Recent progress and open challenges in lattice QCD at finite temperature

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Outline

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Introduction and motivation

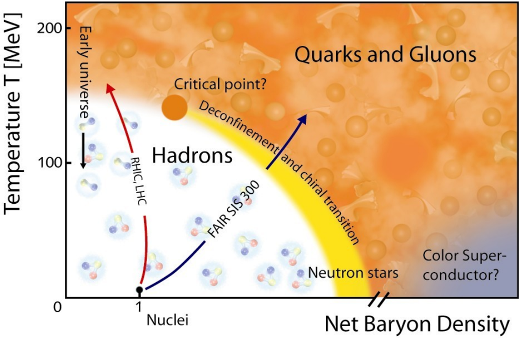
Lattice QCD generalities

Some lattice results in finite-temperature QCD

A look at the future

What?

Strong nuclear interactions under extreme conditions



[N. Cabibbo and G. Parisi, 1975], [J. C. Collins and M. J. Perry, 1975]

Why?

- An important test of QCD, one of the building blocks of the Standard Model
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- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, … **[E. Shuryak, 2009]**
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How?

A minimal summary of experimental QGP production — The stages

1. Heavy nuclei (Au, Pb) accelerated to ultra-relativistic energies; initial, "cold nuclear matter" conditions modelled as a color-glass-condensate (CGC)
2. Collision (central/peripheral); formation of a "glasma"
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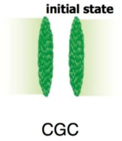

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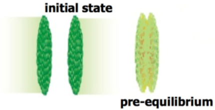

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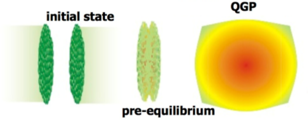

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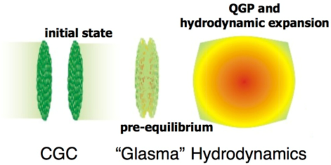

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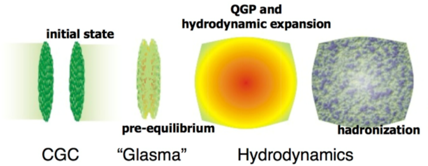

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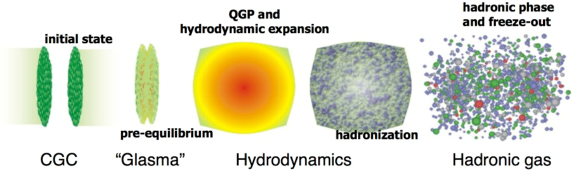

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A minimal summary of experimental QGP production — Observations and challenges for theorists

- Elliptic flow **[ALICE Collaboration, 2010]**
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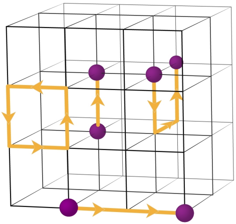
Regularize the QCD path integrals by discretizing the theory on a Euclidean lattice of spacing *a* **[K. G. Wilson, 1974]**



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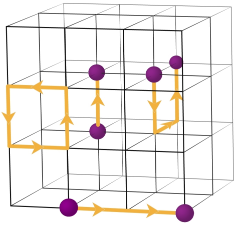

$\psi(x)$

MATTER FIELDS ON SITES

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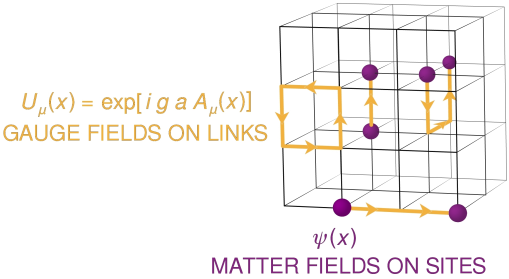

