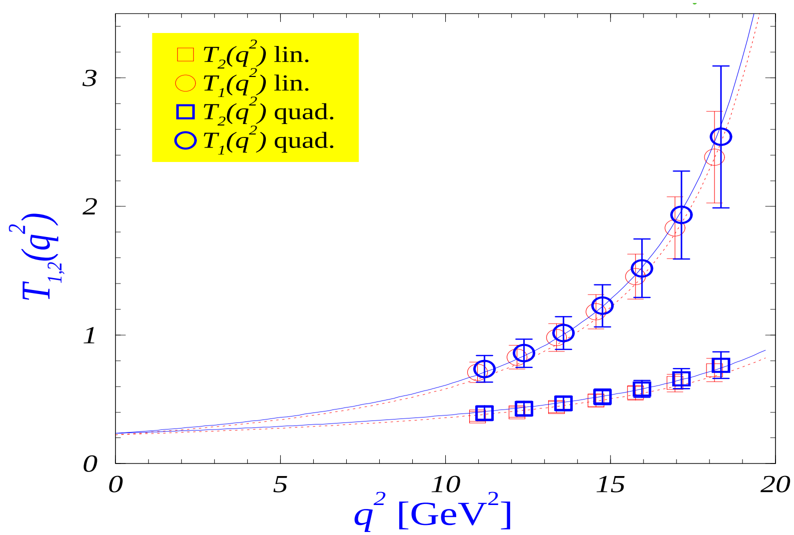
– 1 form factor $f_T(q^2)$ associated to $\langle K(k)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle$

– in HQET, 2 form factors $F_{1,2}(p' \cdot v)$ associated to $\langle \Lambda(p', s')|\bar{s}\Gamma h|\Lambda_h(v, 0, s)\rangle$

$B \to K^*\gamma$: extrapolation of the lattice results to $q^2 = 0$ (emission of a real photon)

$$T_1(q^2) = \frac{T(0)}{\left(1 - q^2/m_{B_s^*}^2\right)\left(1 - \alpha q^2/m_{B_s^*}^2\right)} \quad T_2(q^2) = \frac{T(0)}{\left[1 - q^2/\left(\beta m_{B_s^*}^2\right)\right]}$$ [D. Becirevic, A. Kaidalov, '98]

[D. Becirevic *et al*, '06]

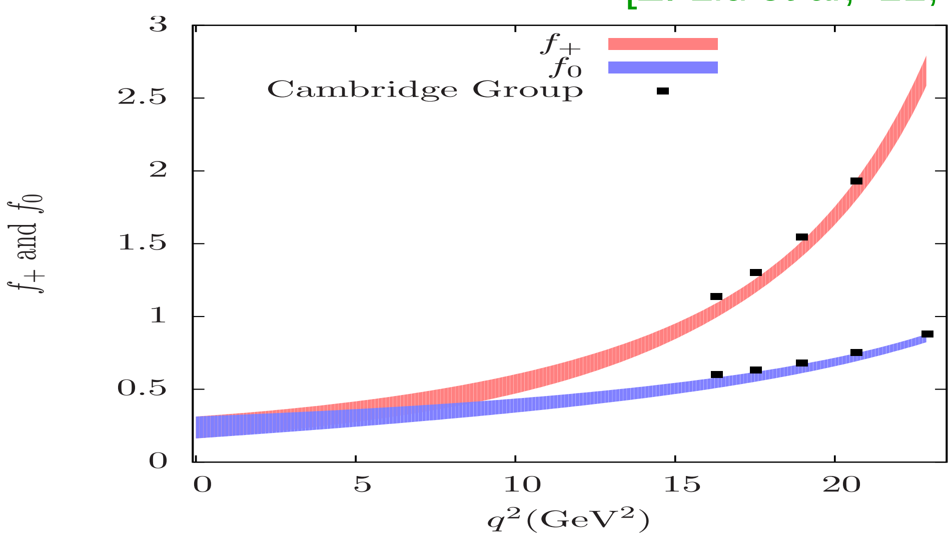


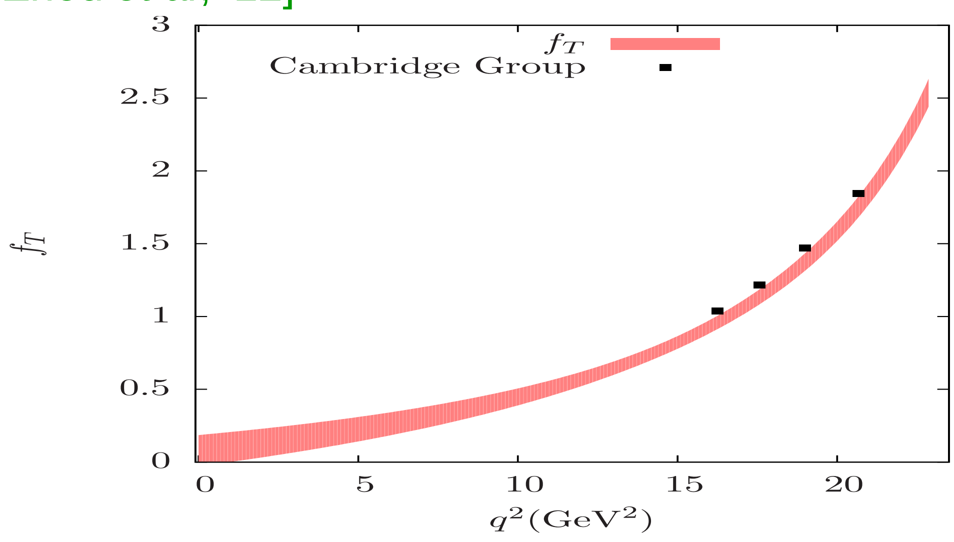
$N_f = 0$: $T(0) = 0.23(3)$ [D. Becirevic *et al*, '06]

$N_f = 2+1$: $T(0) = 0.17(3)$ [Z. Liu *et al*, '11]

$B \to K\ell^+\ell^-$: the lattice sets a normalization point at $q^2_{\rm max}$, the z expansion (for instance) can be used at other q^2

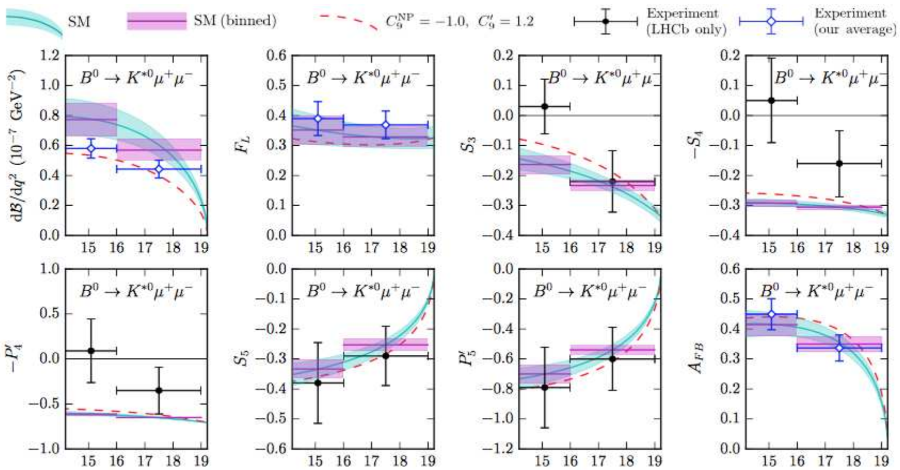
[Z. Liu *et al*, '11; R. Zhou *et al*, '12]



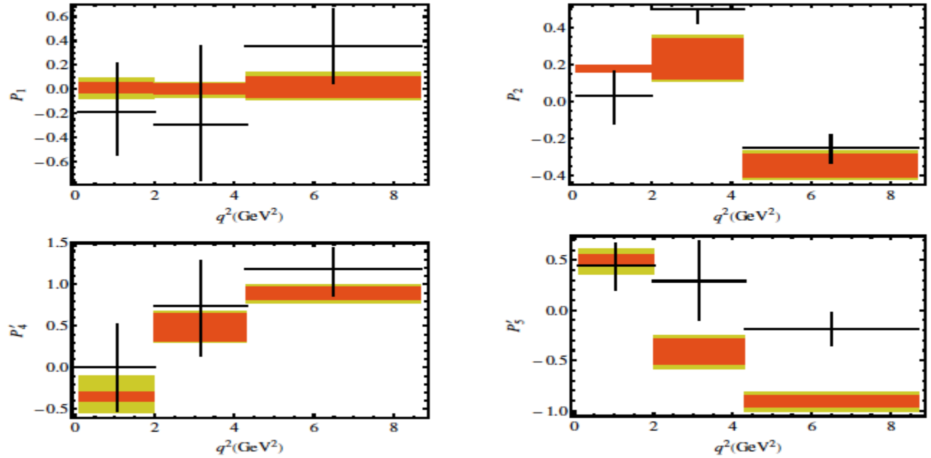


$B \to K^* \mu^+ \mu^-$: has received a lot of attention, discrepancy between theory and experiments in some combinations of observables under deep investigation:

[R. Horgan *et al*, '14]



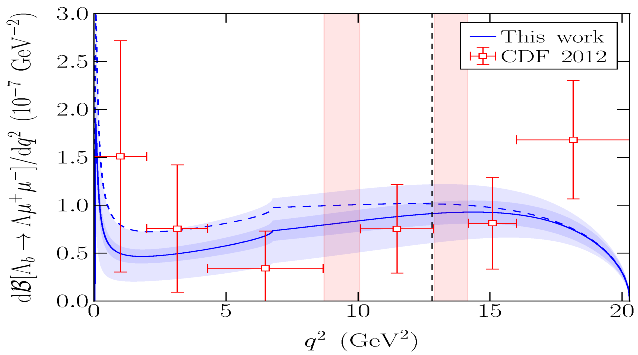
[S. Descotes-Genon *et al*, '14]



Teke into account (factorizable and non factorizable) power corrections in $1/m_b$, as well as $c\bar{c}$ loop effects. The uncertainties are smaller than the disagreement between SM predictions and experiment.

$\Lambda_b \to \Lambda \ell^+ \ell^-$: the matching of HQET to QCD is applied to compute the partial widths. A smooth interpolation is applied in q^2 except in regions of the phase space where long-distance effects are large (charmonium resonances)

[W. Detmold *et al* '12]



So far, no sign of NP seen in $\Lambda_b \to \Lambda \ell^+ \ell^-$.
In December 2012, LHCb data were analysed to confirm that statement.

