Constraining models of Asymptotically Safe inflation

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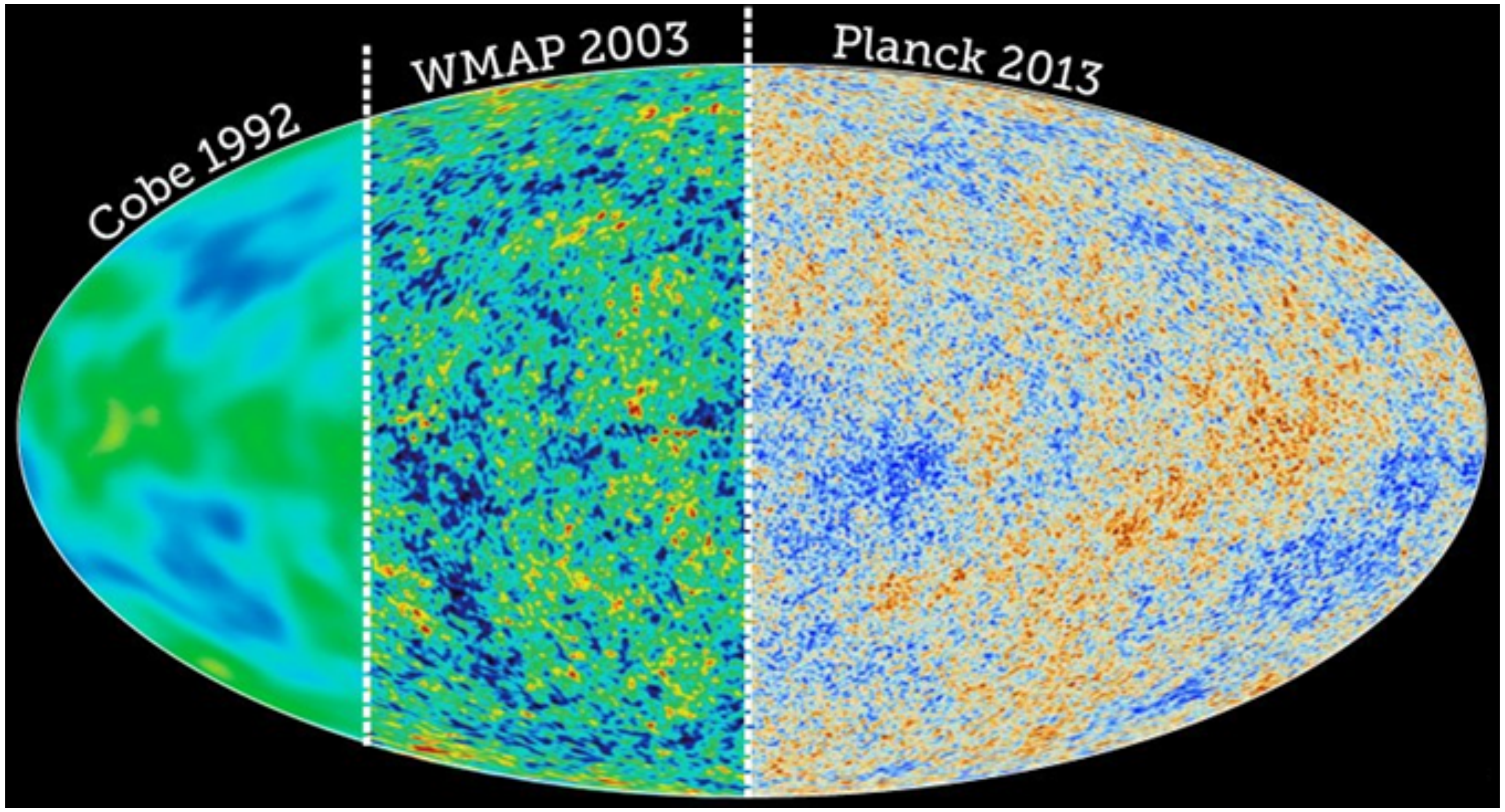
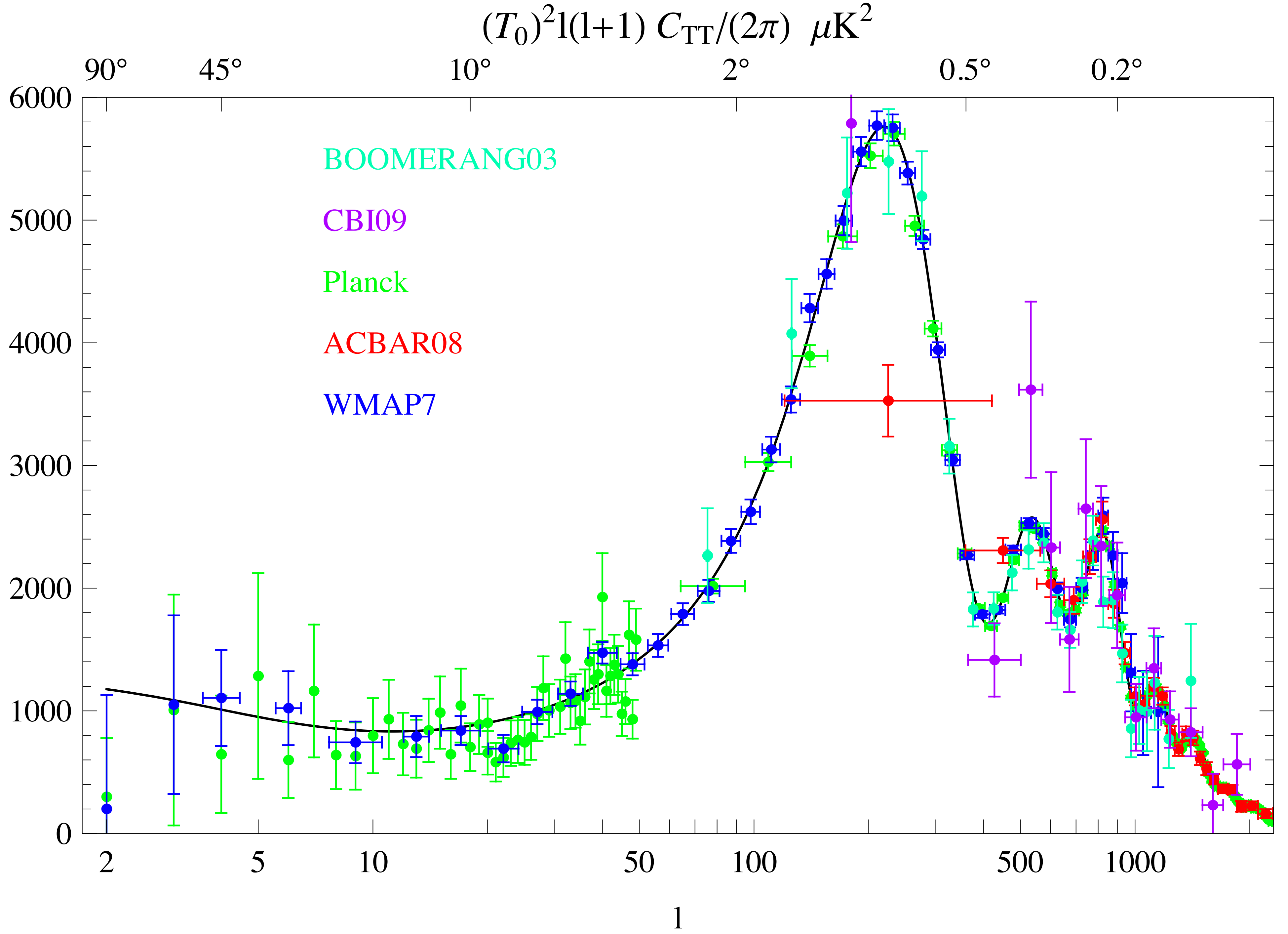
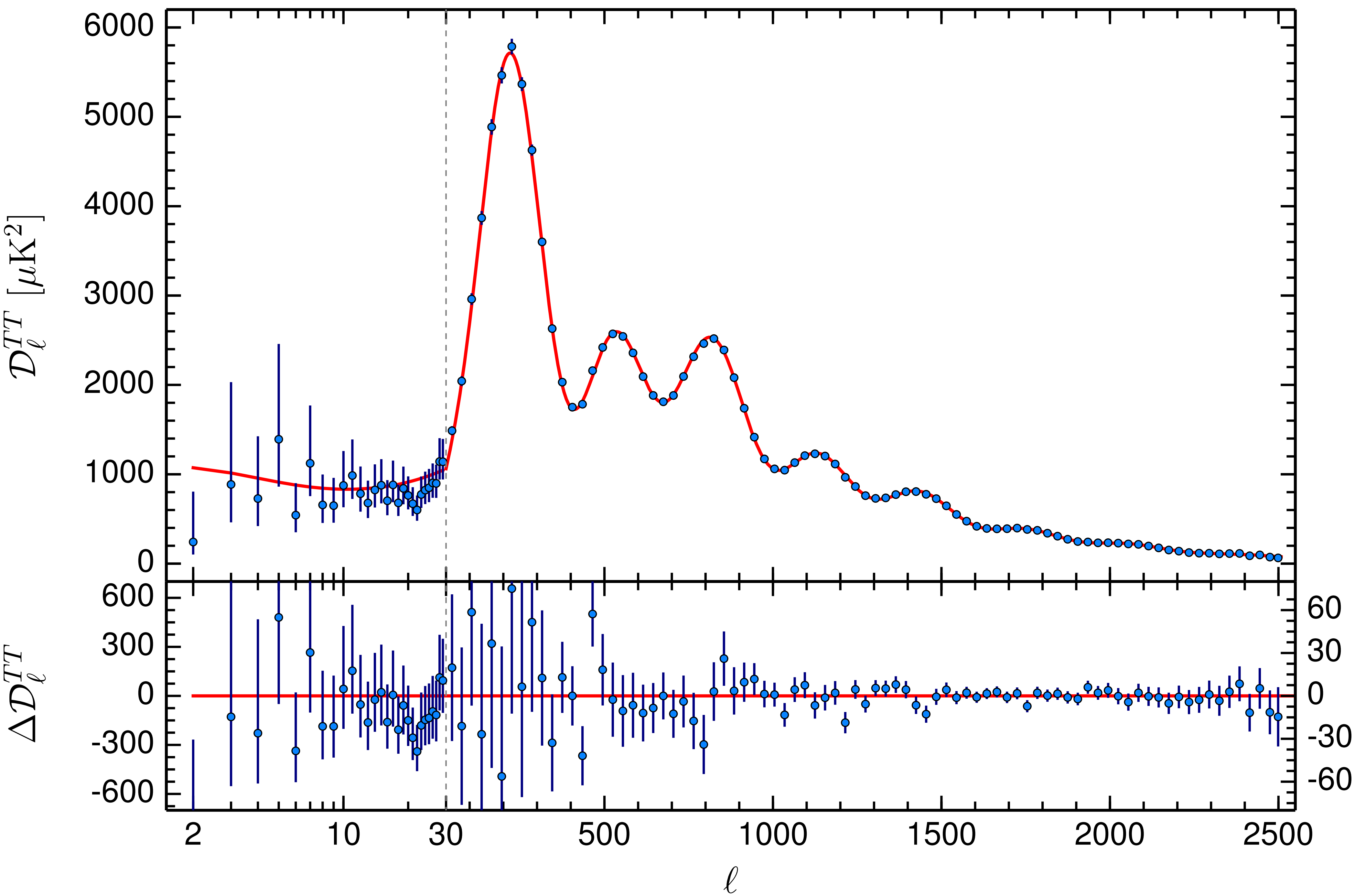
Precision Cosmology