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Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action and observables

$$S = -\frac{1}{g^2} \sum_{\Box} \mathrm{Tr}(U_{\Box} + U_{\Box}^{\dagger}) + \sum_{x,y,f} a^4 \bar{\psi}_f(x) M^f_{x,y} \psi_f(y)$$

$$M^f_{x,y} = m\delta_{x,y} - \frac{1}{2a} \sum_{\mu} \left[(r-\gamma_\mu) U_\mu(x) \delta_{x+a\hat{\mu},y} + (r+\gamma_\mu) U_\mu^{\dagger}(y) \delta_{x-a\hat{\mu},y} \right]$$

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$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}\psi(x) \mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}\psi(x) \mathrm{d}\bar{\psi}(x) \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

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Euclidean QFT also has an interpretation as *thermal QFT at equilibrium*

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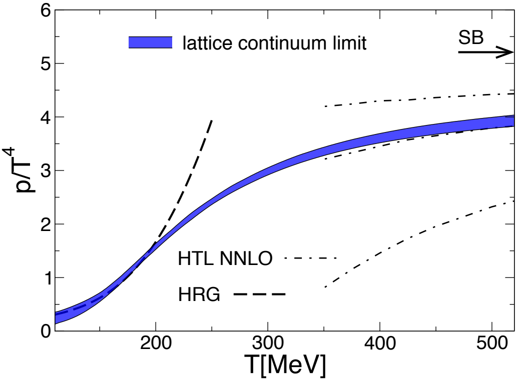
- Equation of state in QCD



[S. Borsányi et al., 2013]

See also **[A. Bazavov et al., 2012]** and **[T. Bhattacharya et al., 2014]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories: the large-N limit

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QCD at large N: Why bother?

The large-N limit of QCD (at fixed $\lambda = g^2 N$) has interesting implications **[G. 't Hooft, 1974]**

It plays a crucial rôle in the holographic gauge/string duality **[J. Maldacena, 1998]**

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$$\lambda = \frac{R^4}{l_s^4} \qquad\qquad \frac{\lambda}{N} = 4\pi g_s$$

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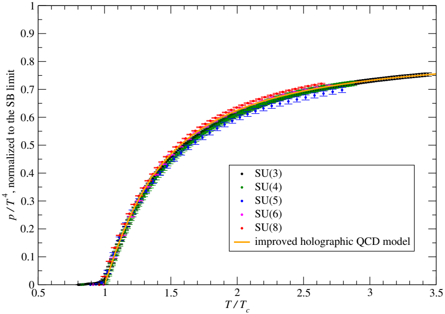


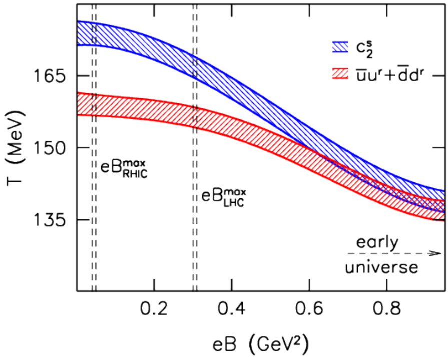
[M. P., 2009]

See also:

- Polyakov loops **[A. Mykkänen, K. Rummukainen and M. P., 2012]** (relevant for phenomenological models **[C. Ratti, M. A. Thaler and W. Wiese, 2006]**, **[H. Hansen et al., 2007]**)
- EoS in G_2 gauge theory **[M. Caselle et al., 2014]**
- EoS in 2+1 dimensions **[M. Caselle et al., 2011]**, **[M. Caselle et al., 2012]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories
- Dependence on (electro)magnetic fields: relevant for electro-weak phase transition in early Universe / peripheral heavy-ion collisions / magnetars