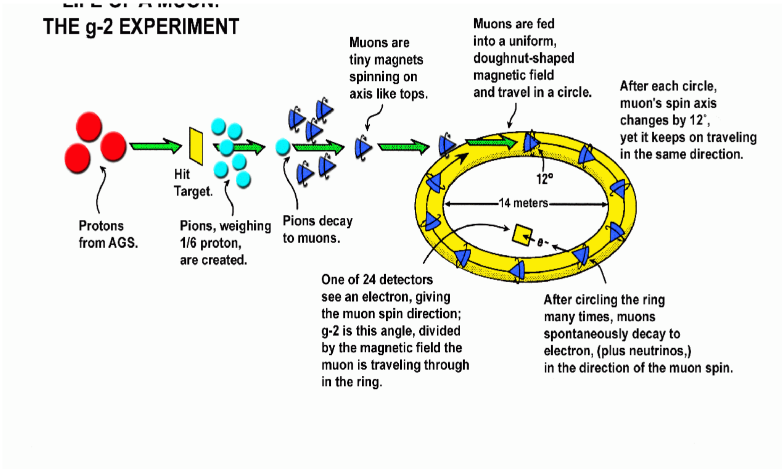
Anomalous magnetic moment of the muon

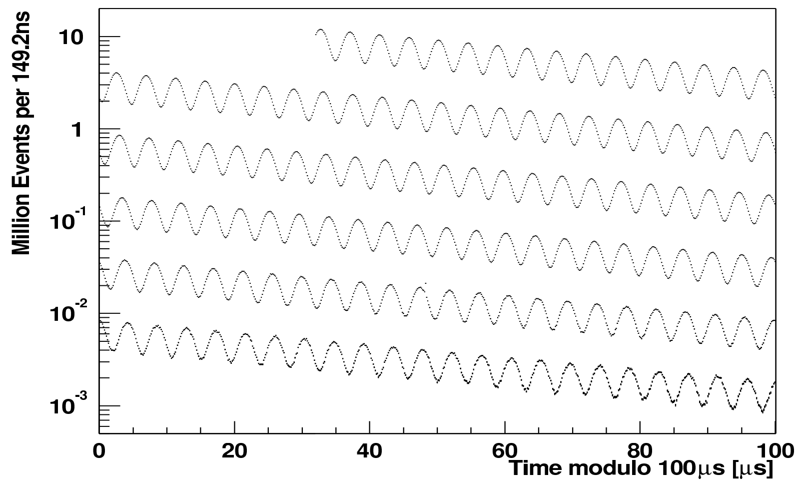
2 ways in the search of New Physics: direct detection at EWSB scale and measurement of indirect effects at the GeV scale.

A typical example: muon $g-2$

$$\vec{\mu}_l = g_l Q \frac{\sigma}{2} \qquad a_l = \frac{g_l - 2}{2}$$

[Muon g-2 Collaboration]

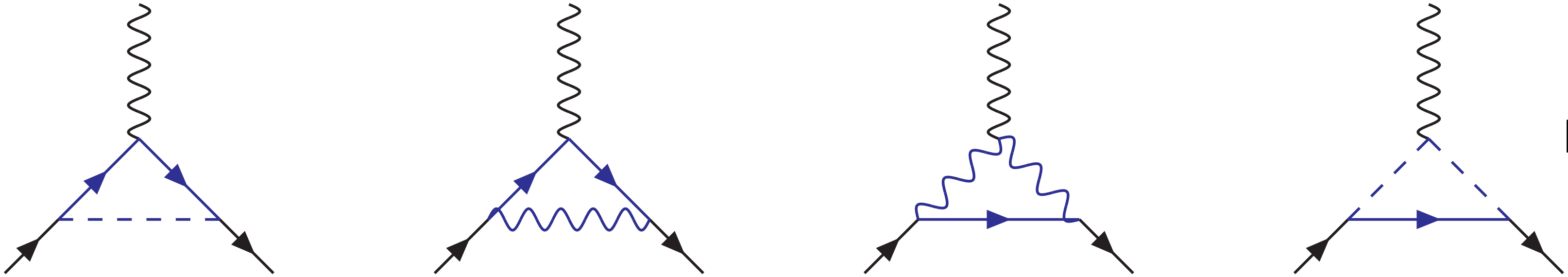




$a_\mu^{\text{exp}} = 1.16592089(63) \times 10^{-11} \quad a_\mu^{\text{SM}} = 1.16591803(49) \times 10^{-11}$

More than 3σ of discrepancy! [F. Jegerlehner and A. Nyffeler, ’09; M. Benayoun *et al*, ’12]

Indication of New Physics?

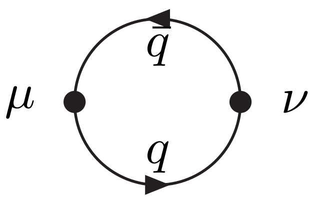


BSM $\mathcal{O}(m_\mu^2/M^2)$

Let's have a look at the SM theoretical error budget:

$a_\mu/10^{-11}$	central value	error
QED	116584719.0	0.2
weak	154.0	1.0
hadronic VP (e^+e^-, τ decay)	6837.0	42.0
light-by-light (model)	115.0	40.0
SM	116591803.0	49.0
exp	116592089.0	63.0

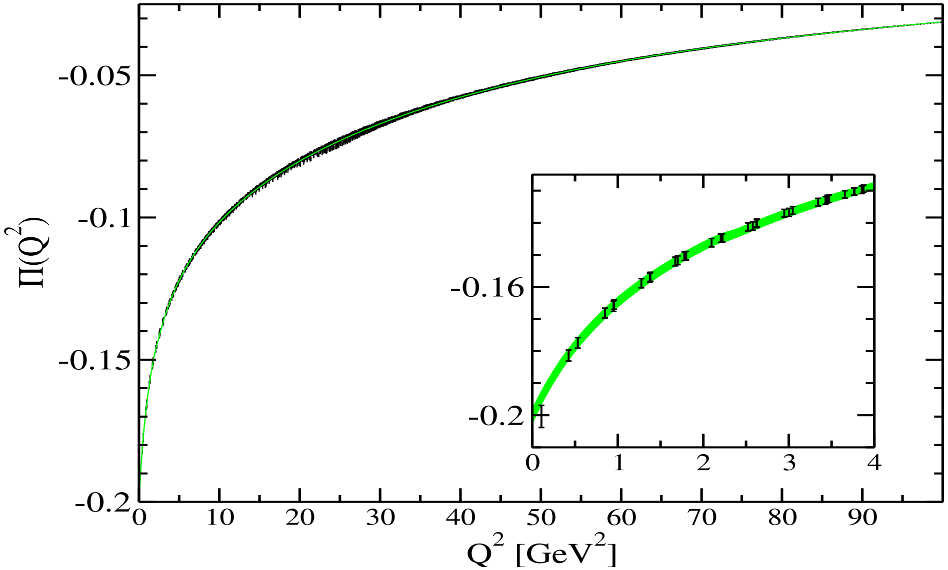
Hadronic contribution to Vacuum Polarisation brings the largest uncertainty.



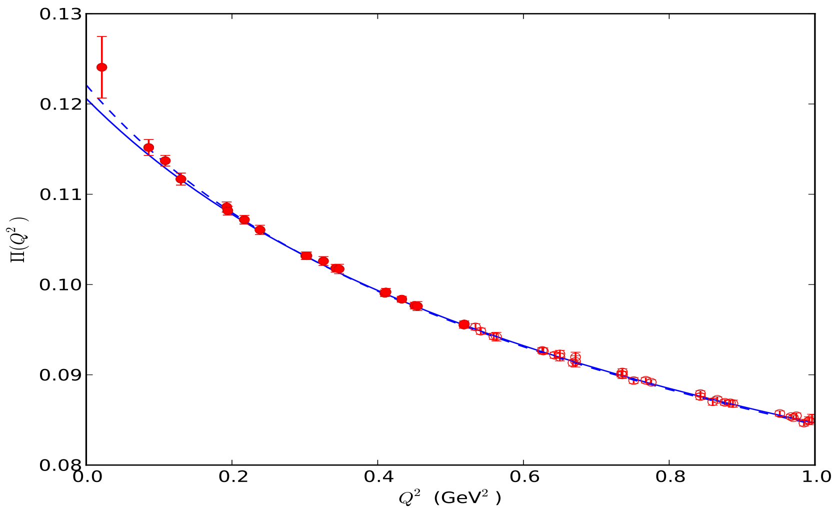
$$\Pi_{\mu\nu}(q^2) = \int e^{-iq\cdot(x-y)} \langle j_\mu(y) j_\nu(x)\rangle_{\rm QCD} \equiv (\delta_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$
$$a_\mu^{\rm hadr} = (\alpha/\pi)^2 \int_0^\infty dq^2 f(q^2)(\Pi(q^2) - \Pi(0))$$

Large contribution from $q^2 = 0$ region. no direct estimate on the lattice $\Longrightarrow$ extrapolate $\Pi(q^2)$.

