

# HYBRID KEYWORD AUCTIONS

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Joint work with Ashish Goel, Stanford University

# ONLINE ADVERTISING

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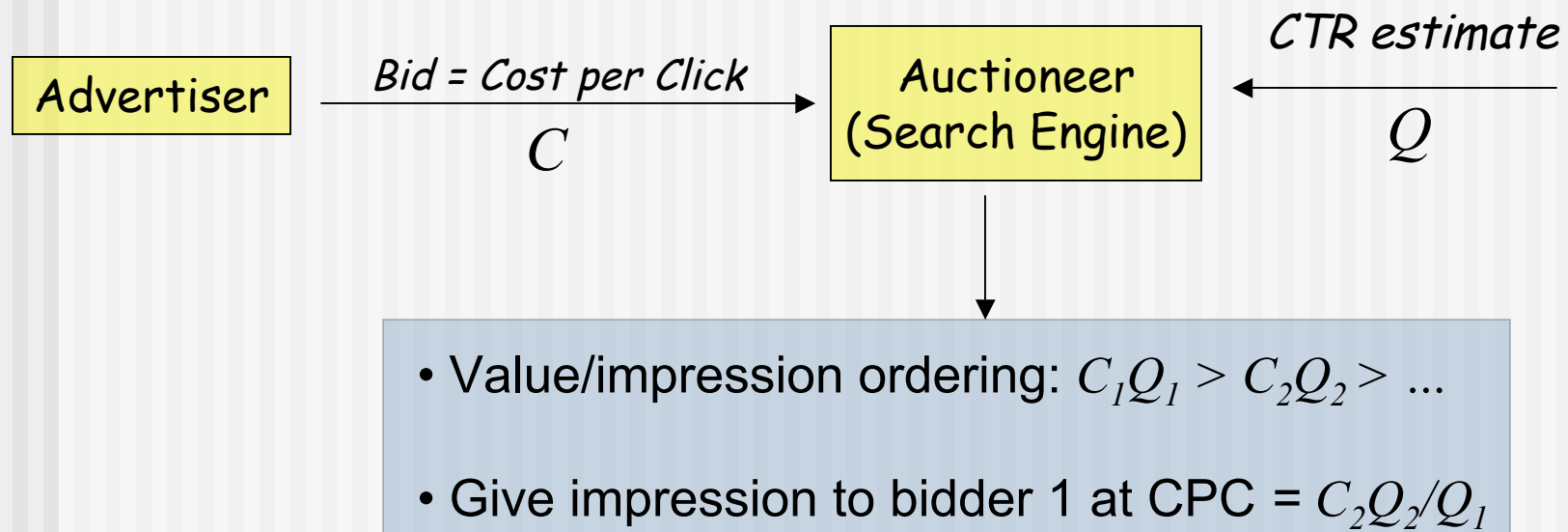
## Pricing Models

- ❑ CPM (Cost per thousand impressions)
- ❑ CPC (Cost per click)
- ❑ CPA (Cost per acquisition)
- ❑ Conversion rates:
  - Click-through-rate (CTR), conversion from clicks to acquisitions, ...

## Differences between these pricing models:

- ❑ Uncertainty in conversion rates:
  - Sparse data, changing rates, ...
- ❑ Stochastic fluctuations:
  - Even if the conversion rates were known exactly, the number of clicks/conversions would still vary, especially for small samples

# SPONSORED SEARCH AUCTION



**VCG Mechanism:** Truthful for a single slot, assuming static CTR estimates

Can be made truthful for multiple slots [Vickrey-Clark-Groves, Myerson81, AGM06]

***This talk will focus on single slot for proofs/examples***

## WHEN DOES THIS WORK WELL?

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- High volume targets (keywords)
  - Good estimates of CTR
- What fraction of targets are high volume?
  - Folklore: a small fraction
  - **Motivating problem:**
    - How to better monetize the low volume keywords?

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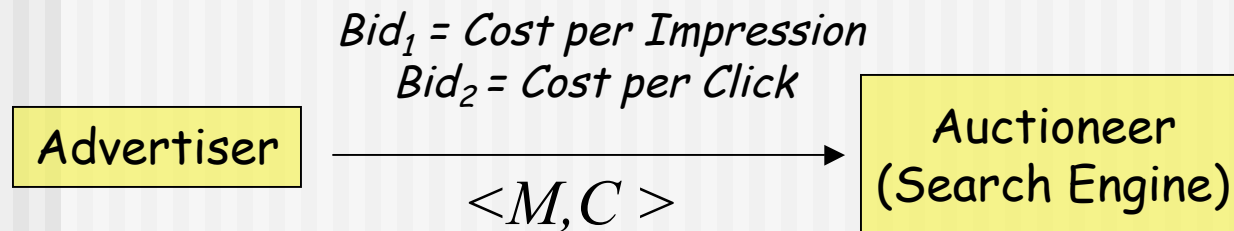
# POSSIBLE SOLUTIONS

