Redistribution in Online Mechanisms

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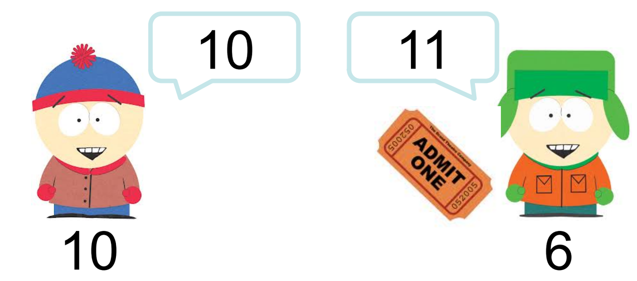
Redistribution Agenda

Allocation Rule $\pi_i(v) = \begin{cases} 1 & \text{if } i \text{ is allocated} \\ 0 & \text{otherwise} \end{cases}$

Utility of agent *i* $u_i = v_i \pi_i(v) - x_i(v)$

Objective: Maximize Social Welfare

Need to Incentivize Truthful Reporting



Truthful Reporting Achievable with Payments

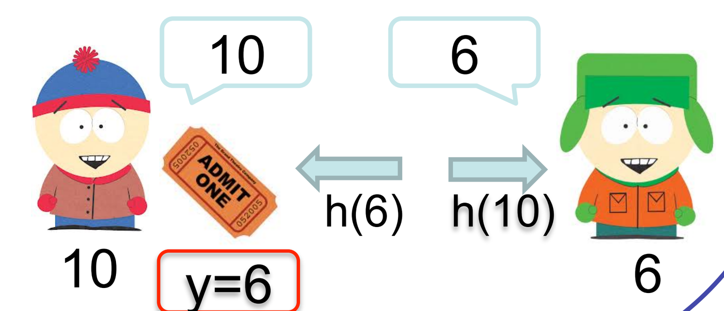
Payment Rule $y_i(v) = \begin{cases} v^c(v_{-i}) & \text{if } \pi_i(v) = 1 \\ 0 & \text{otherwise} \end{cases}$