Vector Meson Dominance [X. Feng *et al*, '11]



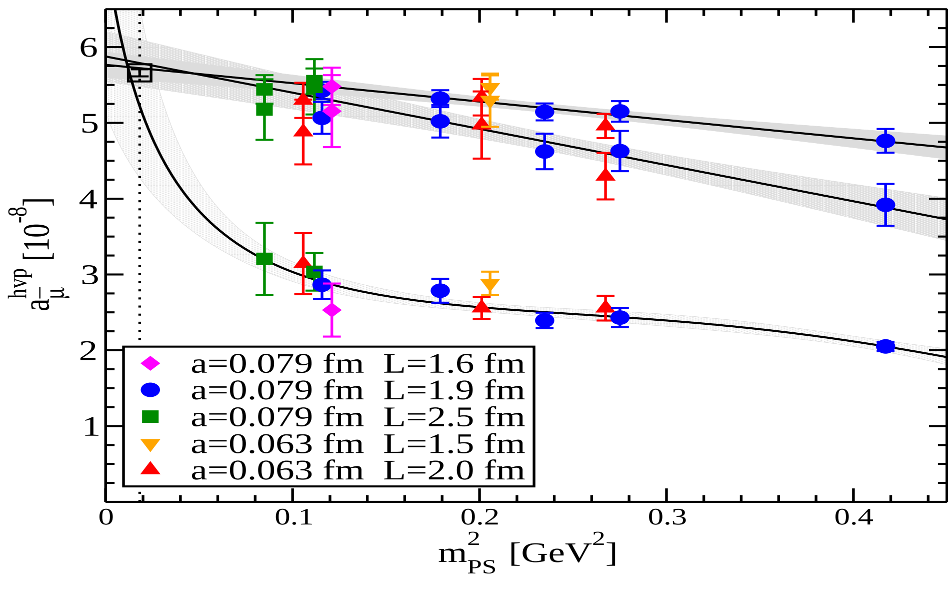
Padé Approximation [T. Blum *et al*, '12]

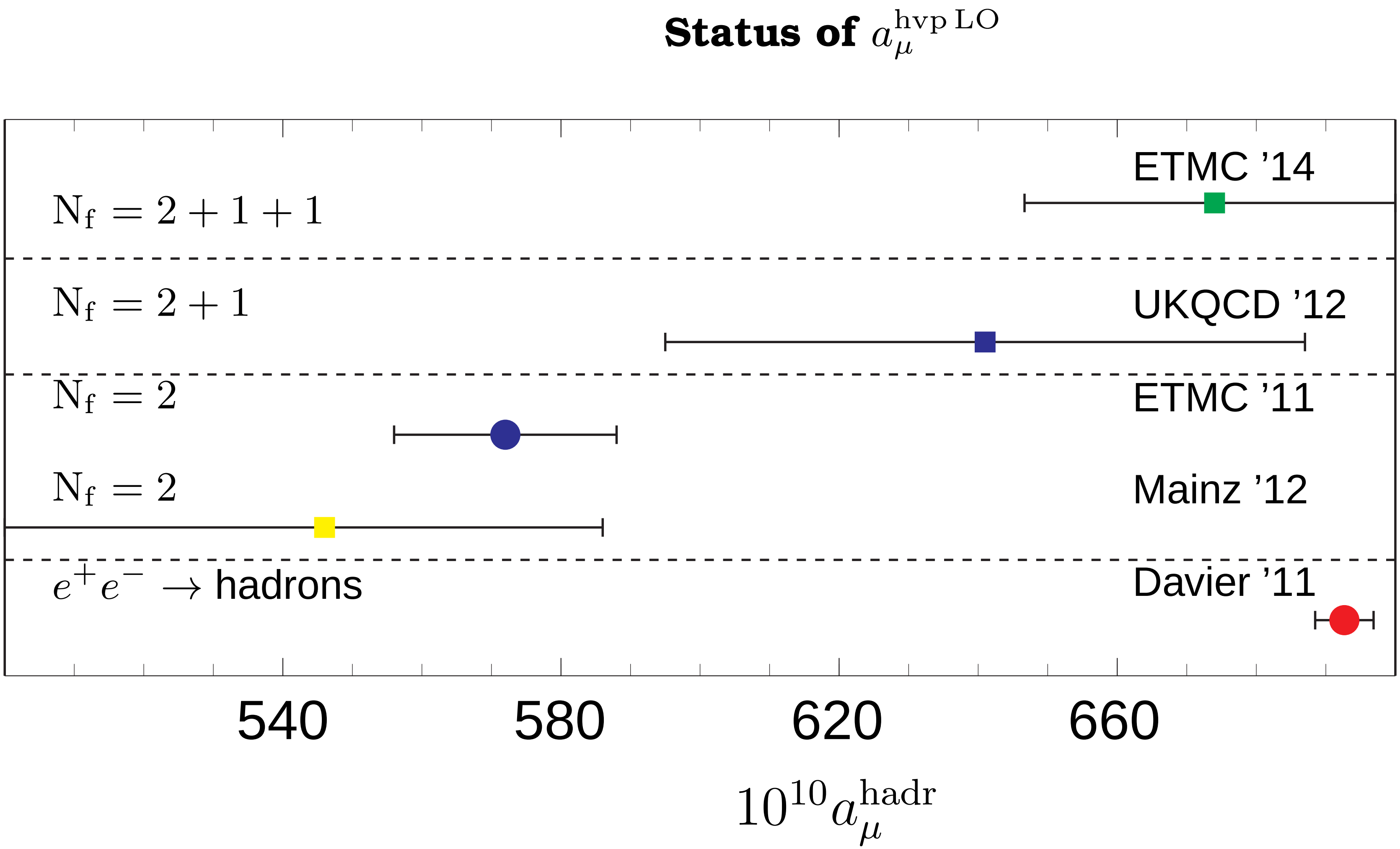


A smooth chiral extrapolation of $a_\mu^{\rm hvp}$ is feasible:

$$a_{\tilde{\mu}}^{\rm hvp} = \int_0^\infty \frac{dq^2}{q^2}\, w(q^2/m_\mu^2\, H_{\rm phys}^2/H^2)\, \Pi_R(q^2) \quad H = m_V, g_V m_V, \ldots \to H_{\rm phys} \text{ at } m_{\rm PS} \to m_\pi$$

[X. Feng *et al*, '11]

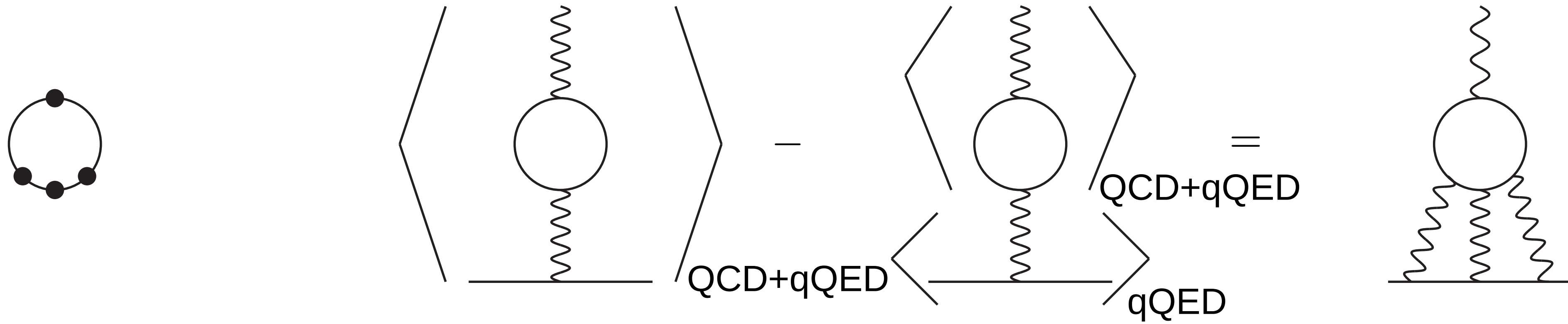




Status of $a_\mu^{\mathrm{hvp\,LO}}$
ETMC '14
$\mathrm{N_f} = 2+1+1$
$\mathrm{N_f} = 2+1$
UKQCD '12
$\mathrm{N_f} = 2$
ETMC '11
$\mathrm{N_f} = 2$
Mainz '12
$e^+e^- \to$ hadrons
Davier '11
540
580
620
660
$10^{10} a_\mu^{\mathrm{hadr}}$

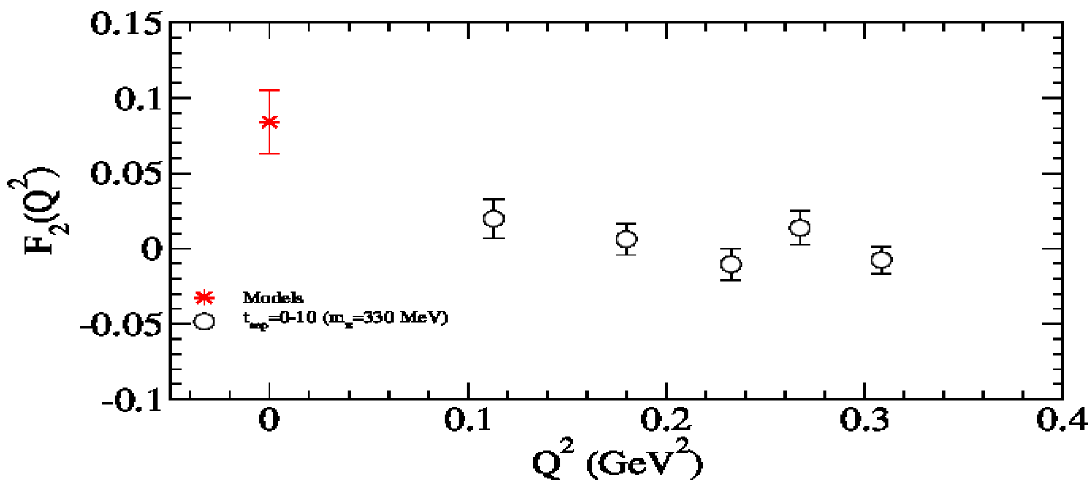
Hadronic light by light

Very few tries to extract light-by-light: 4-pts correlation functions or a tricky combination of correlation functions in QCD+quenched QED [M. Hayakawa *et al*, '05; T. Blum, '12]



$$\langle p', s' | j_\mu | p, s \rangle \equiv -\bar{u}(p', s') \left(F_1(q^2)\gamma_\mu + i \frac{F_2(q^2)}{2m_\mu} \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \right) u(p, s) \quad a_\mu^{\mathrm{hlbl}} = F_2(0)$$

[T. Blum *et al*, '15]

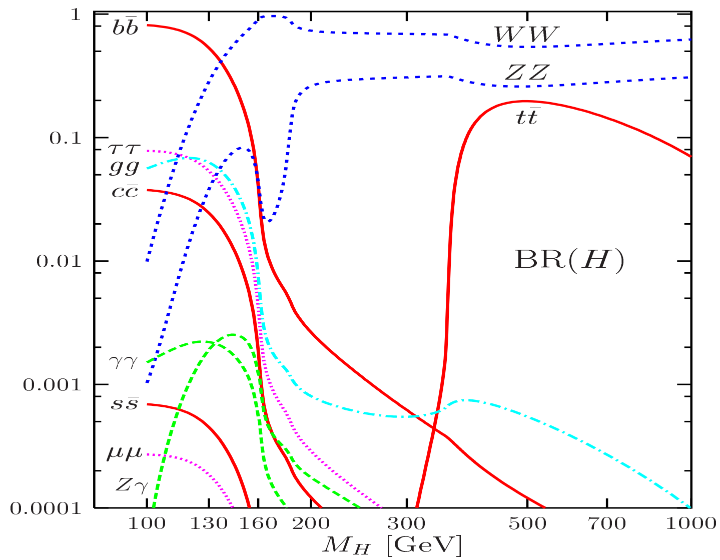


b coupling to the BEH boson

Phenomenological considerations

The main Higgs boson decay channel at the mass scale $m_H = 126$ GeV is $H \to b\bar{b}$.