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- Coarse ad groups to predict CTR:
  - ▣ Use performance of advertiser on possibly unrelated keywords
- Predictive models
  - ▣ Regression analysis/feature extraction
  - ▣ Taxonomies/clustering
  - ▣ Collaborative filtering
- **Our approach:** Devise richer pricing models

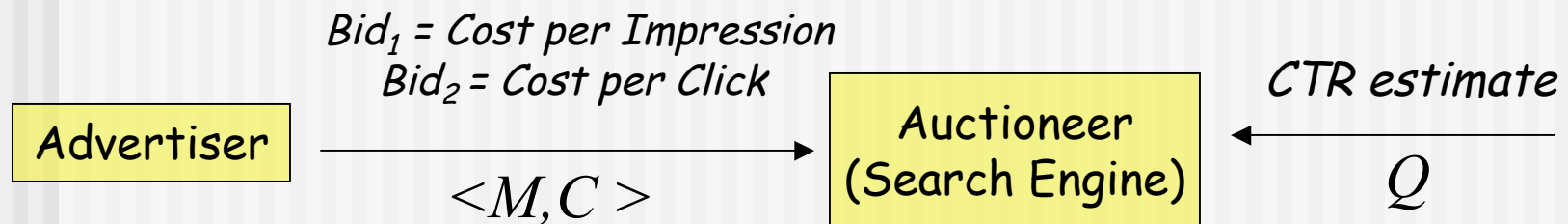
# HYBRID SCHEME

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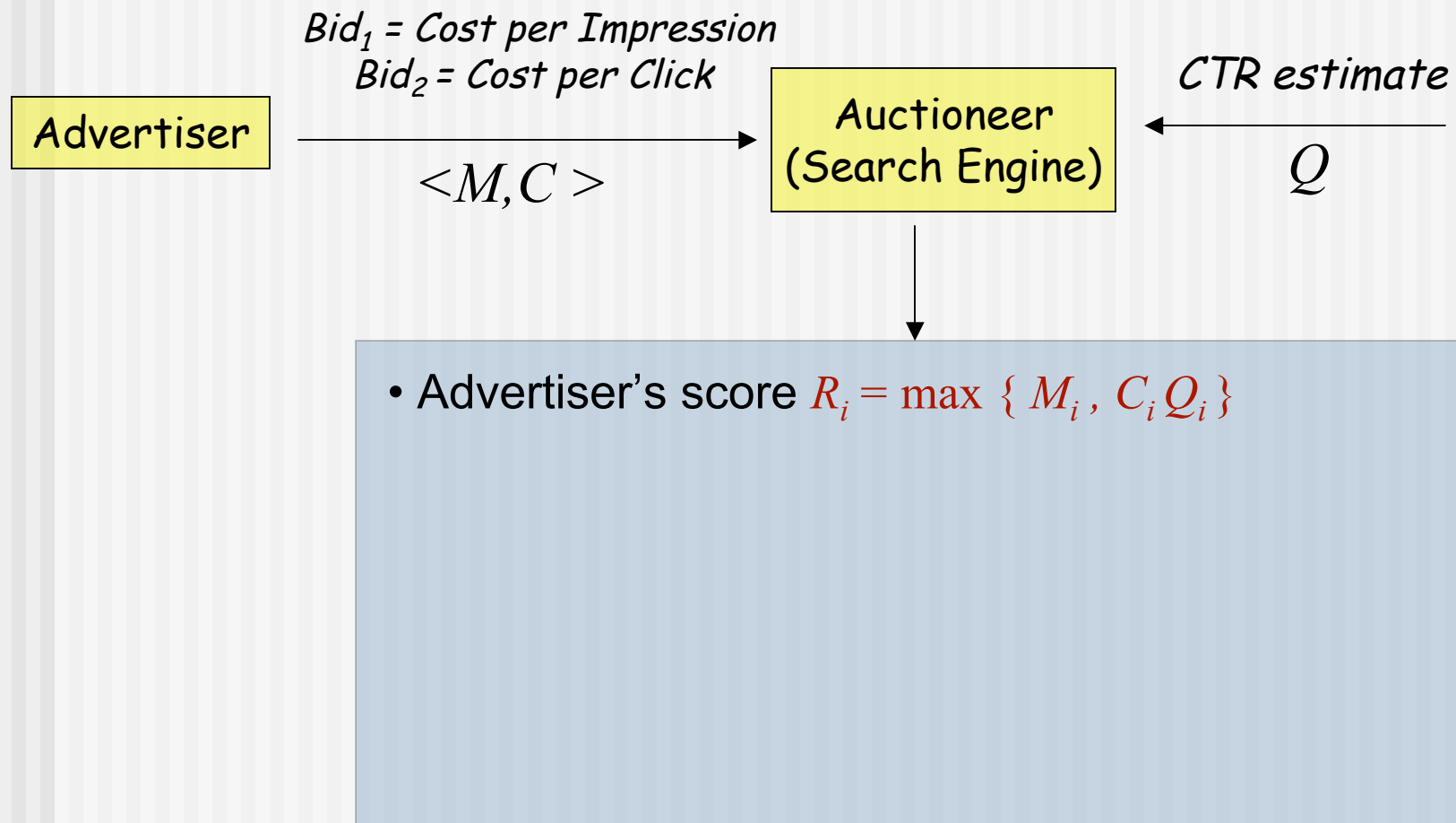