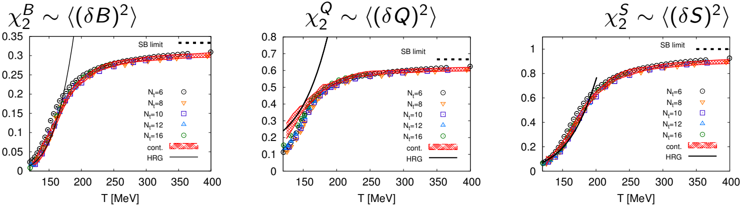
[G. S. Bali et al., 2011]

See also **[M. D'Elia, S. Mukherjee and F. Sanfilippo, 2010]**

Thermodynamics

- Equation of state in QCD and in QCD-like theories
- Dependence on (electro)magnetic fields
- Freeze-out conditions from fluctuations of conserved charges (baryon number B, electric charge Q, strangeness S) **[F. Karsch, 2012]**

$$\chi_{ijk}^{BQS} = \frac{1}{VT^3} \frac{\partial^i}{\partial(\mu_B/T)^i} \frac{\partial^j}{\partial(\mu_Q/T)^j} \frac{\partial^k}{\partial(\mu_S/T)^k} \ln Z$$


$\chi_2^B \sim \langle(\delta B)^2\rangle$ $\chi_2^Q \sim \langle(\delta Q)^2\rangle$ $\chi_2^S \sim \langle(\delta S)^2\rangle$

[S. Borsányi et al., 2011]

See also **[A. Bazavov et al., 2012]**

Consistent determination of freeze-out conditions: $T_{\text{fr}} = 144(10)$ MeV, $\mu_{\text{fr}}^B = 102(6)$ MeV at RHIC (STAR, $\sqrt{s} = 39$ GeV) **[S. Borsányi et al., 2014]**
Comparison with hadron resonance gas model **[P. Alba et al., 2014]** and with statistical hadronization model **[A. Andronic, P. Braun-Munzinger and J. Stachel, 2009]**

Quarkonium melting

A QGP "thermometer" **[T. Matsui and H. Satz, 1986]**

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General strategy for the lattice computation:

- Heavy quarks are, well, heavy
- Compute correlation functions of sources with desired quantum numbers: $G_{\mathrm{E}}(\tau) \simeq \int_{-2M}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \exp(-\omega\tau)\rho(\omega)$
- Invert to extract spectral function $\rho(\omega)$

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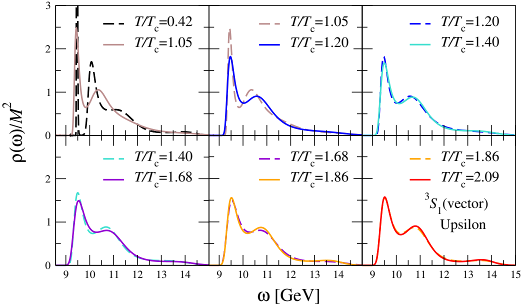

Bottomonium excitation melting **[G. Aarts et al., 2011]**

Transport coefficients

Describe QGP response to long-wavelength / low-frequency perturbations in energy and momentum density and other conserved charges **[H. B. Meyer, 2011]**

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Example: Shear (η) and bulk (ζ) viscosities [P. Romatschke, 2010]

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - \mathbb{P}^{\mu i}\mathbb{P}^{\nu j}\left[\eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3} g_{ij}\partial_k u^k\right) + \zeta g_{ij}\partial_k u^k\right]$$

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Example: Shear viscosity

$$\eta = \pi \lim_{\omega\to 0} \lim_{\mathbf{k}\to 0} \frac{\rho(\omega,\mathbf{k})}{\omega}$$

with ρ the spectral function, related to a suitable (e.g. $T^{\mu\nu}$) Euclidean correlator via

$$G_{\mathrm{E}}(\tau,\mathbf{k}) = \int_0^\infty \mathrm{d}\omega\, \rho(\omega,\mathbf{k}) \frac{\cosh\left[\omega(\tau-\frac{1}{2T})\right]}{\sinh\left(\frac{\omega}{2T}\right)}$$