[A. Djouadi, '05]

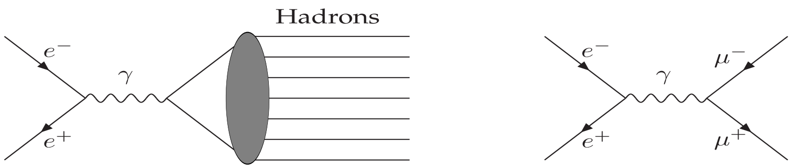


$$\Gamma(H \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} m_H m_b^2(\overline{\mathrm{MS}}, m_H) \left[1 + \underbrace{\Delta_{bb} + \Delta_H^2}_{\text{QCD corr.}} \right]$$

Uncertainty of $\sim$ 2.5% on the width is expected at ILC, the major part coming from m_b.

Analytical extractions of m_b

QCD sum rules and dispersion relations are widely used in the literature [A. Hoang and M. Jamin, '04].



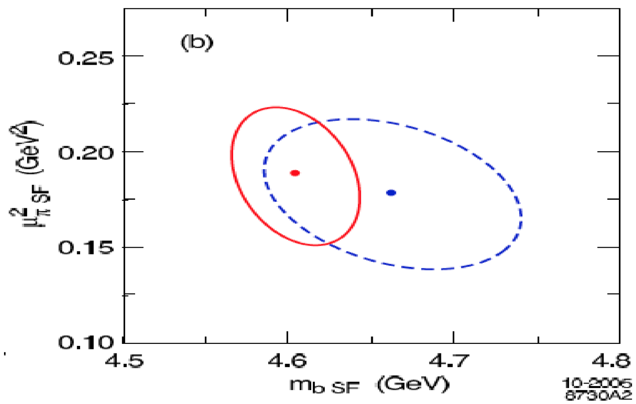
$$R_{bb}(s) = \frac{\sigma(e^+e^- \to b\bar{b}+X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$P_n^{\rm th} = \int \frac{ds}{s^{n+1}} R_{bb}(s) \equiv P_n^{\rm pert} + P_n^{\rm non\ pert}$$

Comparison between $P_n^{\rm th}$ and experimental data gives m_b.

Analysing the Υ spectrum by the $Q\bar{Q}$ potential, using perturbation theory in terms of $\alpha_s(m_b)$, is popular as well [N. Brambilla *et al*, '01].

Inclusive B decays, with the help of Heavy Quark Expansion, offer a further set of m_b estimates, together with V_{cb}, after the fit of experimental data [O. Buchmuller, H. Flächer, '05].



$$\Gamma_{SL}(B \to X_c l\nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{EW})A_{\rm pert}(r,\mu)$$

$$\times \left[z_0(r)\left(1 - \frac{\mu_\pi^2 - \mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{m_b}}{2m_b^2}\right) - 2(1-r)^4 \frac{\mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{m_b}}{m_b^2} \right.$$

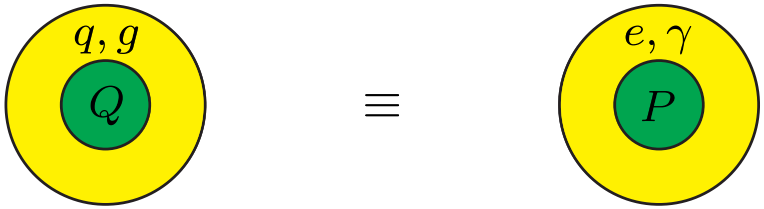
$$\left. + d(r)\frac{\rho_D^3}{m_b^3} + \mathcal{O}(1/m_b^4)\right] \qquad r = m_c^2/m_b^2$$

Heavy Quark Effective Theory

Effective theory "derived" by expanding in $\frac{\Lambda_{QCD}}{m_Q}$ the Lagrangian and currents of QCD.

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(iv\cdot D)h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}^{\rm stat}_{\rm HQET} + \mathcal{O}(\Lambda_{\rm QCD}/m_Q) \quad p_Q = m_Q\, v + k$$

Symmetry SU(2N$_{\rm h}$) for $\mathcal{L}^{\rm stat}_{\rm HQET}$: $\rm flavor \times spin$



Heavy-light meson — Atom of hydrogen

Angular momentum: $J = \frac{1}{2} \oplus j_l$.

Spectroscopy: heavy-light mesons are put together in doublets.

$H = B, D$:

j_l^P	J^P	orbital excitation
$\frac{1}{2}^-$	0^- 1^-	H H^*
$\frac{1}{2}^+$	0^+ 1^+	H_0^* H_1^*
$\frac{3}{2}^+$	1^+ 2^+	H_1 H_2^*

$E(j_l^P) = m_Q + \Lambda_{j_l^P} - \frac{\lambda_1(j_l^P) - 2(J^2 - 1/4 - j_l^2)\lambda_2(j_l^P)}{2m_Q}$:

$\Lambda_{j_l^P}$, $\lambda_1(j_l^P)$ and $\lambda_2(j_l^P) \ll m_Q$ are defined in terms of HQET hadronic matrix elements.

$m_{B^*} - m_B \sim 46$ MeV $\quad m_{D^*} - m_D \sim 142$ MeV

$m_{B^*}^2 - m_B^2 \sim 0.49\ {\rm GeV}^2 \quad m_{D^*}^2 - m_D^2 \sim 0.55\ {\rm GeV}^2$

HQET regularised on the lattice

The goal is to extract B physics quantities from lattice computation using Heavy Quark Effective Theory expanded up to $1/m$.

$$\mathcal{L}^{\mathrm{HQET},1/\mathrm{m}} = \mathcal{L}^{\mathrm{stat}} + m_{\mathrm{bare}}\mathcal{O}^{\mathrm{c.t.}} - \omega_{\mathrm{kin}}\mathcal{O}^{\mathrm{kin}} - \omega_{\mathrm{spin}}\mathcal{O}^{\mathrm{spin}}$$

$$A_0^{\mathrm{HQET},1/\mathrm{m}} = Z_A^{\mathrm{HQET}}[A_0^{\mathrm{stat}} + c_A^{(1)}A_0^{(1)} + c_A^{(2)}A_0^{(2)}]$$

$$\mathcal{L}^{\mathrm{stat}} = \bar{\psi}_h D_0 \psi_h \qquad \mathcal{O}^{c.t.} = \bar{\psi}_h\psi_h \qquad \mathcal{O}^{\mathrm{kin}} = \bar{\psi}_h \mathbf{D}^2\psi_h \qquad \mathcal{O}^{\mathrm{spin}} = \bar{\psi}_h \sigma\cdot\mathbf{B}\psi_h$$

$$A_0^{\mathrm{stat}} = \bar{\psi}_l\gamma_0\gamma^5\psi_h \qquad A_0^{(1)} = \bar{\psi}_l\tfrac{1}{2}\gamma^5\gamma_i(\nabla_i - \overleftarrow{\nabla}_i)\psi_h \qquad A_0^{(2)} = \partial_i[\bar{\psi}_l\gamma_i\gamma^5\psi_h]$$

The HQET integral path is computed by keeping $e^{-(S^{\mathrm{stat}}+S^{\mathrm{YM+light}})}$ as the Boltzmann weight.

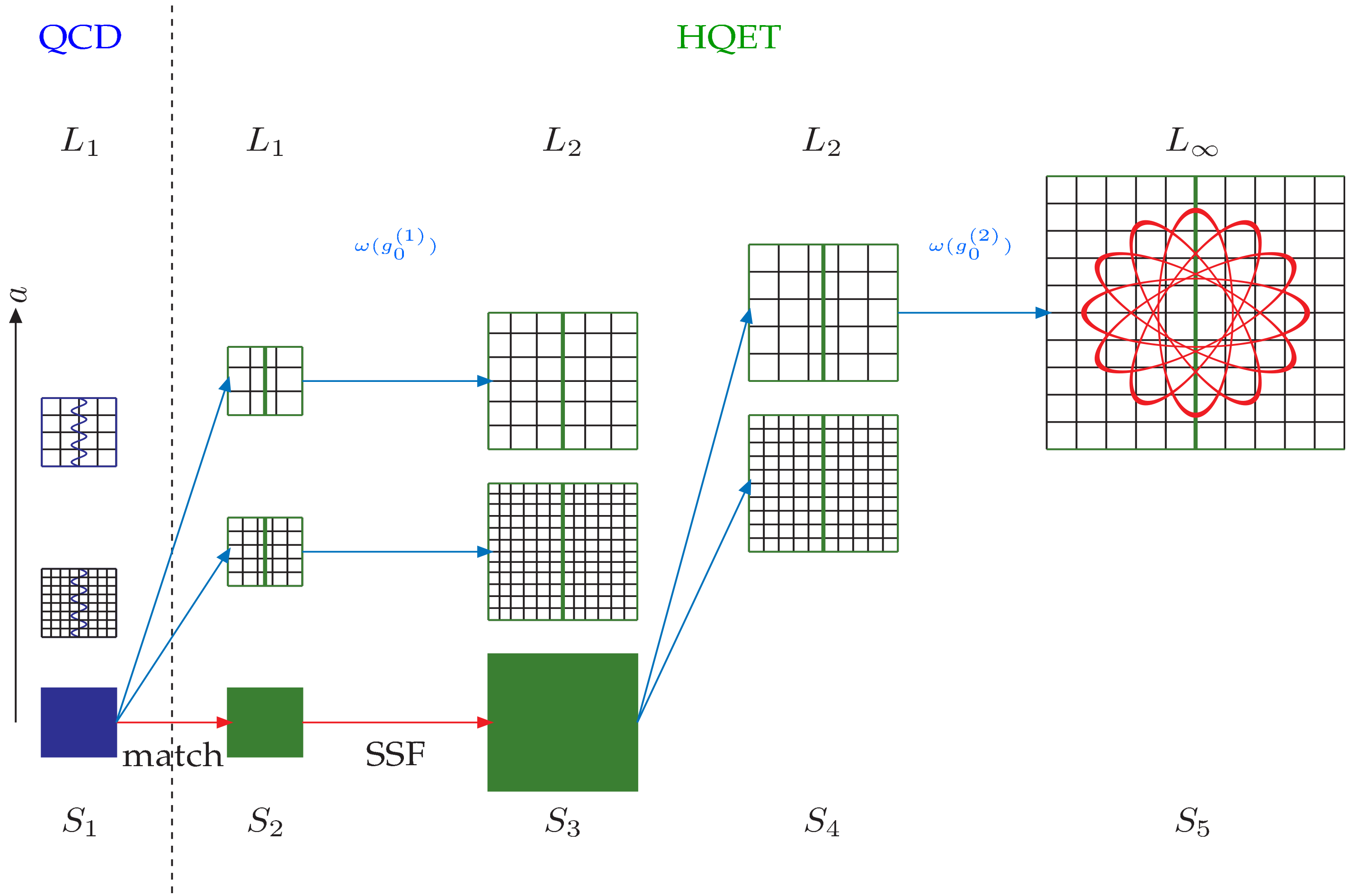
$$\langle O\rangle_{\mathrm{HQET}} \equiv \frac{1}{\mathcal{Z}^{\mathrm{stat}}}\int \mathcal{D}\Phi O e^{-S^{\mathrm{HQET}}-S^{\mathrm{YM+light}}}$$
$$= \langle O\rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}}a^4\sum_x\langle O O_{\mathrm{kin}}(x)\rangle_{\mathrm{stat}} + \omega_{\mathrm{spin}}a^4\sum_x\langle O O_{\mathrm{spin}}(x)\rangle_{\mathrm{stat}}$$

