COALITIONAL GAMES VIA NETWORK FLOWS

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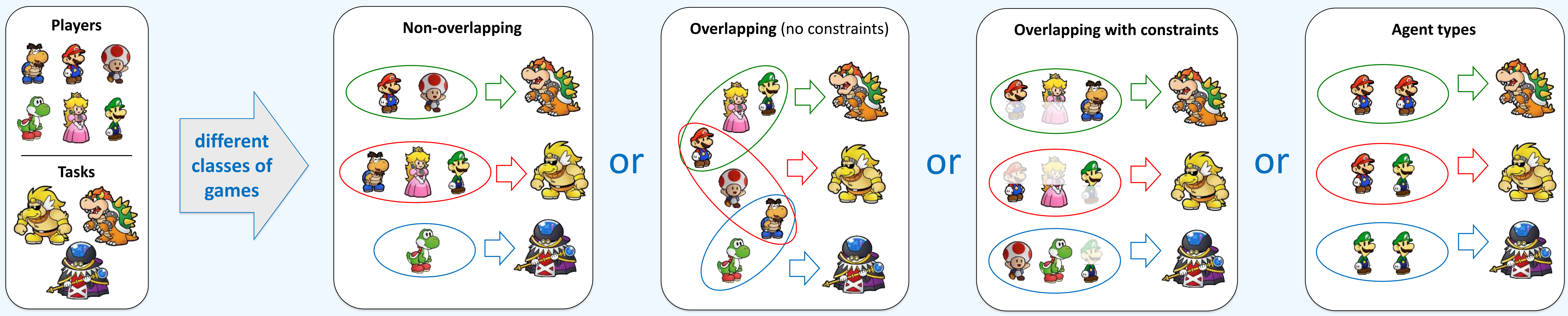
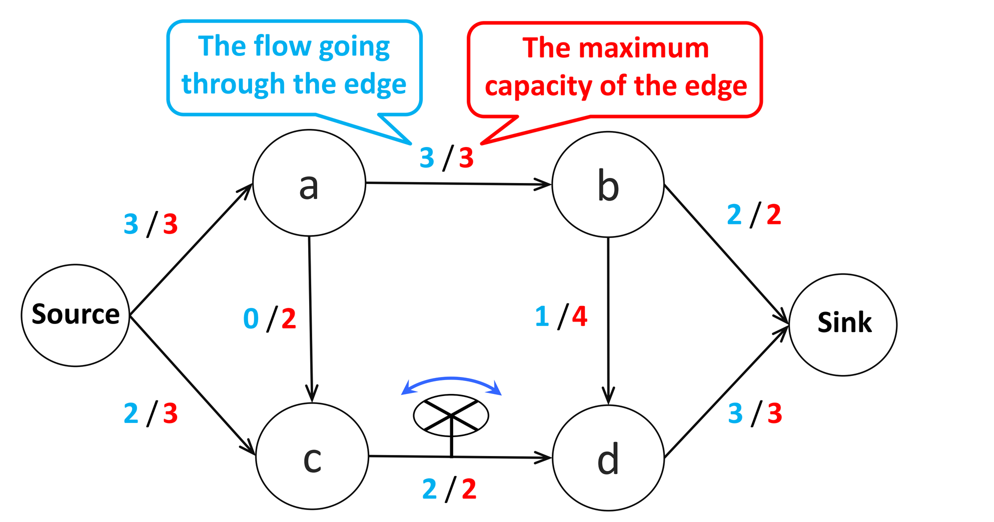
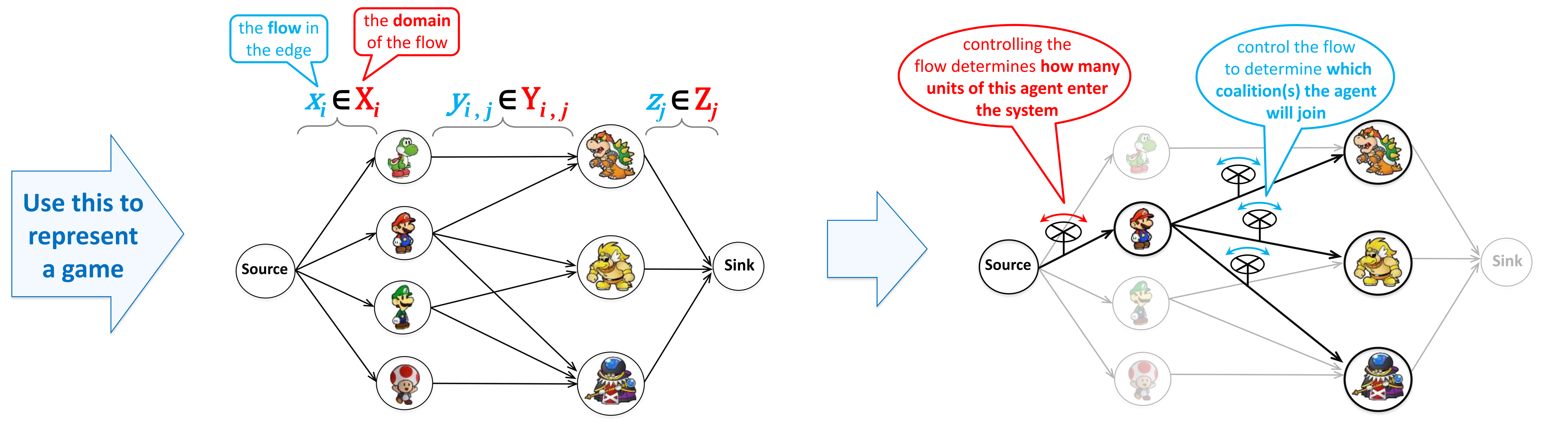


