Towards optimization-based multi-agent collision-avoidance under continuous stochastic dynamics

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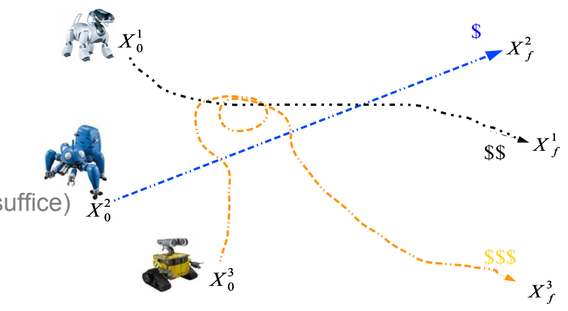
Problem statement

- For all agents a we are **given**:
  - Start / end states: (X_0^a, X_f^a)
  - Continuous dynamics model: D (plan p^a)= uncertain trajectory (stoch. process) (ie: mean & covariance suffice)
  - Cost functions, eg $c^a(p^a)$ = $$



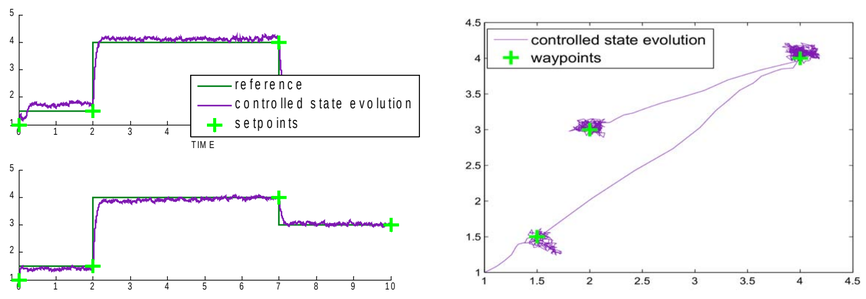
- **Wanted:** Agents' plans: $p^1, p^2, \ldots$

such that:

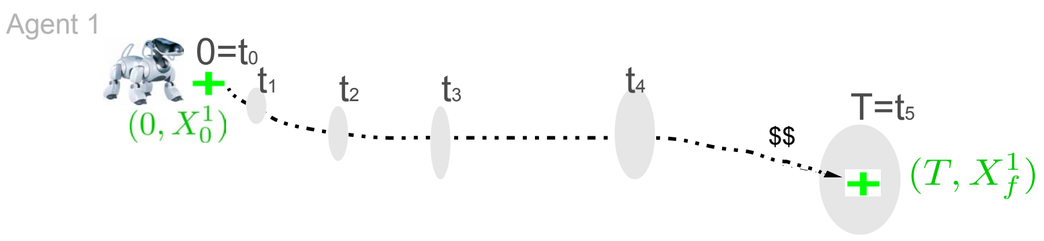
- Pr [exists collision] < Threshold δ
- Goals are met in expectation
- Social cost: $\sum_a c^a(p^a)$ (eg:$$ + $ + $$$) low

Single-agent planning

Agent's dynamics: $SDE: dX(t) = K(\xi(t) - X(t))dt + \sqrt{\nu}dW(t)$



$\xi(t)$: step function given by breakpoints ("setpoints ") setpoints : (time, state) pairs

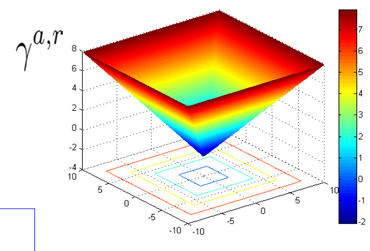


→ represent agent's plan as sequence of setpoints influencing velocity and direction of state evolution to connect start and goal while incurring low cost (in expectation).

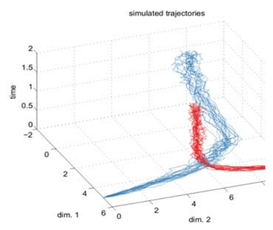
Collision detection

$$\exists j : C_j(t) > 0 \Rightarrow \Pr[Coll^{a,r}(t)] < \delta$$
$$\gamma^{a,r}(t) = \max_j C_j(t)$$

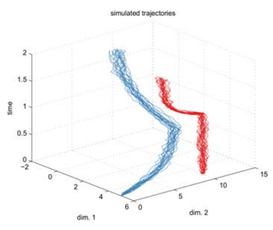
$$\gamma^{a,r}(t) > 0 \Rightarrow Pr[Coll^{a,r}(t)] < \delta$$



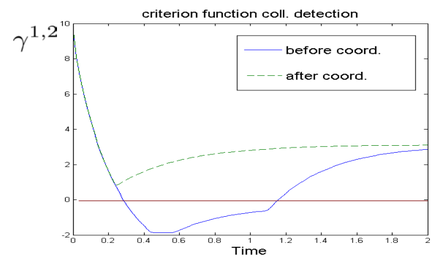
- Collision detection: agent a checks: $\gamma^{a,r}([0,T]) \subset \mathbb{R}_+, \forall r \neq a$



Before coord.