$$\langle F\rangle_{\mathrm{stat}} \equiv \frac{1}{\mathcal{Z}^{\mathrm{stat}}}\int\mathcal{D}\Phi F e^{-S^{\mathrm{stat}}-S^{\mathrm{YM+light}}}$$

$$m_B = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}}E^{\mathrm{kin}} + \omega_{\mathrm{spin}}E^{\mathrm{spin}}$$

$$f_B\sqrt{m_B/2} = Z_A^{\mathrm{HQET}}(1+b_A^{\mathrm{stat}}m_q)p^{\mathrm{stat}}\left(1+\omega_{\mathrm{kin}}p^{\mathrm{kin}} + \omega_{\mathrm{spin}}p^{\mathrm{spin}} + c_A^{(1)}p^{\mathrm{A}^{(1)}}\right)$$

Extraction of m_b in HQET: sketch of the strategy

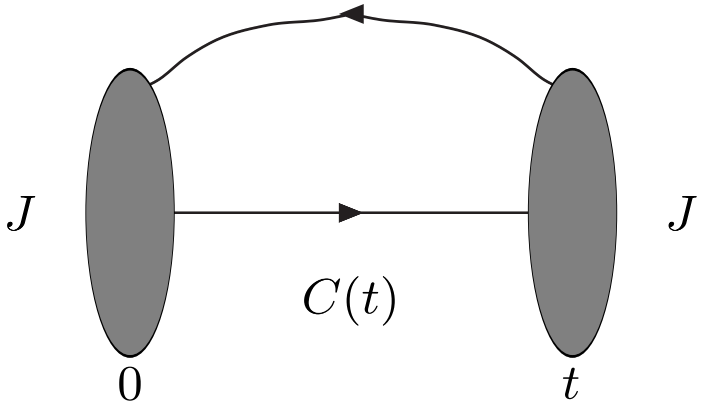


Ultraviolet divergences of HQET are absorbed in the ω_k coefficients, determined from a Schrödinger Functional set up.

Hadronic matrix elements are extracted with a particular care to excited states.

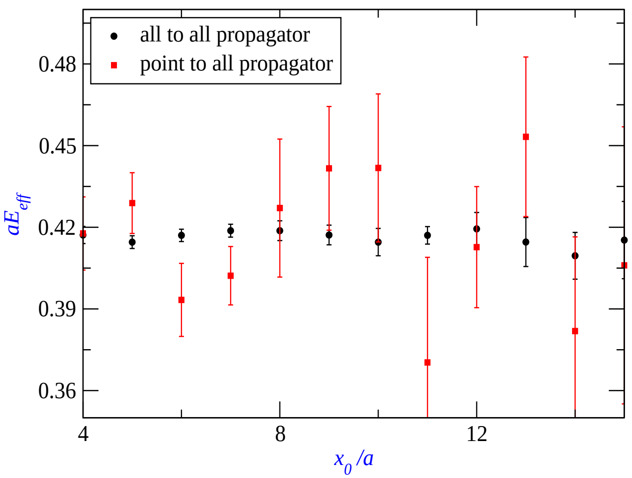
Extraction of HQET hadronic matrix elements (S_5)

$$m_B = m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} E^{\rm kin} + \omega_{\rm spin} E^{\rm spin}$$



$$C_{JJ}(t) = \sum_{\vec{x}} \langle \Omega | \mathcal{T}[J(\vec{x},t) J^\dagger(0)] | \Omega \rangle$$
$$= \sum_n \frac{\mathcal{Z}_n^2 e^{-E_n t}}{2E_n}$$
$$\mathcal{Z}_n = \langle \Omega | J | n \rangle \qquad \langle n | m \rangle = 2E_n \delta_{mn}$$
$$C_{JJ}(t) \xrightarrow{(E_1 - E_0)t \gg 1} \frac{\mathcal{Z}_0^2 e^{-E_0 t}}{2E_0}$$

Issue: at $t \gtrsim 1$ fm, the statistical noise enters severely in competition with the usable signal.



All to all propagators increase dramatically the statistical efficiency [C. Michael and J. Peisa, ’98] [J. Foley et al, ’05]

Example of a B_s meson 2pts correlator

$$aE^{\rm eff}(x_0) = -\ln[C_{PP}(x_0 + a)/C_{PP}(x_0)]$$

$\# = 50 \quad N_f = 0 \quad a \sim 0.1$ fm $\quad L \sim 1.5$ fm $\quad m_q \sim m_s$

We are now in a good position to study the systematic effects induced by excited states.

$m_{B'} - m_B \sim 500$ MeV $\quad m_{B''} - m_{B'} \sim 200$ MeV

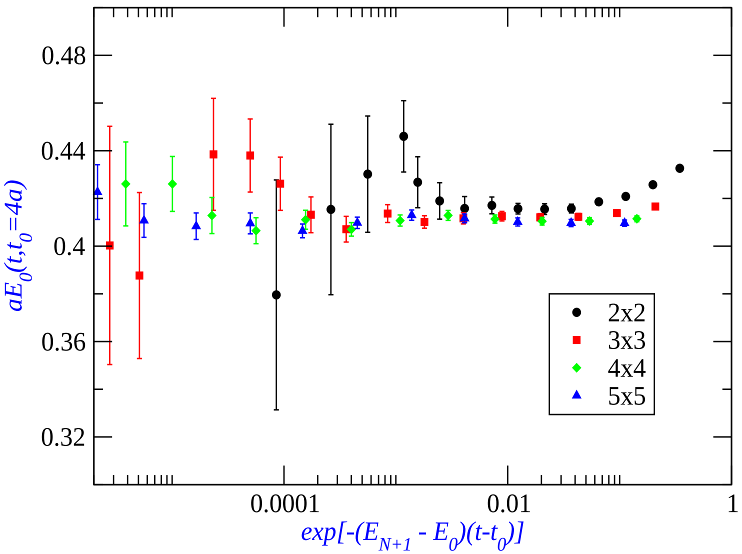
The Variational method

It is an appealing approach to define an operator $O^n_{J^P}$ weakly coupled to other states than $|n\rangle$ [C. Michael, '85] [M. Lüscher and U. Wolff, '90].

– Compute an $N \times N$ matrix of correlators $C^{ij}_{PP}(t) = \sum_{\vec{x},\vec{y}} \langle \Omega | \mathcal{T}[O^i_{J^P}(\vec{x},t) O^j_{J^P}(\vec{y},0)] | \Omega \rangle$ with $O^i_{J^P}(\vec{x},t) = \sum_{\vec{z}} \bar{q}(\vec{x},t) [\Gamma \times \Phi(|\vec{x}-\vec{z}|)]^i_{J^P} q(\vec{z},t)$

– Solve the generalised eigenvalue problem $C^{ij}(t)\, v^j_n(t,t_0) = \lambda_n(t,t_0)\, C^{ij}(t_0)\, v^j_n(t,t_0)$

– $\lambda_n(t,t_0) \sim e^{-E_n\,(t-t_0)}$



$\# = 100 \quad N_f = 0 \quad a \sim 0.1$ fm $\quad L \sim 1.5$ fm $\quad m_q \sim m_s$

The impact of excited states on the ground state effective mass is clearly visible.

We are not sure to keep them under control within 1% unless incorporating in our system the $3^{\rm rd}$ excited state ($E_3 - E_0 \sim 850$ MeV).

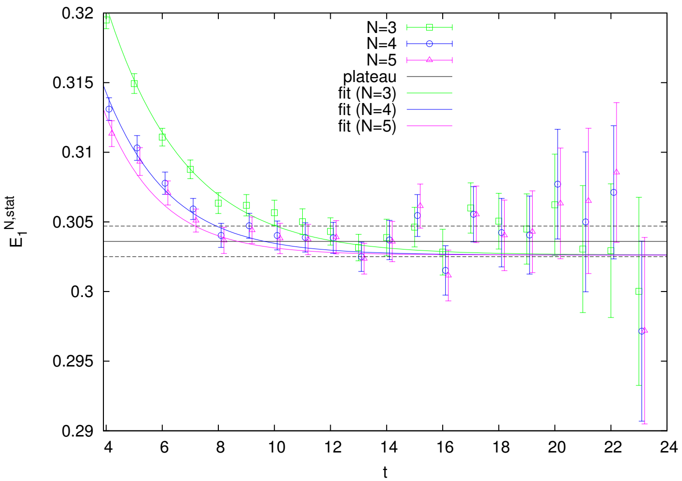
Impossible to do it by a multi-exponential fit without imposing some priors.