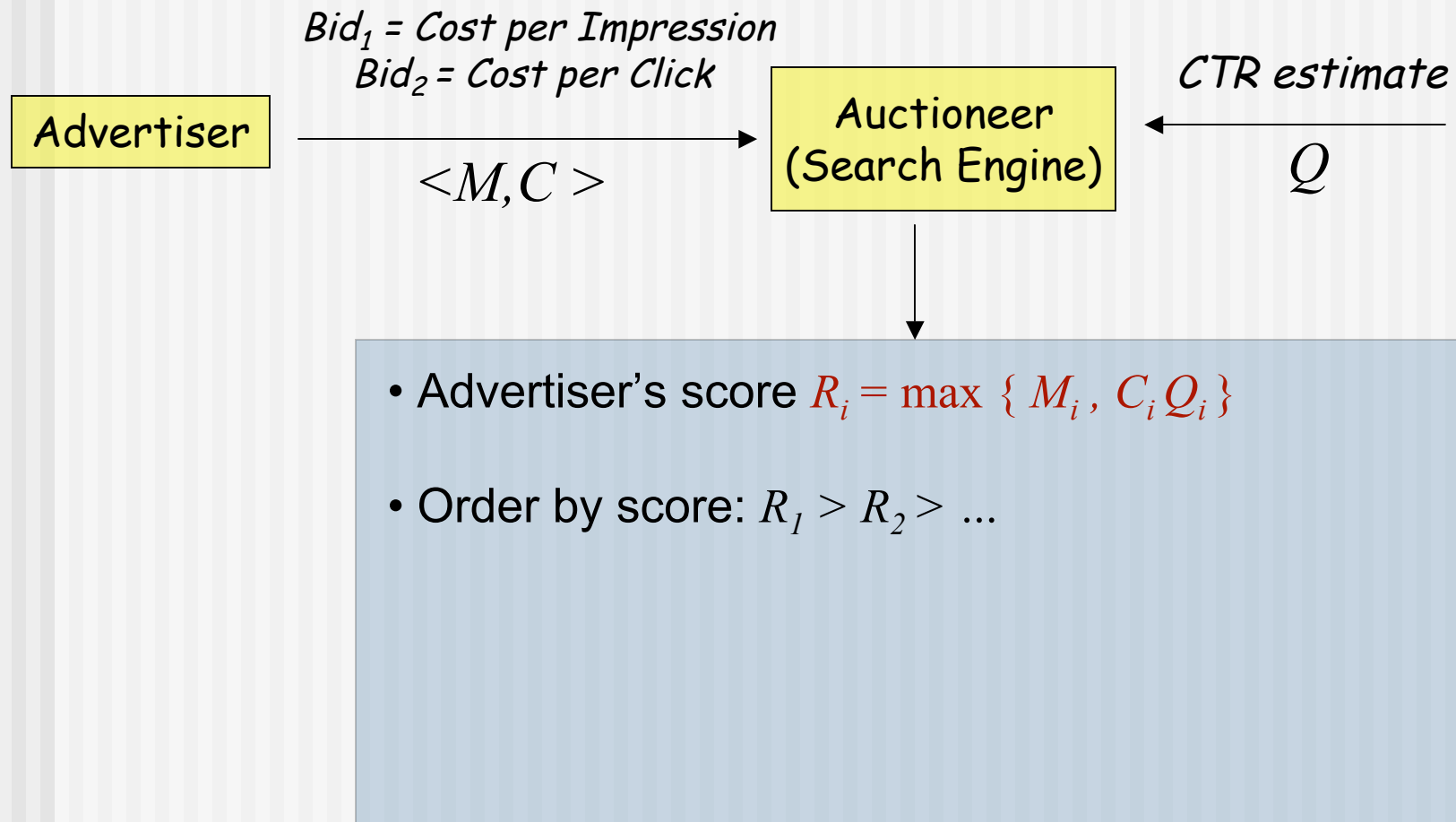
# HYBRID SCHEME



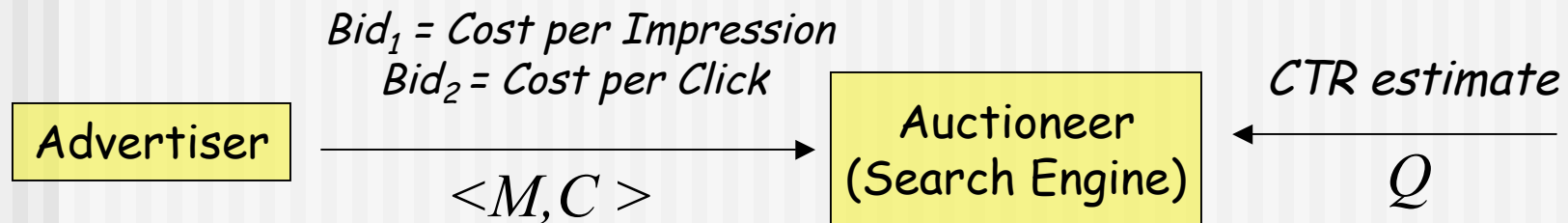
# HYBRID SCHEME



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- Advertiser's score  $R_i = \max \{ M_i, C_i Q_i \}$
- Order by score:  $R_1 > R_2 > \dots$
- Give impression to bidder 1:
  - If  $M_1 > C_1 Q_1$  then charge  $R_2$  per impression
  - If  $M_1 < C_1 Q_1$  then charge  $R_2 / Q_1$  per click

# WHY SUCH A MODEL?

---

- Per-impression bid:
  - ❑ Advertiser's estimate or "belief" of CTR
  - ❑ May or may not be an accurate reflection of the truth
  - ❑ Backward compatible with cost-per-click (CPC) bidding

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- Why would the advertiser know any better?
  - ❑ Advertiser aggregates data from various publishers
  - ❑ Has domain specific models not available to auctioneer
  - ❑ Is willing to pay a premium for internal experiments

# BENEFITS

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## 1. Search engine:

- n Better monetization of low volume keywords

## 2. Advertiser:

- n Opportunity to make the search engine converge to the correct CTR estimate *without paying a premium*

## 3. Technical:

- a) Truthful
- b) Accounts for risk characteristics of the advertiser
- c) Allows users to implement complex strategies

# MULTIPLE SLOTS

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- Show the top  $K$  scoring advertisers
  - Assume  $R_1 > R_2 > \dots > R_K > R_{K+1} \dots$
- Generalized Second Price (GSP) mechanism:
  - For the  $i^{th}$  advertiser, if:
    - If  $M_i > Q_i C_i$  then charge  $R_{i+1}$  per impression
    - If  $M_i < Q_i C_i$  then charge  $R_{i+1} / Q_i$  per click



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- Can also implement VCG [Vickrey-Clark-Groves, Myerson81, AGM06]
  - Need separable CTR assumption
  - Details in the paper

# BAYESIAN MODEL FOR CTR

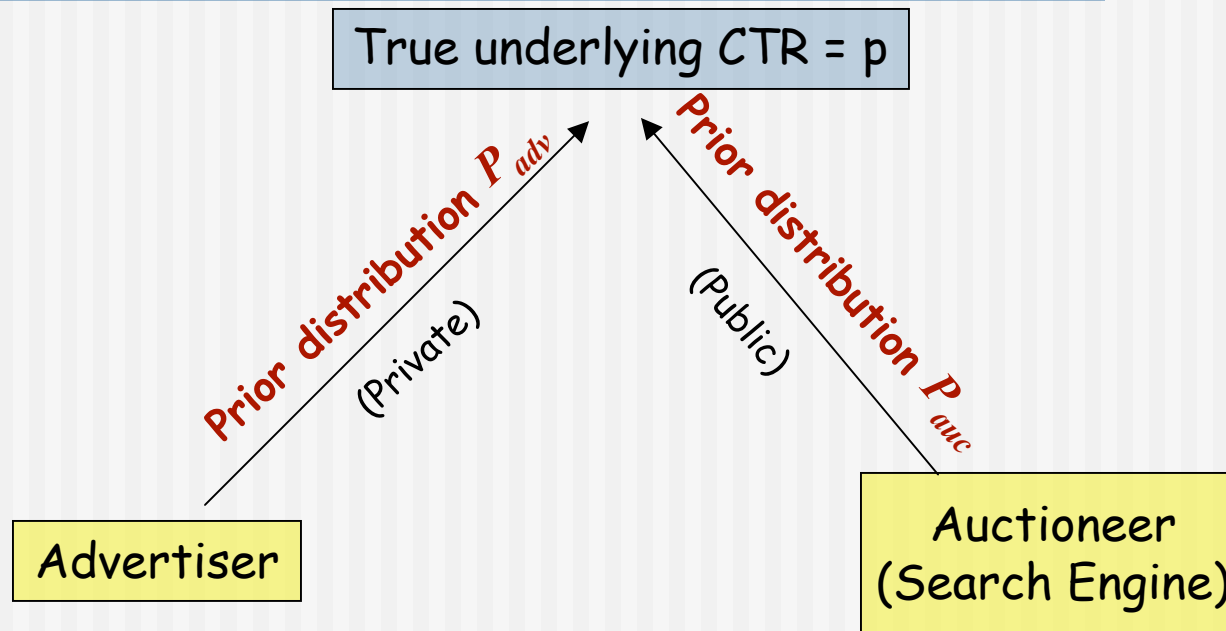
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True underlying CTR =  $p$