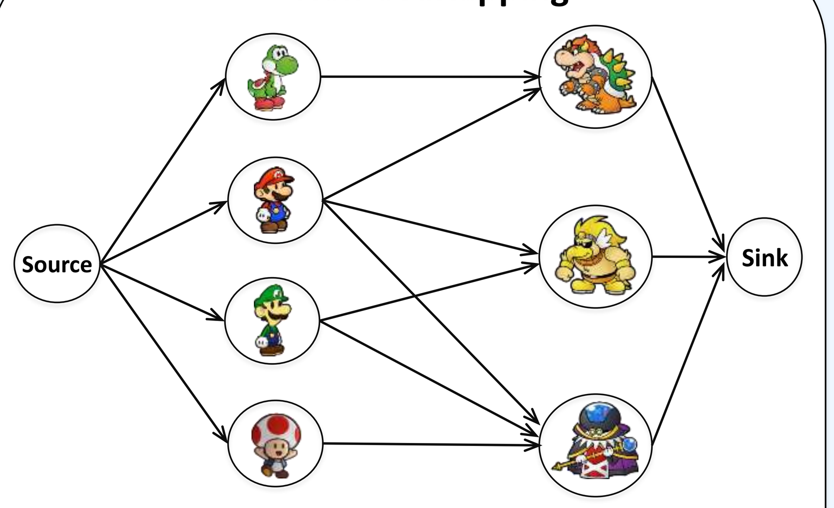
The **Maximum Flow** problem:



Flow Conservation rule:
total flow entering a node = total flow going out of it



Non-overlapping

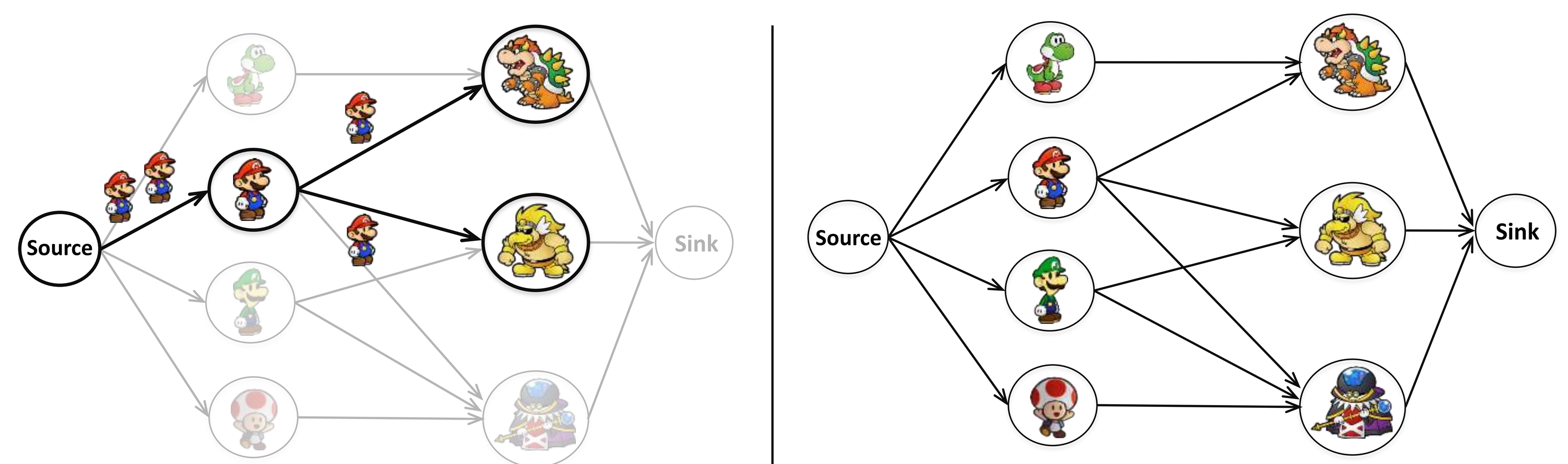


$x_i \in \{1\}$ ⇦ exactly one unit of each agent must enter the system

$y_{ij} \in \{0, 1\}$ ⇦ Every agent can join any coalition once at most

$z_j \in \{0, \ldots, n\}$ ⇦ All agents could end up in one coalition

Overlapping Coalitions



1 agent joining 2 coalitions = 2 copies of the agent, each joining 1 coalition

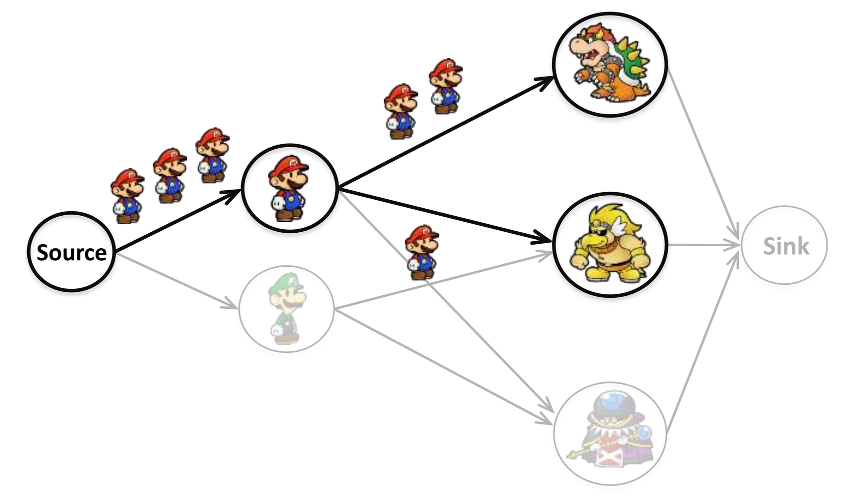
Note: different copies of the agent **cannot** join the same coalition!

$x_i \in \{1, \ldots, |K|\}$ ⇦ an agent can join all coalitions simultaneously

$y_{ij} \in \{0, 1\}$ ⇦ copies of an agent cannot join the same coalition

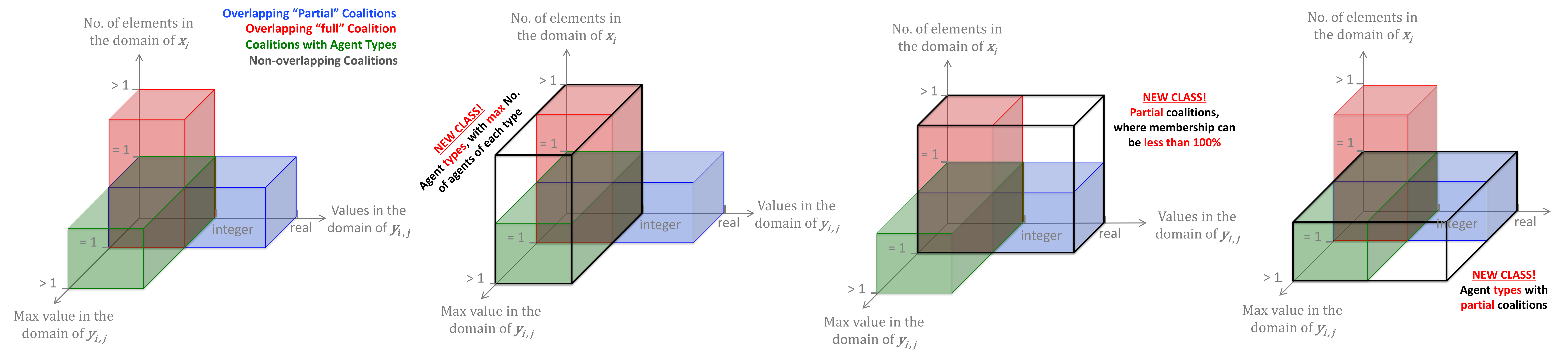
$z_j \in \{0, \ldots, n\}$

Agent types



$x_i \in \{|T_i|\}$ ⇦ all $|T_i|$ copies of a_i must enter the system

$y_{ij} \in \{0, \ldots, |T_i|\}$ ⇦ all copies can join the same coalition



Coalition Structure Generation (CSG)