v^c - critical value is the minimum value agent i can report to be allocated

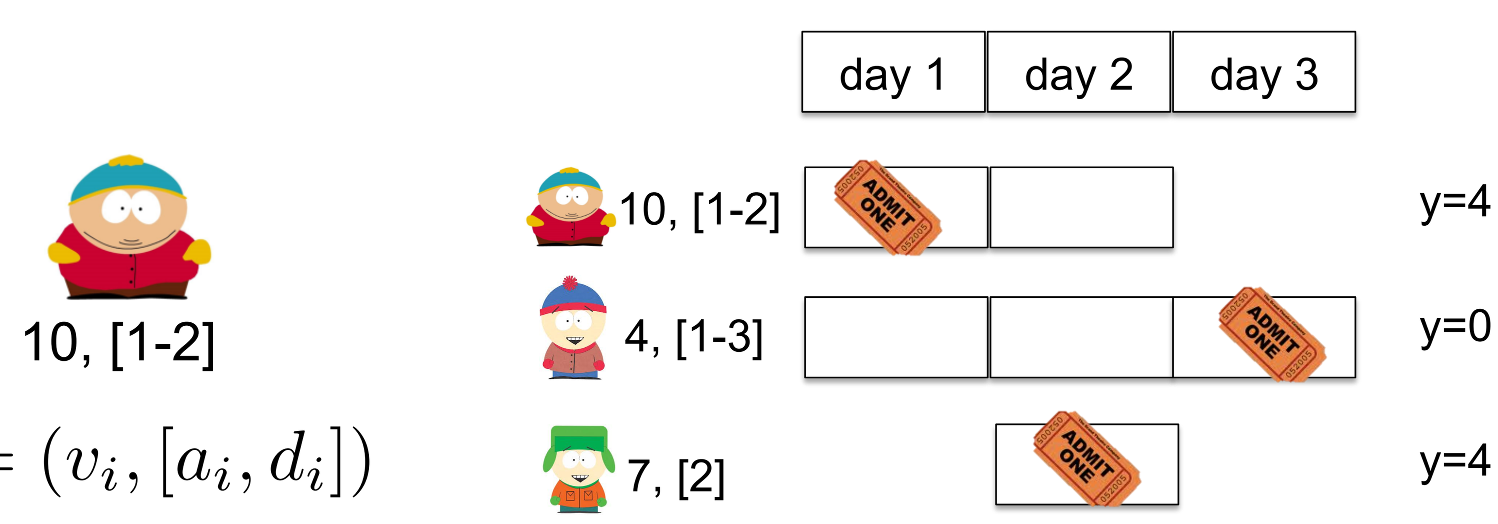
No Auctioneer ➔ Revenue = Agents' Loss

Redistribution Agenda: **optimize over functions h to minimize revenue**

$x_i(v) = y_i(v) - h(v_{-i})$



Online Mechanisms: Allocate One Item Each Day



$\theta_i = (v_i, [a_i, d_i])$

Characterization of Truthful Mechanisms

Mechanism is truthful if and only if the payment is given by the critical value v^c and redistribution h that satisfies

$$h(\theta_{-i}, a_i, d_i) \geq h(\theta_{-i}, a'_i, d'_i) \quad \forall \theta_{-i},\ a'_i \geq a_i,\ d'_i \leq d_i$$