It has been proved in the literature that $aE^{\rm eff}_n(t,t_0) \equiv -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right) = aE_n + \mathcal{O}(e^{-\delta E_n\, t})$ $\delta E_n = \min_m |E_n - E_m|$

Issue if $\delta E_n \lesssim 500$ MeV (Example: $E_{X+\pi+\pi} - E_X$)

Actually the rate of convergence is even faster than $e^{-\delta E_n t}$ under the condition that t_0 is large enough ($t_0 \geq t/2$) [B. B. *et al*, '09]:

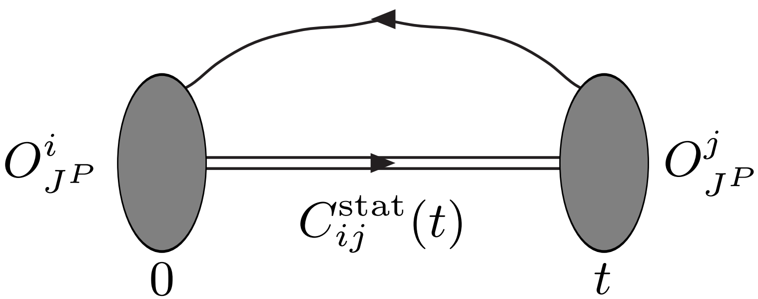
$$aE_n^{\mathrm{eff}}(t,t_0) = aE_n + \mathcal{O}(e^{-\Delta E_{N,n}\, t}) \quad \Delta E_{N,n} = E_{N+1} - E_n$$

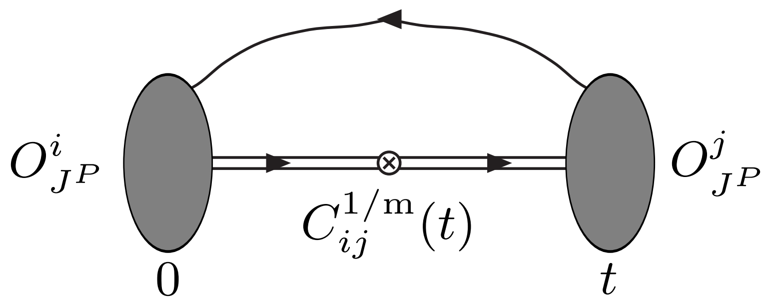


Application to static B_s meson spectroscopy: considering a 5×5 matrix of correlators, one has $\Delta E_{5,0} \sim 1$ GeV.

= 100 $\quad L \sim 1.5$ fm $\quad a \sim 0.07$ fm $\quad m_q \sim m_s$

Thanks to the GEVP analysis one can quantify the systematic error coming from excited states even in a region where the statistical error starts to increase strongly.



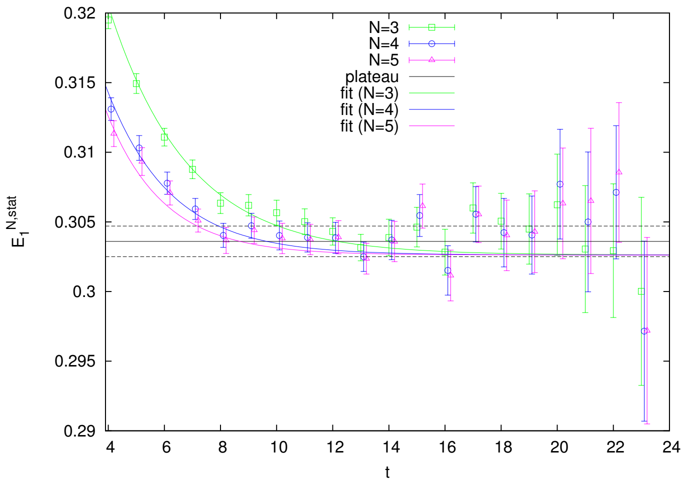


Estimate the $1/m$ corrections in HQET to static energies using GEVP is not an issue; it is enough to determine $\lambda_n^{\mathrm{stat}}$:

$$E_n^{\mathrm{eff}}(t,t_0) = E_n^{\mathrm{eff,stat}}(t,t_0) + \omega E_n^{\mathrm{eff},1/\mathrm{m}}(t,t_0) + O(\omega^2) \tag{1}$$

Actually the rate of convergence is even faster than $e^{-\delta E_n t}$ under the condition that t_0 is large enough ($t_0 \geq t/2$) [B. B. *et al*, '09]:

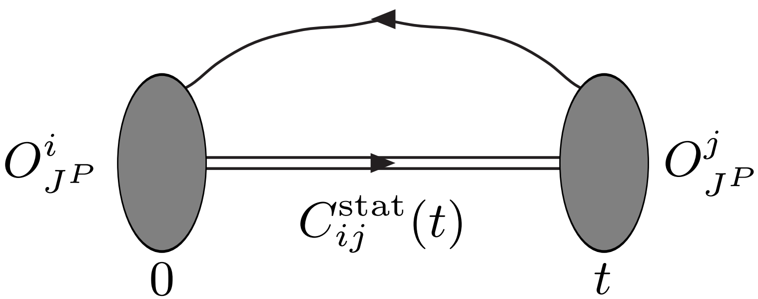
$$aE_n^{\text{eff}}(t, t_0) = aE_n + \mathcal{O}(e^{-\Delta E_{N,n}\, t}) \quad \Delta E_{N,n} = E_{N+1} - E_n$$

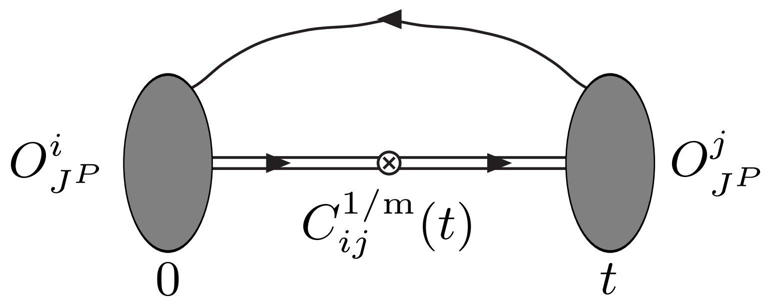


Application to static B_s meson spectroscopy: considering a 5×5 matrix of correlators, one has $\Delta E_{0,5} \sim 1$ GeV.

= 100 $\quad L \sim 1.5$ fm $\quad a \sim 0.07$ fm $\quad m_q \sim m_s$

Thanks to the GEVP analysis one can quantify the systematic error coming from excited states even in a region where the statistical error starts to increase strongly.



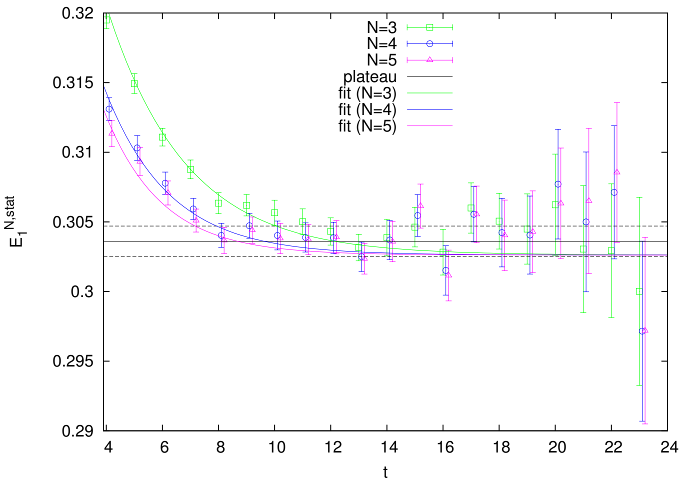


Estimate the $1/m$ corrections in HQET to static energies using GEVP is not an issue; it is enough to determine λ_n^{stat}:

$$aE_n^{\text{eff,stat}}(t, t_0) = -\ln\left(\frac{\lambda_n^{\text{stat}}(t+a, t_0)}{\lambda_n^{\text{stat}}(t, t_0)}\right) \quad E_n^{\text{eff},1/\text{m}}(t, t_0) = \frac{\lambda_n^{1/\text{m}}(t, t_0)}{\lambda_n^{\text{stat}}(t, t_0)} - \frac{\lambda_n^{1/\text{m}}(t+a, t_0)}{\lambda_n^{\text{stat}}(t+a, t_0)}$$

Actually the rate of convergence is even faster than $e^{-\delta E_n t}$ under the condition that t_0 is large enough ($t_0 \geq t/2$) [B. B. *et al*, '09]:

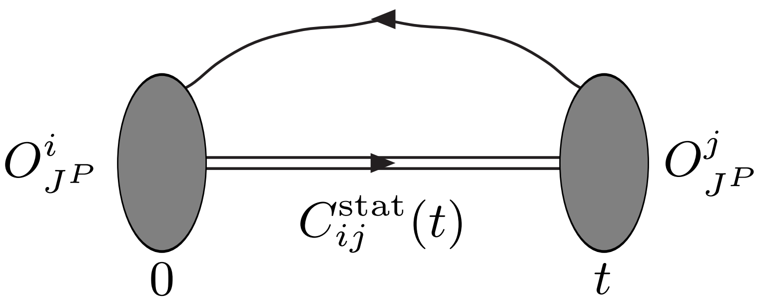
$$aE_n^{\text{eff}}(t, t_0) = aE_n + \mathcal{O}(e^{-\Delta E_{N,n} t}) \quad \Delta E_{N,n} = E_{N+1} - E_n$$

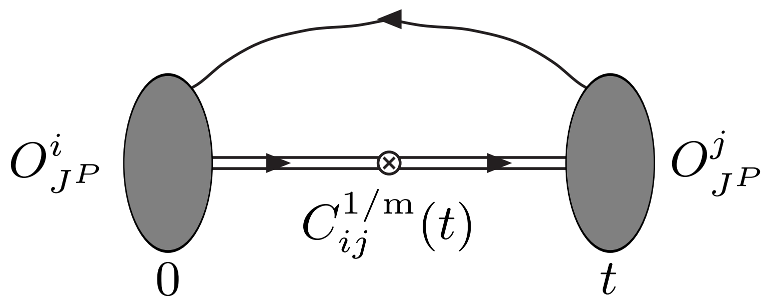


Application to static B_s meson spectroscopy: considering a 5×5 matrix of correlators, one has $\Delta E_{0,5} \sim 1$ GeV.

= 100 $\quad L \sim 1.5$ fm $\quad a \sim 0.07$ fm $\quad m_q \sim m_s$

Thanks to the GEVP analysis one can quantify the systematic error coming from excited states even in a region where the statistical error starts to increase strongly.