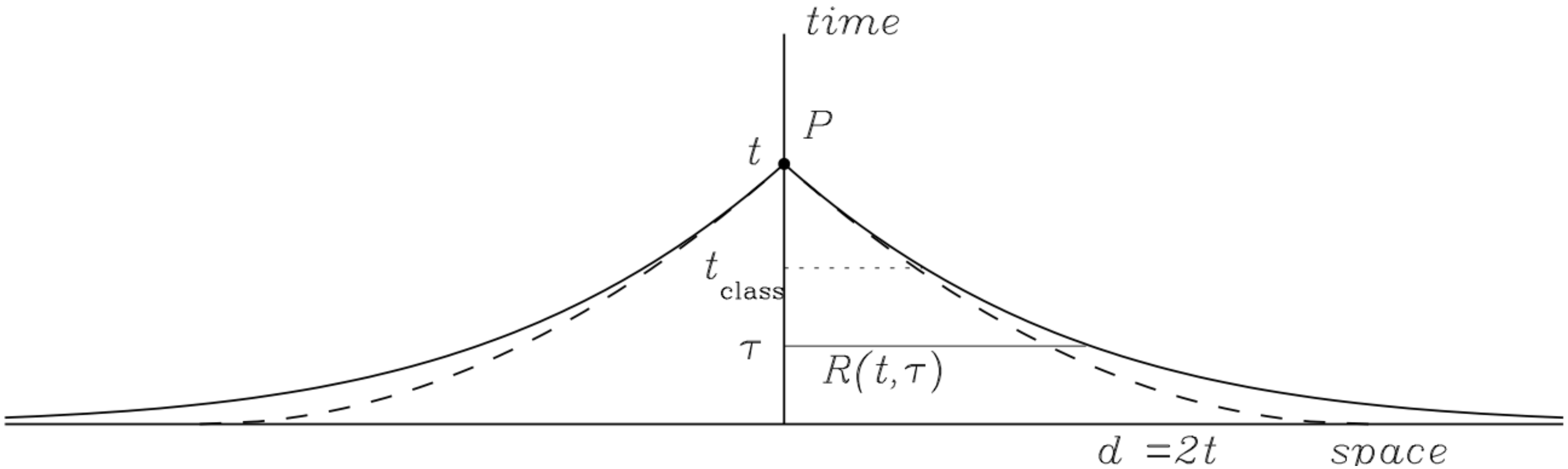


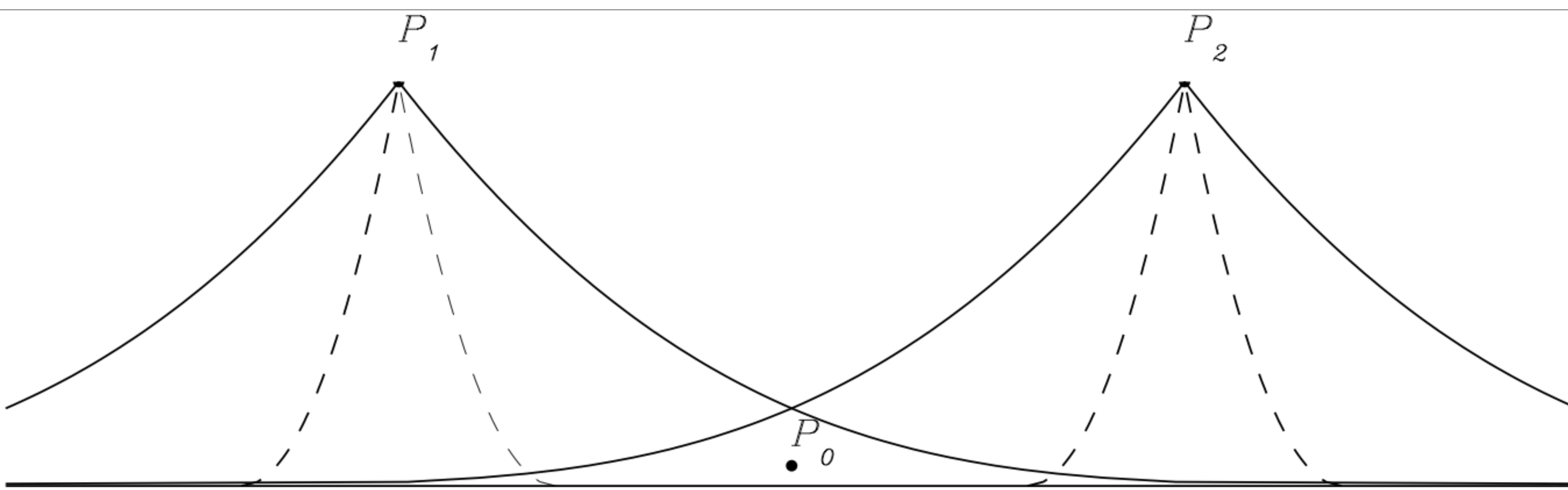


$(T_0)^2 l(l+1)\ C_{TT}/(2\pi)\ \ \mu K^2$
90°
45°
10°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
BOOMERANG03
CBI09
Planck
ACBAR08
WMAP7
2
5
10
50
100
500
1000
l

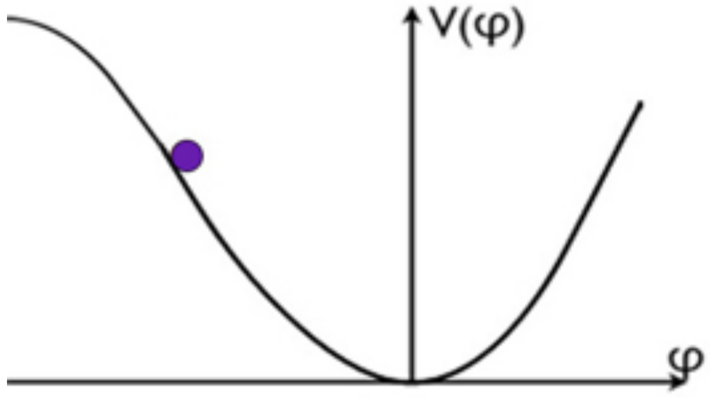


$\mathcal{D}_\ell^{TT}$ [μK^2]
6000
5000
4000
3000
2000
1000
0
$\Delta\mathcal{D}_\ell^{TT}$
600
300
0
-300
-600
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500
ℓ


time
P
t
t class
τ
R(t,τ)
d =2t
space


P 1
P 2
P 0

Quantum generation of initial spectrum



$$(a\delta\phi_k)'' + (k^2 + z''/z)(a\delta\phi_k) = 0$$

$$z = a\dot{\phi}/H$$

$$\mathcal{R} = -H\frac{\delta\phi}{\dot{\phi}}$$

$$\langle 0|\mathcal{R}_k\mathcal{R}_p|0\rangle = \frac{(2\pi)^2}{2k^3}\mathcal{P}_{\mathcal{R}}(k)\delta^3(k+p)$$

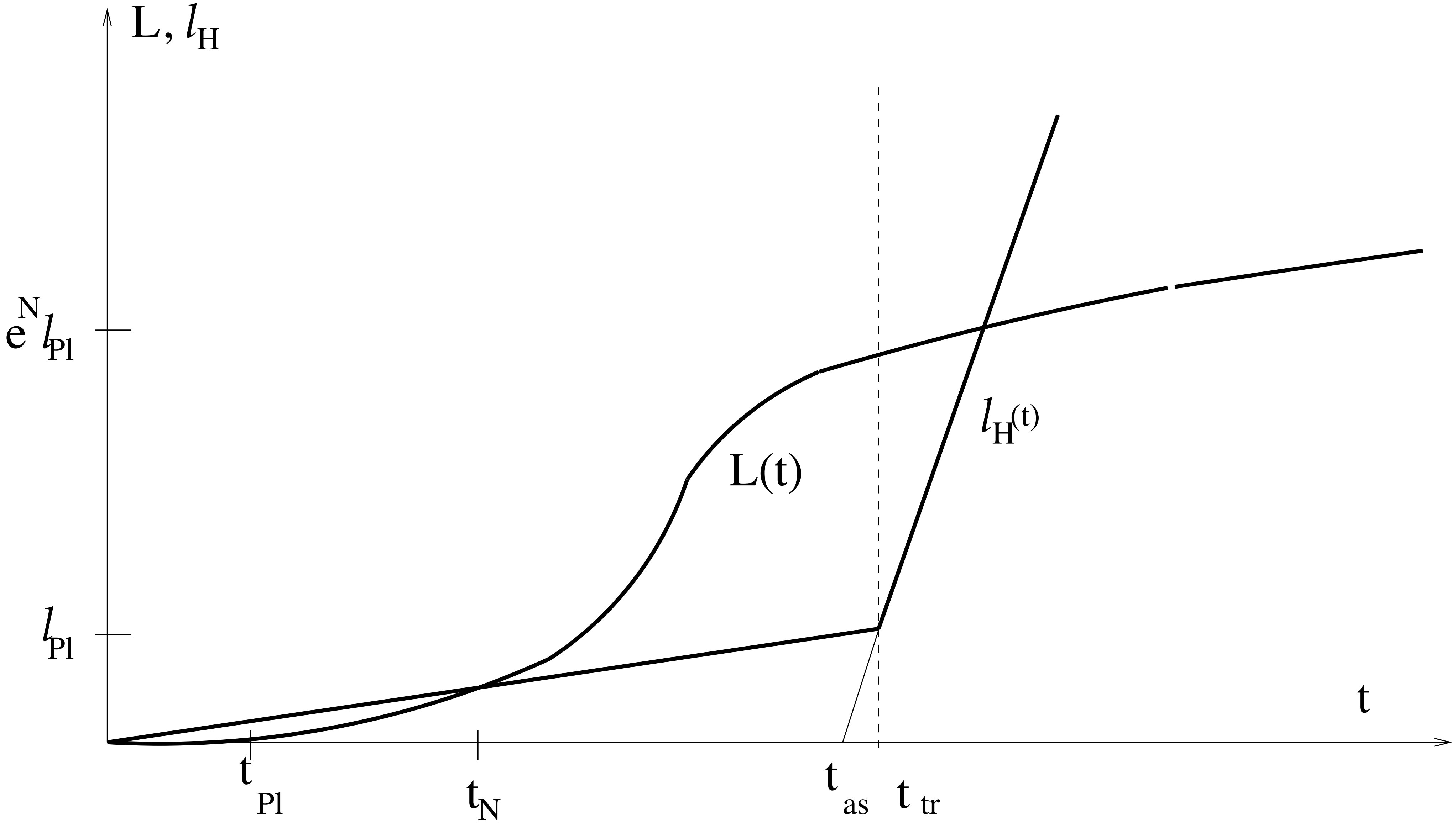
on super-Hubble scale

$$\mathcal{P}_{\mathcal{R}}(k) = A_s\left(\frac{k}{k_0}\right)^{n_s-1+(1/2)(dn_s/d\ln k)\ln(k/k_0)}$$

$$\mathcal{P}_t(k) = A_t\left(\frac{k}{k_0}\right)^{n_t}$$

$$r_{0.05} \equiv A_t/A_s$$

super-planckian initial conditions !



$$H(t) = \frac{\alpha}{t} \quad \alpha > 1$$

The fundamental problem of QG

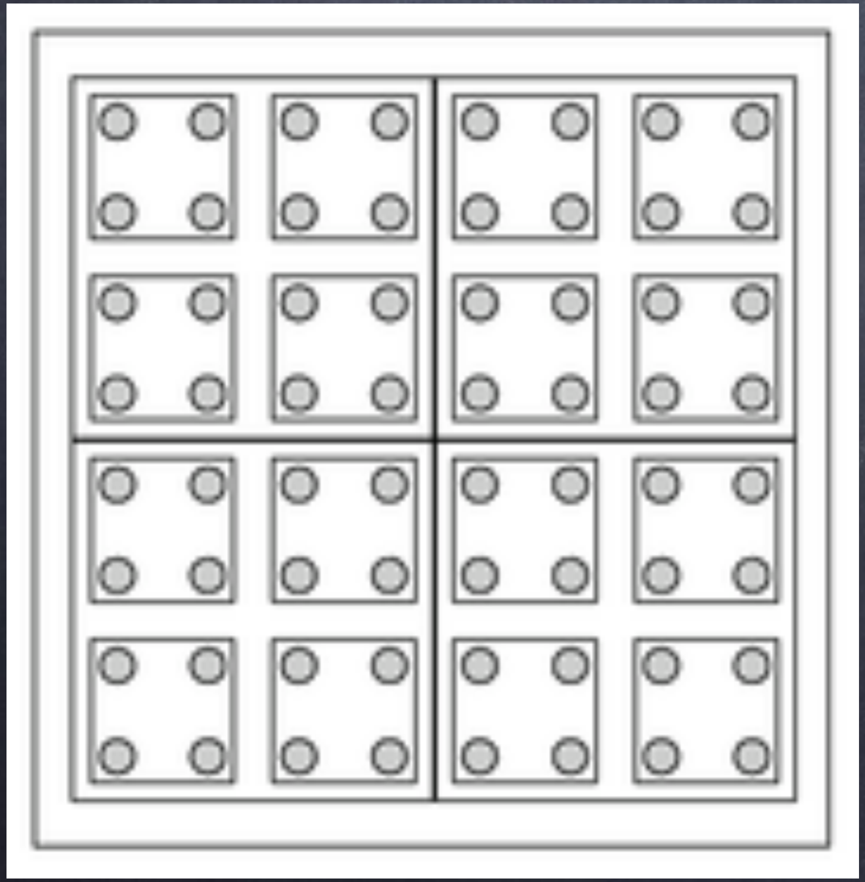
$$\int D[\hat{g}_{\mu\nu}]e^{-S[\hat{g}_{\mu\nu}]}$$