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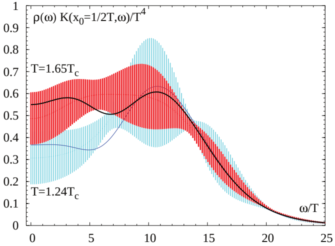
Numerically *very* challenging



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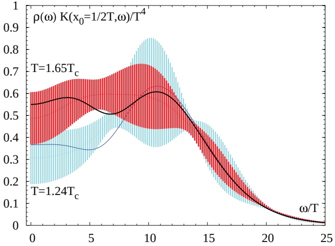

[H. B. Meyer, 2007]

Analytical guidance e.g. from holography? **[G. S. Bali et al., in progress]**

Jet quenching

A hard quark moving through the QGP: average momentum broadening described by jet quenching parameter $\hat{q}$

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 \, C(p_\perp)$$

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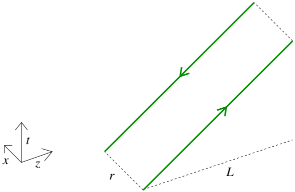
$C(p_\perp)$, the differential parton-plasma constituents collision rate, related to two-point correlator of *light-cone Wilson lines*



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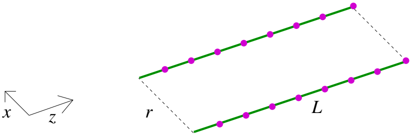

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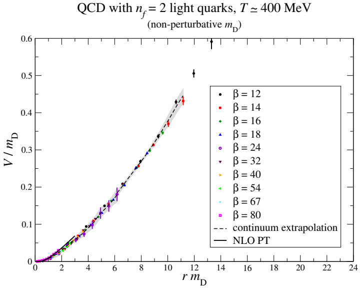
Direct access to collision kernel in coordinate space



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Direct relation to non-perturbative contribution to screening masses **[B. Brandt et al., 2014]**

Outline

Introduction and motivation

Lattice QCD generalities

Some lattice results in finite-temperature QCD

A look at the future

Photon production rates

As electromagnetic probes, photon (and dilepton) rates provide important information on early stages of the nuclear collision **[Ghiglieri et al., 2013]**

$$\frac{\mathrm{d}\Gamma_\gamma}{\mathrm{d}^3\mathbf{k}} = -\frac{1}{(2\pi)^3 2|\mathbf{k}|} W^<(k^0 = k)$$

with $W^<(K)$ the photon polarization

$$W^<(K) = \int \mathrm{d}^4 X \exp\left(iK \cdot X\right) \mathrm{Tr} \rho J^\mu(0) J_\mu(X)$$

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Like for the $\hat{q}$ computation, soft QCD contributions can be computed in a dimensionally reduced effective theory on the lattice

Real-time $Q\bar{Q}$ potential

The *real-time* static quark-antiquark potential $V(r,t)$ in hot QCD is generically *complex* **[M. Laine et al., 2007]**, **[A. Beraudo, J.-P. Blaizot and C. Ratti, 2008] [N. Brambilla et al., 2008] [T. Hayata, K. Nawa and T. Hatsuda, 2012]**

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Outline of lattice strategy:

- Compute Euclidean thermal Wilson loops $W_{\mathrm{E}}(r,\tau)$
- Extract spectral function $\rho(r,\omega)$ by inverting

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- Compute the real-time potential via

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Results exist for $\mathrm{SU}(3)$ Yang-Mills **[A. Rothkopf, T. Hatsuda and S. Sasaki, 2012]**, similar studies in full QCD ongoing **[A. Bazavov, Y. Burnier and P. Petreczky, 2014]**

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Sign problem: At finite quark chemical potential μ, fermionic determinant *complex*

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- Large-N orbifold dualities
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A word of caution: might be a NP-hard problem **[M. Troyer and U.-J. Wiese, 2005]**

Concluding remarks

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Thanks for your attention