Estimate the $1/m$ corrections in HQET to static energies using GEVP is not an issue; it is enough to determine λ_n^{stat}:

$$\frac{\lambda_n^{1/\text{m}}(t, t_0)}{\lambda_n^{\text{stat}}(t, t_0)} = v_{n\,i}^{\text{stat}}(t, t_0) \left[\frac{C_{ij}^{1/\text{m}}(t)}{\lambda_n^{\text{stat}}(t, t_0)} - C_{ij}^{1/\text{m}}(t_0) \right] v_{n\,j}^{\text{stat}}(t, t_0)$$

After an exploratory study led in the quenched approximation [B. B. *et al*, '10], we have followed the same strategy at $\mathrm{N_f} = 2$ [B. B. *et al*, '14].

Simulations S_1, S_2, $S_3 \equiv S_4$ were realized by the ALPHA Collaboration. Parameters of the HMC algorithms were chosen such that nothing insane was observed in the simulations.

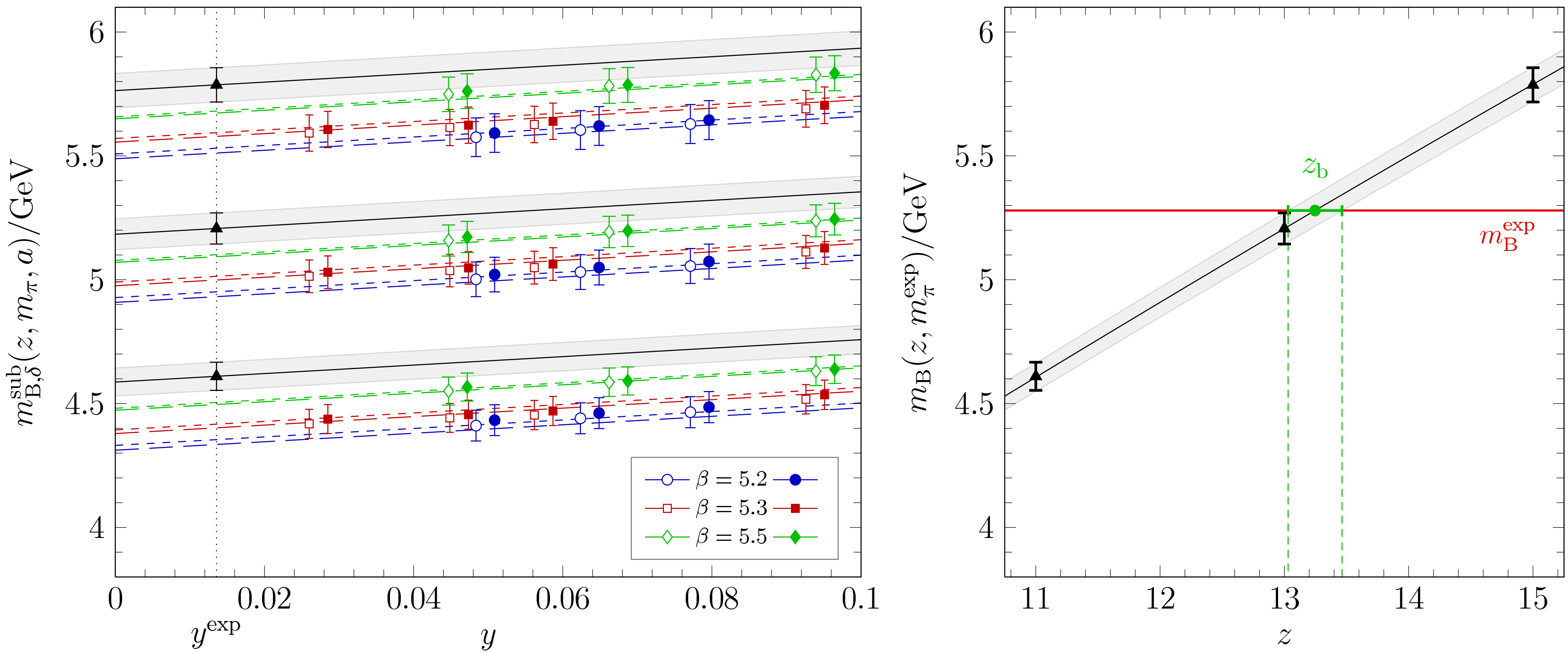
S_5 were made available within the CLS effort.

β	a[fm]	L/a	m_π[MeV]	$m_\pi L$	#cfgs	$\frac{\#\text{cfgs}}{\tau_{\text{exp}}}$
5.2	0.075	32	380	4.7	1012	122
		32	330	4.0	1001	164
		48	280	5.2	636	52
5.3	0.065	32	440	4.7	1000	120
		48	310	5.0	500	30
		48	270	4.3	602	36
		64	190	4.1	410	17
5.5	0.048	48	440	5.2	477	4.2
		48	340	4.0	950	38
		64	270	4.2	980	20

Some attention has been paid to the autocorrelations induced by the coupling of observables to the slow modes of the Markov chain, that decay in $e^{\tau_{\text{MC}}/\tau_{\text{exp}}}$ [S. Schaefer *et al*, '10].

Chiral and continuum limit extrapolations of m_B are performed to get m_b^{RGI}. Several heavy quark masses m_h are considered on the QCD side of the whole program $\Longrightarrow$ effective couplings $\omega(m_h)$ and meson masses $m_B(m_h)$.

$$y = \frac{m_\pi^2}{8\pi f_\pi^2} \qquad z = L_1 m_h^{\mathrm{RGI}}$$



We obtain: $m_b^{\overline{\mathrm{MS}},\mathrm{N_f}=2}(m_b) = 4.21(11)$ GeV

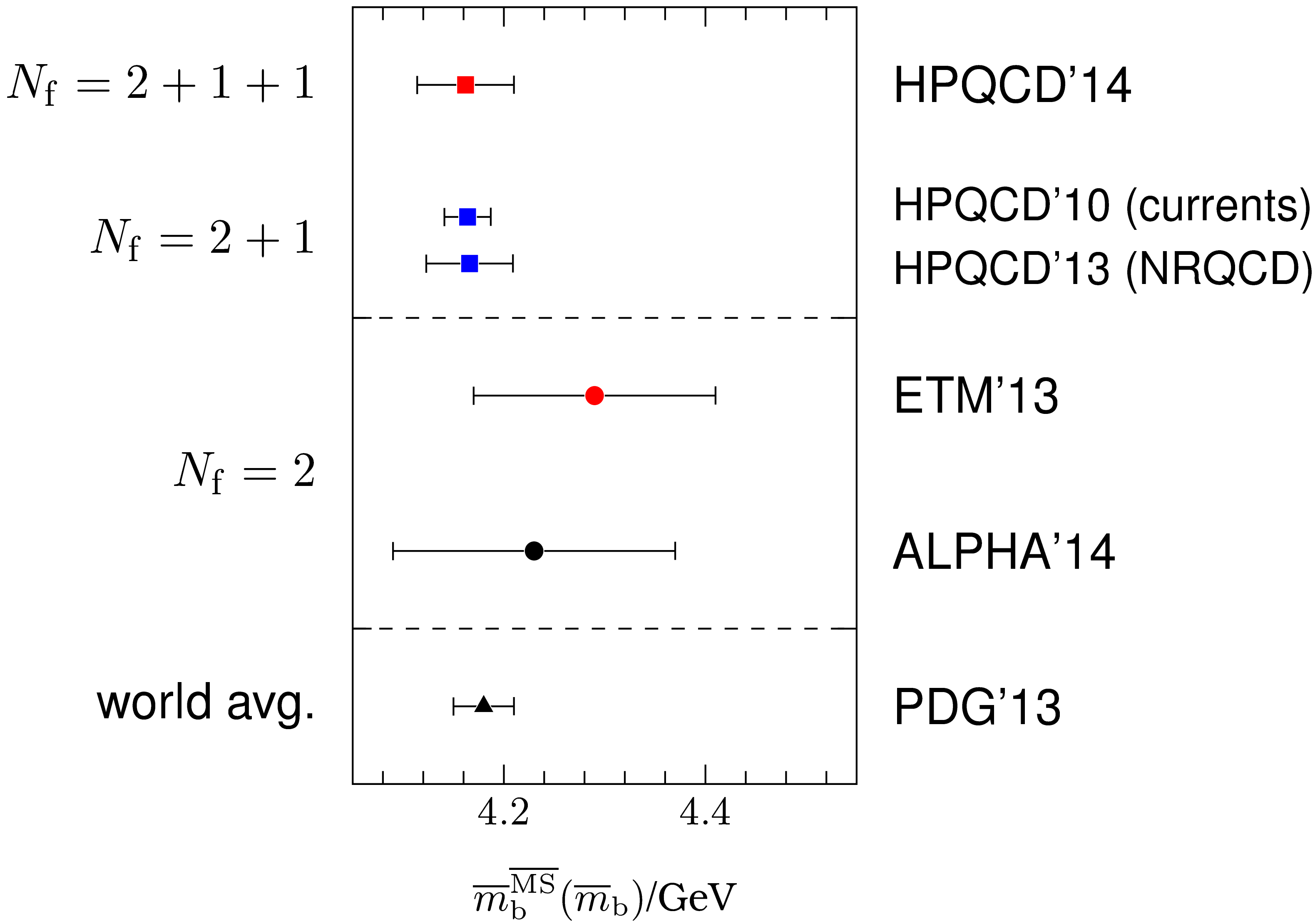
Error budget:

– 2% from statistics, chiral extrapolation (NLO vs. LO) and continuum extrapolation

– 1% from Z_m^{RGI}

Conclusions of our study

Collection of lattice results and PDG average:



Conclusions of our study

Collection of lattice results and PDG average:

$\mathrm{N_f}$	m_b^{RGI}	$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$	$m_b^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	$\Lambda^{\overline{\mathrm{MS}}}$ [MeV]
0	6.76(9)	4.35(5)	4.87(8)	0.238(19)
2	6.57(17)	4.21(11)	4.88(15)	0.310(20)
5	7.50(8)	4.18(3)	4.91(5)	0.212(8)

– Weak $\mathrm{N_f}$ dependence of m_b in $[2\,\mathrm{GeV}, m_b]$, as observed for other quark masses: matching of effective theories performed in the low energy region (m_B^{exp} for m_b, f_K or f_π for a, m_π for $m_{u/d}$).

– Discrepancies in m_b^{RGI}: $\mathrm{N_f}$ dependence of the RG functions and $\Lambda^{\overline{\mathrm{MS}}}$; reinforcement between $\mathrm{N_f} = 5$ and $\mathrm{N_f} = 2$, partial compensation between $\mathrm{N_f} = 2$ and $\mathrm{N_f} = 0$.