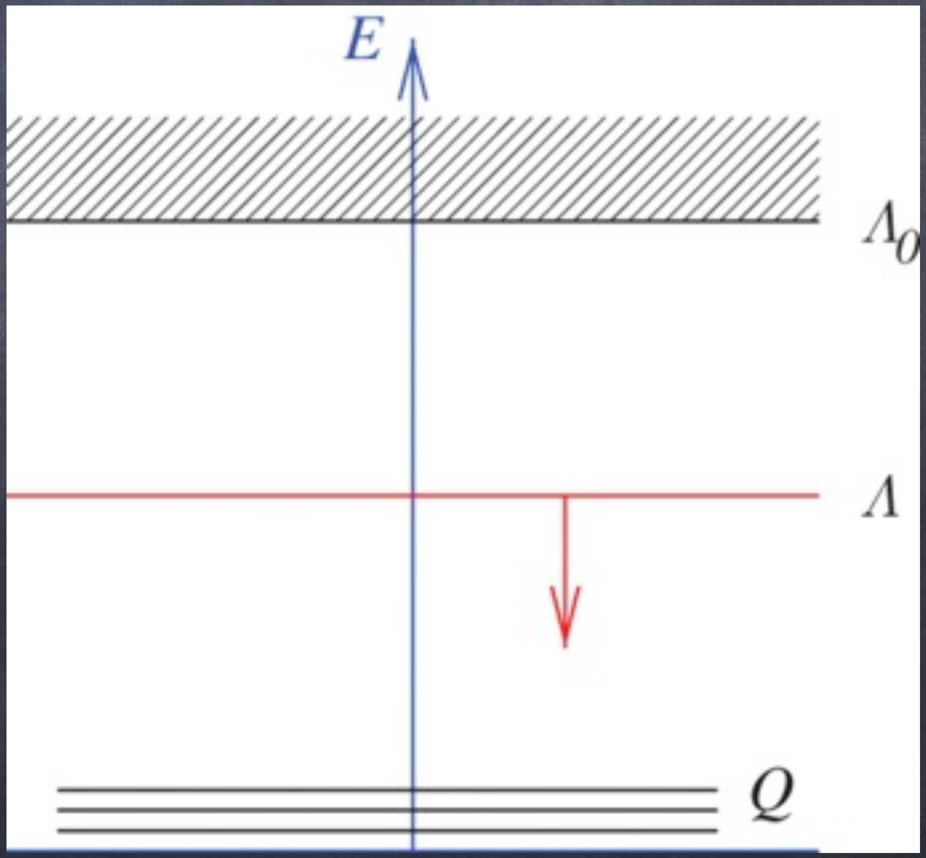
$$D[\hat{g}_{\mu\nu}] \equiv \prod_{x\in M} \prod_{\mu,\nu} dg_{\mu\nu}(x)$$

Wilsonian renormalization

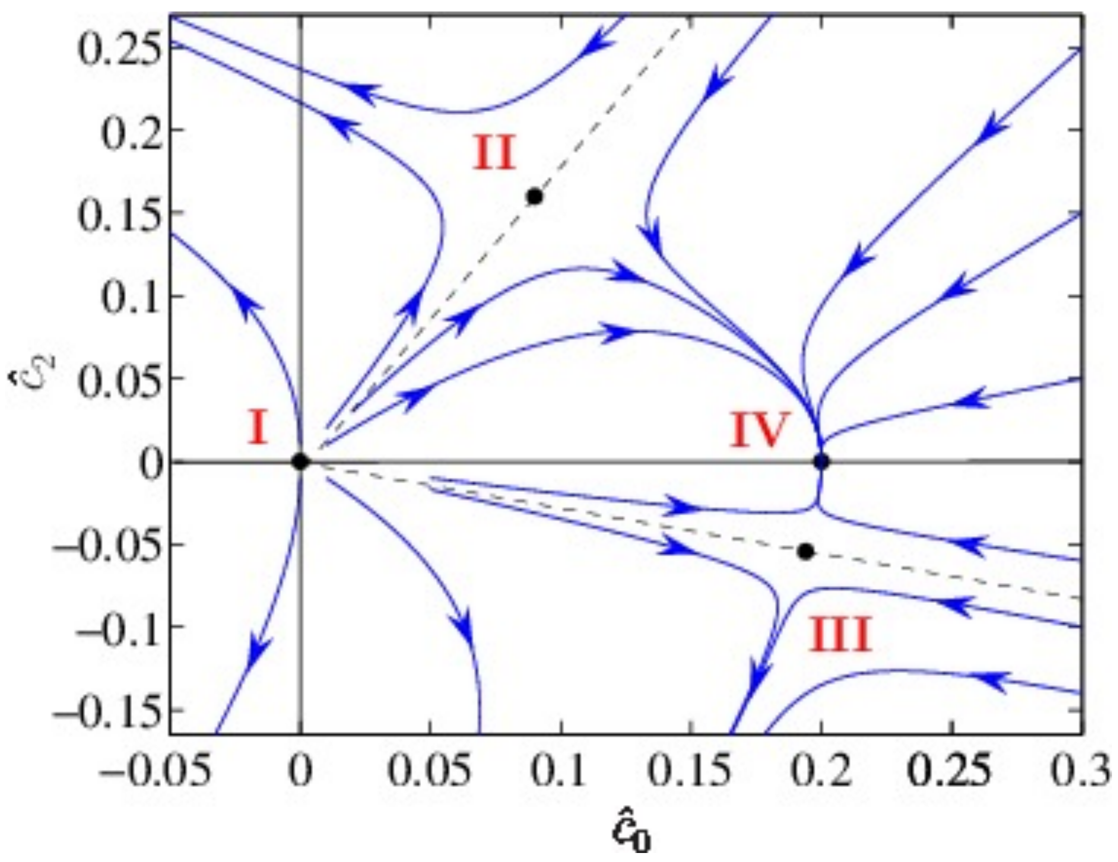
$$e^{\Gamma_k[\Phi]} = \int D[\varphi]\delta(\Phi - C(\varphi))e^{-S[\varphi]}$$

$$C(\varphi) \sim \langle\varphi\rangle_k$$





The quest for fixed points!



Self-similarity

Wilson renormalization generates a flow in energy (k) for infinitely many couplings

$$\lambda_i^*$$

$$\lambda_i(k) = \lambda_i^* + \sum_n c_n\, V_i^n\, k^{\vartheta_n} + \text{subleading}$$

$\vartheta_n > 0$ relevant operator

$\vartheta_n < 0$ irrelevant operator

Predictability means a finite number of positive relevant operators

The Einstein-Hilbert truncation

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g} \left[-R + 2\Lambda(k) \right] + S^{\rm gf} + S^{\rm gh}$$

microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

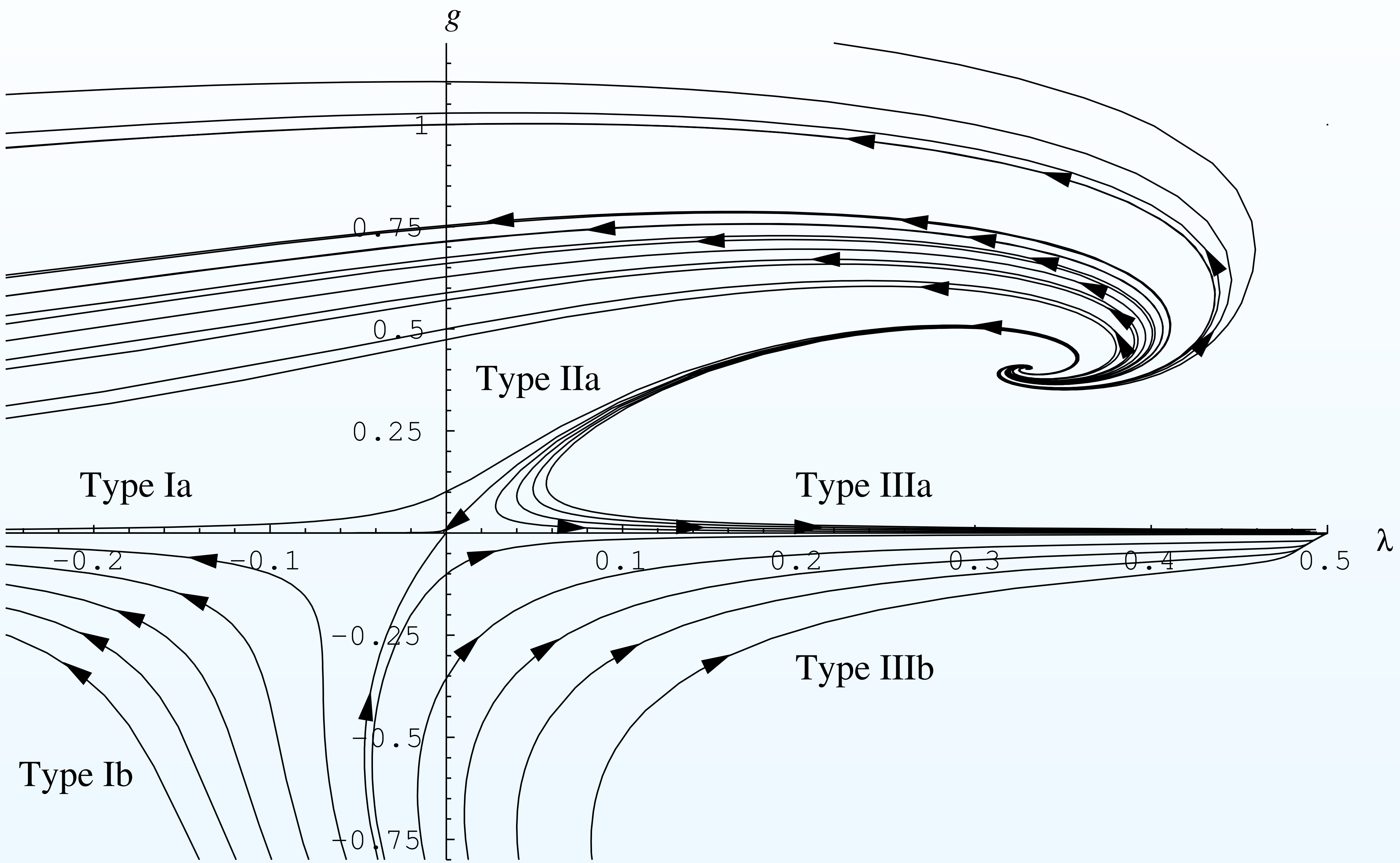
$$\beta_g(g^*, \lambda^*) = 0 \, , \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

- Gaussian Fixed Point:
  - at $g^* = 0, \lambda^* = 0 \Longleftrightarrow$ free theory
  - saddle point in the g-λ-plane
- non-Gaussian Fixed Point ($\eta_N^* = -2$):
  - at $g^* > 0, \lambda^* > 0 \Longleftrightarrow$ "interacting" theory
  - UV attractive in g_k, λ_k

Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

Einstein-Hilbert-truncation: the phase diagram

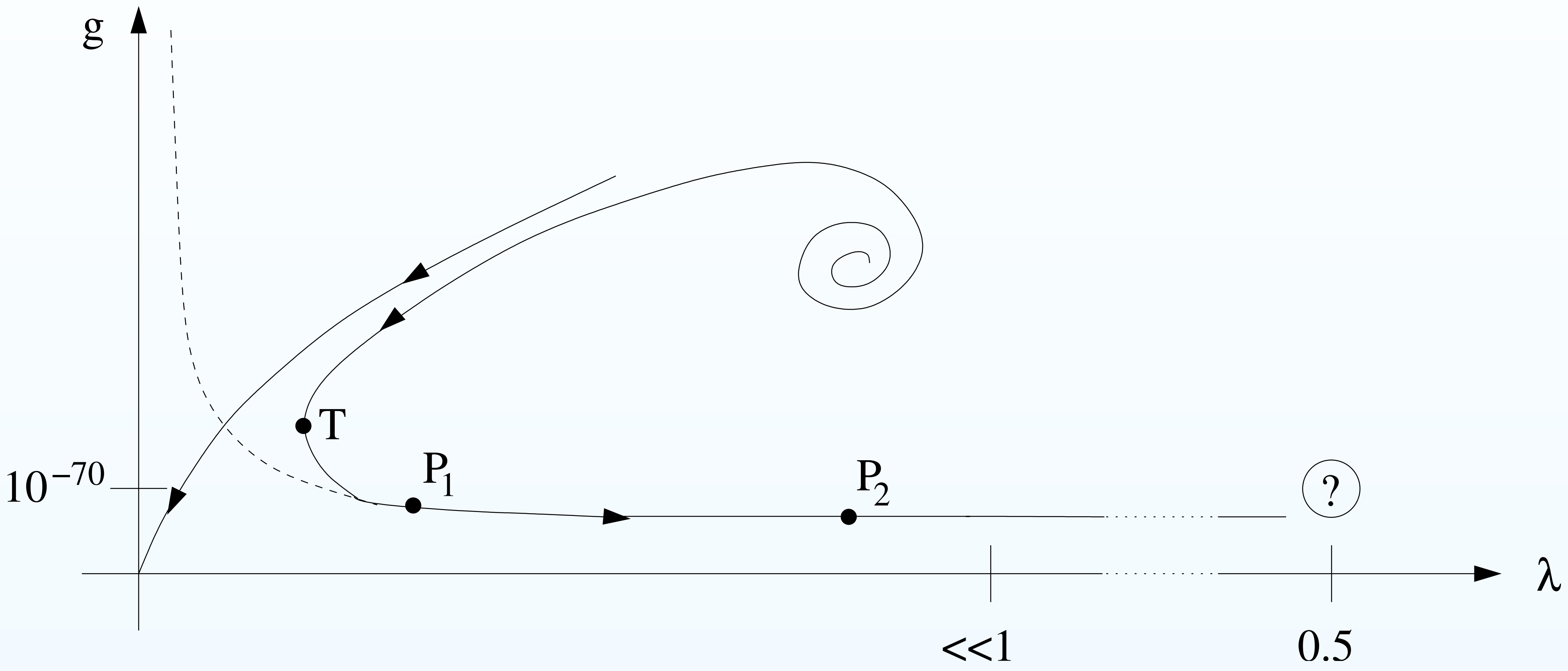
M. Reuter, F. S., Phys. Rev. D **65** (2002) 065016, hep-th/0110054



The RG trajectory realized in Nature

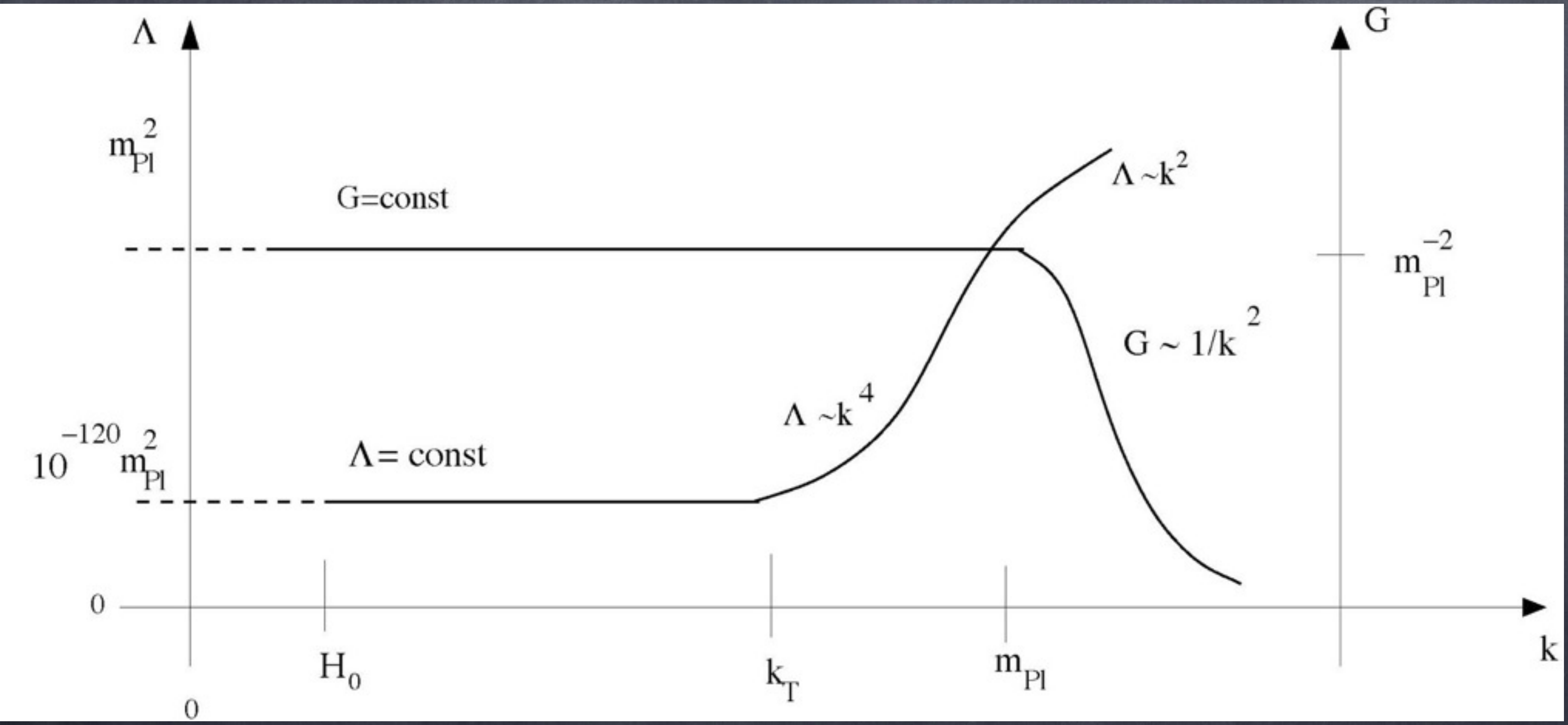
M. Reuter, H. Weyer, JCAP 0412 (2004) 001, hep-th/0410119

measurement of G_N, Λ in classical regime:

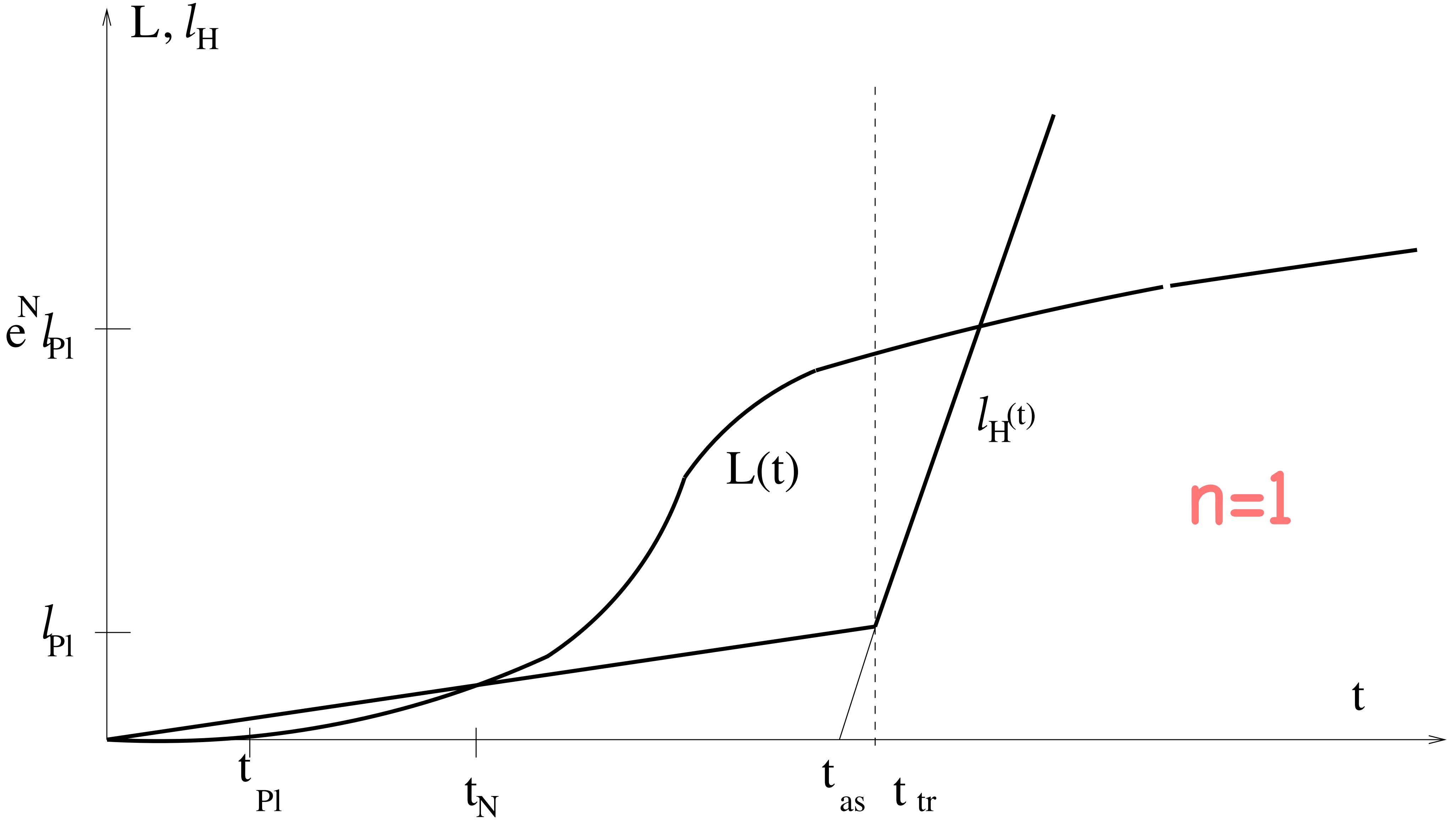


- originates at NGFP (quantum regime: $G(k) = k^{2-d} g_*, \Lambda(k) = k^2 \lambda_*$)
- passing *extremely* close to the GFP
- long classical GR regime (classical regime: $G(k) = \text{const}, \Lambda(k) = \text{const}$)
- $\lambda \lesssim 1/2$: IR fixed point?

Complete cosmic history



AB & M Reuter, 2003, 2007, 2008. AB, Esposito, Rubano, 2003,2004,2005; AB, Contillo, Percacci 2010



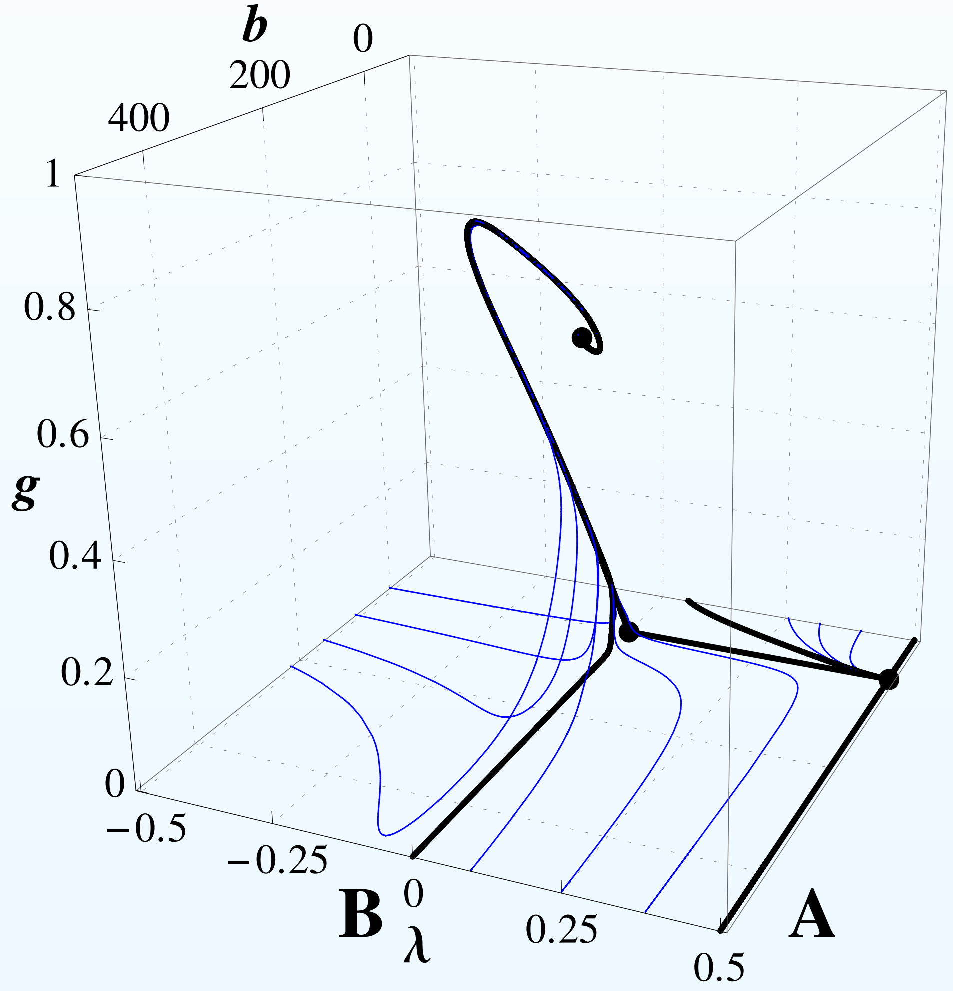
A.B. & M. Reuter, JCAP 2007

Charting the RG-flow of the R^2-truncation

O. Lauscher, M. Reuter, Phys. Rev. D66 (2002) 025026, hep-th/0205062
S. Rechenberger, F.S., Phys. Rev. D86 (2012) 024018, arXiv:1206.0657