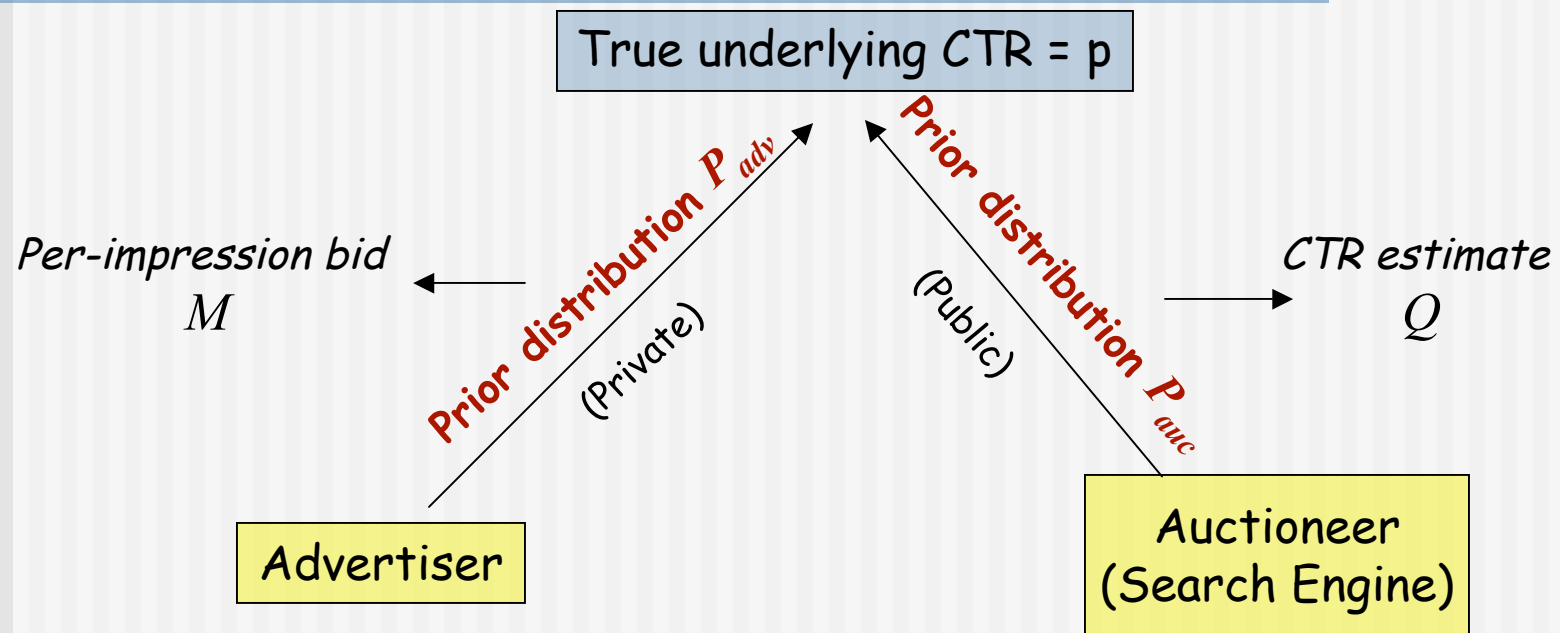
Advertiser

Auctioneer  
(Search Engine)