– Reliability of using $m_b(\mu \sim 2\,\mathrm{GeV})$ for predictions from theories with $\mathrm{N_f} < 5$.

– m_b appropriately determined from the different approaches; error budget are such that in more coming works with $\mathrm{N_f} = 2+1$ or $2+1+1$, a competitive number can be obtained, as far as Higgs physics and, in particular, the $H \to b\bar{b}$ channel, is concerned.

Outlook

- Lattice community does make an important effort to compute from first principles of quantum field theory hadronic quantities with a competitive accuracy with respect to experimental measurements.

- We provide theoretical inputs to constrain NP scenarios from flavour physics, i.e. from low energy processes that are under study at LHCb and, soon, at Super-Belle: kaon decays, $\Delta F = 2$ oscillations, rare $\Delta F = 1$ decays, anomalous magnetic moment of the muon.

- A lot of other phenomenological topics were not covered here: isospin breaking corrections, $b \to c$ transitions, unstable particles,...

- We provide theoretical inputs for Higgs physics as well: the main decay channel, $H \to b\bar{b}$, is parametrized by m_b.

Schrödinger Functional

Partition function: $\mathcal{Z}[C,C'] = \langle C'|e^{-H\,T}|C\rangle$ [K. Symanzik, '81]

$C(x_0=0)$ and $C'(x_0=T)$ are 2 field configurations that are given.

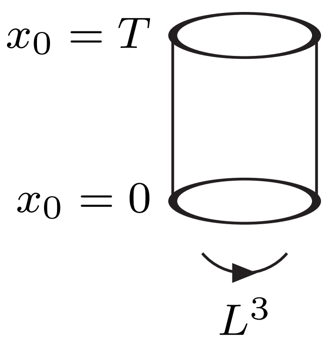
The Schrödinger Functional is renormalisable with Yang-Mills theories. [M. Lüscher et al, '92]

The associated renormalisation scheme is of finite volume kind and regularisation independent:

$$\Gamma(\Phi_{\rm cl}) \equiv -\ln \mathcal{Z}[C,C'] = g_0^{-2}\Gamma_0[\Phi_{\rm cl}] + \Gamma_1[\Phi_{\rm cl}] + g_0^2\Gamma_2[\Phi_{\rm cl}] + \ldots \qquad \left.\frac{\delta S}{\delta \Phi}\right|_{\Phi=\Phi_{\rm cl}} = 0$$

$$C^{(')} \equiv C^{(')}(\eta) \qquad \bar{g}^2(L) = \left[\frac{\partial \Gamma_0(\Phi_{\rm cl})}{\partial \eta}\right] \Big/ \left[\frac{\partial \Gamma(\Phi_{\rm cl})}{\partial \eta}\right] \qquad \bar{g}^2(L) = \left\langle \frac{\partial S}{\partial \eta}\right\rangle$$

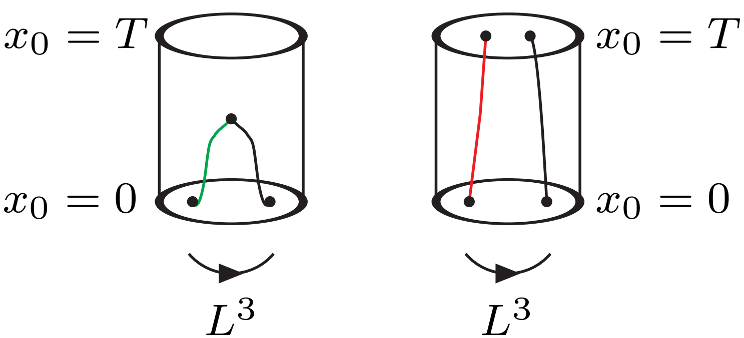
SF is renormalisable with QCD as well. [S. Sint, '93]



$$P_+\psi(x)|_{x_0=0} = \rho(\vec{x}) \qquad P_-\psi(x)|_{x_0=T} = \rho'(\vec{x}) \qquad \psi(x+L\hat{k}) = e^{i\theta_k}\psi(x)$$

$$\langle O\rangle = \left.\left(\tfrac{1}{\mathcal{Z}}\int[\mathcal{D}U][\mathcal{D}\psi][\mathcal{D}\bar{\psi}]\,O e^{-S(U,\psi,\bar{\psi})}\right)\right|_{\rho=\bar{\rho}=\rho'=\bar{\rho}'=0}$$

The Dirac operator has no zero mode in the chiral limit.



Computation of boundary to bulk and boundary to boundary correlators

Small volume part of the strategy (S_1)

Bare couplings of the HQET Lagrangian and currents are determined by imposing in a small volume $L_1 \sim$ 0.5 fm several matching conditions between correlators defined in QCD and their HQET counterpart:

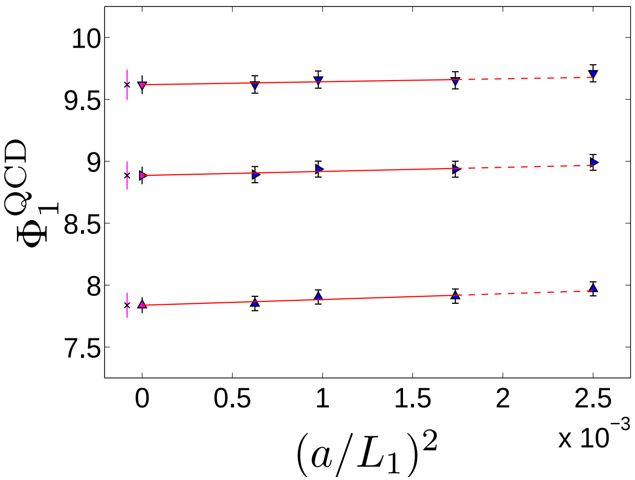
$$\Phi_\alpha^{\mathrm{QCD,\ cont}} = f_{\alpha\beta}[\omega(g_0^{(1)})]\Phi_\beta^{\mathrm{HQET}}(g_0^{(1)})$$

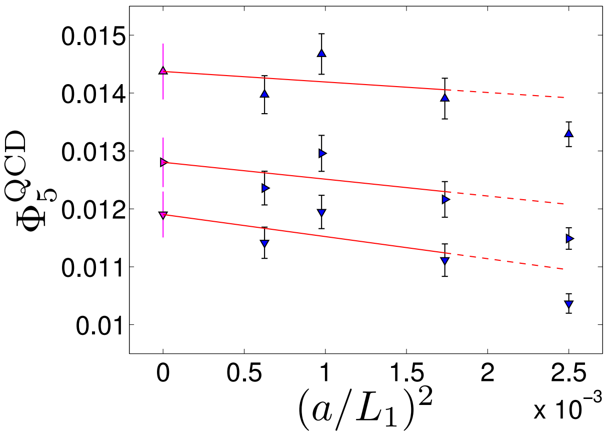
$$\Phi_{AA}(t) \equiv Z_A^2 \sum_{\vec{x}} \langle (\bar{\psi}_b \gamma_0 \gamma^5 \psi_l)(\vec{x}, t)(\bar{\psi}_l \gamma_0 \gamma^5 \psi_b)(0)\rangle$$

$$\Phi_{AA}(t) = e^{-m_{\mathrm{bare}} t}(Z_A^{\mathrm{HQET}})^2 \left[\Phi_{AA}^{\mathrm{stat}}(t) + \omega_{\mathrm{kin}} \Phi_{AA}^{\mathrm{kin}}(t) + \omega_{\mathrm{spin}} \Phi_{AA}^{\mathrm{spin}}(t) + C_A^{(1)}[\Phi_{A\delta A}(t) + \Phi_{\delta AA}(t)]\right]$$

In such a small volume it is possible to simulate the b quark in QCD; at this stage of the program we are only concerned by the short-distance regime and absorption of UV divergences.

Extrapolation to the continuum limit of $\Phi_1^{\mathrm{QCD}} \equiv$ "m_{B_s}" and $\phi_5^{\mathrm{QCD}} \equiv$ "$m_{B_s^*} - m_{B_s}$" ($\mathrm{N_f} = 0$)



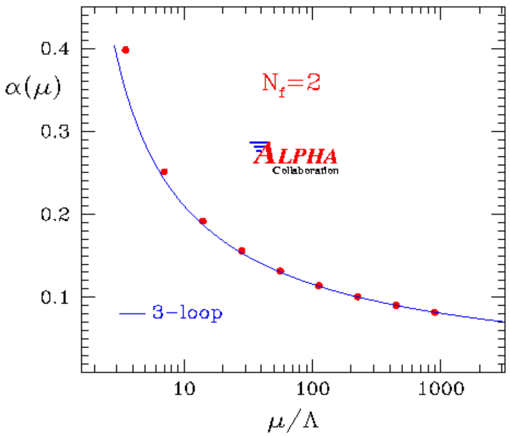


Step scaling in volume (S_2, S_3) and matching (S_4)

Then one uses Step Scaling functions to let the observables evolve from the volume L_1 to a volume $L_{\rm inf} = s^k L_1$ where long-distance physics dominates and where one extracts hadronic quantities

$$\Phi_i^{\rm QCD,cont}(sL) = \lim_{a^{(1)}\to 0} \Sigma_{ij}(g_0(a^{(1)}), L, sL)\Phi_j^{\rm QCD,cont}(L)$$

$$\Sigma_{ij}(g_0(a^{(1)}), L, sL) = \frac{f_{ik}[\omega(g_0^{(1)})]\Phi_k^{\rm HQET}(g_0^{(1)}, sL)}{f_{jl}[\omega(g_0^{(1)})]\Phi_l^{\rm HQET}(g_0^{(1)}, L)}$$



This approach with SSF's is very popular: it has been successfully used to measure the running of the strong coupling constant $\bar{g}^2(\mu = 1/L)$ up to the perturbative regime.

[M. Della Morte et al, '04]

$$\Phi_\alpha^{\rm QCD,\ cont}(sL) \quad = \quad f'_{\alpha\beta}[\omega(g_0^{(2)})]\Phi_\beta^{\rm HQET}(g_0^{(2)}, sL)$$

HQET parameters $\omega(g_0(a^{(2)}))$ are obtained at a second set of lattice spacings $\{a^{(2)}\}$. All the strategy is based on the fact that simulations are realized with L/a always in the range [10 – 40].