Extending Einstein-Hilbert truncation with higher-derivative couplings

$$\Gamma_k^{\rm grav}[g] = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G_k} \left(-R + 2\Lambda_k\right) + \frac{1}{b_k} R^2 \right]$$



Asymptotically Safe Inflation (Weinberg 2010)

$$I_\Lambda[g] = -\int d^4x\sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_{2a}(\Lambda) R^2$$
$$+ g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu} + \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots\Big]$$

Consider a general truncation

Optimal cutoff: radiative corrections just beginning to be important and higher order terms just beginning to be less important

Objective: to obtain a dS solution which is unstable but lasts N>60 e-folds

Difficulties: too much fine tuning, strong dependence on FP quantities (see Tye & Xu, PRD 2010)

It would be desirable to have a framework where only the information from the critical exponents play a significant role

How to extract physical information ?

RG-improvement of the standard QCD Lagrangian

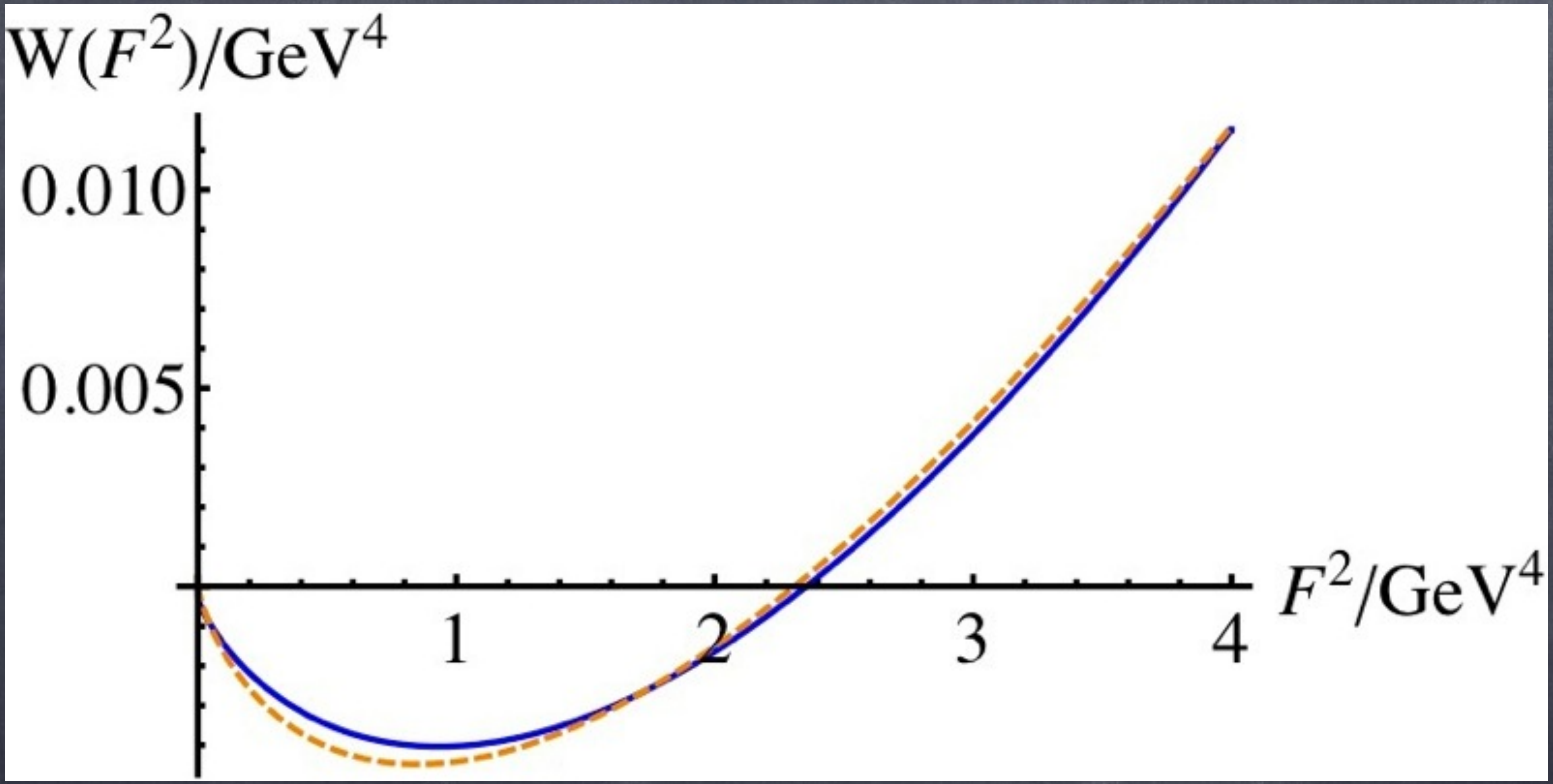
$$\mathcal{L}_{\rm eff}^{\rm QCD} = \frac{\mathcal{F}}{2g^2_{\rm running}} \qquad g^2_{\rm running} = \frac{2g^2(\mu^2)}{1 + \frac{1}{4}\, b\, g^2(\mu^2) \log\left(\frac{\mathcal{F}}{\mu^4}\right)}$$

$$k^2 \propto \mathcal{F}^{1/2}$$

It correctly reproduces a linear potential between to static quarks and various other IR properties!

Migdal 1973, Pagels and Tomboulis, 1978, NPB, Adler, 1982

RG-improved QCD



Eichhorn, Gies and Pawlowski, 2011

If one must choose between rigour and meaning, I shall unhesitatingly choose the latter. (René Thom)

Apply the same approach in QEG

Einstein-Hilbert truncation: $$\mathcal{L}^{\mathrm{EH}} = \frac{1}{16\pi G}(R - 2\Lambda)$$

Linearized flow around NGFP:

$$(\lambda, g)^{\mathbf{T}} = (\lambda_*, g_*)^{\mathbf{T}} + 2\{[\mathrm{Re}C\cos(\theta'' t) + \mathrm{Im}C\sin(\theta'' t)]\mathrm{Re}\,V$$
$$+[\mathrm{Re}C\cos(\theta'' t) - \mathrm{Im}C\sin(\theta'' t)]\mathrm{Im}\,V\}\,e^{-\theta' t}$$

$$t = \ln(k/k_0) \qquad \theta = \theta' \pm i\theta''$$

(AB, PRD 2012)

Substitute this solution in the EH Lagrangian after identifying k with the field strength

$$\mathcal{L}_{\rm eff}^{\rm QEG}(R) = R^2 + bR^2 \cos\left[\alpha \log\left(\frac{R}{\mu}\right)\right] \left(\frac{R}{\mu}\right)^{\beta}$$

$\alpha = \theta''/2,\ \beta = -\theta' < 0$

μ is a renormalization scale

$$2.1 < \theta' < 3.4, \qquad 3.1 < \theta'' < 4.3$$

Dietz and Morris, 2013

$$f(R) = R^2 + R + A\ R \cos\log R^2 + B\ R \sin\log R^2$$

$$\dot{H}^2 + 6^\beta b \cos\left[\alpha \ln\left(\frac{6(2H^2+\dot{H})}{\mu}\right)\right]\left(\frac{2H^2+\dot{H}}{\mu}\right)^\beta (2\beta H^4 + (4\alpha^2 - 6$$

$$-\beta(9+4\beta))H^2\dot{H} + (1+\beta)\dot{H}^2 + (\alpha - (1+\beta)(2+\beta))H\ddot{H})$$

$$= 2H(3H\dot{H} + \ddot{H}) + 6^\beta b\alpha \sin\left[\alpha \ln\left(\frac{6(2H^2+\dot{H})}{\mu}\right)\right]\left(\frac{2H^2+\dot{H}}{\mu}\right)^\beta$$

$$(2H^4 - (9+8\beta)H^2\dot{H} + \dot{H}^2 - (3+2\beta)H\ddot{H})$$

look for de Sitter solutions:

$$\bar{H} = \sqrt{\frac{\mu}{12}}\,\exp\,\left[\frac{1}{2\alpha}\left(\tan^{-1}\frac{\beta}{\alpha} + n\pi\right)\right], \quad n \in \mathbb{Z}$$

Look for unstable solutions with growth time >>1/H so that inflation comes to an end after enough e-folds

$$H(t) = \bar{H} + \delta \exp(\xi \bar{H} t) \qquad \xi^2 + \xi\, 3\, e^{\frac{n\pi}{2\alpha}} + A = 0$$

$$1/\xi \approx e^{-n\pi/\theta''}$$

The stability of the solutions does not depend on μ

For negative values of n, A is always negative !

Polynomial expansion of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede, '07]
[P. Machado, F. Saueressig, '07]

Flow equation for $f(R)$-gravity:

$$\Gamma_k^{\rm grav}[g] = \int d^4x \sqrt{g}\, f_k(R)$$

- complicated partial differential equation governing k-dependence of $f_k(R)$

UV properties of RG flow:

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} \bar{u}_n R^n + \ldots$
- expand flow equation $\Longrightarrow$ β-functions for $g_n = \bar{u}_n k^{2n-4}$

$$k\partial_k g_n = \beta_{g_n}(g_0, g_1, \ldots)\,, \quad n = 0, \ldots, N$$

- reduces search for NGFP to algebraic problem

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots)\,, \quad i = 0, \ldots, N$$

- NGFP can be traced through extensions of truncation subspace

N	g_0^*	g_1^*	g_2^*	g_3^*	g_4^*	g_5^*	g_6^*
1	0.00523	-0.0202					
2	0.00333	-0.0125	0.00149				
3	0.00518	-0.0196	0.00070	-0.0104			
4	0.00505	-0.0206	0.00026	-0.0120	-0.0101		
5	0.00506	-0.0206	0.00023	-0.0105	-0.0096	-0.00455	
6	0.00504	-0.0208	0.00012	-0.0110	-0.0109	-0.00473	0.00238

NGFP is stable under extension of truncation subspace

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \dots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \dots) \,, \quad i = 0, \dots, N$$

- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\theta_{0,1}$	Im $\theta_{0,1}$	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.38	2.17					
2	1.26	2.44	27.0				
3	2.67	2.26	2.07	-4.42			
4	2.83	2.42	1.54	-4.28	-5.09		
5	2.57	2.67	1.73	-4.40	-3.97 + $4.57i$	-3.97 - $4.57i$	
6	2.39	2.38	1.51	-4.16	-4.67 + $6.08i$	-4.67 - $6.08i$	-8.67

NGFP is stable under extension of truncation subspace

good evidence: fundamental theory has finite number of relevant parameters

New approach

Basic idea: consider only linearized flow of the relevant operators

$$L = \frac{1}{16\pi G_k}\left(R - 2\Lambda_k\right) - \beta_k R^2$$

Approximate solutions of the non-perturbative β-functions in the crossover region between the NGFP and GFP:

$$G_k = \frac{G(\mu)}{1 + \omega G(\mu)(k^2 - \mu^2)}$$

$$\lambda = \lambda_0 + C\left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_1}{2}} \qquad \beta = \beta_0 + B\left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_3}{2}}$$

with $\theta_1 > 0$ and $\theta_3 > 0$.

- For irrelevant operators $\theta_i < 0$ and their contribution can be neglected as a first approximation.