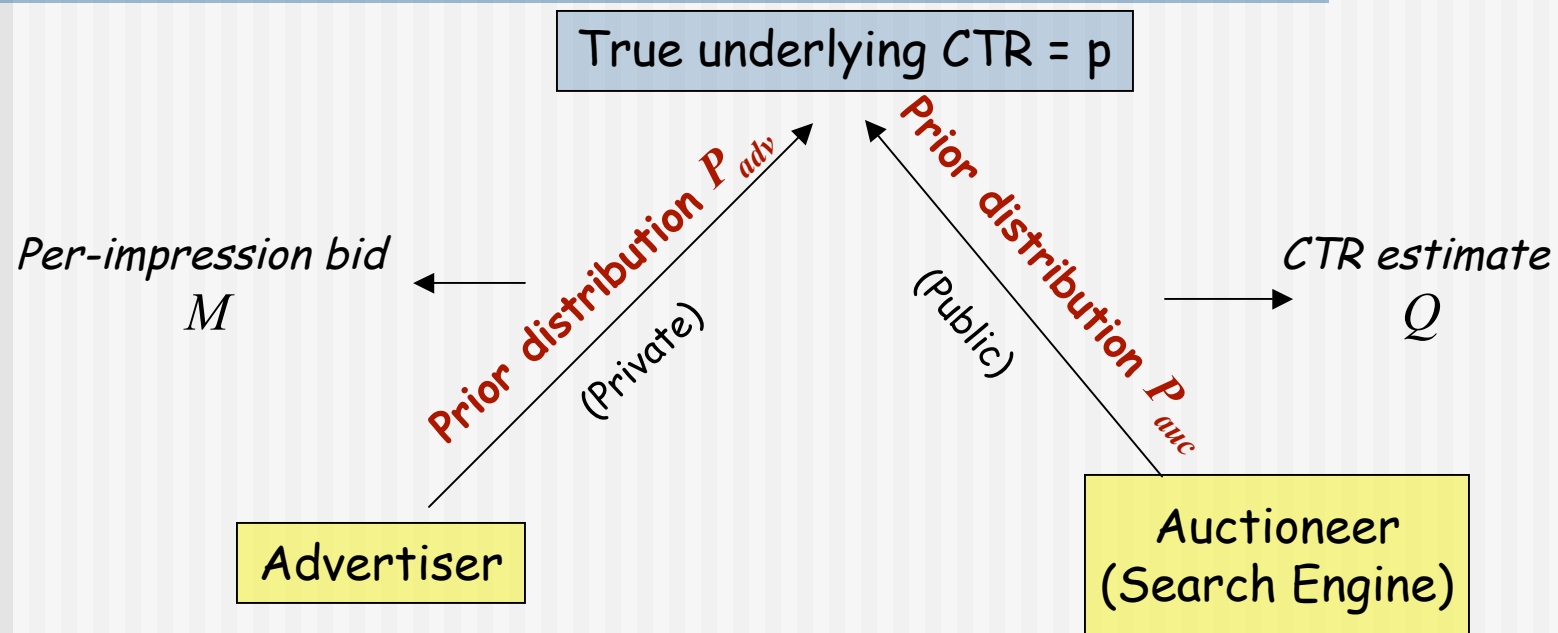
# BAYESIAN MODEL FOR CTR



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Each agent optimizes based on its current “belief” or prior:  
Beliefs updated with every impression  
Over time, become sharply concentrated around true CTR

# WHAT IS A PRIOR?

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- Simply models asymmetric information
  - Sharper prior  $\Rightarrow$  More certain about true CTR  $p$
  - $E[\text{Prior}]$  need not be equal to  $p$
- Main advantage of per-impression bids is when:
  - Advertiser's prior is sharper than auctioneer's
  - **Limiting case:** Advertiser certain about CTR  $p$
- Priors are only for purpose of analysis
  - Mechanism is well-defined regardless of modeling assumptions

# TRUTHFULNESS

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- Advertiser assumes CTR follows distribution  $P_{adv}$
- Wishes to maximize expected profit at current step
  - ▣  $\mathbf{E}[P_{adv}] = x$  = Expected belief about CTR
  - ▣ Utility from click =  $C$
  - ▣ Expected profit =  $Cx$  - Expected price

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Let  $Cy$  = Per impression bid  
 $R_2$  = Highest other score  
If  $\max(Cy, CQ) < R_2$  then Price = 0  
Else:  
    If  $y < Q$  then: Price =  $x R_2 / Q$   
    If  $y > Q$  then: Price =  $R_2$



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**Bidding  $(Cx, C)$  is the dominant strategy**