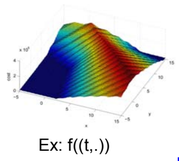
After coord.



Collision resolution

Question: How to find a new setpoint (t,s) to avoid collision?

Answer 1: choose new setpoint **free**ly as argmin of cost function f:



$$f((t,s)) = < c(p_{\uparrow(t,s)}) > + w_\beta \beta(p_{\uparrow(t,s)})$$

plan updated by setpoint (t,s)

large

hinge-loss penalizing collision:

$$\beta(p_{\uparrow(t,s)}) = \max\{0, -\min_t \max_{r \neq a} \gamma^{a,r}_{\uparrow(t,s)}(t)\}$$

→ "**FREE**" method.

Answer 2: choose new setpoint to make agent wait.

→ "**WAIT**" method.

Multi-agent planning

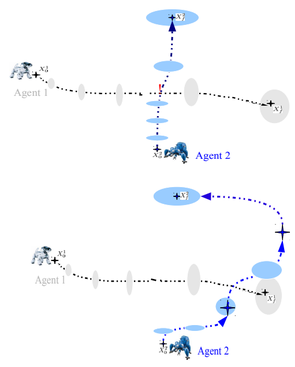
- **Multi-agent planning: many single agent problems connected by collision constraints.**

input : Agents $\mathfrak{a} \in \mathfrak{A}$, cost functions $c^{\mathfrak{a}}$, dynamics, initial start and goal states, initial plans $p^1, \ldots, p^{|\mathfrak{A}|}$.
output: collision-free plans $p^1, \ldots, p^{|\mathfrak{A}|}$.

for $\mathfrak{a} = 2 : |\mathfrak{A}|$ **do**
  repeat
    $[\text{flag}^{\mathfrak{a}}, \mathfrak{C}^{\mathfrak{a}}, t_{\text{coll}}] \leftarrow$ `CollDetect` $^{\mathfrak{a}}(\mathfrak{a}, \{1, \ldots, \mathfrak{a}-1\})$
    if $\text{flag}^{\mathfrak{a}} = 1$ **then**
      $p^{\mathfrak{a}} \leftarrow$ `Avoid` $^{\mathfrak{a}}(\mathfrak{C}^{\mathfrak{a}}, t_{\text{coll}})$
    end
  until $\text{flag}^{\mathfrak{a}} = 0$;
  `Broadcast` $^{\mathfrak{a}}(p^{\mathfrak{a}})$
end

Algorithm 2: Fixed priorities coordination method (FP) (written in a sequentialized form). Collisions are resolved by choosing new setpoints to enforce collision avoidance. $\mathfrak{C}^{\mathfrak{a}}$: set of agents detected to be in conflict with $\mathfrak{a}$. $\text{flag}^{\mathfrak{a}}$: collision detection flag (=0, iff no collision detected). t_{coll}: earliest time where a collision was detected. `Avoid`: collision resolution method updating the plan by a single new setpoint according to WAIT or FREE (ref. (II), Sec.

- **Solution: add new setpoints incrementally until all collisions with higher-ranking agents are avoided.**

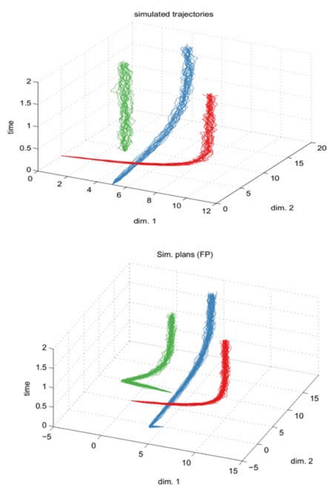


initial plans

after collision avoidance

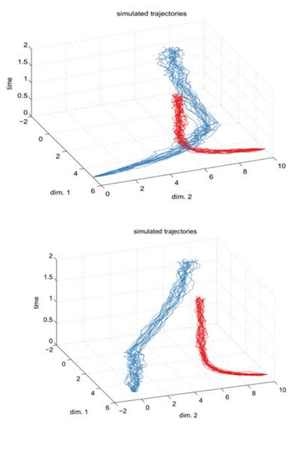
- **Needed: collision detection & collision resolution modules.**

Simulations



NONE (uncoord.)

Colliding trajectories: 51 %



NONE (uncoord.)

Colliding trajectories: 78 %

FREE

Colliding trajectories: 0 %

WAIT

Colliding trajectories: 0 %

Agent ranking: 1 > 2 > 3

Summary & Ongoing work

Summary:

- **Successful collision avoidance.**
- **Goals reached.**

Ongoing Work:

- **Parallelization.**
- **Learning dynamics.**
- **Auctions.**
- **Probabilistic guarantees for minimization / root detection.**
- **Static obstacles.**