New approach

Substituting the previous relations in the original lagrangian we obtain the following effective lagrangian (we used $\theta_1 \approx 2$, $\theta_3 \approx 1$ and $k^2 = \xi R$.)

$$-b_0\mu^2 R\sqrt{\frac{R}{\mu^2}} - \frac{c_0\mu^2\xi}{8\pi G_0} + \frac{23c_0\mu^4\xi^2}{36\pi^2} - \frac{23c_0\mu^2\xi^2 R}{36\pi^2} - \frac{\lambda_0\xi R}{8\pi G_0} + \frac{R}{16\pi G_0} - \beta_0 R^2 -$$

$$\frac{23\lambda_0\xi^2 R^2}{36\pi^2} + \frac{23\xi R^2}{72\pi^2} + \frac{23\lambda_0\mu^2\xi^2 R}{36\pi^2} - \frac{23\mu^2\xi R}{72\pi^2}$$

Effective Action for the Planck scale

$$S = \frac{1}{2\kappa^2}\int\sqrt{-g}d^4x\left\{R+\frac{1}{6m^2}R^2+\frac{\lambda}{3m\sqrt{3}}R\sqrt{R}-\Lambda m^2\right\}$$

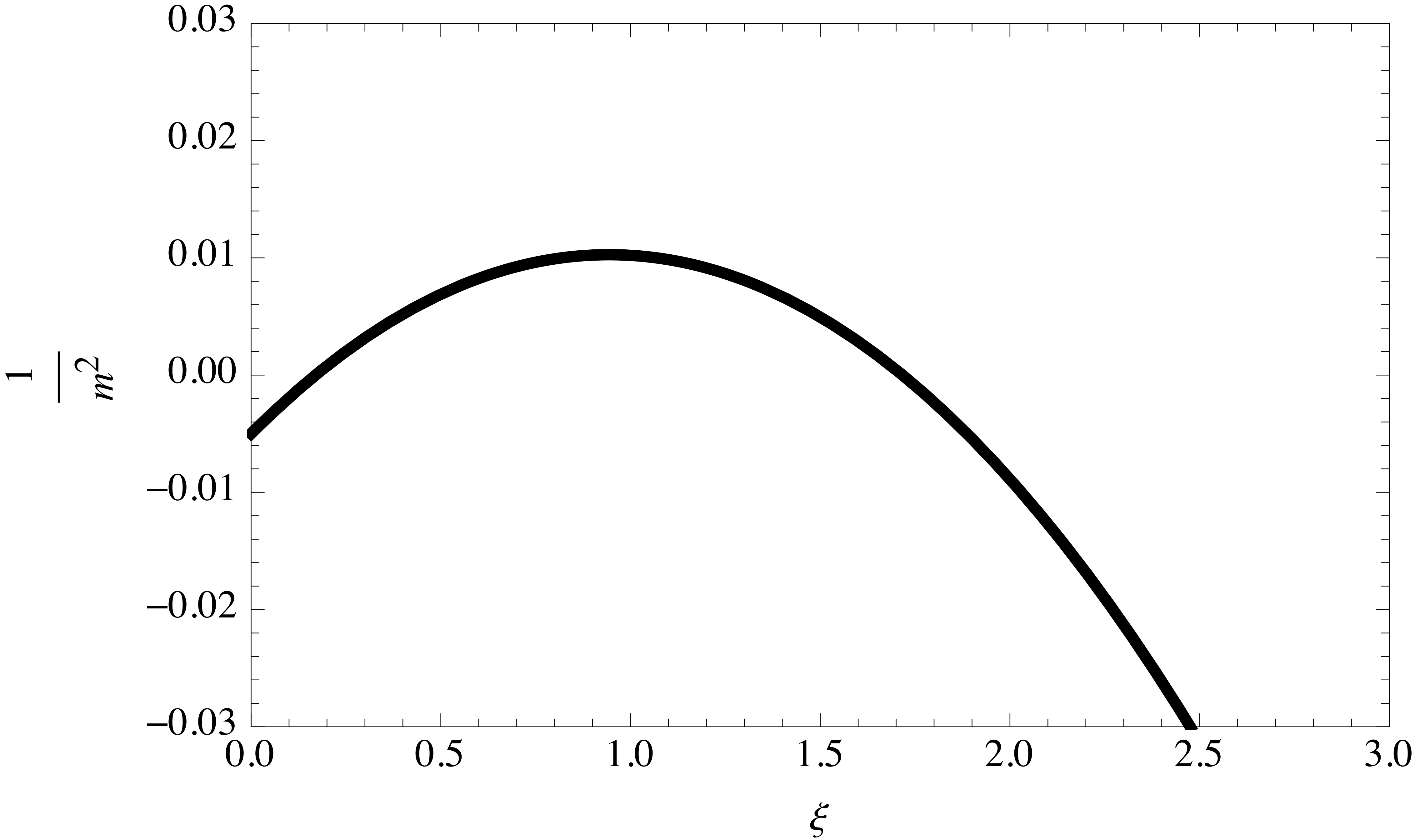
- The Newton's constant reads

$$\kappa = -\frac{9\pi G_0}{46G_0\mu^2\xi\left(2c_0\xi-2\lambda_0\xi+1\right)+9\pi\left(2\lambda_0\xi-1\right)}$$

  It is positive for $\xi > 1/2\lambda_0$ and $C < \frac{-1+2\xi\lambda_0}{2\xi}$.

- Moreover, the scalar degree of freedom reads

$$\frac{1}{m^2} = -\beta_0 - \frac{23\lambda_0\xi^2}{36\pi^2}+\frac{23\xi}{72\pi^2}$$



0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
1/m^2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ξ

Conformal frame representation of f(R)

Let's start with the general $f(R)$ gravity in which the action takes the form

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R), \tag{1}$$

where

$$f(R) = R + F(R), \tag{2}$$

and $\kappa^2 = 8\pi G_N$. If $F_{,RR} \neq 0$, the above action is equivalent to

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\varphi R - U(\varphi)\right], \tag{3}$$

where

$$\varphi \equiv 1 + F_{,\chi}(\chi), \quad U(\varphi) = \frac{(\varphi - 1)\chi(\varphi) - F(\chi(\varphi))}{2\kappa^2}. \tag{4}$$

Conformal frame representation of f(R)

We can do a conformal transformation to Einstein frame whose metric $g^E_{\mu\nu}$ is related to $g_{\mu\nu}$ by

$$g^E_{\mu\nu} = \varphi g_{\mu\nu}, \tag{5}$$

and the action in the Einstein frame becomes

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{1}{2\kappa^2} R_E - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \tag{6}$$

where

$$V(\phi) = \frac{1}{2\kappa^2 \varphi^2} \left[(\varphi - 1)\chi(\varphi) - F(\chi(\varphi)) \right], \tag{7}$$

and φ is related to the canonical field ϕ by

$$\varphi = e^{\sqrt{2/3}\kappa\phi}. \tag{8}$$

Conformal frame representation of f(R)

For the inflation govern by the action (6), the slow-roll parameters ϵ and η are respectively given by

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'_\phi}{V}\right)^2 = \frac{\varphi^2}{3}\left(\frac{V'_\varphi}{V}\right)^2, \tag{9}$$

and

$$\eta \equiv \frac{1}{\kappa^2}\frac{V''_\phi}{V} = \frac{2}{3}\frac{\varphi V'_\varphi + \varphi^2 V''_\varphi}{V}. \tag{10}$$

The number of e-folds before the end of inflation is related to the value of φ_N by

$$N \simeq \int_{t_N}^{t_{\rm end}} H dt = \frac{3}{2}\int_{\varphi_{\rm end}}^{\varphi_N} \frac{V}{V'_\varphi}\frac{d\varphi}{\varphi^2}. \tag{11}$$

Spectral index in the conformal frame

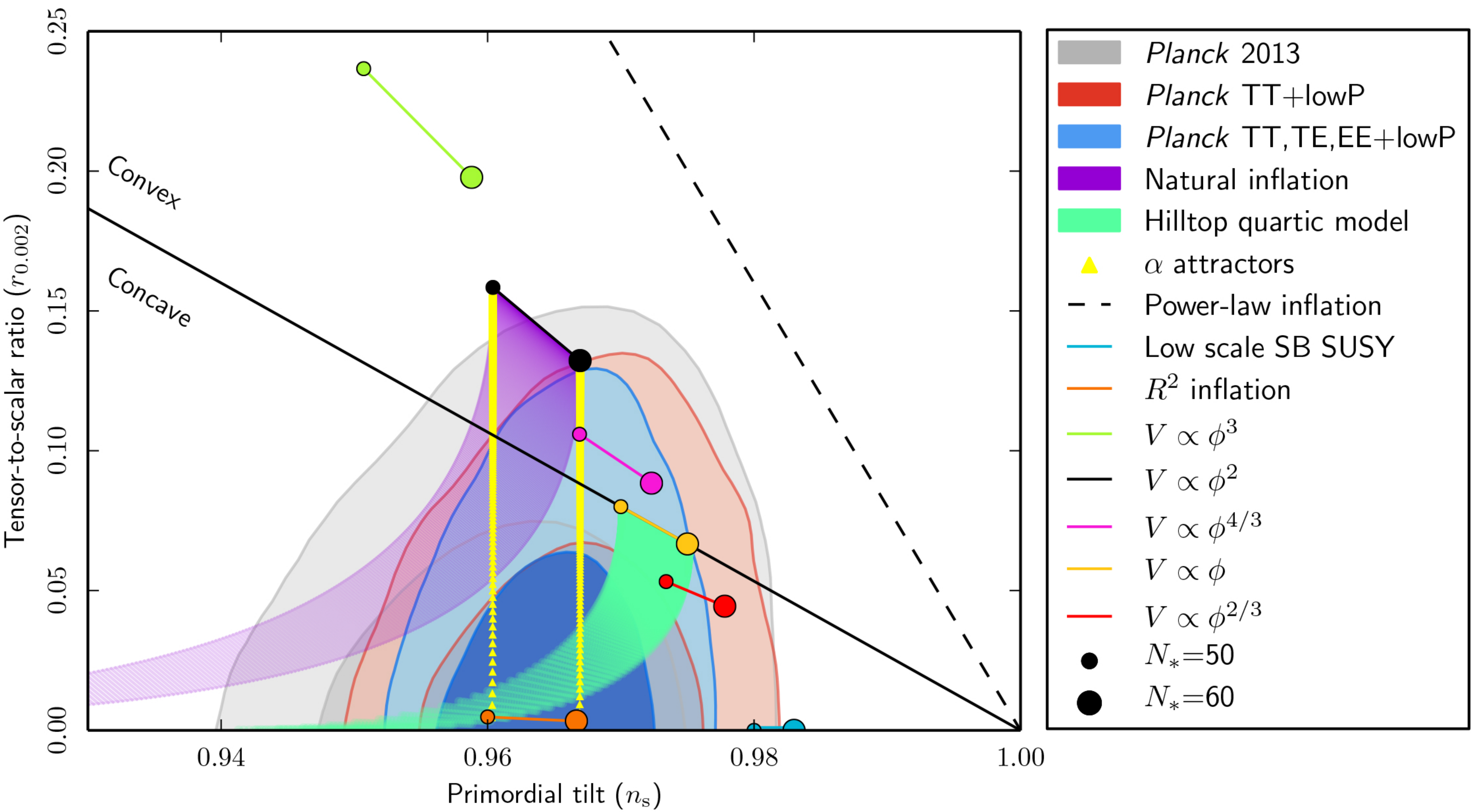
The amplitude of the primordial scalar power spectrum takes the form

$$\Delta_{\mathcal{R}}^2 = \frac{\kappa^4 V}{24\pi^2 \epsilon}, \tag{12}$$

and the spectral index n_s and the tensor-to-scalar ratio r are the the standard ones for the slow-roll inflation:

$$n_s = 1 - 6\epsilon + 2\eta, \tag{13}$$

$$r = 16\epsilon \tag{14}$$

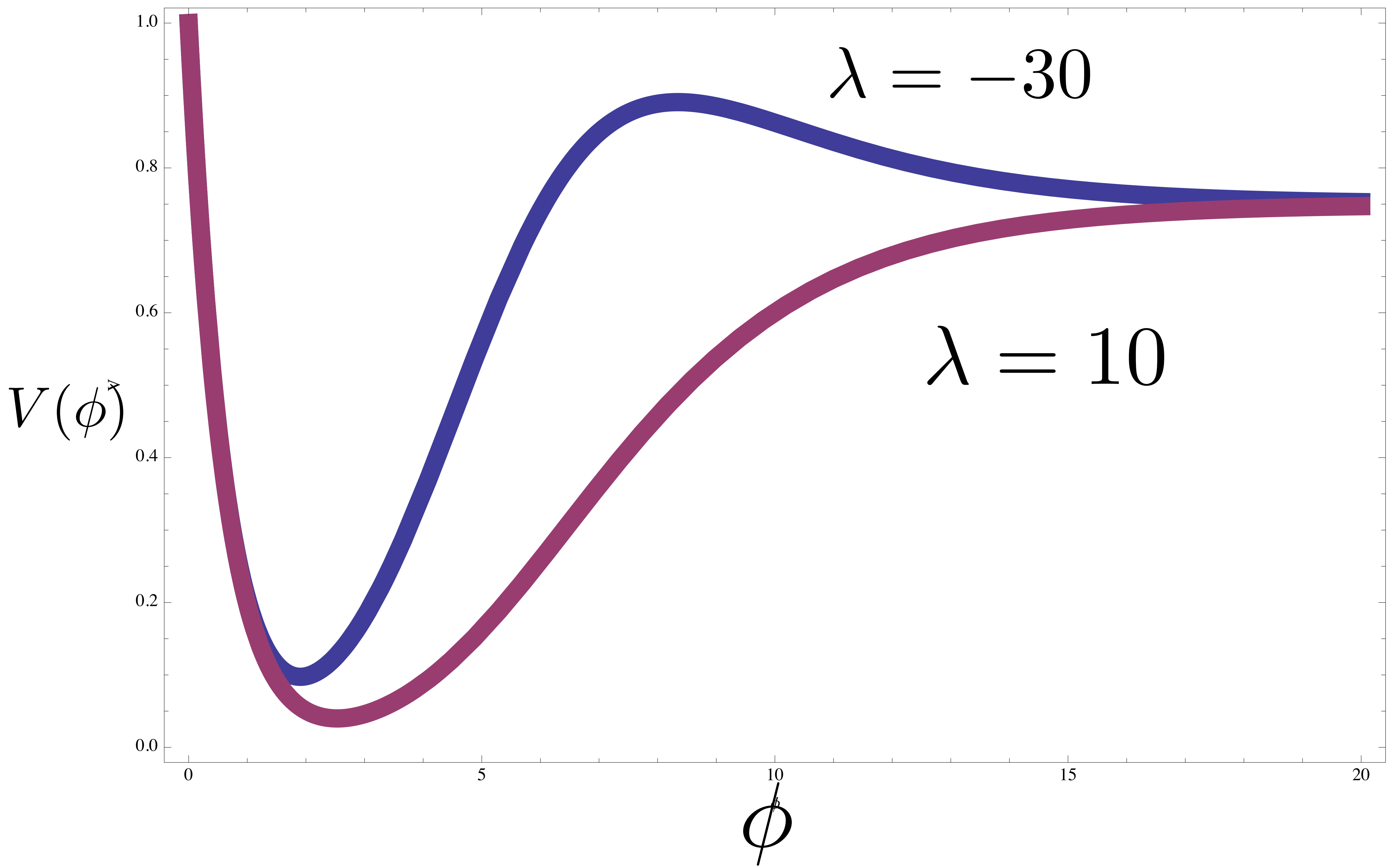


Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Inflationary potential in the Einstein frame

$$V(\phi) = \frac{e^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{128\kappa^3 M^2}\Bigg[- 3\kappa^3\lambda^4 M^6 - 24\kappa^3\lambda^2 M^6 e^{\sqrt{\frac{2}{3}}\kappa\phi} + 24\kappa^3\lambda^2 M^6$$
$$+64\kappa^3\Lambda M^6 - 192\kappa^3 M^6 e^{\sqrt{\frac{2}{3}}\kappa\phi} + 96\kappa^3 M^6 e^{2\sqrt{\frac{2}{3}}\kappa\phi}$$
$$+96\kappa^3 M^6 - 2\sqrt{2}\lambda\left(\kappa^2 M^4\left(8e^{\sqrt{\frac{2}{3}}\kappa\phi} + \lambda^2 - 8\right) - \sqrt{\kappa^4\lambda^2 M^8\left(16e^{\sqrt{\frac{2}{3}}\kappa\phi} + \lambda^2 - 16\right)}\right)^{3/2}$$
$$+3\kappa\lambda^2 M^2\sqrt{\kappa^4\lambda^2 M^8\left(16e^{\sqrt{\frac{2}{3}}\kappa\phi} + \lambda^2 - 16\right)}\Bigg] \tag{1}$$

where we have introduced the normalization scale M so that $m = M^2\kappa$



$\lambda = -30$
$\lambda = 10$
$V(\phi)$
ϕ
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20

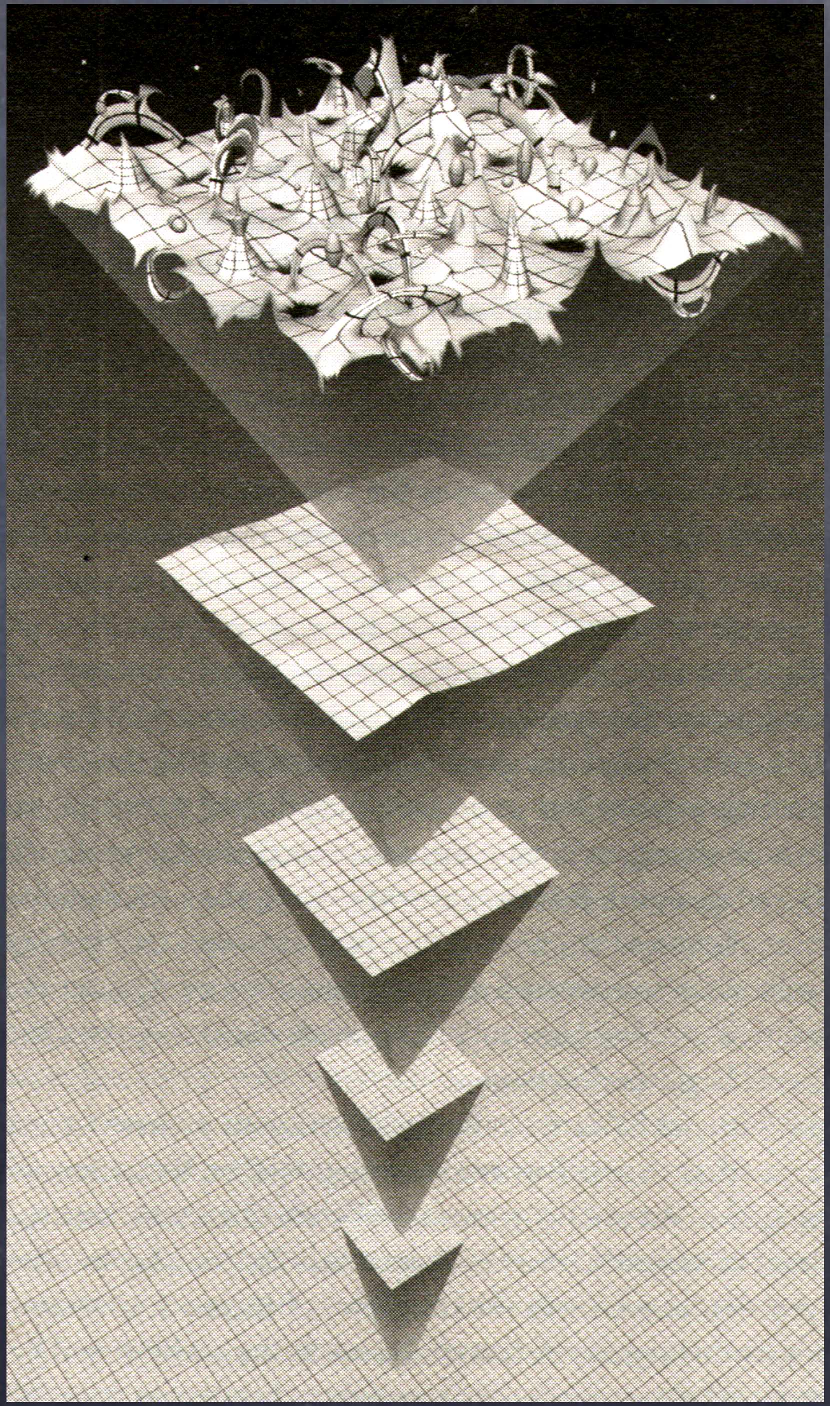
Results from Planck 2015: $ns = 0.968 \pm 0.006$, $r < 0.11$

λ	n_s	r
60	0.958321	0.0191536
20	0.961746	0.0161616
14	0.961300	0.0164368
12	0.963479	0.0146538
10	0.964575	0.0137433
8	0.962512	0.0151532
6	0.964905	0.0130958
4	0.966939	0.0111435
2	0.966797	0.0094684
-2	0.871784	0.0111355
-4	0.785626	0.0510945
-8	0.682099	0.1807045

Conclusions

- AS inflation emerges naturally from the structure of the UV critical surface
- It is in agreement with Planck 2015 data release depending on the value of lambda
- Present CMB data can put important constraints on the structure of the effective lagrangian of the Planck scale

Is the AS - vacuum "flat" ?



Asymptotic Safety predicts the presence of at least another relevant direction in the UV critical manifold