Regardless of  $Q$  used by auctioneer

Regardless of  $P_{adv}$  and true CTR  $p$

**Elicits advertiser's "expected belief" about the CTR!**

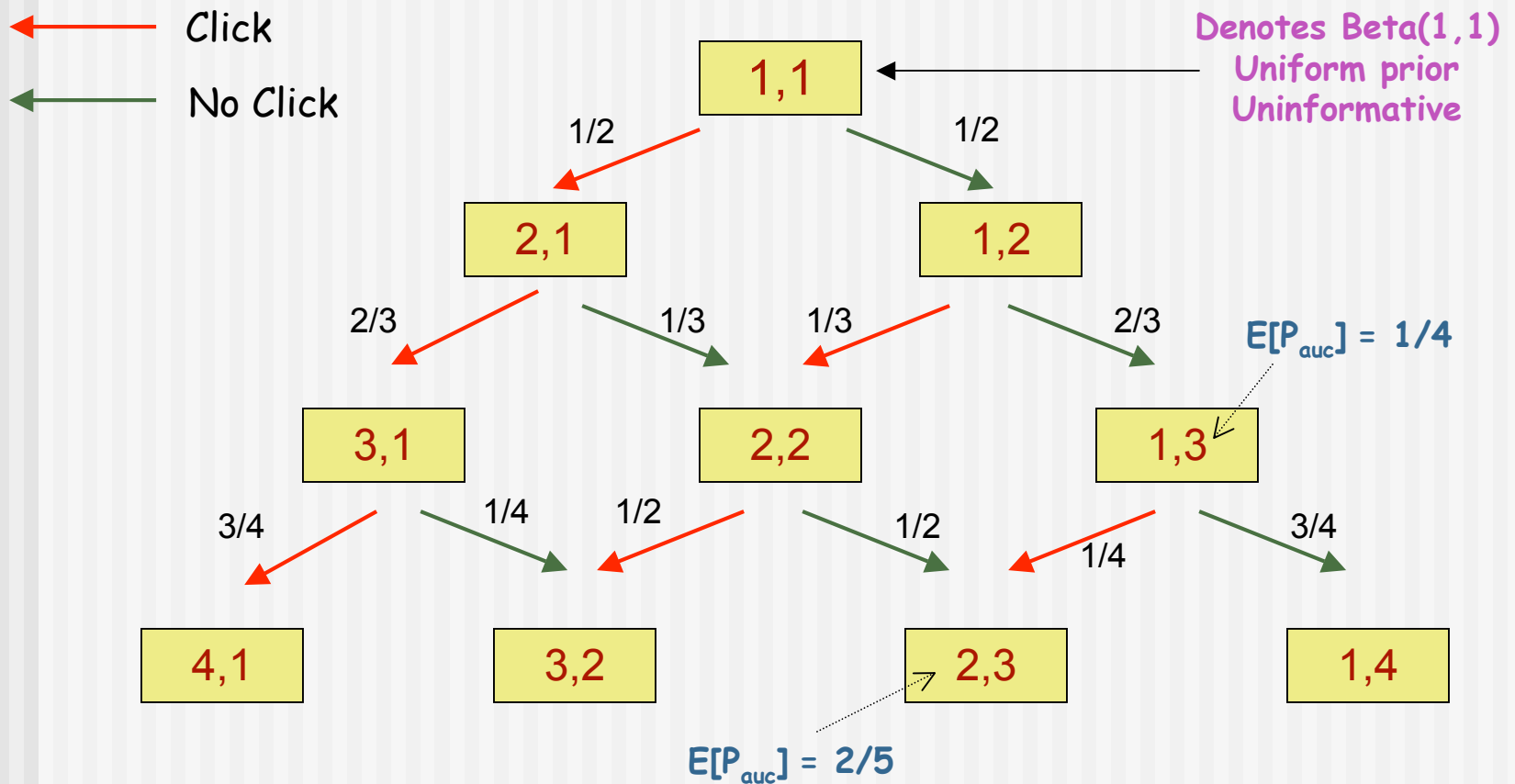
Holds in many other settings (more later)

# CONJUGATE BETA PRIORS

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- $P_{auc}$  for advertiser  $i = \text{Beta}(\alpha, \beta)$ 
  - $\alpha, \beta$  are positive integers
  - Conjugate of Bernoulli distribution (CTR)
  - Expected value =  $\alpha / (\alpha + \beta)$
- Bayesian prior update:
  - Probability of a click at the next step is:  $\alpha / (\alpha + \beta)$
  - If click, new  $P_{auc}$  (posterior) =  $\text{Beta}(\alpha+1, \beta)$
  - If no click, new  $P_{auc}$  (posterior) =  $\text{Beta}(\alpha, \beta+1)$

# EVOLUTION OF BETA PRIORS



# PROPERTIES

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- Larger  $\alpha, \beta \Rightarrow$  Sharper concentration around  $p$ 
  - **Uninformative prior:**  $Beta(1,1) = Uniform[0,1]$
- $Q = \mathbf{E}[P_{auc}] = \alpha / (\alpha + \beta)$ 
  - Encodes auctioneer's "belief"
  - Could be different from true CTR  $p$

# CERTAIN ADVERTISER

- Knows true CTR  $p$  and bids rationally ( $M_i = p_i$ )
  - $P_{adv} = p_i$  with probability 1
  - $P_{auc} = \text{Beta}(\alpha_i, \beta_i)$  and  $Q_i = \mathbf{E}[P_{auc}] = \alpha_i / (\alpha_i + \beta_i)$
- Revenue properties of auctioneer:
  - **Worst case:** 63% of CPC scheme
  - **Canonical case:**  $\log n$  times better than CPC scheme
- Flexibility for advertiser:
  - Can make  $P_{auc}$  converge to  $p$  without losing revenue
  - But pays huge premium for achieving this in CPC auction

# BETTER MONETIZATION

- Illustrative Scenario: Low volume keywords
  - $n$  advertisers, all click-utilities  $C = 1$
  - All  $P_{auc} = \text{Beta}(1, \log n)$  so that  $\mathbf{E}[P_{auc}] = Q \approx 1 / \log n$ 
    - High variance prior
    - Some  $p_i$  close to 1 with high probability
  - Per-impression bid will elicit this high  $p_i$
  - CPC auction allocates slot to a random advertiser
- **Theorem:** Hybrid auction can generate  **$\log n$  times more revenue** for auctioneer than existing CPC auction

# FLEXIBILITY FOR ADVERTISERS

- Suppose advertiser certain about CTR =  $p$ 
  - Assume  $C = 1$  and  $Q < p$
  - Bids truthfully and wins on per impression bids
- **Hybrid scheme:** Charged at most  $p$  per impression
  - Impressions shown repeatedly
  - Auctioneer's belief  $P_{auc}$  will converge to have mean  $p$
  - Now, advertiser switches to CPC bidding
- Assume auctioneer's prior is  $Beta(\alpha, \beta)$ 
  - $Q = \alpha / (\alpha + \beta) < p$

# FLEXIBILITY FOR ADVERTISERS

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- If CTR converges in  $T$  impressions resulting in  $N$  clicks:
  - $(\alpha + N)/(\alpha + \beta + T) \geq p$
  - Since  $Q = \alpha/(\alpha + \beta) < p$ , this implies  $N \geq T p$
  - Value gain =  $N$ ; Payment for  $T$  impressions at most  $T * p$
  - Hence, no loss in revenue to advertiser!
  
- In the existing CPC auction:
  - The advertiser would have to pay a huge premium for getting impressions and making the CTR converge



# UNCERTAIN ADVERTISERS

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- Advertiser should “pay premium” for CTR  $p$  resolving to a high value
  - What should her bidding strategy be?
  - Does it lead to a socially optimal mechanism?
- Key contribution:
  - Defining a Bayesian model for repeated auctions
  - Dominant strategy exists!

# SEMI-MYOPIC ADVERTISER

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- Maximizes discounted utility in contiguous time horizon in which she wins the auction
  - State of other advertisers stays the same during this time
  - Once she stops getting impressions, cannot predict future
    - ... since future will depend on private information of other bidders!
  - Circumvents negative results in economics literature
- Private information with advertiser:
  - Discount factor  $\gamma$ , value  $C_i$  and prior  $P_{adv}$
  - Discount factor models varying optimization horizons
    - Strategic vs. myopic

# DOMINANT HYBRID STRATEGY

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- Bidder always has a dominant hybrid strategy
  - ▣ **Bidding Index:** Computation similar to the Gittins index
  - ▣ Bidder can optimize her utility by dynamic programming

# DOMINANT HYBRID STRATEGY

- Bidder always has a dominant hybrid strategy
  - **Bidding Index:** Computation similar to the Gittins index
  - Bidder can optimize her utility by dynamic programming
- **Socially optimal** in many reasonable scenarios:
  - **Myopic advertiser:** Has  $\gamma_i = 0$ ; trusts auctioneer's prior:
    - Pure per-click bidding implements the **Gittins index policy**
  - If advertiser is *certain* of CTR, and  $Q_i$  is an underestimate:
    - Bidding index = Per-impression bidding, which is socially optimal
  - Implementation needs *both* per-impression and per-click bids

# SUMMARY

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- Allow both per-impression and per-click bids
  - Same ideas work for CPM/CPC + CPA
- Significantly higher revenue for auctioneer
- Easy to implement
  - Hybrid advertisers can co-exist with pure per-click advertisers
  - Easy path to deployment/testing
- Many variants possible with common structure:
  - Optional hybrid bids
  - Use the “max” operator to compute score

# OPEN QUESTIONS

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- Some issues that may be exacerbated:
  - **Whitewashing**: Re-entering when CTR is lower than the default
  - **Fake Clicks**: Bid per impression initially and generate false clicks to drive up CTR estimate  $Q$ 
    - Switch to per click bidding when slot is “locked in” by the high  $Q$
- Analysis of semi-myopic model
  - Other applications of separate beliefs?
- Connections of Bayesian mechanisms to:
  - Regret bounds and learning [Nazerzadeh, Saberi, Vohra '08]
  - Best-response dynamics [Edelman, Ostrovsky, Schwarz '05]