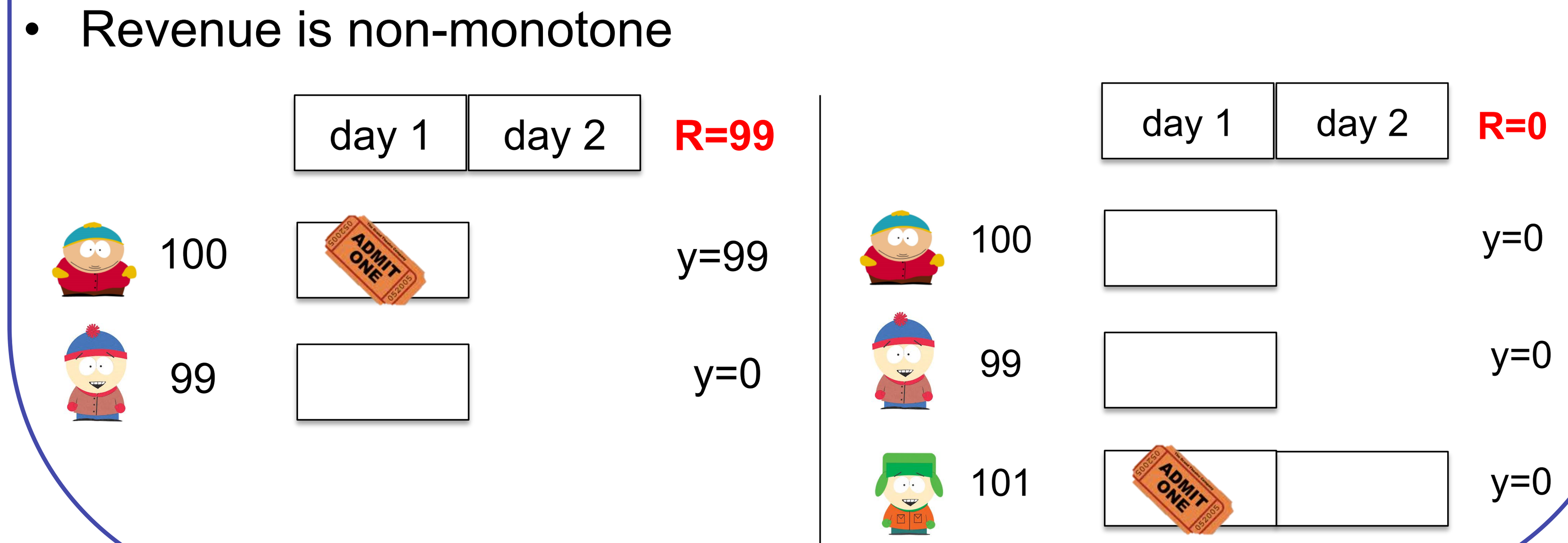
Evaluation Metric

Objective value: $\min_{\theta} \frac{H(\theta)}{R(\theta)}$ $\quad H(\theta) = \sum_{i \in N} h_i(\theta_{-i}, a_i, d_i)$ $\quad R(\theta) = \sum_{i \in N} y_i(\theta)$

In general online cases, with redistribution rule of Bailey/Cavallo, the objective value is 0.

Observations

- Agent *i* can decrease the revenue by at most the highest single payment among agents in the market without *i*
- Revenue is non-monotone



Our Redistribution Functions

Redistribution al the Last Period

Redistribution Rule (Bailey/Cavallo) $h(\theta_{-i}) = \frac{R(\theta_{-i})}{n}$

Objective value: $r(m, n, \bar{v}) \geq \frac{n - 2T}{n}(R(\theta) - \bar{v})$

$\bar{v}$ is the highest value a single agent may have

Redistribution at Each Agent's Departure

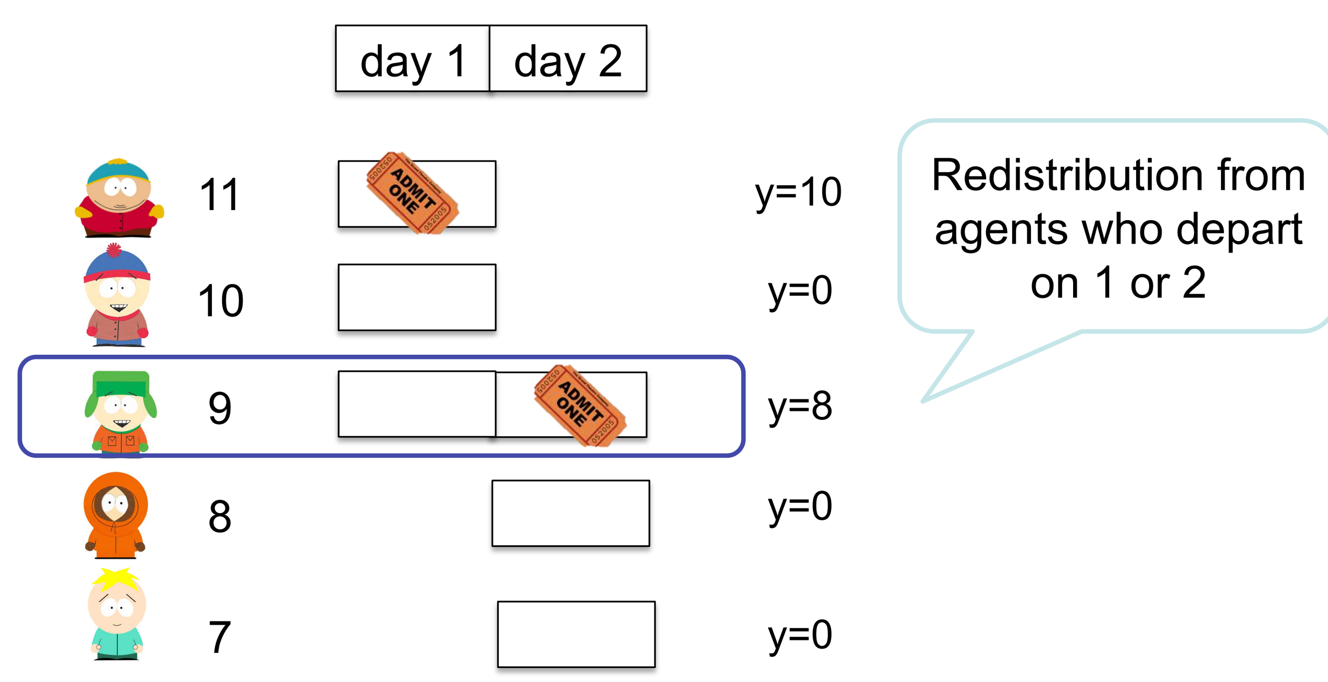
"Charge" the rebate of *i* against payments of agents who contribute to it

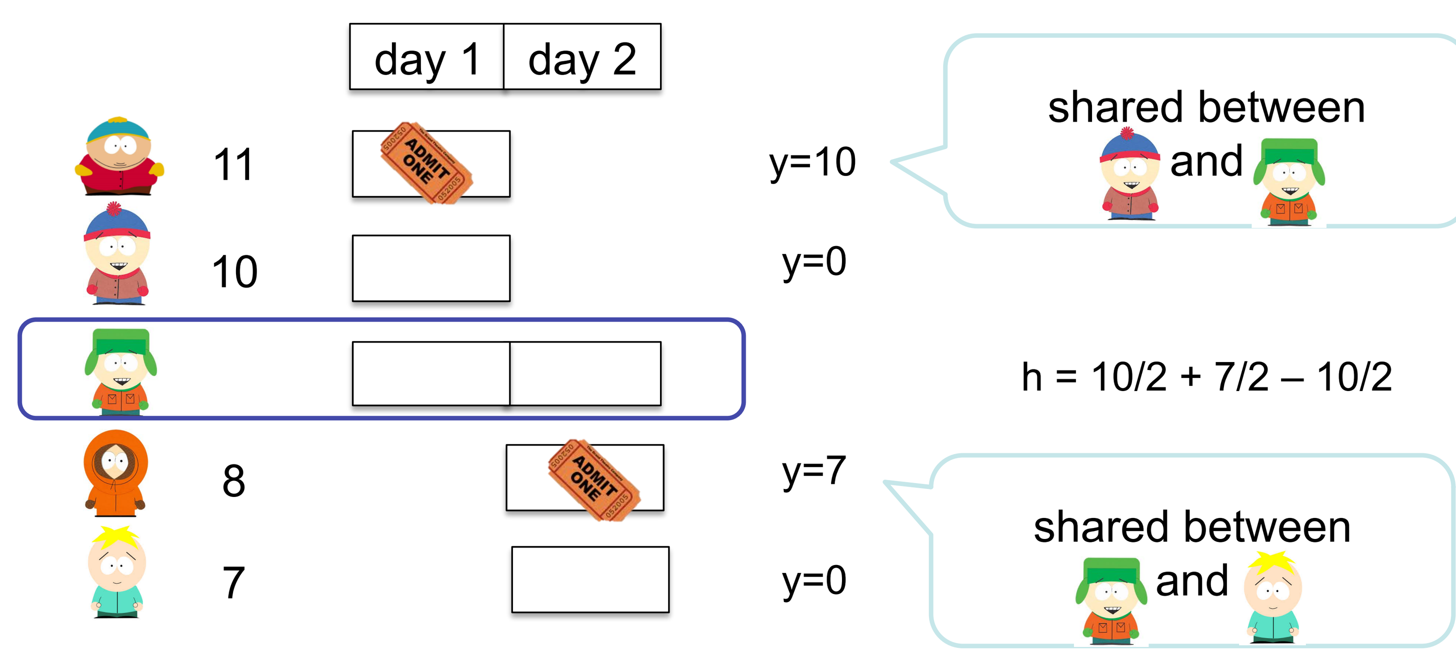
Redistribution Rule

$$h^d(\theta_{-i}, a_i, d_i) = \left(\sum_{j \in \text{to}_i} \frac{y_j(\theta_{-i})}{|\text{from}_j|} \right) - \max_{j \in \text{to}_i} \frac{y_j(\theta_{-i})}{|\text{from}_j|}$$

to$_i$:
- agents who depart when agent *i* is active
- agents charged by agent *i*

from$_j$:
- Agents who are active when agent *j* departs
- Agents who charge agent *j*

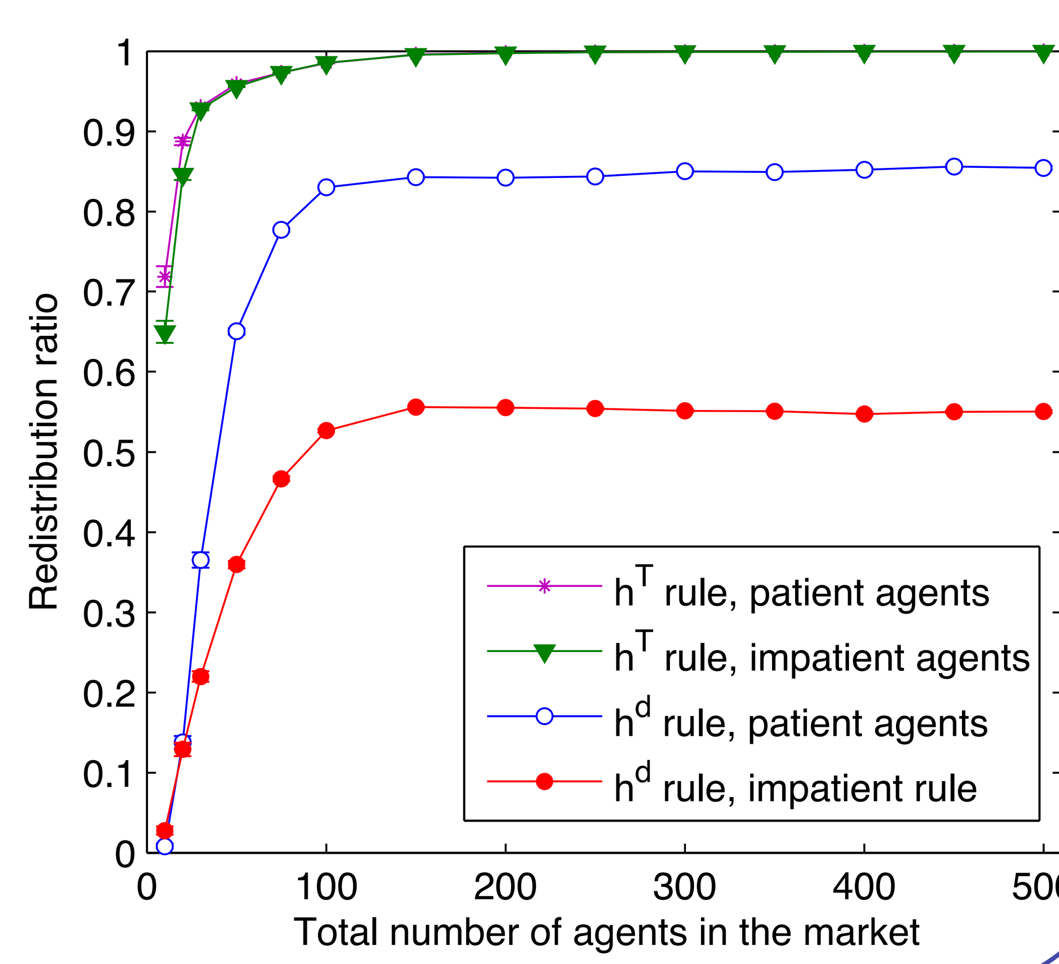




The redistribution function assures No-Subsidy

Experiments

- T=100 days
- Patient agents present for U(1,20) days
- Impatient agents present for U(1,4) days



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