$$L = \frac{1}{16\pi G}(-R + 2\Lambda) + \beta R^2 \qquad \beta > 0$$

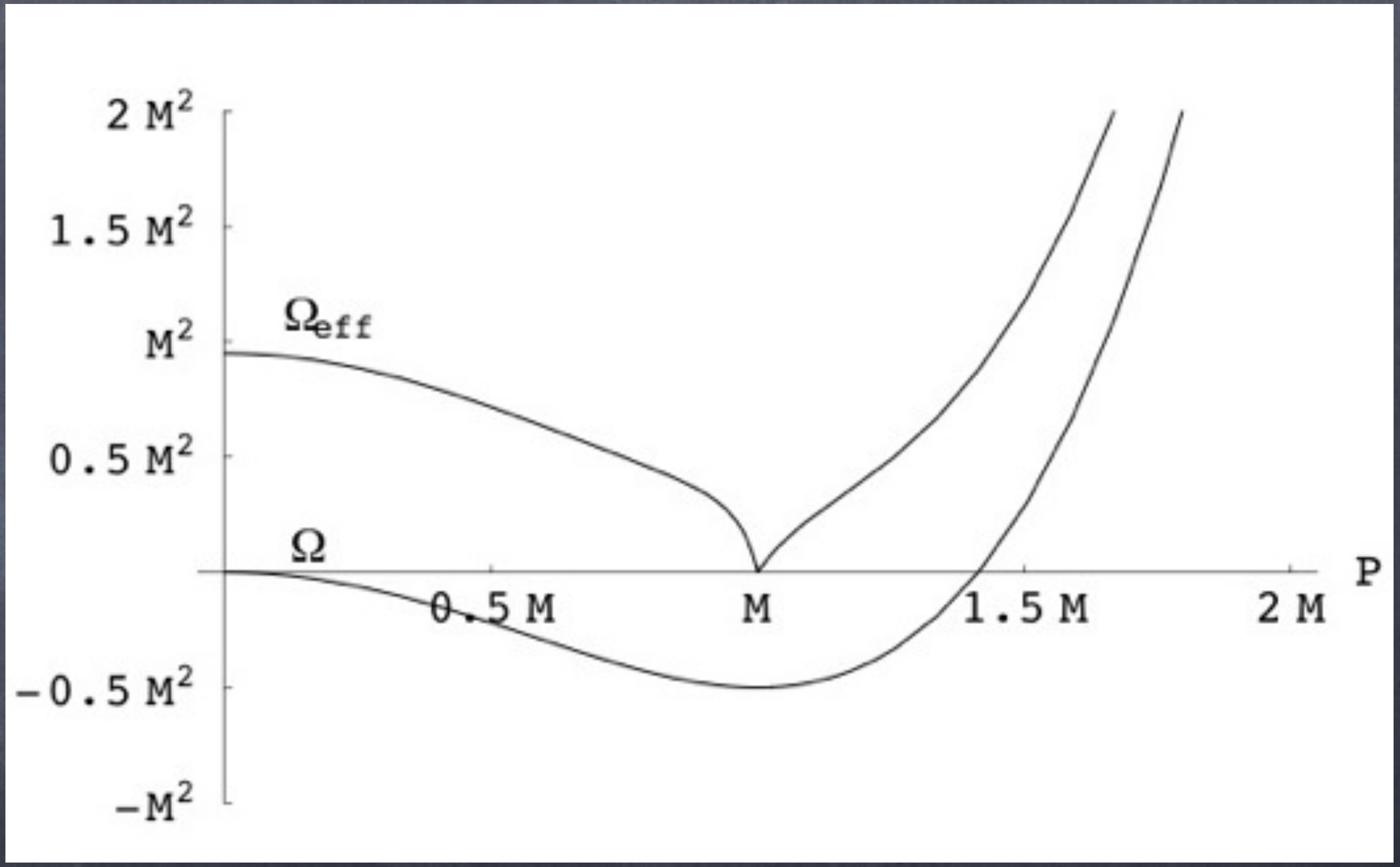
- Ref: Codello, Percacci, Rahmede 2007

Kinetic "condensate" model

$\langle 0|\partial_\mu\phi\partial^\mu\phi|0\rangle \neq 0$

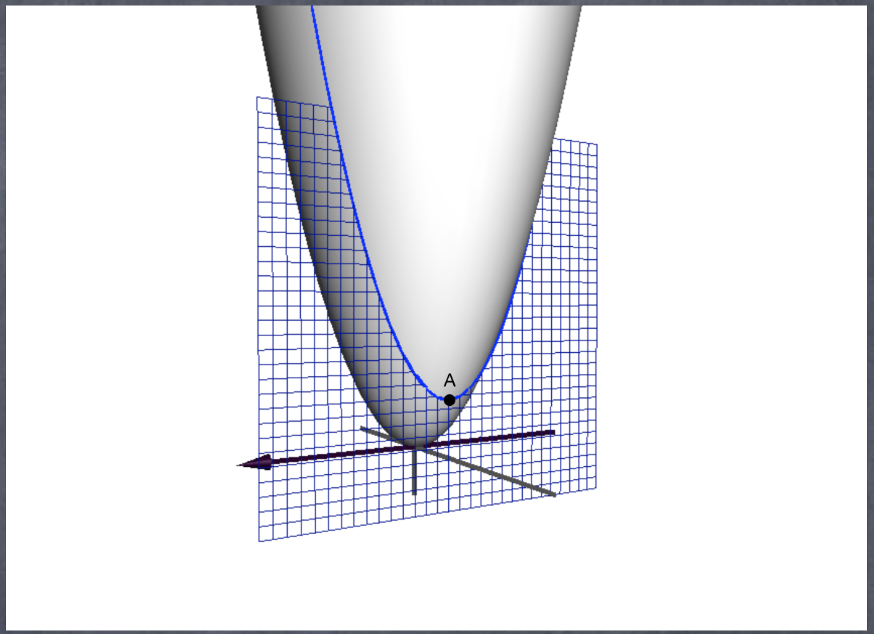
$$S[g] = S_{\rm EH} + \beta\int d^4x\sqrt{\beta}R^2 \Rightarrow \Omega(-\Box) = \Box + \Box\Box/2M^2$$

$$\Omega(p^2) = -p^2 + \frac{(p^2)^2}{2M^2}$$



Lauscher, Reuter and Wetterich, 2000

Conformally reduced R+R2 gravity



$$g_{\mu\nu} = \tfrac{1}{3}(4\pi G)\, \mathrm{e}^{2\sigma(x)}\, \delta_{\mu\nu}$$

$$S[\sigma] = \int d^4x \left\{ \tfrac{1}{2}\mathrm{e}^{2\sigma}\left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right) + 36\beta\left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right)^2 + \tfrac{u}{4!}\,\mathrm{e}^{4\sigma}\right.$$

$$\mathrm{e}^{2\sigma}\left[\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right] + \frac{u}{6}\,\mathrm{e}^{4\sigma} + 72\beta\left[\Box\Box\sigma + 2(\partial_\mu\partial_\nu\sigma)(\partial^\mu\partial^\nu\sigma) - 2(\Box\sigma)(\Box\sigma)\right.$$

$$\left. -2(\partial_\mu\sigma)(\partial^\mu\sigma)\Box\sigma - 4(\partial_\mu\sigma)(\partial_\nu\sigma)(\partial^\mu\partial^\nu\sigma)\right] = 0$$

Lattice-regulated model in 2-dim

$$S[\sigma(x)] = \sum_x \Big\{ \frac{u}{4!}\, e^{4\sigma(x)}$$

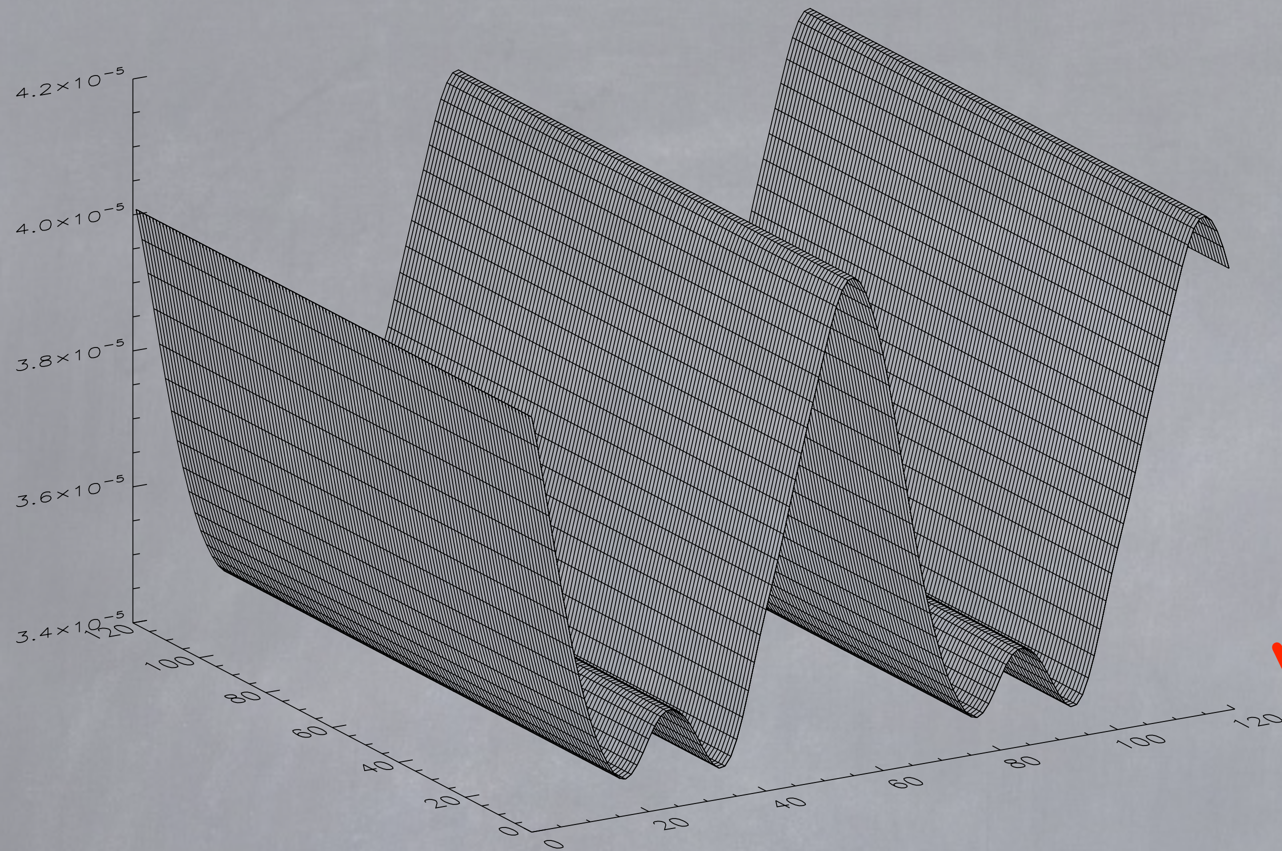
$$+\sum_\mu \frac{1}{2}\Big[e^{2\sigma(x)} (\sigma(x+e_\mu) + \sigma(x-e_\mu) - 2\sigma(x) + (\sigma(x+e_\mu) - \sigma(x))^2)$$

$$+\sum_\nu 36\beta(\sigma(x+e_\mu) + \sigma(x-e_\mu) - 2\sigma(x))(\sigma(x+e_\nu) + \sigma(x-e_\nu) - 2\sigma(x))$$

$$+(\sigma(x+e_\mu) + \sigma(x-e_\mu) - 2\sigma(x))(\sigma(x+e_\nu) - \sigma(x))$$

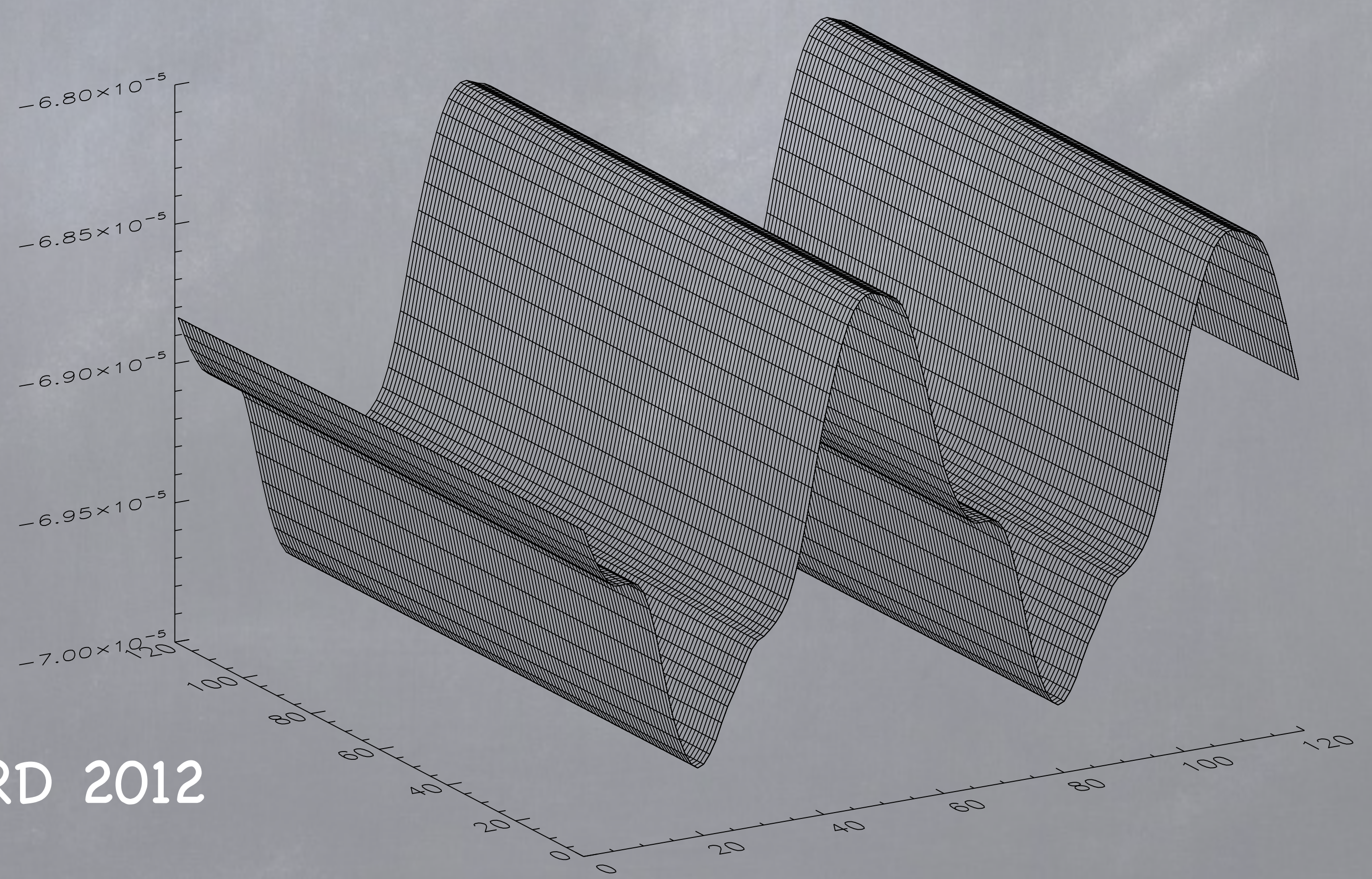
$$+(\sigma(x+e_\nu) + \sigma(x-e_\nu) - 2\sigma(x))(\sigma(x+e_\mu) - \sigma(x))$$

$$+(\sigma(x+e_\mu) - \sigma(x))(\sigma(x+e_\nu) - \sigma(x))^2 \Big]\Big\}$$



"Lasagne"-type of vacuum!

2-D simulations



Bonanno & Reuter, PRD 2012