We focus on the non-overlapping case, without agent types

$$CSG := \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \sum_{i=1}^{n}\sum_{j=1}^{q} d_{ij} y_{ij} + \sum_{j=1}^{q} g_j(z_j)$$

⇦ The value of the coalition structure

$$\text{s.t.} \quad z_j = \sum_{i=1}^{n} y_{ij}, \quad \forall\, j = 1, \ldots, q,$$

$$x_i = \sum_{j=1}^{q} y_{ij}, \quad \forall\, i = 1, \ldots, n,$$

⇦ The flow conservation rules

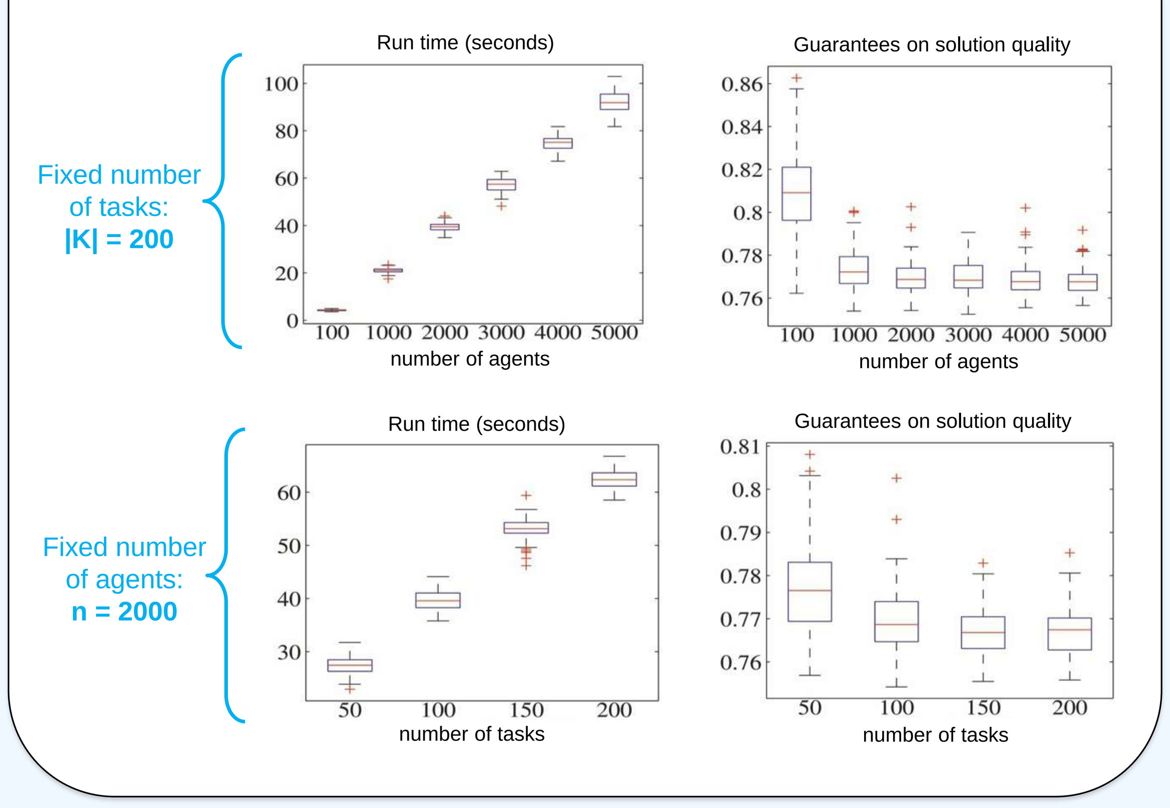
$$\boldsymbol{x} = \{1\}^n, \boldsymbol{y} \in \{0,1\}^{n \times q}, \boldsymbol{z} \in \{0, \ldots, n\}^q.$$

⇦ The capacities of edges

Approximation:

1. Solve a Lagrangian Dual Problem to obtain an upper bound
2. From the solution in Step 1, construct a feasible solution to the primal problem
3. Repeat 1 and 2 to tighten the bound and improve the solution

Empirical Evaluation (non-overlapping, no agent types)



Future Work

- Study different synergy functions
- Study solution concepts (the Core, Shapley Value, Kernel, …)
- Introduce *new components* to the representation: